Sage Quick Reference: Abstract Algebra

B. Balof, T. W. Judson, D. Perkinson, R. Potluri

version 1.0, Sage Version 5.0.1

latest version: `http://wiki.sagemath.org/quickref`

Based on work by P. Jipsen, W. Stein, R. Beezer

Basic Help

com⟨`tab`⟩ complete *command*

`a.`⟨`tab`⟩ all methods for object `a`

`<command>?` for summary and examples

`<command>??` for complete source code

`*`*foo*`*?` list all commands containing *foo*

`_` underscore gives the previous output

`www.sagemath.org/doc/reference` online reference

`www.sagemath.org/doc/tutorial` online tutorial

`load foo.sage` load commands from the file `foo.sage`

`attach foo.sage`
loads changes to `foo.sage` automatically

Lists

`L = [2,17,3,17]` an ordered list

`L[i]` the ith element of L
Note: lists begin with the 0th element

`L.append(x)` adds x to L

`L.remove(x)` removes x from L

`L[i:j]` the i-th through $(j-1)$-th element of L

`range(a)` list of integers from 0 to $a-1$

`range(a,b)` list of integers from a to $b-1$

`[a..b]` list of integers from a to b

`range(a,b,c)`
every c-th integer starting at a and less than b

`len(L)` length of L

`M = [i^2 for i in range(13)]`
list of squares of integers 0 through 12

`N = [i^2 for i in range(13) if is_prime(i)]`
list of squares of prime integers between 0 and 12

`M + N` the concatenation of lists `M` and `N`

`sorted(L)` a sorted version of `L` (`L` is not changed)

`L.sort()` sorts `L` (`L` is changed)

`set(L)` an unordered list of unique elements

Programming Examples

Print the squares of the integers $0, \ldots, 14$:

```
for i in range(15):
    print i^2
```

Print the squares of those integers in $\{0, \ldots, 14\}$ that are relatively prime to 15:

```
for i in range(13):
    if gcd(i,15)==1:
        print i^2
```

Preliminary Operations

`a = 3; b = 14`

`gcd(a,b)` greatest common divisor a, b

`xgcd(a,b)`
triple (d, s, t) where $d = sa + tb$ and $d = \gcd(a, b)$

`next_prime(a)` next prime after a

`previous_prime(a)` prime before a

`prime_range(a,b)` primes p such that $a \leq p < b$

`is_prime(a)` is a prime?

`b % a` the remainder of b upon division by a

`a.divides(b)` does a divide b?

Group Constructions

Permutation multiplication is left-to-right.

`G = PermutationGroup([[(1,2,3),(4,5)],[(3,4)]])`
perm. group with generators $(1, 2, 3)(4, 5)$ and $(3, 4)$

`G = PermutationGroup(["(1,2,3)(4,5)","(3,4)"])`
alternative syntax for defining a permutation group

`S = SymmetricGroup(4)` the symmetric group, S_4

`A = AlternatingGroup(4)` alternating group, A_4

`D = DihedralGroup(5)` dihedral group of order 10

`Ab = AbelianGroup([0,2,6])` the group $\mathbb{Z} \times \mathbb{Z}_2 \times \mathbb{Z}_6$

`Ab.0, Ab.1, Ab.2` the generators of `Ab`

`a,b,c = Ab.gens()`
shorthand for `a = Ab.0; b = Ab.1; c = Ab.2`

`C = CyclicPermutationGroup(5)`

`Integers(8)` the group $\mathbb{Z}_8$

`GL(3,QQ)` general linear group of 3×3 matrices

`m = matrix(QQ,[[1,2],[3,4]])`

`n = matrix(QQ,[[0,1],[1,0]])`

`MatrixGroup([m,n])`
the (infinite) matrix group with generators `m` and `n`

`u = S([(1,2),(3,4)]); v = S((2,3,4))` elements of `S`

`S.subgroup([u,v])`
the subgroup of `S` generated by `u` and `v`

`S.quotient(A)` the quotient group `S/A`

`A.cartesian_product(D)` the group `A`×`D`

`A.intersection(D)` the intersection of groups `A` and `D`

`D.conjugate(v)` the group $\mathtt{v}^{-1}\mathtt{Dv}$

`S.sylow_subgroup(2)` a Sylow 2-subgroup of `S`

`D.center()` the center of `D`

`S.centralizer(u)` the centralizer of `x` in `S`

`S.centralizer(D)` the centralizer of `D` in `S`

`S.normalizer(u)` the normalizer of `x` in `S`

`S.normalizer(D)` the normalizer of `D` in `S`

`S.stabilizer(3)` subgroup of `S` fixing 3

Group Operations

`S = SymmetricGroup(4); A = AlternatingGroup(4)`

`S.order()` the number of elements of `S`

`S.gens()` generators of `S`

`S.list()` the elements of `S`

`S.random_element()` a random element of `S`

`u*v` the product of elements `u` and `v` of `S`

`v^(-1)*u^3*v` the element $\mathtt{v}^{-1}\mathtt{u}^3\mathtt{v}$ of `S`

`u.order()` the order of `u`

`S.subgroups()` the subgroups of `S`

`S.normal_subgroups()` the normal subgroups of `S`

`A.cayley_table()` the multiplication table for `A`

`u in S` is `u` an element of `S`?

`u.word_problem(S.gens())`
write `u` as a product of the generators of `S`

`A.is_abelian()` is `A` abelian?

`A.is_cyclic()` is `A` cyclic?

`A.is_simple()` is `A` simple?

`A.is_transitive()` is `A` transitive?

`A.is_subgroup(S)` is `A` a subgroup of `S`?

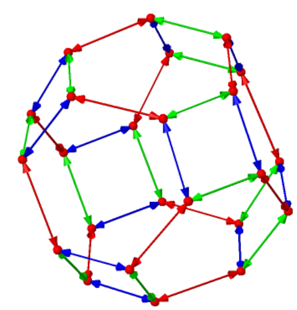
`A.is_normal(S)` is `A` a normal subgroup of `S`?

`S.cosets(A)` the right cosets of `A` in `S`

`S.cosets(A,'left')` the left cosets of `A` in `S`

`g = S.cayley_graph()` Cayley graph of `S`

`g.show3d(color_by_label=True, edge_size=0.01, vertex_size=0.03)` see below:



Ring and Field Constructions

`ZZ` integral domain of integers, $\mathbb{Z}$

`Integers(7)` ring of integers mod 7, $\mathbb{Z}_7$

`QQ` field of rational numbers, $\mathbb{Q}$

`RR` field of real numbers, $\mathbb{R}$

`CC` field of complex numbers, $\mathbb{C}$

`RDF` real double field, inexact

`CDF` complex double field, inexact

`RR` 53-bit reals, inexact, not same as `RDF`

`RealField(400)` 400-bit reals, inexact

`ComplexField(400)` complexes, too

`ZZ[I]` the ring of Gaussian integers

`QuadraticField(7)` the quadratic field, $\mathbb{Q}(\sqrt{7}\,)$

`CyclotomicField(7)`
smallest field containing $\mathbb{Q}$ and the zeros of x^7-1

`AA, QQbar` field of algebraic numbers, $\overline{\mathbb{Q}}$

`FiniteField(7)` the field $\mathbb{Z}_7$

`F.<a> = FiniteField(7^3)`
finite field in a of size 7^3, $\mathrm{GF}(7^3)$

`SR` ring of symbolic expressions

`M.<a>=QQ[sqrt(3)]` the field $\mathbb{Q}[\sqrt{3}]$, with $a=\sqrt{3}$.

`A.<a,b>=QQ[sqrt(3),sqrt(5)]`
the field $\mathbb{Q}[\sqrt{3},\sqrt{5}]$ with $a=\sqrt{3}$ and $b=\sqrt{5}$.

`z = polygen(QQ,'z'); K = NumberField(x^2 - 2,'s')`
the number field in s with defining polynomial x^2-2

`s = K.0` set `s` equal to the generator of `K`

`D = ZZ[sqrt(3)]`

`D.fraction_field()`
field of fractions for the integral domain `D`

Ring Operations

Note: Operations may depend on the ring

`A = ZZ[I]; D = ZZ[sqrt(3)]` some rings

`A.is_ring()` is A a ring?

`A.is_field()` is A a field?

`A.is_commutative()` is A commutative?

`A.is_integral_domain()`
`True` is A an integral domain?

`A.is_finite()` is A is finite?

`A.is_subring(D)` is A a subring of D?

`A.order()` the number of elements of A

`A.characteristic()` the characteristic of A

`A.zero()` the additive identity of A

`A.one()` the multiplicative identity of A

`A.is_exact()`
`False` if `A` uses a floating point representation

`a, b = D.gens(); r = a + b`

`r.parent()` the parent ring of r (in this case, `D`)

`r.is_unit()` is r a unit?

Polynomials

`R.<x> = ZZ[ ]` `R` is the polynomial ring $\mathbb{Z}[x]$

`R.<x> = QQ[ ]; R = PolynomialRing(QQ,'x'); R = QQ['x']`
`R` is the polynomial ring $\mathbb{Q}[x]$

`S.<z> = Integers(8)[ ]` `S` is the polynomial ring $\mathbb{Z}_8[z]$

`S.<s, t> = QQ[ ]` `S` is the polynomial ring $\mathbb{Q}[s,t]$

`p = 4*x^3 + 8*x^2 - 20*x - 24`
a polynomial in `R` $(=\mathbb{Q}[x])$

`p.is_irreducible()` is p irreducible over $\mathbb{Q}[x]$?

`q = p.factor()` factor p

`q.expand()` expand `q`

`p.subs(x=3)` evaluates p at $x=3$

`R.ideal(p)` the ideal in R generated by p

`R.cyclotomic_polynomial(7)`
the cyclotomic polynomial $x^6+x^5+x^4+x^3+x^2+x+1$

`q = x^2-1`

`p.divides(q)` does p divide q?

`p.quo_rem(q)`
the quotient and remainder of `p` upon division by `q`

`gcd(p, q)` the greatest common divisor of p and q

`p.xgcd(q)` the extended gcd of p and q

`I = S.ideal([s*t+2,s^3-t^2])`
the ideal $(st+2, s^3-t^2)$ in `S` $(=\mathbb{Q}[s,t])$

`S.quotient(I)` the quotient ring, S/I

Field Operations

`A.<a,b>=QQ[sqrt(3),sqrt(5)]`

`C.<c> = A.absolute_field()`
"flattens" a relative field extension

`A.relative_degree()`
the degree of the relative extension field

`A.absolute_degree()`
the degree of the absolute extension

r = a + b; `r.minpoly()`
the minimal polynomial of the field element `r`

`C.is_galois()` is `C` a Galois extension of Q?