# Homework 1

- (1) Prove that the DTFT of *a*$^{|n|}$ is as follows:

$$a^{|n|} \Leftrightarrow \frac{1-a^2}{1-2a\cos w+a^2}$$

  - Solution:

$$\sum_{n=-\infty}^{\infty} a^{|n|} e^{-j\omega n}$$

$$=\sum_{n=0}^{\infty} a^{n} e^{-j\omega n}+\sum_{n=-\infty}^{0} a^{-n} e^{-j\omega n}-a^0 e^0$$

$$=\frac{1}{1-ae^{-j\omega}}+\frac{1}{1-ae^{j\omega}}-1$$

$$=\frac{1-a^2}{1-2ae^{\omega}+a^2}$$

# Homework 1

- (2) Find the DTFT of the following two functions:

$$1.\ \ x_1(n) = 10(0.5)^n \cos\left(0.2\pi n + \frac{\pi}{3}\right)u(n)$$

$$2.\ \ x_2(n) = n(0.2)^n u(n)$$

# Homework 1

- 2.1 Solution:

$$\sum_{n=-\infty}^{\infty} x_1(n)e^{-j\omega n}$$

$$=\sum_{n=0}^{\infty} 10(0.5)^n \cos(0.2\pi n+\frac{\pi}{3})e^{-j\omega n}$$

$$=\sum_{n=0}^{\infty} 10(0.5)^n (\frac{1}{2}e^{j(0.2\pi n+\frac{\pi}{3})}+\frac{1}{2}e^{-j(0.2\pi n+\frac{\pi}{3})})e^{-j\omega n}$$

$$=\sum_{n=0}^{\infty} 5(0.5)^n e^{j(0.2\pi n+\frac{\pi}{3}-\omega n)}+\sum_{n=0}^{\infty} 5(0.5)^n e^{-j(0.2\pi n+\frac{\pi}{3}+\omega n)}$$

$$=5e^{j\frac{\pi}{3}}\sum_{n=0}^{\infty}(0.5e^{j(0.2\pi-\omega)})^n+5e^{-j\frac{\pi}{3}}\sum_{n=0}^{\infty}(0.5e^{-j(0.2\pi+\omega)})^n$$

$$=5e^{j\frac{\pi}{3}}(\frac{1}{1-0.5e^{j(0.2\pi-\omega)}})+5e^{-j\frac{\pi}{3}}(\frac{1}{1-0.5e^{-j(0.2\pi+\omega)}})$$

# Homework 1

- 2.2 Solution:

$$\sum_{n=-\infty}^{\infty} x_2(n)e^{-j\omega n} = \sum_{n=0}^{\infty} n(0.2)^n e^{-j\omega n}$$

Since $\sum_{n=0}^{\infty}(0.2)^n e^{-j\omega n} = 1 + 0.2e^{-j\omega} + (0.2e^{-j\omega})^2 + \cdots = \frac{1}{1-0.2e^{-j\omega}}$

and $nx[n] \Leftrightarrow j\frac{d\mathrm{X}(e^{j\omega})}{d\omega}$,

we have $\sum_{n=0}^{\infty} n(0.2)^n e^{-j\omega n} = j\frac{d(1-0.2e^{-j\omega})^{-1}}{d\omega} = -j\frac{0.2je^{-j\omega}}{(1-0.2e^{-j\omega})^2} = \frac{0.2e^{-j\omega}}{(1-0.2e^{-j\omega})^2}$