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1.

	public	private
(1)	5.54	5.63
(2)	6.05	5.98

若把預測值設為前一小時的PM2.5，RMSE為6.4
用前9H的PM2.5做feature為6.05提升不大，由此可知時間
遠近影響很大，加其它feature的提升也不是很顯著

2. 去除PM2.5值 < 2 或 > 100的 data
去除風向和小時平均風向這2個變數：RMSE從 7.47 → 4.70
使用前4小時來預測就好：RMSE = 3.9
用5小時以上RMSE明顯上升
testing data 若有漏缺，補上該變數的平均值

1-(a)

$$L_{ssq}(w,b) = \frac{1}{10}\left[(1.2-w-b)^2+(2.4-2w-b)^2+(3.5-3w-b)^2+(4.1-4w-b)^2+(5.6-5w-b)^2\right]$$

求 $\frac{\partial L_{ssq}}{\partial w} = 0$

$$\Rightarrow \frac{1}{5}\left[(w+b-1.2)+(4w+2b-4.8)+(9w+3b-10.5)+(16w+4b-16.6)+(25w+5b-28)\right]$$

$$\Rightarrow \frac{1}{5}(55w+15b-61.1)=0$$

求 $\frac{\partial L_{ssq}}{\partial b} = 0$

$$\Rightarrow \frac{1}{5}(5b+15w-16.8)=0$$

$$\begin{cases} 55w+15b-61.1=0 \\ 45w+15b-50.4=0 \end{cases}$$

$$10w = 10.7$$

$$\begin{cases} w=1.07 \\ b=0.15 \end{cases}$$ ※

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1-(b) $L_{sq}(w,b) = \frac{1}{2N}\sum_{i=1}^{N}(y_i - (w^T x_i + b))^2$

求 $\frac{\partial L}{\partial b} = 0$

$\Rightarrow \frac{-1}{N}\sum_{i=1}^{N}(y_i - w^T x_i - b) = 0 \Rightarrow \sum y_i - \sum w^T x_i - \sum b = 0$

$\Rightarrow N \cdot b = \sum y_i - w^T \sum x_i \Rightarrow b = \bar{y} - w^T \bar{x}$ ※

求 $\frac{\partial L}{\partial w} = 0$

代换

$\Rightarrow \frac{-1}{N}\sum_{i=1}^{N}(y_i - w^T x_i - b)x_i = 0 \Rightarrow \sum y_i x_i - w^T \sum x_i^2 - \boxed{b}\sum x_i = 0$

$\Rightarrow \sum y_i x_i - w \sum x_i^2 - (\bar{y} - w^T \bar{x})\sum x_i = 0$

$\Rightarrow w \sum (x_i^T \bar{x} - x_i^2) = \bar{y}\sum x_i - \sum y_i x_i = \sum x_i(\bar{y} - y_i)$

$\Rightarrow w = \frac{\sum_{i=1}^{N} x_i(\bar{y} - y_i)}{\sum_{i=1}^{N} x_i^T(\bar{x} - x_i)}$ ※

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$1-(c)\ L_{reg}(w,b) = \frac{1}{2N}\sum_{i=1}^{N}(y_i-(w^Tx_i+b))^2+\frac{\lambda}{2}\|w\|^2\ ,\ \lambda\geq 0$

求 $\frac{\partial L_{reg}}{\partial b}=0 \Rightarrow$ 同 $1-(b)$: $b=\bar{y}-w^T\bar{x}$ #

求 $\frac{\partial L_{reg}}{\partial w}=0$

$\Rightarrow \sum y_ix_i - w\sum x_i^2-(\bar{y}-w^T\bar{x})\sum x_i+\lambda w=0$

$\Rightarrow w\left(\lambda+\sum(x_i^T\bar{x}-x_i^2)\right)=\sum x_i(\bar{y}-y_i)$

$\Rightarrow w=\dfrac{\sum_{i=1}^{N}x_i(\bar{y}-y_i)}{\lambda+\sum_{i=1}^{N}x_i^T(\bar{x}-x_i)}$ #

$2.\ L_{noise}(w,b)=\frac{1}{2N}\sum_{i=1}^{N}(y_i-w^T(x_i+\eta_i)-b)^2\ ,\ \sigma^2=E[\eta_i^2]$

求 $\frac{\partial L_{noise}}{\partial b}=0 \Rightarrow \frac{-1}{N}\sum_{i=1}^{N}(y_i-w^Tx_i-w^T\eta_i-b)=0$

$\Rightarrow \sum y_i - w^T\sum x_i - w^T\overset{=0}{\boxed{\sum\eta_i}}-N\cdot b=0 \Rightarrow b=\bar{y}-w^T\bar{x}$

求 $\frac{\partial L_{noise}}{\partial w}=0 \Rightarrow w=\dfrac{\sum(x_i+\eta_i)(y_i-\bar{y})}{\sum(x_i+\eta_i)^T(x_i+\eta_i-\bar{x})}$

$w=\dfrac{\sum x_i(y_i-\bar{y})-\bar{y}\sum\eta_i+\sum\eta_iy_i}{\sum x_i^T(x_i-\bar{x})+x_i^T\eta_i+\eta_i^Tx_i+\eta_i^T\eta_i-\bar{x}\eta_i^T}$

$w=\dfrac{\sum x_i(y_i-\bar{y})-0+0}{\sum x_i^T(x_i-\bar{x})+0+0+\sigma^2-0}$ ，等價於 $1-(c)$ 取 $\lambda=\sigma^2$

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3-(a) $\sum_{i=1}^{N} g_k(x_i) y_i = A$

$$\begin{cases} e_0 = \frac{1}{N} \sum y_i^2 \\ e_k = \frac{1}{N} \sum (g_k(x_i) - y_i)^2 = \frac{1}{N} \sum \left((g_k(x_i))^2 - 2 y_i g_k(x_i) + y_i^2 \right) \\ s_k = \frac{1}{N} \sum (g_k(x_i))^2 \end{cases}$$

$$e_k = s_k - 2A + e_0 \Rightarrow A = \frac{s_k + e_0 - e_k}{2} \quad ※$$

3-(b) $L = \min \left[\frac{1}{N} \sum_{i=1}^{N} \left(\sum_{k=1}^{K} \alpha_k g_k(x_i) - y_i \right)^2 \right]$

$$\frac{\partial L}{\partial \alpha_k} = \frac{2}{N} \sum_{i=1}^{N} (\alpha_k g_k(x_i) - y_i) g_k(x_i) = 0$$

$$\Rightarrow \alpha_k g_k(x_i) - y_i = 0 \quad , \quad g_k(x_i) = 0$$

$$\Rightarrow \alpha_k = \frac{y_i}{g_k(x_i)} \quad ※$$