# Regime Switching, Learning, and the Great Moderation

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# Purpose



- How much does "bad luck" explain changing volatility when adaptive expectations react to suspicions of structural changes.
- **Great Inflation / Great Moderation:** decade of high inflation and volatility followed by seemingly permanent reduction in macroeconomic volatility.
- Bad luck explanation: changes in volatility is due to *exogenous* changes in variance of structural shocks.
- Adaptive expectations:
  - Agents do not know reduced form parameters governing evolution of the economy.
  - Agents estimate a reduced form VAR(1) by least squares.
  - Agents endogenously give more recent observations more weight.

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- Volatile periods were hit with bad shocks.
- Sims and Zha (AER, 2006): evidence points in favor of bad shocks over changes in policy.
- Stock and Watson (2003): improved policy accounts for only small part of volatility slowdown.
- Justiniano and Primiceri (AER, 2008): evidence points in favor of time evolving variance of structural shocks.

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- Oraphanides and Williams (JEDC, 2005): Monetary authority was optimizing, but mis-informed.
- Primiceri (QJE, 2006): Monetary authority mis-informed, expectations improved with time.
- Milani (JME, 2007): Agents learn, little evidence of difference in policy parameters.
- Milani (2008): Time varying expectations explains time-varying volatility.
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# New Keynesian Model



- Consumer behavior:
  - Choose consumption and labor to maximize utility.
  - Habit formation: utility on consumption depends on past consumption.
- Producer behavior:
  - Intermediate goods are produced with labor in monopolistically competitive markets.
  - Intermediate goods subject to Calvo (1983) price friction.
  - Price indexation: when not re-optimizing prices, price can adjust according to past inflation.
- Taylor (1993) type monetary policy:
  - Nominal interest rate responds to inflation, expected future output gap, and past interest rate.

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# Optimal Consumer Behavior



$$\tilde{\lambda}_t = E_t \tilde{\lambda}_{t+1} + \hat{r}_t - E_t \pi_{t+1} - r_t^n,$$

$$\tilde{\lambda}_t = \frac{1}{(1-\beta\eta)(1-\eta)} \left[ \beta\eta E_t \tilde{y}_{t+1} - (1+\beta\eta^2)\tilde{y}_t + \eta \tilde{y}_{t-1} \right]$$

- Variables:
  - $\tilde{\lambda}_t$: marginal utility of income.
  - $\tilde{y}_t$: output gap.
  - $\hat{r}_t$: nominal interest rate.
  - $\pi_t$: inflation.
  - $r_t^n$: "natural rate" shock, deviation of interest rate from flexible price outcome.
- Parameters:
  - $\eta \in [0, 1)$: degree of habit formation.
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# Producer Behavior



- Phillips curve:

$$\pi_t = \frac{1}{1+\beta\gamma}\left[\gamma\pi_{t-1} + \beta E_t\pi_{t+1} + \kappa(\tilde{y}_t - \mu\tilde{\lambda}_t) + u_t\right]$$

- Cost push shock: $u_t$.
- Parameters:
  - $\gamma \in [0,1)$: degree of price indexation.
  - $\kappa \in (0,\infty)$: reduced form parameter inversely related to degree of price flexibility.

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# Monetary policy



- Nominal interest rate responds to expected output gap and inflation:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r)\left(\psi_\pi E_t \pi_{t+1} + \psi_y E_t \tilde{y}_{t+1}\right) + \epsilon_{r,t}$$

- Monetary policy shock: $\epsilon_{r,t}$.
  - $\psi_\pi \in (0, \infty)$: feedback on inflation.
  - $\psi_y \in (0, \infty)$: feedback on output.
  - $\rho_r \in (0, 1)$: smoothing parameter.

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# Structural Shocks



- Natural interest rate shock:

$$r_t^n = \rho_n r_{t-1}^n + \epsilon_{n,t}$$

- Cost push shock:

$$u_t = \rho_u u_{t-1} + \epsilon_{u,t}$$

- Monetary policy shock is not autoregressive.

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# Regime Switching Volatility



- Variance of shocks switches between high and low regimes:

$$Var\begin{bmatrix}\epsilon_{n,t}(s_t)\\ \epsilon_{u,t}(s_t)\\ \epsilon_{r,t}(s_t)\end{bmatrix} = \left\{\begin{array}{ll}\begin{bmatrix}\sigma^2_{n,L} & 0 & 0\\ 0 & \sigma^2_{u,L} & 0\\ 0 & 0 & \sigma^2_{r,L}\end{bmatrix}, & \text{if } s_t = L\\ \begin{bmatrix}\sigma^2_{n,H} & 0 & 0\\ 0 & \sigma^2_{u,H} & 0\\ 0 & 0 & \sigma^2_{r,H}\end{bmatrix}, & \text{if } s_t = H\end{array}\right\}$$

- $s_t = L \rightarrow$ low volatility regime, $s_t = H \rightarrow$ high volatility regime.
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# Markov Switching



- Regimes evolve according to a Markov Chain with transition matrix:

$$P = \begin{bmatrix} p_L & 1 - p_H \\ 1 - p_L & p_H \end{bmatrix}$$

- Probabilities:
  - $p_L$: probability of remaining in low volatility regime.
  - $p_H$: probability of remaining in high volatility regime.
- State evolves according to: $S_{t+1} = PS_t$
  - Where $S_t' = [P(s_t = L) \; P(s_t = H)]$.

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# Learning



- Log-linearized New Keynesian model has the structural form:

$$\Omega_0 x_t = \Omega_1 x_{t-1} + \Omega_2 E_t^* x_{t+1} + \Omega_3 E_t^* x_{t+2} + \Psi z_t$$

- All observable by the agents: $x_t = [\tilde{y}_t \ \pi_t \ \hat{r}_t]$
- Shocks not observable to agents that learn: $z_t = [r_t^n \ u_t \ \epsilon_{r,t}]'$
- Rational expectations solution:

$$E_t x_{t+1} = G x_t + H z_t$$

- Agents estimate $G$ by least squares, data on $z_t$ is not available.
  - Regressors: constant, first lag of $x_t$.

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- Log-linearized New Keynesian model has the structural form:

$$\Omega_0 x_t = \Omega_1 x_{t-1} + \Omega_2 E_t^* x_{t+1} + \Omega_3 E_t^* x_{t+2} + \Psi z_t$$

- All observable by the agents: $x_t = [\tilde{y}_t \ \pi_t \ \hat{r}_t]$
- Shocks not observable to agents that learn: $z_t = [r_t^n \ u_t \ \epsilon_{r,t}]'$
- Rational expectations solution:

$$E_t x_{t+1} = G x_t + H z_t$$

- Agents estimate $G$ by least squares, data on $z_t$ is not available.
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# Recursive algorithm



- It can be shown that least squares implies recursion:

$$\hat{G}_t^* = \hat{G}_{t-1}^* + g_t(x_{t-1} - \hat{G}_{t-1}^* x_{t-2}^*) x_{t-2}^*{}' R_t^{-1}$$

$$R_t = R_{t-1} + g_t(x_{t-2}^* x_{t-2}^*{}' - R_{t-1})$$

- OLS: $g_t = 1/(t-1)$
  - Learning dynamics disappear over time.
  - No suspicion of structural change.
  - Extremely slow to adjust to structural changes.
- Constant gain learning: $g_t = g$ for all $t$.
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# Dynamic Learning Gain



- Agents use decreasing gain unless previous $J = 8$ periods forecast errors exceed a threshold.
- Threshold $= \nu_t =$ historical average absolute value forecast error.
- Agents only suspect structural change when forecast errors are exceptionally high.

$$g_t^{-1} = \begin{cases} g_{t-1}^{-1} + 1 & \text{if } \frac{1}{J}\sum_{j=1}^{J}\frac{1}{n}\sum_{v=1}^{n}\left|x_{t-j}(v) - \hat{G}_{t-j}^{*}(v)x_{t-j-1}^{*}\right| < \nu_t \\ g^{-1} & \text{otherwise} \end{cases}$$

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# Approach



- Use a standard, commonly estimated monetary model: New Keynesian Model.
  - Three equation model with optimizing consumers, sticky prices, monetary policy.
  - Stochastic shocks: demand shock, cost-push shock, monetary policy shock.
- Augment New Keynesian model with dynamic gain learning.
- Also estimate model under RE and constant gain learning.
- Markov switching process for variances of structural shocks.

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# Questions



- Does the dynamic gain model predict a lower likelihood economy was in volatile regime?
  - Spoiler: No.
- When is the economy in the volatile regime?
  - Spoiler: All models predict dates surrounding NBER recessions of 1970s.
- Does the dynamic gain model predict lower variances for volatile regime shocks?
  - Spoiler: Yes.
- When are agents using larger learning gain?
  - Spoiler: During most of the 1970s.
- What expectations mechanism fits the data best?
  - Spoiler: Rational expectations, constant gain learning, decreasing learning have nearly identical fit.

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# Estimation Procedure



- Maximum Likelihood
  - Use Kim and Nelson (1999) method.
- Quarterly data from 1960:Q1 through 2008:Q1
  - Output gap: measured by Congressional Budget Office.
  - CPI inflation rate.
  - Federal funds rate.
- Pre-sample period to initialize expectations: 1954:Q3 - 1959:Q4.
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# Parameter Estimates



| | Parameter | Rational Expectations | Dynamic Gain | Constant Gain |
|---|---|---|---|---|
| $\sigma_{n,L}$ | Nat. Rate (Low) | 0.1768 (0.3720) | 0.0454 (0.0217) | 0.0931 (0.0572) |
| $\sigma_{u,L}$ | Cost Push (Low) | 0.0023 (0.0001) | 0.0045 (0.0004) | 0.0042 (0.0001) |
| $\sigma_{r,L}$ | MP Shock (Low) | 0.0013 (0.0001) | 0.0012 (0.0000) | 0.0012 (0.0000) |
| $\sigma_{n,H}$ | Nat. Rate (High) | 0.4295 (0.9056) | 0.0966 (0.0485) | 0.1794 (0.1144) |
| $\sigma_{u,H}$ | Cost Push (High) | 0.0044 (0.0004) | 0.0092 (0.0010) | 0.0085 (0.0005) |
| $\sigma_{r,H}$ | MP Shock (High) | 0.0070 (0.0005) | 0.0064 (0.0003) | 0.0056 (0.0002) |
| $p_L$ | P(Remain Low) | 0.9609 (0.0224) | 0.9724 (0.0097) | 0.9780 (0.0109) |
| $p_H$ | P(Remain High) | 0.8099 (0.0578) | 0.8924 (0.0264) | 0.9412 (0.0159) |
| $g$ | Learning Gain | – | 0.0045 (0.0007) | 0.0000 (0.0018) |

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Expectations are not adaptive.

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Regimes are highly persistent.

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| $\sigma_{u,L}$ | Cost Push (Low) | 0.0023 (0.0001) | 0.0045 (0.0004) | 0.0042 (0.0001) |
| $\sigma_{r,L}$ | MP Shock (Low) | 0.0013 (0.0001) | 0.0012 (0.0000) | 0.0012 (0.0000) |
| $\sigma_{n,H}$ | Nat. Rate (High) | 0.4295 (0.9056) | 0.0966 (0.0485) | 0.1794 (0.1144) |
| $\sigma_{u,H}$ | Cost Push (High) | 0.0044 (0.0004) | 0.0092 (0.0010) | 0.0085 (0.0005) |
| $\sigma_{r,H}$ | MP Shock (High) | 0.0070 (0.0005) | 0.0064 (0.0003) | 0.0056 (0.0002) |
| $p_L$ | P(Remain Low) | 0.9609 (0.0224) | 0.9724 (0.0097) | 0.9780 (0.0109) |
| $p_H$ | P(Remain High) | 0.8099 (0.0578) | 0.8924 (0.0264) | 0.9412 (0.0159) |
| $g$ | Learning Gain | – | 0.0045 (0.0007) | 0.0000 (0.0018) |

Learning predicts smaller variances of the natural rate shock.

# Parameter Estimates



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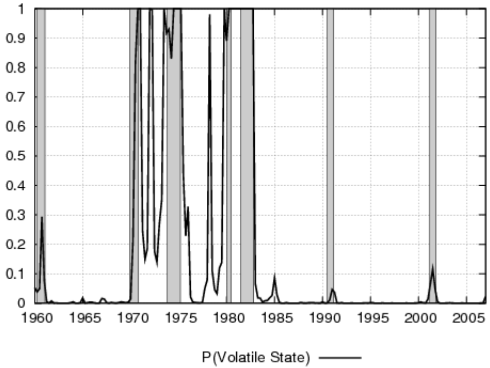
# Regime-Switching Volatility



Rational Expectations
Probability Economy is in the Volatile Regime

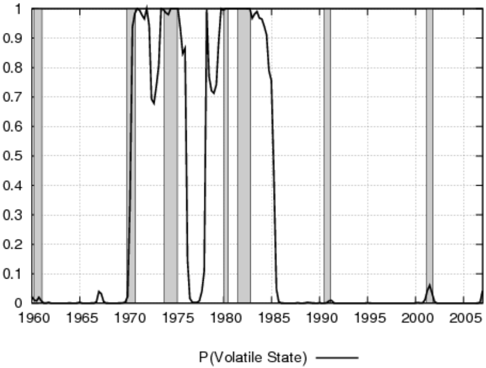



Expected 7.77 volatile years

# Regime-Switching Volatility



Constant Gain Learning
Probability Economy is in the Volatile Regime

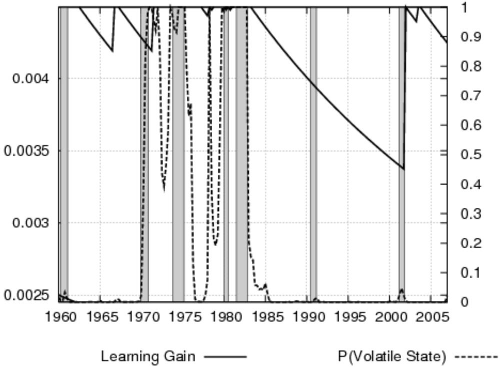



Expected 12.26 volatile years

# Regime-Switching Volatility



Dynamic Gain Learning
Probability Economy is in the Volatile Regime
and Evolution of the Learning Gain




Expected 9.17 volatile years

# Forecast Errors Comparison



| | Rational Expectations | Dynamic Gain | Constant Gain |
|---|---|---|---|
| RMSE Output Gap | 3.12 | 3.13 | 3.18 |
| RMSE Inflation | 4.41 | 4.69 | 4.69 |
| RMSE Federal Funds Rate | 5.01 | 5.05 | 5.09 |
| AR(1) Output Variance | 0.0904 (0.0730) | 0.1715 (0.0722) | 0.1240 (0.0728) |
| AR(1) Inflation Variance | 0.1760 (0.0716) | 0.1364 (0.0699) | 0.1073 (0.0653) |
| AR(1) Fed Funds Variance | 0.3851 (0.0670) | 0.3798 (0.0659) | 0.3798 (0.0636) |

- Rational Expectations actually (very slightly) fits data better than learning models.
- All models show some persistence in volatility of forecast errors.
- Models especially fail to explain changing volatility of the federal funds rate.

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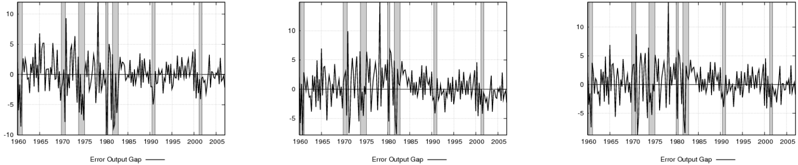
# Forecast Errors: Output Gap



Rational Exp. (1.0) | Constant Gain (0.86) | Dynamic Gain (0.82)

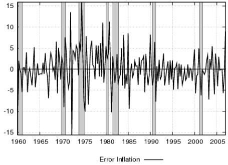



- (Correlation with Rational Expectations)
- All models made similar errors
- Most volatile during recessions in 1970s, early 1980s

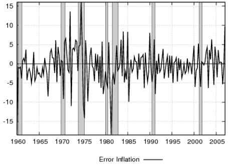
# Forecast Errors: Inflation



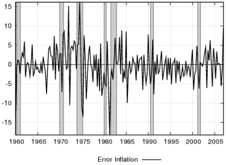
Rational Exp. (1.0)




Constant Gain (0.85)




Dynamic Gain (0.80)

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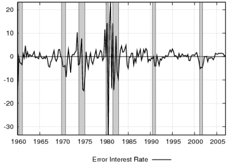
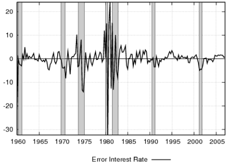
# Forecast Errors: Federal Funds Rate



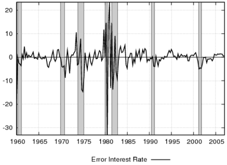
Rational Exp. (1.0)




Constant Gain (0.99)




Dynamic Gain (0.99)




- (Correlation with Rational Expectations)
- Essentially identical errors.
- Do not explain change in policy in early 1980s.

# Conclusions



- When allowing for regime-switching volatility, there is little evidence of adaptive expectations.
- Constant gain learning and dynamic gain learning both produce less volatility for the natural rate shock.
- Learning frameworks actually deliver a higher prediction for the time spent in volatile regime.
- All models make similar forecast errors at similar points in sample.
- Rational expectations model actually yields smallest in-sample MSE.

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