1. Introduction

In 2020, the Federal Reserve laid out an average inflation targeting (AIT) monetary policy framework where inflation could temporarily deviate from the Fed's target in the short run, as long as the average level of inflation in the medium to long run remained consistent with the Fed's target. If inflation remained consistently below its target for some period, it could be followed by a period where inflation would remain above its target.

Recent research has been examining a range of issues related to AIT, including welfare implications and optimal monetary policy (e.g. Budianto et al., 2020; Eo and Lie, 2020, Nessén and Vestin, 2005), how AIT affects inflation expectations (e.g. Coibion et al., 2020; Hoffmann et al., 2022), how AIT affects macroeconomic stability (Piergallini, 2022), and implications for boundedly-rational expectations on macroeconomic outcomes (eg: Honkapohja and McClung, 2021; Budianto et al., 2020). A central question that pertains to the literature on the AIT framework is the window for how the 'average' level of inflation is determined. The papers cited above use backward-looking inflation averages for the monetary policy target, but the Fed's average may be based on past values of inflation, expectations of future values for inflation, or some combination. In an official statement by the Federal Open Market Committee states,

> "In order to anchor longer-term inflation expectations at this level, the Committee seeks to achieve inflation that averages 2 percent over time, and therefore judges that, following periods when inflation has been running persistently below 2 percent, appropriate monetary policy will likely aim to achieve inflation moderately above 2 percent for some time."[1]

This implies a time frame for the average target that is a combination of both past inflation and future inflation.

We examine a model of average inflation targeting within the context of a standard three-equation New Keynesian model. We construct a measure of the inflation target that is a weighted average of past observations of inflation, current inflation, and expectations for future values of inflation. We evaluate conditions on monetary policy and the target window to assure determinacy. A rational expectations model is considered determinant when there is exactly one solution for the mathematical the expected value for all variables and one reduced

[1] See https://www.federalreserve.gov/monetarypolicy/files/FOMC_LongerRunGoals.pdf (accessed on January 5, 2023.)

form solution to the model, that together with the realization of structural shocks of the model, determine the outcome for the variables in the model. When there is indeterminacy, there are infinitely many solutions for a mathematical expectation for the variables in the model, each resulting in a different reduced form solution to the model. The continuum of rational expectation solutions can be expressed as a function of "sunspot" shocks. The outcome for the variables in the model is a function of both the realization of the structural and sunspot" shocks, leading to excess macroeconomic volatility. Woodford (1987) describes sunspot shocks neatly as random effects having nothing to do with the fundamentals of the model, but are shocks to agents' expectations that are self-fulfilling. For an empirical investigation for how U.S. monetary policy may have led to sunspot shocks and excessive macroeconomic volatility, see, for example, Lubik and Schorfheide (2004).

2. Model

This paper builds upon a standard three-equation New Keynesian model along the lines of Clarida et al. (1999).

2.1. Baseline Framework

The IS equation is derived from consumer utility maximization and states that the current output gap depends on expectations of next period's output gap, and is negatively related to the real interest rate:

$$x_t = x_{t+1}^e - \frac{1}{\sigma}\left(r_t - \pi_{t+1}^e - r^n\right) + \xi_t^x, \tag{1}$$

where x_t denotes the output gap (given by the difference between the log of output and its natural rate), r_t is the nominal interest rate, π_t the inflation rate, $r^n = 1/\beta - 1$ the natural rate of interest and $\beta \in (0,1)$ is the household's discount factor, and x_{t+1}^e and π_{t+1}^e represent private sector expectations on next period's output gap and inflation rate, respectively. The preference parameter, σ, is inversely related to consumers' intertemporal elasticity of substitution, and ξ_t^x, represents a demand shock. A fraction of agents, $\lambda \in [0,1)$, form naïve expectations, so aggregate expectations are given by,

$$\begin{aligned} x_{t+1}^e &= \lambda x_t + (1-\lambda)\,\mathbb{E}_t\, x_{t+1}, \\ \pi_{t+1}^e &= \lambda \pi_t + (1-\lambda)\,\mathbb{E}_t\, \pi_{t+1}. \end{aligned} \tag{2}$$

Expectations are fully rational when $\lambda = 0$. We explore the implications for indeterminacy when not all agents are fully rational.

The second equation is the Phillips Curve which states that inflation depends on the expectation of next period's inflation and the output gap:

$$(\pi_t - \pi^*) = \beta(\pi^e_{t+1} - \pi^*) + \kappa x_t + \xi^\pi_t, \tag{3}$$

where π^* is the long-run steady state inflation rate, ξ^π_t is an exogenous cost shock, and κ is a reduced form parameter that is inversely related to the degree of price stickiness.[2]

The third relationship governs monetary policy:

$$r_t = (1 - \rho_r)(r^n + \pi^*) + \rho_r r_{t-1} + (1 - \rho_r)\left[\psi_\pi(\pi^A_t - \pi^*) + \psi_x x_t\right] + \epsilon^r_t, \tag{4}$$

where ρ_r captures persistence, and ψ_π and ψ_x represent policy responses to inflation and the output gap, respectively. The average inflation target is given by π^A_t and ϵ^r_t is a monetary policy shock.

2.2. Average Inflation Targeting

Monetary policy targets an average value of inflation "over time" that may include backward- and forward-looking terms for inflation. Let the average inflation target be given by,

$$\pi^A_t = \gamma\pi^B_t + (1 - \gamma)\pi^F_t, \tag{5}$$

where $\gamma \in [0, 1]$ is the relative weight given to past average inflation, π^B_t, versus expected future average inflation, π^F_t.

It is typical in the average inflation targeting literature to define the average inflation target as an arithmetic mean over a defined target "window" with a specific, finite number of quarters in the window. However, the Federal Reserve describes its behavior as targeting average inflation "over time" and does not use the word "window" to describe this goal. It is reasonable to suppose that a weighted average may be an appropriate representation monetary policy, where realizations of inflation more close to the present day have a more relevancy for monetary plicy than inflation outcomes from the more distant past. Suppose

[2] In a typical model, $\kappa = (1/\omega)(1 - \omega)(1 - \omega\beta)$, where $\omega \in (0, 1)$ is the fraction of firms that do not re-optimize their prices each period. Smets and Wouters (2007) estimate $\omega \approx 0.66$.

the target for past average inflation is given by,

$$\pi_t^B = \delta_B \pi_t + (1 - \delta_B)\pi_{t-1}^B, \tag{6}$$

where $\delta_B \in (0, 1)$ is the weight given to the most recent observation. We include the current value for inflation, π_t, in this "backward-looking" window. Repeated substitution reveals the nature with which the weights decline geometrically with time:

$$\pi_t^B = \delta_B \sum_{j=0}^{\infty} (1 - \delta_B)^j \pi_{t-j}, \tag{7}$$

where $\delta_B(1-\delta_B)^j$ is the weight on an observation of inflation j periods in the past, $\sum_j \delta_B(1-\delta_B)^j = 1$, and $\lim_{j\to\infty} \delta_B(1-\delta_B)^j = 0$. Smaller values for δ_B imply more weight put on past observations, and so can be viewed as longer backward-looking windows for average inflation targeting. This continuous nature for thinking about the average avoids the awkwardness implied by a finite equally-weighted window, where information on inflation has no relevancy on one side of the window's limit, and has as much relevancy as the present-day on the inside of the limit. Since δ_B is the weight on the current observation for the calculation of the average, and since the weight on an observation in an equally-weighted arithmetic mean is the inverse of the sample size, a weight of δ_B can be thought of as an approximation of monetary policy behavior using an equally-weighted finite window of length $1/\delta_B$ periods. The continuous nature of δ_B will allow us to more fully explore regions of determinacy.

Considering the forward-looking nature of the target average, similarly let the expected average future inflation given by,

$$\pi_t^F = \delta_F \, \mathbb{E}_t \, \pi_{t+1} + (1 - \delta_F) \, \mathbb{E}_t \, \pi_{t+1}^F, \tag{8}$$

where $\delta_F \in (0, 1)$ is the weight given to next period's expected inflation. The forward-looking average is a sum of only expected future outcomes. Repeated substitution reveals,

$$\pi_t^F = \delta_F \sum_{j=0}^{\infty} (1 - \delta_F)^j E_t \pi_{t+1+j}, \tag{9}$$

where the weight on expected inflation rate j periods in the future, $\delta_F(1 - \delta_F)^j$, declines geometrically with the distance into the future, $\sum_j \delta_F(1-\delta_F)^j = 1$, and $\lim_{j\to\infty} \delta_F(1-\delta_F)^j = 0$. The allow the weight δ_F to be different than δ_B, as the forward-looking time horizon for the average inflation target may be different than the backward-looking time horizon. The

Fed could have a longer forward horizon to give more time and flexibility to bring inflation to its target. Or the Fed could have a shorter forward-horizon, given the greater uncertainty for inflation in the more distant future. The value $1/\delta_F$ approximates the length of an equally-weighted finite forward-looking window. We vary the parameters $\{\delta_B, \delta_F, \gamma, \lambda, \psi_\pi, \psi_x, \rho_r\}$ and explore the implications for determinacy below. Note that a standard Taylor-type rule emerges as a special case with $\gamma = 1.0$ and $\delta_B = 1.0$.

2.3. Full Model

Following Sims (2002), the model can be expressed as,

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi z_t + \Pi \eta_t \tag{10}$$

where y_t is a vector that includes x_t, π_t, r_t, π_t^A, π_t^B, and π_t^F; z_t is a vector of the shocks, ξ_t^x, ξ_t^π, and ξ_t^r; and $\eta_t \equiv y_t - E_{t-1} y_t$ equals the ex-post rational expectations forecast errors. We use the method in Sims (2002) to solve the model and identify if the model is determinant or indeterminant for various parameter values.

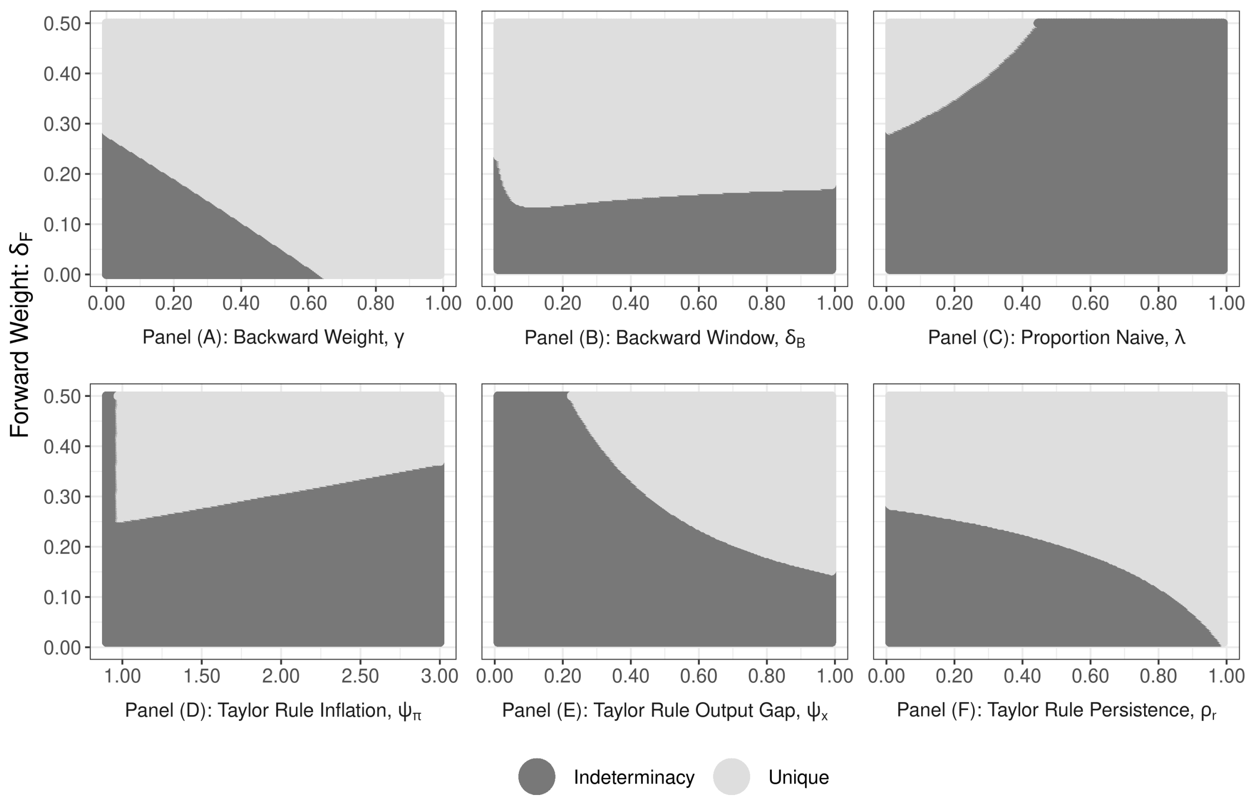
Table 1: Parameter Calibrations

Description	Parameter	Value
Discount rate (quarterly)	β	0.99
Inverse intertemporal elasticity	σ	0.72
Phillips curve coefficient	κ	0.178
Steady state inflation rate (quarterly)	π^*	0.005

Baseline Parameters	Parameter	Value(s)
AIT weight past inflation	γ	$\{0.0, 0.25\}$
Backward-looking weight	δ_B	1.0
Monetary policy: average inflation	ψ_π	1.5
Monetary policy: output gap	ψ_x	0.5
Monetary policy: persistence	ρ_r	0.0

Parameter calibrations are given in Table 1. Values for σ and κ are set to estimates from Smets and Wouters (2007). We set $\pi^* = 0.005$ so that the annualized long-run inflation level is 2%.

We explore the determinacy regions for δ_F, the weight placed on the expected value for the next period's inflation in the forward-looking window. We investigate how the regions of



Notes: Parameters not varying in each graph are given in Table 1. In Panel (B), the baseline parameter for γ is 0.25, implying a 25% weight given to the backward-looking window. In all other panels, γ is set to 0.0, implying purely forward-looking windows.

Figure 1: Regions of Determinacy for Forward-Looking Windows

determinacy differ with calibrations for the weight placed on past inflation in the AIT window, γ; the weight placed on the most recent inflation observation in the backward-looking window, δ_B; and the Taylor rule coefficients, ψ_π, ψ_x, and ρ_r. The baseline parameters given in Table 1 represent the calibrations we use when not varying each of those particular parameters. We use $\gamma = 0.0$ for all calibrations not involving the backward-looking parameter, δ_B, implying monetary policy is purely forward looking. When exploring determinacy ranges for δ_B, we use a weight $\gamma = 0.25$. We set the baseline values for $\psi_x = 0.5$, $\psi_\pi = 1.5$, and $\rho = 0.0$.

3. Results

Figure 1 shows the regions of determinacy for different values of the forward-looking weight, δ_F, depending on six other parameters in the model. Given the inverse relationship between the weight on an individual observation and the length of a finite window, larger values for

δ_F imply shorter forward-looking windows. The largest value considered, 0.5, approximates a two-quarter window.

Panel (A) demonstrates the importance of using current or past values of inflation in the target window. When $\gamma = 0.0$, no weight is put on past or current inflation, and the window is purely forward-looking. The smallest value for δ_F that delivers determinacy in this scenario is 0.28, so the largest possible forward-looking window is approximately 3.57 quarters. When $\gamma \geq 0.63$, all possible forward-looking windows yield determinate solutions. This implies, though, that the target window has at least a 63% weight on the current inflation rate, and therefore at most a 37% weight on future inflation.

Panel (B) shows how the length of the backward-looking window affects determinacy. The minimal combinations of values for δ_B and δ_F that achieve determinacy are each 0.14, implying the longest the forward-looking and backward-looking windows can be are approximately 7.14 quarters. Panel (C) reveals that the presence of naïve agents have crucial implications for determinacy. When more than 40% of agents form naïve expectations, no purely forward-looking window for AIT leads to determinacy.

Panels (D), (E), and (F) show how the length of the forward-looking window depends on the Taylor Rule coefficients. Larger response to inflation lead to more restrictive forward windows. Larger responses to the output gap are also important for determinacy. Values of $\psi_x \geq 0.2$ are necessary and larger values allow for longer forward windows. Monetary policy persistence also plays important role in that the stronger is persistence, the longer can be the forward looking window.

4. Conclusion

Forward-looking AIT has important implications for monetary policy to avoid issues of indeterminacy. We find large ranges of indeterminacy, especially when a large portion of aggregate expectations are naïve, when little weight is put on the output gap, and when the forward window is greater than two years. Our findings suggest that the Fed can assure determinacy with a high rate of monetary policy persistence or with a target window that puts significant weight on current and past inflation.

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