Math 4/510: Dynamical Systems and Control

University of Oregon, Fall 2024

Time: TΘ, 12-1:20pm
Place: Tykeson 240
Instructor: James Murray (`jmurray9@uoregon.edu`)
Instructor office hours: 10am-12pm Wednesdays, Huestis 224

Prerequisites: MATH 281 and MATH 341. Students should also have some experience with programming in Python or another programming language at the level of an introductory class in programming (although we will provide resources for students who are new to coding to get up and running with a bit of extra effort in the early weeks of the term).

Textbooks: The main textbook for this course will be

- (AM) Feedback Systems: An Introduction for Scientists and Engineers, by Åström and Murray.

The book is available free here: https://fbswiki.org. We will also be using the following supplemental texts by the same authors, available from the same website:

- (LST) Linear Systems Theory, by Åström and Murray.
- (OBC) Optimization-Based Control, by Åström and Murray.

The following text may be of interest for students who would like a more complete treatment of optimal control theory:

- (Kirk) Optimal Control Theory, by Donald Kirk.

Finally, the following text (available for free at http://www.sontaglab.org/) covers many of the same topics as the main texts listed above, but with more emphasis on mathematical rigor and less emphasis on applications:

- (Sontag) Mathematical Control Theory: Deterministic Finite Systems, by Eduardo Sontag.

Description: This course will cover the mathematics of control theory, which provides a framework for driving dynamical systems such as bicycles and robots in optimal ways. Beginning with relatively simple linear systems, we will develop the mathematical tools needed to extend our investigations to systems that are noisy, nonlinear, and/or partially observed. Assuming basic familiarity with calculus, linear algebra, and differential equations, this will lead us to introduce some more-sophisticated tools including dynamical systems theory, stochastic processes, and calculus of variations. This will be an applied math course, in the sense that we will use practical problems to motivate the mathematical tools that we learn. Students will come away from this class with an understanding of the core principles of control theory, including positive and negative feedback, robustness to noise, and the trade-off between stability and maneuverability. Through mathematical exercises and scientific programming exercises in Python, students will learn how to apply these principles in examples drawn from engineering, biology, neuroscience, and bicycling.

Learning outcomes: By the end of this course, students will have obtained the following skills:

- Develop the ability to explain the following basic principles of control theory and apply them to solving problems: stability, feedback, optimization, controllability, stochasticity, and robustness.
- Demonstrate understanding of core concepts from the following mathematical topics and apply these tools to solving problems in control theory: dynamical systems theory, stochastic processes, and calculus of variations.
- Develop basic competence in numerical computing and implementing control algorithms in Python.

Class format: This class will be held in person. Class time will consist of blackboard lectures combined with interactive coding demonstrations. The coding demonstrations will be based on tutorial Python notebooks, which can be found on the following webpage:

https://github.com/murray-lab/control

Students should bring a laptop to class on days when coding tutorials are planned. (The instructor will announce this in the prior class and/or on Canvas.)

Homework: Homework assignments will be announced on Canvas and posted on the Github page given above and will consist of a mix of pen-and-paper calculations together with implementations and applications of control algorithms using Python notebooks. Homework assignments will be submitted via Canvas, and solutions to homework assignments will be posted to Canvas approximately one week after each assignment due date. Please do not share these solutions with anyone who is not currently enrolled in the class. Details about how to complete and turn in homework assignments can be found on the course's Canvas page. The lowest homework score will be dropped in computing the student's final grade for the course.

During the term, each student will be granted a total of 10 days to be late on homework assignments. This means that one assignment may be turned in 10 days late without penalty during the term, or five assignments may be turned in two days late, or any other combination. After the 10 days have been used up, late assignments will no longer be accepted and will receive a grade of zero.

Students are welcome and encouraged to discuss and work together on their homework assignments in small groups. So that all students have a chance to participate, this works best in groups of 2-4 students. Each student must complete and submit their work independently, however, and under no circumstances should a student copy work directly from another student.

Attendance: Attendance, while strongly encouraged, is not required for this course.

Expected classroom behavior: Students are expected to behave respectfully toward each other and toward the instructor during class time. This includes refraining from using cell phones during lectures.

Evaluation: Homework assignments will be a mix of pen-and-paper calculations and Python notebooks. Midterm and final exams will be taken on paper and in person. Final grades will be weighted as 70% homework, 15% midterm, and 15% final exam. If a student is not able to complete either exam at the scheduled time, they should provide the instructor with advance notice and arrange a make-up period for the exam to be taken.

Schedule: This course will cover the following topics, with each topic taking approximately 1-2 weeks. The following schedule is approximate and subject to change.

- Introduction to scientific computing in Python.
- Introductory examples (AM Ch. 3-4). Inverted pendulum (AM Sec. 3.2); bicycle (AM Sec. 4.2).
- Feedback principles (AM Ch. 2). Open loop vs. closed loop, negative and positive feedback effects on linearity, sensitivity, and robustness (AM Sec. 2.1); transfer functions for linear dynamical systems including P control, PI control, and positive feedback (AM Sec. 2.2-3, 2.5).
- Dynamical behavior of ordinary differential equations (AM Ch. 5). Solving differential equations (AM Sec. 5.1); equilibrium points and limit cycles (AM Sec. 5.2); stability (AM Sec. 5.3); Lyapunov stability analysis (AM Sec. 5.4).
- Control in linear systems (AM Ch. 6). Linearity and time invariance (AM Sec. 6.1); the matrix exponential (AM Sec. 6.2); input/output response via the convolution equation (AM Sec. 6.3); linearization (AM Sec. 6.4).

- State feedback (AM Ch. 7). Reachability (aka controllability, AM Sec. 7.1); stabilization by state feedback (AM Sec. 7.2); linear quadratic regulators (AM Sec. 7.5).
- Stochastic systems (OBC Ch. 5). Random variables (OBC Sec. 5.1); random processes (OBC Sec. 5.2-3); linear stochastic systems with Gaussian noise (OBC Sec. 5.4).
- Output feedback (AM Ch. 8). Observability (AM Sec. 8.1); state estimation (AM Sec. 8.2); control using estimated state (AM Sec. 8.3); Kalman filtering in discrete time (AM Sec. 8.4); state space controller design (8.5).
- Optimal control (OBC Ch. 3). Optimization theory (OBC Sec. 3.1, Kirk Ch. 4); Hamiltonian formulation of optimal control (OBC Sec. 3.2, Kirk Ch. 5); examples of optimal control (OBC Sec. 3.3); LQR derivation (OBC Sec. 3.5, Kirk Sec. 5.2); dynamic programming (OBC Sec. 3.7, Kirk Ch. 3, Kirk Sec. 7.1-2).

Artificial Intelligence: Generative AI algorithms such as Chat-GPT have recently emerged as a powerful technology that can be highly useful for the sorts of coding problems that we will be doing in class. Students are welcome to make use of such tools in completing homework assignments, though they should be mindful of the fact that they will learn the material best by struggling with it themselves and not leaning excessively on AI to do the work for them at every opportunity. AI tools may not be used during the midterm and final exams.

What follows is a general description of various University of Oregon policies, not specific to this course.

Academic Disruption due to Campus Emergency: In the event of a campus emergency that disrupts academic activities, course requirements, deadlines, and grading percentages are subject to change. Information about changes in this course will be communicated as soon as possible by email, and on Canvas. If we are not able to meet face-to-face, students should immediately log onto Canvas and read any announcements and/or access alternative assignments. Students are also expected to continue coursework as outlined in this syllabus or other instructions on Canvas. In the event that the instructor of this course has to quarantine, this course may be taught online during that time.

Academic Misconduct: The University Student Conduct Code defines academic misconduct. It can be found at https://conduct.uoregon.edu. Students are prohibited from committing or attempting to commit any act that constitutes academic misconduct. By way of example, students should not give or receive (or attempt to give or receive) unauthorized help on assignments or examinations without express permission from the instructor. Students should properly acknowledge and document all sources of information (e.g. quotations, paraphrases, ideas) and use only the sources and resources authorized by the instructor. If there is any question about whether an act constitutes academic misconduct, it is the students' obligation to clarify the question with the instructor before committing or attempting to commit the act. Additional information about a common form of academic misconduct, plagiarism, is available at https://researchguides.uoregon.edu/citing-plagiarism.

Accessible Education: The University of Oregon is working to create inclusive learning environments. Please notify me if there are aspects of the instruction or design of this course that result in disability-related barriers to your participation. You are also encouraged to contact the Accessible Education Center in 360 Oregon Hall at 541-346-1155 or uoaec@uoregon.edu.

Basic Needs: Any student who has difficulty affording groceries or accessing sufficient food to eat every day, or who lacks a safe and stable place to live and believes this may affect their performance in the course is urged to contact the Dean of Students Office (346-3216, 164 Oregon Hall) for support. This UO webpage includes resources for food, housing, healthcare, childcare, transportation, technology,finances, and legal support: https://blogs.uoregon.edu/basicneeds/food.

Inclement Weather: It is generally expected that class will meet unless the University is officially closed for inclement weather. If it becomes necessary to cancel class while the University remains open, this will be announced on Canvas and by email. Updates on inclement weather and closure are also communicated in other ways described here: https://hr.uoregon.edu/about-hr/campus-notications/inclement-weather.

Mental Health and Wellness: Life at college can be very complicated. Students often feel overwhelmed or stressed, experience anxiety or depression, struggle with relationships, or just need help navigating challenges in their life. If you're facing such challenges, you don't need to handle them on your own—there's help and support on campus.

As your instructor, if I believe you may need additional support, I will express my concerns, the reasons for them, and refer you to resources that might be helpful. It is not my intention to know the details of what might be bothering you, but simply to let you know I care and that help is available. Getting help is a courageous thing to do—for yourself and those you care about.

University Health Services help students cope with difficult emotions and life stressors. If you need general resources on coping with stress or want to talk with another student who has been in the same place as you, visit the Duck Nest (located in the EMU on the ground floor) and get help from one of the specially trained Peer Wellness Advocates. Find out more at health.uoregon.edu/ducknest.

University Counseling Services (UCS) has a team of dedicated staff members to support you with your concerns, many of whom can provide identity-based support. All clinical services are free and confidential. Find out more at counseling.uoregon.edu or by calling 541-346-3227 (anytime UCS is closed, the After-Hours Support and Crisis Line is available by calling this same number).

Reporting Obligations: I am a designated reporter. For information about my reporting obligations as an employee, please see Employee Reporting Obligations on the Office of Investigations and Civil Rights Compliance (OICRC) website. Students experiencing sex or gender-based discrimination, harassment or violence should call the 24-7 hotline 541-346-SAFE (7244) or visit https://safe.uoregon.edu for help. Students experiencing all forms of prohibited discrimination or harassment may contact the Dean of Students Office at 5411-346-3216 or the non-confidential Title IX Coordinator/OICRC at 541-346-3123 to request information and resources. Students are not required to participate in an investigation to receive support, including requesting academic supportive measures. Additional resources are available at https://investigations.uoregon.edu/how-get-support. I am also a mandatory reporter of child abuse. Please find more information at Mandatory Reporting of Child Abuse and Neglect (https://hr.uoregon.edu/policies-leaves/general-information/mandatory-reporting-child-abuse-and-neglect).