Como fazemos $A = LU$ devemos encontrar as matrizes L (Triangular Inferior) e U (Triangular Superior). Assim:

$$\overbrace{\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}}^{L} \overbrace{\begin{bmatrix} 3 & 2 & 4 \\ 1 & 1 & 2 \\ 4 & 3 & -2 \end{bmatrix}}^{A}$$

$$\underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}}_{U}$$

a) Primeira linha de U:

$1 \times u_{11} + 0 \times 0 + 0 \times 0 = 3$
$u_{11} = 3$

$1 \times u_{12} + 0 \times u_{22} + 0 \times 0 = 2$
$u_{12} = 2$

$1 \times u_{13} + 0 \times u_{23} + 0 \times u_{33} = 4$
$u_{13} = 4$

b) Primeira coluna de L:

$l_{21} \times u_{11} + 1 \times 0 + 0 \times 0 = 1$

$l_{21} \times 3 + 0 + 0 = 1$

$l_{21} = \dfrac{1}{3}$

$l_{31} \times u_{11} + l_{32} \times 0 + 1 \times 0 = 4$

$l_{31} \times 3 + 0 + 0 = 4$

$l_{31} = \dfrac{4}{3}$

c) Segunda linha de U:

$l_{21} \times u_{12} + 1 \times u_{22} + 0 \times 0 = 1$

$\dfrac{1}{3} \times 2 + u_{22} + 0 = 1$

$u_{22} = 1 - \dfrac{2}{3}$

$u_{22} = \dfrac{1}{3}$

$l_{21} \times u_{13} + 1 \times u_{23} + 0 \times u_{33} = 2$

$\dfrac{1}{3} \times 4 + u_{23} + 0 = 2$

$u_{23} = 2 - \dfrac{4}{3}$

$u_{23} = \dfrac{2}{3}$

d) Segunda coluna de L:

$l_{31} \times u_{12} + l_{32} \times u_{22} + 1 \times 0 = 3$

$\dfrac{4}{3} \times 2 + l_{32} \times \dfrac{1}{3} + 0 = 3$

$\dfrac{l_{32}}{3} = 3 - \dfrac{8}{3}$

$l_{32} = 1$

a) Terceira linha de U:

$l_{31} \times u_{13} + l_{32} \times u_{23} + 1 \times u_{33} = -2$

$\dfrac{4}{3} \times 4 + 1 \times \dfrac{2}{3} + u_{33} = -2$

$u_{33} = -2 - 6$

$u_{33} = -8$

$$\overbrace{\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & 1 & 0 \\ \frac{4}{3} & 1 & 1 \end{bmatrix}}^{L} \times \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \qquad \overbrace{\begin{bmatrix} 3 & 2 & 4 \\ 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & -8 \end{bmatrix}}^{U} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

- $\boldsymbol{L.y = b}$

$$\begin{cases} y_1 + 0y_2 + 0y_3 = 1 \\ \frac{1}{3}y_1 + y_2 + 0y_3 = 2 \\ \frac{4}{3}y_1 + y_2 + y_3 = 3 \end{cases}$$

$$y_1 = 1$$

$$\frac{1}{3}y_1 + y_2 = 2$$
$$y_2 = 2 - \frac{1}{3}$$
$$y_2 = \frac{5}{3}$$

$$\frac{4}{3} \times 1 + \frac{5}{3} + y_3 = 3$$
$$y_3 = 3 - 3$$
$$y_3 = 0$$

- $\boldsymbol{U.x = y}$

$$\begin{cases} 3x_1 + 2x_2 + 4x_3 = 1 \\ \frac{1}{3}x_2 + \frac{2}{3}x_3 = \frac{5}{3} \\ -8x_3 = 0 \end{cases}$$

$$x_3 = 0$$

$$\frac{1}{3}x_2 + \frac{2}{3} \times 0 = \frac{5}{3}$$
$$x_2 = 3 \times \frac{5}{3}$$
$$x_2 = 5$$

$$3x_1 + 2 \times 5 + 4 \times 0 = 1$$
$$3x_1 = 1 - 10$$
$$x_1 = -3$$

Então, temos como solução do sistema: $\overline{x} = \{-3, 5, 0\}^t$