Regime Switching, Learning, and the Great Moderation

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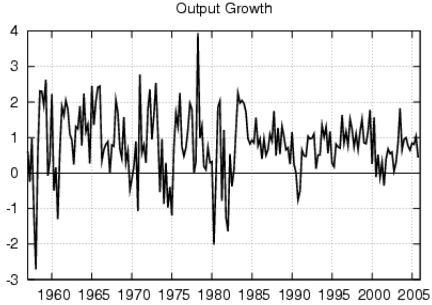
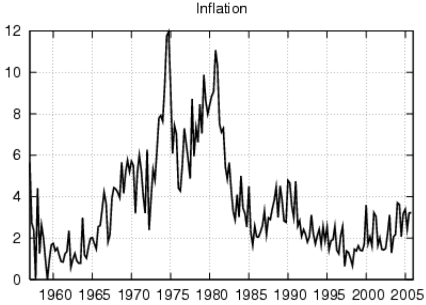
Time Varying Volatility





- Good vs. bad policy.
  - Lubik and Schorfheide (2004): find monetary policy was destabilizing pre-Volker.
  - Milani (2005): accounting for learning, little evidence of a change in monetary policy
  - Primiceri (2005): Monetary authority was optimizing, but mis-informed.
- Bad luck: bad periods were hit with bad shocks.
  - Sims and Zha (2006): evidence points in favor of bad shocks.
  - Bullard and Singh (2007): bad luck + Bayesian learning.
- Learning
  - It is possible for learning *alone* to generate time-varying volatility.
  - Depends on how you specify learning process.

- Incorporate bad luck and learning into a New Keynesian model.
- Estimate how much volatility is explained by these sources.
- Marcet and Nicolini (2003) learning.
    - Agents run OLS to form expectations (decreasing learning gain).
    - If agents suspect structural change, switch to a high learning gain.
- Bad luck: regime switching in variance of structural shocks.
    - Low volatility regime
    - High volatility regime

- Continuum of consumer types, each type with a specific labor skill.
- Continuum of monopolistically competitive intermediate goods firms.
- Calvo (1983) pricing.
- No capital.
- Taylor rule monetary policy.
- Internal habit formation in preferences (mechanical persistence).
- Inflation indexation (mechanical persistence).
- Serially correlated non-policy shocks: technology and preference.

- Preferences:

$$E_0^* \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\frac{1}{\sigma}} \xi_t \left(c_t - \eta c_{t-1}\right)^{1-\frac{1}{\sigma}} - \frac{1}{1+\frac{1}{\mu}} n_t(i)^{1+\frac{1}{\mu}} \right] \quad (1)$$

- Budget constraint:

$$c_t + b_t(i) = \frac{1+r_{t-1}}{1+\pi_t} b_{t-1}(i) + \frac{w_t(i)}{p_t} n_t(i) + \Pi_t - \tau_t \quad (2)$$

- Notation
  - E_t^* denotes a non-rational expectation.
  - ξ_t: preference shock.
  - $\eta \in (0,1)$: degree of habit formation.
  - $n_t(i)$ consumer type *i*s choice of labor supply.
  - $w_t(i)$ nominal wage.
  - p_t, π_t: price and inflation rate of final good.

- Intermediate good production: $y_t(i) = z_t n_t(i)$
- Calvo (1983) pricing: only a constant fraction, ω of firms can re-optimize their price each period.
- Those who cannot: $p_t(i) = p_{t-1}(i) + \gamma\pi_{t-1}$
- Those who can, maximize:

$$E_t^* \sum_{T=0}^{\infty} (\omega\beta)^T \frac{\lambda_{t+T}}{\lambda_t} \left\{ \left(\frac{p_t(i)\pi_{t+T}^*}{p_{t+T}} \right) y_{t+T}(i) - \Psi\left[y_{t+T}(i)\right] \right\} \tag{3}$$

- γ: degree of inflation indexation.
- Indexation adjustment through period $t+T$:

$$\pi_{t+T}^* = \prod_{j=1}^{T} (1 + \gamma\pi_{t+j-1}) \tag{4}$$

- Serially correlated shocks:

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t}(s_t) \tag{5}$$

$$\xi_t = \rho_\xi \xi_{t-1} + \epsilon_{\xi,t}(s_t) \tag{6}$$

- Innovations for a given regime are mean zero and iid.

$$Var\left(\begin{bmatrix} \epsilon_{z,t}(s_t) \\ \epsilon_{\xi,t}(s_t) \end{bmatrix}\right) = \begin{cases} \begin{bmatrix} \sigma^2_{z,1} & 0 \\ 0 & \sigma^2_{\xi,1} \end{bmatrix}, & \text{if } s_t = 1 \\ \begin{bmatrix} \sigma^2_{z,2} & 0 \\ 0 & \sigma^2_{\xi,2} \end{bmatrix}, & \text{if } s_t = 2 \end{cases} \tag{7}$$

- Two state Markov chain
- Let p_i denote the probability of staying in regime i from $t-1$ to t.
- Transition matrix:

$$P = \begin{bmatrix} p_1 & 1-p_1 \\ 1-p_2 & p_2 \end{bmatrix}. \tag{8}$$

- Log-linearized New Keynesian model has the structural form:

$$\Omega_0 x_t = \Omega_1 x_{t-1} + \Omega_2 E_t^* x_{t+1} + \Psi \epsilon_t(s_t) \tag{9}$$

- All observable by the agents: $x_t = [\hat{y}_t \ \pi_t \ \hat{r}_t \ \hat{z}_t \ \hat{\xi}_t]'$
- Shocks not observable: $\epsilon_t(s_t) = [\epsilon_{z,t}(s_t) \ \epsilon_{\xi,t}(s_t) \ \epsilon_{r,t}]'$
- Rational expectations solution:

$$E_t x_{t+1} = G x_t, \tag{10}$$

- Agents estimate G by least squares.
  - Use as regressors a constant, and all past observations of x_t.

- It can be shown that least squares implies recursion:

$$\hat{G}_t^* = \hat{G}_{t-1}^* + g_t(x_{t-1} - \hat{G}_{t-1}^* x_{t-2}^*) x_{t-2}^{*}{}' R_t^{-1} \tag{11}$$

$$R_t = R_{t-1} + g_t(x_{t-2}^* x_{t-2}^{*}{}' - R_{t-1}) \tag{12}$$

- OLS: $g_t = 1/(t-1)$
  - Learning dynamics disappear over time.
  - No suspicion of structural change.
  - Extremely slow to adjust to structural changes.
- Constant gain learning: $g_t = g$ for all t.
  - Bullard and Duffy (2005)
  - Time varying volatility is likely small.

- Marcet and Nicolini (2003): learning is endogenous.
- Let $\alpha_t \equiv 1/g_t$. In OLS case α_t is sample size.

$$\alpha_t = \begin{cases} \alpha_{t-1}+1 & \text{if } \frac{1}{J}\sum_{j=1}^{J}\frac{1}{n}\sum_{v=1}^{n}\left|\frac{x_{t-j}(v)-\hat{G}^*_{t-j}(v)x^*_{t-j-1}}{\hat{G}^*_{t-j}(v)x^*_{t-j-1}}\right| < \nu \\ \alpha & \text{otherwise} \end{cases} \tag{13}$$

- Notation:
  - n is the number of variables in x_t.
  - $x_{t-j}(v)$: vth element of x_{t-j}
  - $\hat{G}^*_{t-j}(v)$: vth row of $\hat{G}^*_{t-j}(v)$
  - $\alpha \equiv 1/g$ is the constant gain.
  - $\nu \in (0,\infty)$ is a threshold level.

- State equation:

$$x_t = b_t + F_t x_{t-1} + v_t(s_t)$$

$$b_t = \Omega_0^{-1}\Omega_2 \left(I + \hat{G}_t \right) \hat{g}_{0,t}$$

$$F_t = \Omega_0^{-1} \left(\Omega_1 + \Omega_2 \hat{G}_t^2 \right) \tag{14}$$

$$v_t(s_t) = \Omega_0^{-1}\Psi\epsilon_t(s_t)$$

$$Var\left[v_t(s_t)\right] = \left(\Omega_0^{-1}\Psi\right) Var\left[\epsilon_t(s_t)\right] \left(\Omega_0^{-1}\Psi\right)'$$

- Observation equations:

$$\begin{aligned} GDP_t &= 100\hat{y}_t, \\ INF_t &= \pi^* + 400\pi_t, \\ FF_t &= r^* + 400\hat{r}_t, \end{aligned} \tag{15}$$

- New Keynesian model parameters.
- Variance of the shocks in each state: $\sigma^2_{z,1}, \sigma^2_{\xi,1}$, and $\sigma^2_{z,1}, \sigma^2_{\xi,1}$.
- Probabilities of switching between states: p_1, p_2.
- Learning parameters: g and ν.

- Kim (1994), Kim and Nelson (1999): form the likelihood.
  - Kalman filter for state-space models.
  - Hamilton (1989) filter for regime switching.
- Plan to estimate:
  - Smoothed estimates of the regime probabilities for each period.
  - Learning gain for each period.
  - Smoothed estimates of the shocks.
  - Out-of-sample forecasts of the model.

Interesting questions

- With endogenous gain learning, how sizable of an increase in shock volatility is necessary to explain volatile periods such as the 1970s.
- When did regime changes occur?
- When did learning gain changes occur?
- How do these answers compare to constant gain learning, decreasing gain learning, and rational expectations.
- How well does do competing specifications fit the data.
    - Different assumptions on learning gain.
    - Fixed state regime.