Implications for Determinacy with Average Inflation Targeting

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Statement on Longer-Run Goals and Monetary Policy Strategy by the FOMC:

> *"In order to anchor longer-term inflation expectations at this level, the Committee seeks to achieve inflation that averages 2 percent over time, and therefore judges that, following periods when inflation has been running persistently below 2 percent, appropriate monetary policy will likely aim to achieve inflation moderately above 2 percent for some time."*

August 27, 2020 - Jackson Hole, Wyoming

"... our new statement indicates that we will seek to achieve inflation that **averages** *2 percent over time. Therefore, following periods when inflation has been running below 2 percent, appropriate monetary policy will likely aim to achieve inflation moderately above 2 percent for some time.*

In seeking to achieve inflation that averages 2 percent over time, **we are not tying ourselves to a particular mathematical formula that defines the average**. *Thus, our approach could be viewed as a flexible form of average inflation targeting."- Jerome Powell*

Research Question

Question(s): What is the impact of Average Inflation Targeting - AIT?

- How is the 'average' measure of inflation constructed?
  - Is it an (weighted) average of past inflation terms?
  - Is it an (weighted) average of expected future inflation?
  - Is the measure a hybrid?
- Are there implications for (in)determinacy?
- Does the length of the 'window' used to construct the average impact stability?
- If we consider a hybrid, what happens if the window lengths (forward vs backwards) are asymmetric?
- What is the impact of a monetary policy shock under AIT?

Literature

AIT

- Welfare Implications and Stability - Budianto et al. (2020), Eo and Lie (2020), Piergallini (2022)
- Impact on Inflation expectations - Coibion et al. (2020), Hoffmann et al. (2022)
- Impact on boundedly rational expectations - Honkapohja and McClung (2021), Budianto et al. (2020)

Monetary policy rules

- Indeterminacy and Stability of policy rules - Clarida et al. (2000), Lubik and Schorfheide (2004), Evans and McGough (2005), Nessén and Vestin (2005), Castelnuovo and Fanelli (2015), Mertens and Williams (2019), Svensson (2020)

New Keynesian Framework

There are 3 key equations of interest in the NK framework with monetary policy

- The key equations include:
  - The IS Curve - derived from the Household's utility maximization problem
  - The Phillips Curve - derived from the Firm's problem
  - The monetary policy rule (e.g. a Taylor-type rule)
- We log-linearize the model around the (long-run) steady state.

New Keynesian Framework

- The IS Equation:

$$x_t = x^e_{t+1|t} - \frac{1}{\sigma}\left(r_t - \pi^e_{t+1|t} - r^n\right) + \xi^x_t, \tag{1}$$

- We assume that a fraction of agents, $\lambda \in [0, 1)$, form naïve expectations, so aggregate expectations are given by,

$$\begin{aligned} x^e_{t+1} &= \lambda x_t + (1-\lambda)\,\mathbb{E}_t\, x_{t+1}, \\ \pi^e_{t+1} &= \lambda \pi_t + (1-\lambda)\,\mathbb{E}_t\, \pi_{t+1}. \end{aligned} \tag{2}$$

- Expectations are fully rational when $\lambda = 0$. We allow $\lambda \neq 0$.

New Keynesian Framework

- The Phillips Curve:

$$(\pi_t - \pi^*) = \beta(\pi^e_{t+1|t} - \pi^*) + \kappa x_t + \xi^\pi_t, \tag{3}$$

- The monetary policy rule:

$$\begin{aligned} r_t = {} & (1-\rho_r)(r^n + \pi^*) + \rho_r r_{t-1} \\ & + (1-\rho_r)\left[\psi_\pi(\pi^A_t - \pi^*) + \psi_x x_t\right] + \epsilon^r_t, \end{aligned} \tag{4}$$

Average Inflation Targeting

- We assume that monetary policy targets an average value of inflation over a target window that may include backward- and forward-looking terms for inflation.
- The average inflation target is:

$$\pi_t^A = \gamma \pi_t^B + (1 - \gamma)\pi_t^F, \tag{5}$$

where $\gamma \in [0, 1]$ is the relative weight given to past average inflation, π_t^B, versus expected future average inflation, π_t^F.

Backwards window for AIT

- The past average inflation, π_t^B, is given by:

$$\pi_t^B = \delta_B \pi_t + (1 - \delta_B)\pi_{t-1}^B, \tag{6}$$

where $\delta_B \in (0, 1)$ is the weight given to the most recent observation.

- Substituting recursively, we obtain:

$$\pi_t^B = \delta_B \sum_{j=0}^{\infty} (1 - \delta_B)^j \pi_{t-j},$$

where $\sum_j \delta_B (1 - \delta_B)^j = 1$, and $\lim_{j \to \infty} \delta_B (1 - \delta_B)^j = 0$.

- A weight of δ_B approximates monetary policy behavior using an equally-weighted finite window of length $1/\delta_B$ periods.

Forward window for AIT

- Similarly, the forward window includes expected future average inflation, π_t^F:

$$\pi_t^F = \delta_F \, \mathbb{E}_t \, \pi_{t+1} + (1 - \delta_F) \, \mathbb{E}_t \, \pi_{t+1}^F \tag{7}$$

$$\implies \pi_t^F = \delta_F \sum_{j=0}^{\infty} (1 - \delta_F)^j \, \mathbb{E}_t \, \pi_{t+1+j}$$

  where $\delta_F \in (0, 1)$ is the weight given to next period's expected inflation and $\sum_j \delta_F (1 - \delta_F)^j = 1$, and $\lim_{j \to \infty} \delta_F (1 - \delta_F)^j = 0$.

- Similarly, $1/\delta_F$ approximates the length of an equally-weighted finite forward-looking window.

Summary of Key Parameters

- We vary the parameters below and explore the implications for determinacy:
  - $\{\delta_B, \delta_F\}$ - the weights on the previous/next period's (expected) inflation
  - γ - the relative weight on past vs expected future inflation
  - λ - the share of the population that form naïve expectations
  - $\{\psi_\pi, \psi_x\}$ - the weights on the inflation and output gap terms in the policy rule
  - ρ_r - the persistence of monetary policy
- Note that a standard Taylor-type rule emerges as a special case with $\gamma = 1.0$ and $\delta_B = 1.0$.

Key Model Equations I

1. The IS Equation:

$$x_t = x^e_{t+1|t} - \frac{1}{\sigma}\left(r_t - \pi^e_{t+1|t} - r^n\right) + \xi^x_t,$$

2. The Phillips Curve:

$$(\pi_t - \pi^*) = \beta(\pi^e_{t+1|t} - \pi^*) + \kappa x_t + \xi^\pi_t,$$

3. Evolution of the expected output gap

$$x^e_{t+1} = \lambda x_t + (1-\lambda)\,\mathbb{E}_t\, x_{t+1}$$

4. Evolution of the expected inflation

$$\pi^e_{t+1} = \lambda \pi_t + (1-\lambda)\,\mathbb{E}_t\, \pi_{t+1}$$

Key Model Equations II

5. The monetary policy rule:

$$r_t = (1-\rho_r)(r^n + \pi^*) + \rho_r r_{t-1} + (1-\rho_r)\left[\psi_\pi(\pi_t^A - \pi^*) + \psi_x x_t\right] + \epsilon_t^r,$$

6. The average inflation target:

$$\pi_t^A = \gamma \pi_t^B + (1-\gamma)\pi_t^F,$$

7. Past average inflation:

$$\pi_t^B = \delta_B \pi_t + (1-\delta_B)\pi_{t-1}^B,$$

8. Expected future average inflation:

$$\pi_t^F = \delta_F \, \mathbb{E}_t \, \pi_{t+1} + (1-\delta_F) \, \mathbb{E}_t \, \pi_{t+1}^F$$

Our Approach

- Following Sims (2002), the model can be expressed as:

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi z_t + \Pi \eta_t \tag{8}$$

where y_t is a vector that includes x_t, π_t, r_t, π_t^A, π_t^B, and π_t^F; z_t is a vector of the shocks, ζ_t^x, ζ_t^π, and ζ_t^r; and $\eta_t \equiv y_t - E_{t-1} y_t$ equals the ex-post rational expectations forecast errors.

- We use the method in Sims (2002) to explore regions of indeterminacy.
  - i.e. do the rank conditions hold for our system of equations?

Calibration

Description	Parameter	Value
Discount rate (quarterly)	β	0.99
Inverse intertemporal elasticity	σ	0.72
Phillips curve coefficient	κ	0.178
Steady state inflation rate (quarterly)	π^*	0.005

Baseline Parameters	Parameter	Value(s)
AIT weight past inflation	γ	$\{0.0, 0.25\}$
Backward-looking weight	δ_B	1.0
Monetary policy: average inflation	ψ_π	1.5
Monetary policy: output gap	ψ_x	0.5
Monetary policy: persistence	ρ_r	0.0

Regions of Determinacy for Forward Looking Windows

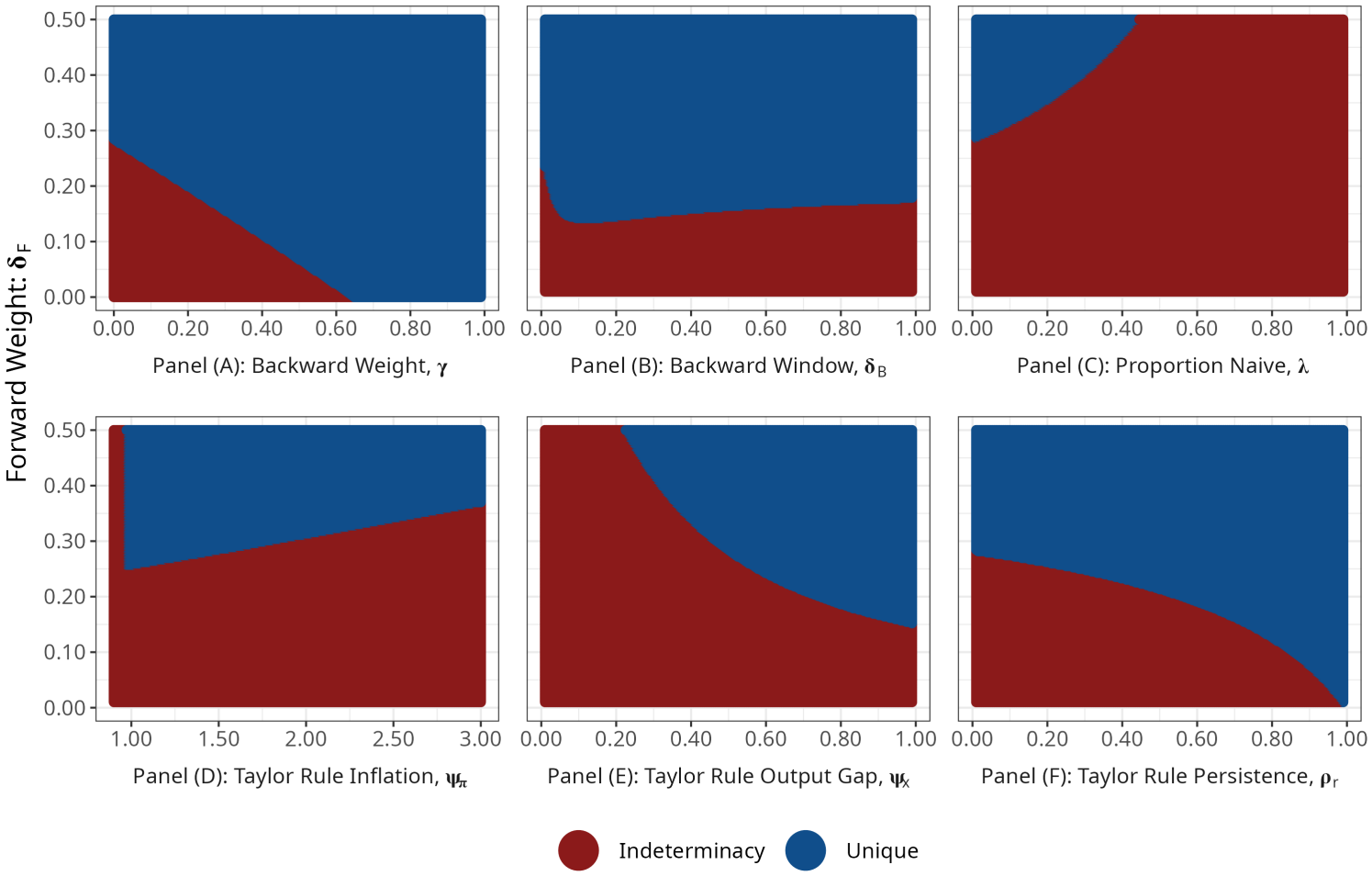
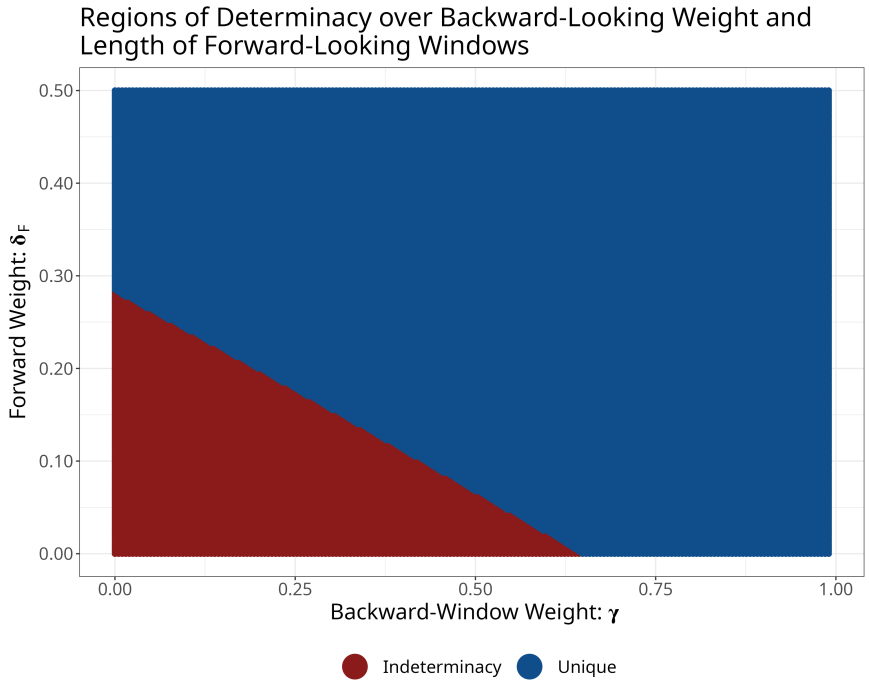


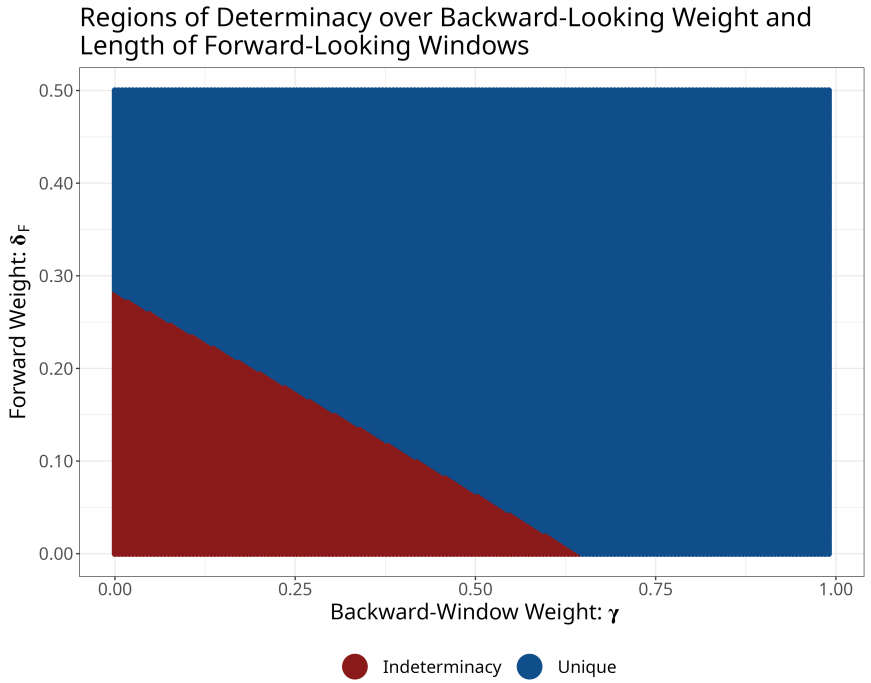
Figure: In Panel (B), $\gamma = 0.25$, implying a 25% weight given to the backward-looking window. In all other panels, $\gamma = 0$, implying purely forward-looking windows.

Varying the relative weight on past inflation, γ



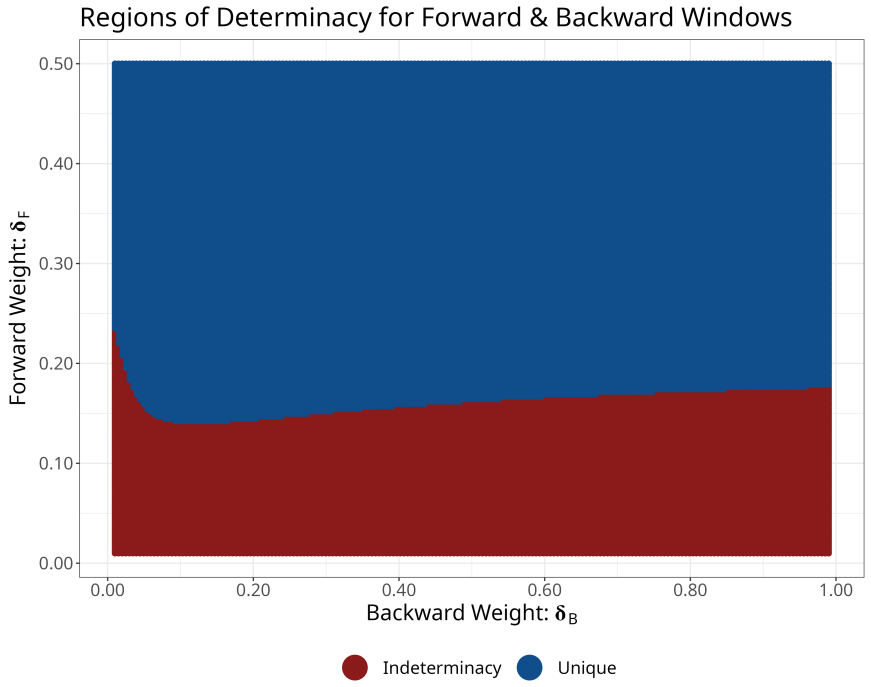
1. In panel (A), $\delta_F = 0.28$ is the smallest value that delivers determinacy in this scenario (the largest possible forward-looking window is approximately 3.57 quarters).

Varying the relative weight on past inflation, γ



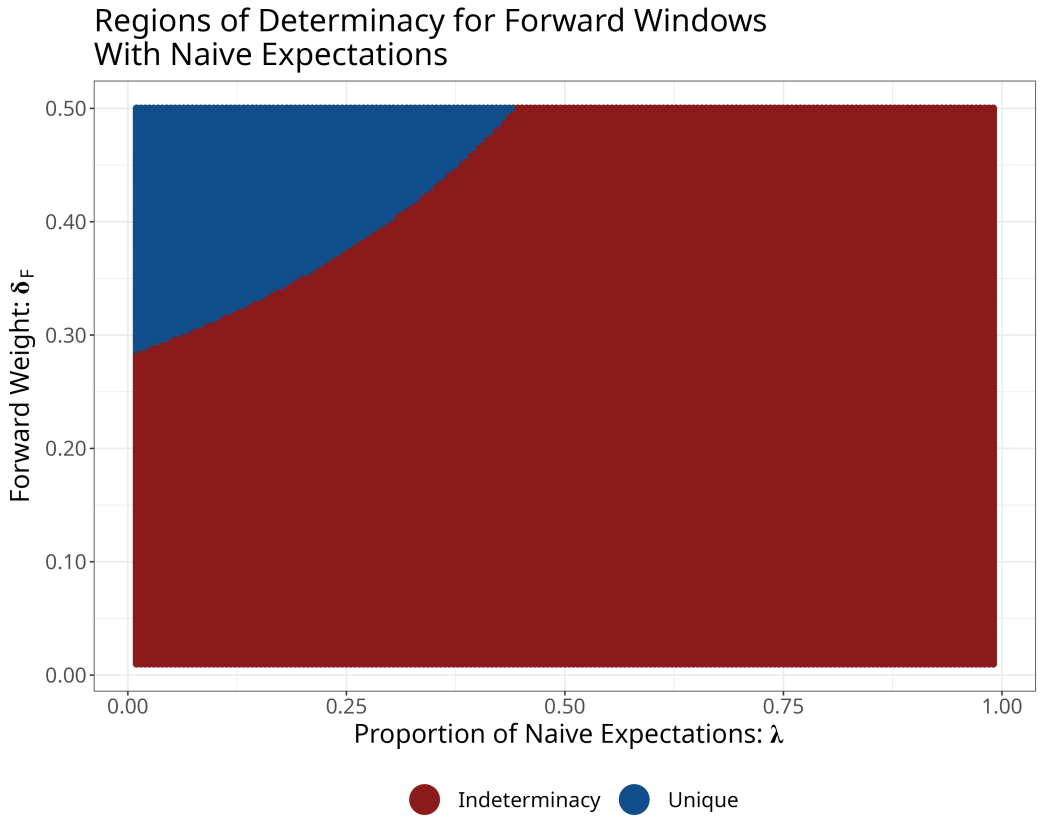
2. When $\gamma \geq 0.63$, all possible forward-looking windows yield determinate solutions.
   - This implies, though, that the target window has at least a 63% weight on the current inflation rate, and therefore at most a 37% weight on future inflation.

Varying the backward-window weight, δ_B



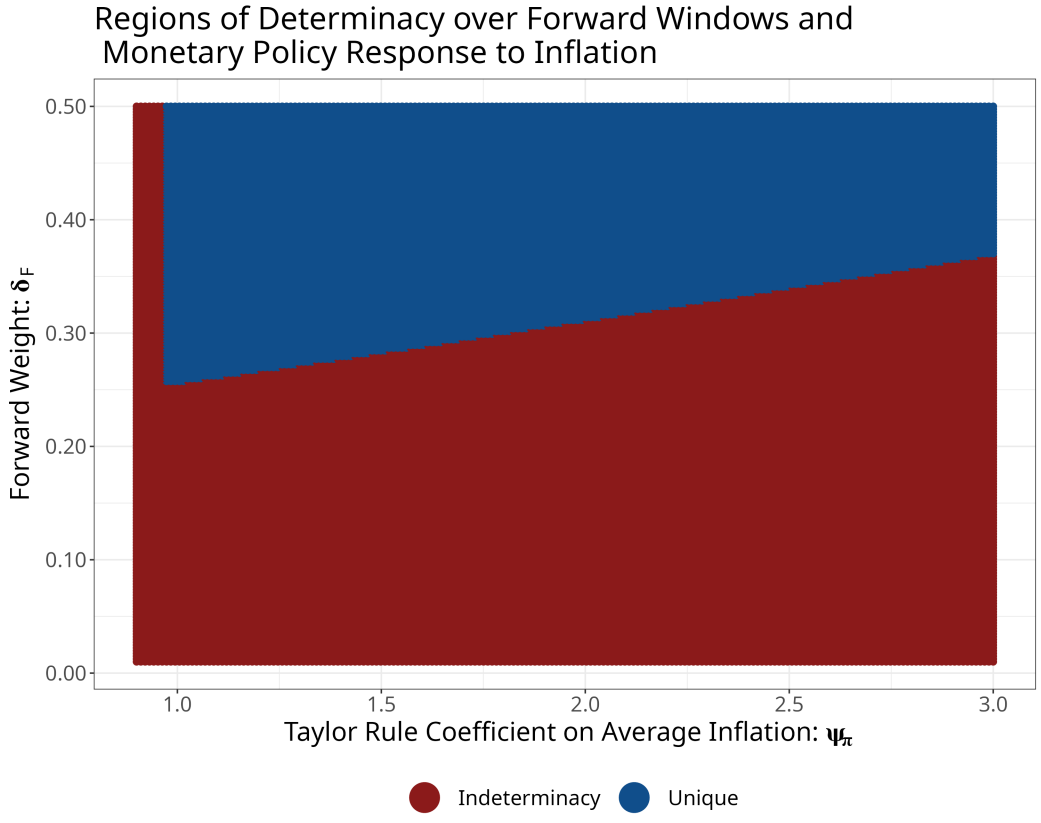
3. The minimal combinations of values for δ_B and δ_F that achieve determinacy are each 0.14, implying the longest the forward-looking and backward-looking windows can be are approximately 7.14 quarters.

Varying the Proportion of naïve expectations, λ



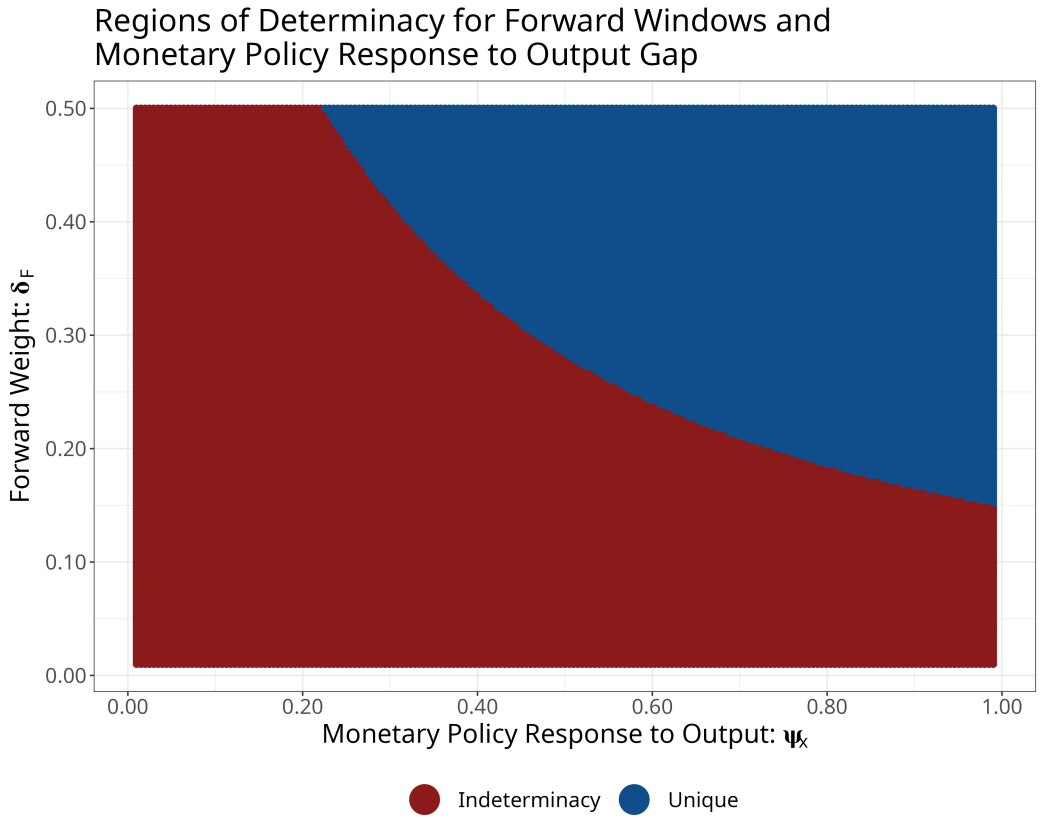
4. When more than 40% of agents form naïve expectations, no purely forward-looking window for AIT leads to determinacy.

Varying the weight on the inflation term, ψ_π



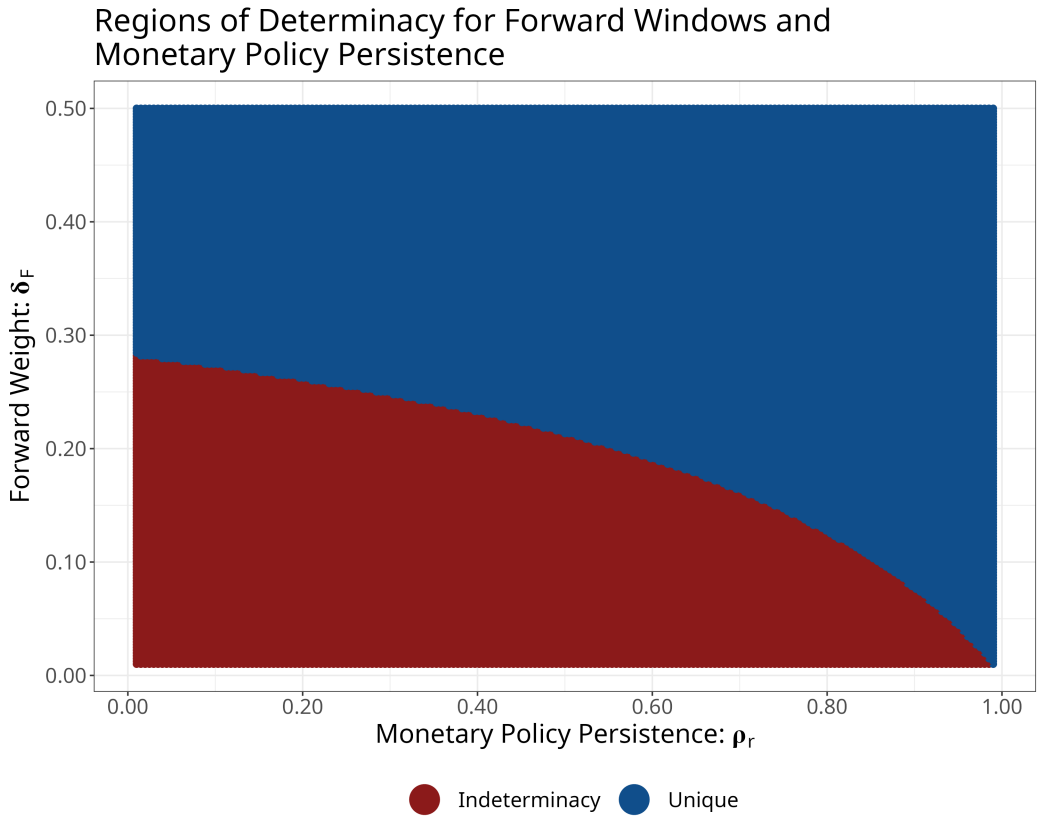
5. In general, a larger response to inflation leads to more restrictive (shorter) forward windows

Varying the weight on the output-gap term, ψ_x



6. $\psi_x \geq 0.2$ is necessary for determinacy and larger values allow for longer forward windows.

Varying the weight on policy persistence, ρ_r



7. The stronger the persistence of monetary policy, the longer can be the forward looking window.

Work to be done

Looking ahead ...

- Explore the rank condition to see what/how causes system to change from determinate to indeterminate (and vice versa) as model parameters change
- Impact of a monetary policy shock under AIT
- Impact on central bank credibility (crediblity/price shocks)
- Are there thresholds on how far a central bank can deviate from the long run target (... before monetary policy becomes time-inconsistent)?

The End!

Thank you for your questions and comments.

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