Implications for Determinacy with Average Inflation Targeting

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Abstract

We use a three-equation New Keynesian model to explore the implications of backward- and forward-looking windows when constructing a measure for an average inflation target within a monetary policy rule, and investigate the conditions for determinacy. A uniquely determined equilibrium rules out an environment subject to sunspot shocks that can lead to self-fulfilling shocks for inflation expectations. We find that the length of the forward-looking window under average inflation targeting (AIT) is limited to approximately eight-quarters or less to assure determinacy. We further demonstrate how limitations on the length of the forward window depends on other parameters in the model, including other AIT policy parameters and parameters governing how agents form expectations.

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1. Introduction

At the 2020 Jackson Hole symposium, Fed Chairman Jerome Powell laid out the new monetary policy framework where inflation could temporarily deviate from the Fed's inflation target in the short run, as long as the average level of inflation in the medium to long run remained consistent with the Fed's target. Consequently under Average Inflation Targeting (AIT), if inflation were to remain consistently below its target for some period, it would be

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followed by a period where inflation would remain above its target, and vice versa. In this way, one aim of adopting an AIT framework was to help provide a nominal anchor where long-run inflation expectations were consistent with the central bank's target.

Since the Fed's announcement, academic research began examining a range of issues under AIT, such as the welfare implications (e.g. Budianto et al., 2020; Eo and Lie, 2020), how adoption of AIT has affected inflation expectations (e.g. Coibion et al., 2020; Hoffmann et al., 2022), and the implications for boundedly-rational expectations on macroeconomic outcomes under AIT (eg: Honkapohja and McClung, 2021; Budianto et al., 2020). A central question that pertains to the literature on the AIT framework is how the 'average' level of inflation is determined. The average may be based purely on past values of inflation, expectations of future values for inflation, or some combination of the two.

We examine this issue within the context of a standard three-equation New Keynesian model. We construct a measure of the average inflation target that is a weighted average of past observations of inflation, current inflation, and expectations for the future path of inflation. We evaluate conditions on monetary policy and the inflation target window for determinacy and indeterminacy. With indeterminacy, the economy is subject to sunspot shocks, where self-fulfilling shocks to inflation expectations can lead to excess volatility in inflation and the business cycle (see, for example, Lubik and Schorfheide, 2004). This paper identifies the boundaries for average inflation targeting windows for determinate outcomes, and how these boundaries depend on the other parameters in the model.

2. Model

This paper builds upon a standard representative agent New Keynesian model along the lines of Clarida et al. (1999). This standard model consists of infinitely-lived utility-maximizing households, monopolistically-competitive firms, imperfectly flexible prices, and monetary policy.

2.1. Baseline Framework

There are three key equations of interest. Once log-linearized, the dynamic IS curve derived from the consumer's utility maximization problem states that the current output gap depends on expectations of next periods output gap, and is negatively related to the difference between

the ex-ante real interest rate and the natural rate of interest:

$$x_t = x_{t+1|t}^e - \frac{1}{\sigma}\left(r_t - \pi_{t+1|t}^e - r^n\right) + \xi_t^x \tag{1}$$

where in period t, x_t denotes the output gap (given by the difference between the log of output and its natural rate), r_t is the nominal interest rate, π_t the inflation rate, $r^n = 1/\beta - 1$ the natural rate of interest and where $\beta \in (0,1)$ is the household's discount factor; $x_{t+1|t}^e$ and $\pi_{t+1|t}^e$ represent expectations of private sector agents on next period's output gap and inflation rate respectively. The preference parameter, σ, is inversely related to consumers' intertemporal elasticity of substitution, and ξ_t^x, represents a demand shock. A fraction of agents, $\lambda \in [0,1)$, form naïve expectations, so in aggregate, expectations are given by:

$$x_{t+1}^e = \lambda x_t + (1-\lambda)\,\mathbb{E}_t\,x_{t+1}$$

$$\tag{2}$$

$$\pi_{t+1}^e = \lambda \pi_t + (1-\lambda)\,\mathbb{E}_t\,\pi_{t+1}$$

Expectations are fully rational when $\lambda = 0$. We explore the implications for indeterminacy below when not all expectations are fully rational.

The second key equation is the New Keynesian Phillips Curve (NKPC), which states that inflation depends on the expectation of next period's inflation, and the output gap:

$$(\pi_t - \pi^*) = \beta(\pi_{t+1|t}^e - \pi^*) + \kappa x_t + \xi_t^\pi \tag{3}$$

where π^* is the long-run steady state inflation rate, ξ_t^π is an exogenous cost shock and κ is a reduced form parameter that is inversely related to the degree of price stickiness.[1]

The third relationship is an interest rate rule that describes the conduct of monetary policy:

$$r_t = (1-\rho_r)(r^n + \pi^*) + \rho_r r_{t-1} + (1-\rho_r)\left[\psi_\pi(\pi_t^A - \pi^*) + \psi_x x_t\right] + \epsilon_t^r \tag{4}$$

where ρ_r represents the parameter used to smooth the path of interest rates; parameters ψ_π and ψ_x represent monetary policy responses to inflation and the output gap, respectively. The variable, π_t^A represents the Fed's average inflation target. The shock term, ϵ_t^r, denotes a monetary policy shock.

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[1]In a typical microfounded New Keynesian model, $\kappa = (1/\omega)(1 - \omega)(1 - \omega\beta)$, where $\omega \in (0,1)$ is the fraction of firms that do not re-optimize their prices each period. Smets and Wouters (2007) estimate ω to be approximately 0.66.

2.2. Average Inflation Targeting

Under AIT the central bank targets an average value of inflation instead of the current inflation rate. The target window may include both backward and forward-looking terms for inflation, and the relative weights for each may be different. Suppose the average inflation target is given by,

$$\pi_t^A = \gamma \pi_t^B + (1 - \gamma) \pi_t^F, \tag{5}$$

where $\gamma \in [0, 1]$ is the relative weight given to past inflation versus expected future inflation, π_t^B is the backward-looking average inflation, and π_t^F is the forward-looking average inflation. π_t^B is given by:

$$\pi_t^B = \delta_B \pi_t + (1 - \delta_B) \pi_{t-1}^B, \tag{6}$$

where $\delta_B \in (0, 1)$ is the weight given to the most recent observation. We include the current value for inflation, π_t, in this "backward-looking" window. Repeated substitution reveals π_t^B the nature with which the weights declining geometrically with time,

$$\pi_t^B = \delta_B \sum_{j=0}^{\infty} (1 - \delta_B)^j \pi_{t-j}, \tag{7}$$

where $\delta_B (1 - \delta_B)^j$ is the weight on an observation of inflation j periods in the past, such that $\sum_j \delta_B (1 - \delta_B)^j = 1$ and $\lim_{j \to \infty} \delta_B (1 - \delta_B)^j = 0$. Smaller values for δ_B can be viewed as longer backward-looking windows for average inflation targeting and a weight of δ_B approximates monetary policy behavior using a equally-weighted finite window of length $1/\delta_B$ periods.

Similarly, suppose π_t^F is given by:

$$\pi_t^F = \delta_F \, \mathbb{E}_t \, \pi_{t+1} + (1 - \delta_F) \, \mathbb{E}_t \, \pi_{t+1}^F, \tag{8}$$

where $\delta_F \in (0, 1)$ is the weight given to next periods expected inflation. The forward-looking average is a sum of only expected future outcomes. Repeated substitution of equation (8) similarly reveals the nature the weights on future values for inflation:

$$\pi_t^F = \delta_F \sum_{j=0}^{\infty} (1 - \delta_F)^j E_t \pi_{t+1+j}, \tag{9}$$

where the weight on expected inflation rate j periods in the future, $\delta_F (1 - \delta_F)^j$, declines geometrically with the distance into the future; also $\sum_j \delta_F (1 - \delta_F)^j = 1$ and $\lim_{j \to \infty} \delta_F (1 - \delta_F)^j = 0$. $1/\delta_F$ can be considered to approximate the length of an equally-weighted finite

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forward-looking window.

The continuous nature of the forward-looking and backward-looking weighted averages are both mathematically convenient and arguably more realistic than rolling-window arithmetic mean. Equations (6) and (8) are difference equations over a single period. The parameters, δ_B and δ_F can be varies on a continuous scale, instead of making discrete choices for rolling window lengths. A standard Taylor-type rule, like equation (4), is also a special case of the larger model, with $\gamma = 1.0$ and $\delta_B = 1.0$. By considering values for these parameters below 1.0, we can conveniently describe how AIT frameworks can differ from the standard monetary policy rule.

It may be more realistic to suppose that the central bank puts more weight on more recent past observations than on farther observations. It may be more realistic to suppose too that the central bank puts more weight on expectations for the near future than expectations for the distant future, where inflation forecasts may be less reliable. Similarly, the Fed may put more weight on more recent past inflation than on the history of inflation in the more distant past.

2.3. Full Model

Following Sims (2002), the model can be expressed in the compact form,

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi z_t + \Pi \eta_t \tag{10}$$

where y_t is a vector of the endogenous variables in the model, including x_t, π_t, r_t, π_t^A, π_t^B, and π_t^F; z_t is a vector of the three exogenous shocks t, ξ_t^x, ξ_t^π, and ξ_t^r; and η_t are the ex-post rational expectations forecast errors, $\eta_t \equiv y_t - E_{t-1} y_t$. Sims (2002) describes a method to solve the model and the conditions for existence and uniqueness of the solution. We use this method to explore parameter values for monetary policy behavior that lead to indeterminacy (i.e. existence of an infinite number of solutions).

We calibrate the model using the parameters in Table 1. The values for σ and κ are set to the median posterior estimates from Smets and Wouters (2007). We set $\pi^* = 0.005$ so that the annualized long-run inflation level is 2%, the Federal Reserve's explicit target.

In the results section below, we explore determinacy regions for different values of δ_F, the weight placed on the expected value for the next period's inflation in the forward-looking window. We explore how regions of determinacy for this parameter differs with calibrations

Table 1: Parameter Calibrations

Description	Parameter	Value
Discount rate (quarterly)	β	0.99
Inverse intertemporal elasticity	σ	0.72
Phillips curve coefficient	κ	0.178
Steady state inflation rate (quarterly)	π^*	0.005

Baseline Parameters	Parameter	Value(s)
AIT weight on past inflation	γ	$\{0.0, 0.25\}$
Backward-looking weight	δ_B	1.0
Monetary policy response to average inflation	ψ_π	1.5
Monetary policy response to output gap	ψ_x	0.5
Monetary policy persistence	ρ_r	0.81

for the weight placed on past inflation in the AIT window, γ, the weight placed on the most recent inflation observation in the backward-looking window, δ_B, and the Taylor rule coefficients, ψ_π, ψ_x, and ρ_r. The baseline parameters given in Table 1 represent the calibrations we use when not varying each of those particular parameters. We use $\gamma = 0.0$ for all calibrations not involving the backward-looking parameter, δ_B, implying monetary policy is purely forward looking. When exploring determinacy ranges for δ_B, we use a weight $\gamma = 0.25$. We follow Taylor (1993) and use baseline parameters $\psi_x = 0.5$ and $\psi_\pi = 1.5$. We use a high degree of inflation persistence for a baseline ($\rho = 0.81$), using the estimated value in Smets and Wouters (2007).

3. Results

Figure 1 shows the regions of determinacy for different values of the forward-looking weight, δ_F, depending on four other parameters in the model. On the vertical axis in each panel are values for δ_F from 0.0 to 0.5. Given the inverse relationship between the weight on an individual observation and the length of a finite window, larger values for δ_F can be thought of as shorter forward-looking windows. The largest value considered, 0.5, approximates a forward-looking window of two quarters.

The first panel demonstrates the importance of using current or past values for inflation in the target window. When $\gamma = 0.0$, no weight is put on past or current inflation, and the window is purely forward-looking. The smallest value for δ_F that delivers indeterminacy in this scenario is 0.12, so the largest possible forward-looking window is approximately 8.33
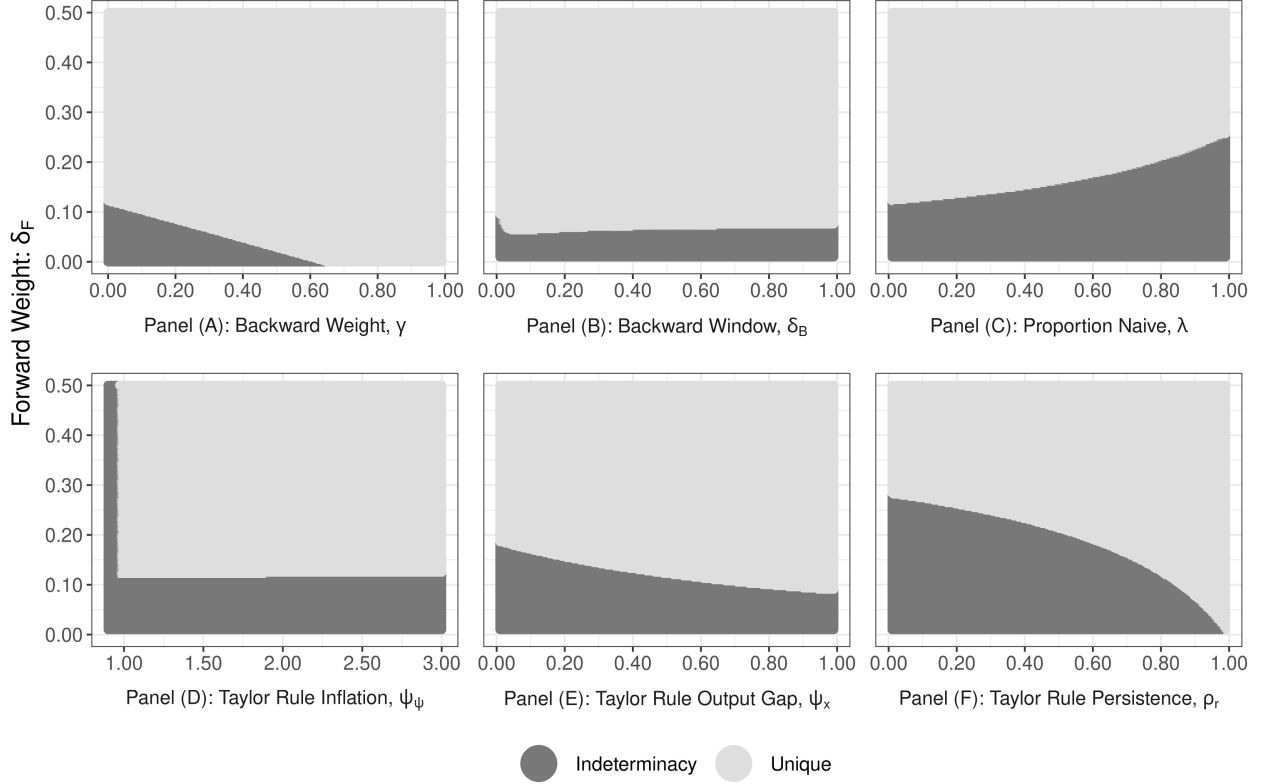
Notes: Larger values for δ_F imply shorter forward-looking windows. Other parameters not varying in each graph are given in Table 1. In Panel (B), the baseline parameter for γ is 0.25, implying a 25% weight given to the backward-looking window versus the forward-looking window. In all other panels, the baseline parameter for γ is equal to 0.0, implying purely forward-looking windows.

Figure 1: Regions of Determinacy for Forward-Looking Windows

quarters, or just over two years. When $\gamma \geq 0.63$, all possible forward-looking windows yield determinate solutions. This implies, though, that the average inflation window has at least a 63% weight on the current inflation rate, and therefore at most a 37% weight on the forward window.

Panel (B) shows how the length of the backward-looking window affects determinacy. From Panel (A), we know that large weights placed on the backward-looking window can yield determinacy.[2] We use a baseline value of $\gamma = 0.25$ and find minimal variability in values of δ_F that yield determinacy. The most extreme value for δ_F yielding indeterminacy is 0.0933, implying that with 25% weight put on the backward-window, the backward window can be as long as possible, and the forward-looking window can be as long as about $1/0.0933 = 10.7$ quarters.

Panel (C) shows that larger proportions of naïve agents lead to shorter lengths of forward-looking windows for determinacy. At the extreme, when nearly all agents are naïve, the smallest value δ_F can take is 0.255, so the longest a forward-looking window can be in such a scenario is approximately 3.92 quarters, just less than one year.

Panes (D), (E), and (F) show how the length of the forward-looking window depends on the Taylor Rule coefficients. Panel (D) shows that determinacy varies little with ψ_π. Panel (E) shows that as the Federal Reserve responds more strongly to the output gap, the longer it can make its forward-looking window (i.e. the smaller it can allow δ_F). Panel (F) shows the important role persistence in monetary policy plays. The stronger is the level of persistence, the longer can be the forward looking window. At the extreme case when $\rho_r = 0.0$, the smallest δ_F can be and still yield a determinant solution is 0.282, implying forward-looking windows be no longer than approximately 3.55 quarters.

4. Conclusion

The Federal Reserve's decision to engage in average inflation targeting has implications for monetary policy to avoid problems of indeterminacy, especially regarding the weight that the Fed puts on expected future values of inflation in its average inflation target. When little to no weight is put on past inflation, we find evidence that the Fed should restrict its forward looking window to less than two year to assure determinacy. When there is

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[2]Not shown in the figure, we verified that when $\gamma = 1.0$, implying that monetary policy has no forward-looking component, all values for $\delta_B \in [0,1]$ yield unique solutions.

little persistence in the federal funds rate, we find evidence that the Fed should restrict its forward looking window to less than one year. We also find that when a large portion of economic agents form naïve expectations, the Fed should also restrict its forward-looking window to less than one year. When there is a high rate of persistence in monetary policy or the average inflation target is largely backward-looking, there are large ranges for average inflation targeting policy that lead to determinacy.

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