Regime Switching, Learning, and the Great Moderation

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1 Introduction

1.1 Purpose

Purpose

- How much does "bad luck" explain changing volatility when adaptive expectations react to suspicions of structural changes.

- **Great Inflation / Great Moderation:** decade of high inflation and volatility followed by seemingly permanent reduction in macroeconomic volatility.

- Bad luck explanation: changes in volatility is due to *exogenous* changes in variance of structural shocks.

- Adaptive expectations:

  - Agents do not know reduced form parameters governing evolution of the economy.
  - Agents estimate a reduced form VAR(1) by least squares.
  - Agents endogenously give more recent observations more weight.

1.2 Bad Luck

Bad Luck

- Volatile periods were hit with bad shocks.

- Sims and Zha (AER, 2006): evidence points in favor of bad shocks over changes in policy.

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- Stock and Watson (2003): improved policy accounts for only small part of volatility slowdown.

- Justiniano and Primiceri (AER, 2008): evidence points in favor of time evolving variance of structural shocks.

1.3 Adaptive Expectations

Adaptive Expectations

- Oraphanides and Williams (JEDC, 2005): Monetary authority was optimizing, but mis-informed.

- Primiceri (QJE, 2006): Monetary authority mis-informed, expectations improved with time.

- Milani (JME, 2007): Agents learn, little evidence of difference in policy parameters.

- Milani (2008): Time varying expectations explains time-varying volatility.

- Bullard and Singh (2007): bad luck + Bayesian learning.

2 Framework

2.1 New Keynesian Model

New Keynesian Model

- Consumer behavior:

  - Choose consumption and labor to maximize utility.
  - Habit formation: utility on consumption depends on past consumption.

- Producer behavior:

  - Intermediate goods are produced with labor in monopolistically competitive markets.
  - Intermediate goods subject to Calvo (1983) price friction.
  - Price indexation: when not re-optimizing prices, price can adjust according to past inflation.

- Taylor (1993) type monetary policy:

  - Nominal interest rate responds to inflation, expected future output gap, and past interest rate.

Optimal Consumer Behavior

$$\tilde{\lambda}_t = E_t\tilde{\lambda}_{t+1} + \hat{r}_t - E_t\pi_{t+1} - r_t^n,$$

$$\tilde{\lambda}_t = \frac{1}{(1-\beta\eta)(1-\eta)}\left[\beta\eta E_t\tilde{y}_{t+1} - (1+\beta\eta^2)\tilde{y}_t + \eta\tilde{y}_{t-1}\right]$$

- Variables:

  - $\tilde{\lambda}_t$: marginal utility of income.
  - $\tilde{y}_t$: output gap.
  - $\hat{r}_t$: nominal interest rate.
  - π_t: inflation.
  - r_t^n: "natural rate" shock, deviation of interest rate from flexible price outcome.

- Parameters:

  - $\eta \in [0,1)$: degree of habit formation.
  - $\beta \in (0,1)$: discount rate.

Producer Behavior

- Phillips curve:

$$\pi_t = \frac{1}{1+\beta\gamma}\left[\gamma\pi_{t-1} + \beta E_t\pi_{t+1} + \kappa(\tilde{y}_t - \mu\tilde{\lambda}_t) + u_t\right]$$

- Cost push shock: u_t.

- Parameters:

  - $\gamma \in [0,1)$: degree of price indexation.
  - $\kappa \in (0,\infty)$: reduced form parameter inversely related to degree of price flexibility.

Monetary policy

- Nominal interest rate responds to expected output gap and inflation:

$$\hat{r}_t = \rho_r\hat{r}_{t-1} + (1-\rho_r)\left(\psi_\pi E_t\pi_{t+1} + \psi_y E_t\tilde{y}_{t+1}\right) + \epsilon_{r,t}$$

- Monetary policy shock: $\epsilon_{r,t}$.

  - $\psi_\pi \in (0,\infty)$: feedback on inflation.
  - $\psi_y \in (0,\infty)$: feedback on output.
  - $\rho_r \in (0,1)$: smoothing parameter.

Structural Shocks

- Natural interest rate shock:

$$r_t^n = \rho_n r_{t-1}^n + \epsilon_{n,t}$$

- Cost push shock:

$$u_t = \rho_u u_{t-1} + \epsilon_{u,t}$$

- Monetary policy shock is not autoregressive.

2.2 Regime Switching Volatility

Regime Switching Volatility

- Variance of shocks switches between high and low regimes:

$$Var \begin{bmatrix} \epsilon_{n,t}(s_t) \\ \epsilon_{u,t}(s_t) \\ \epsilon_{r,t}(s_t) \end{bmatrix} = \left\{ \begin{array}{l} \begin{bmatrix} \sigma_{n,L}^2 & 0 & 0 \\ 0 & \sigma_{u,L}^2 & 0 \\ 0 & 0 & \sigma_{r,L}^2 \end{bmatrix}, \text{ if } s_t = L \\ \begin{bmatrix} \sigma_{n,H}^2 & 0 & 0 \\ 0 & \sigma_{u,H}^2 & 0 \\ 0 & 0 & \sigma_{r,H}^2 \end{bmatrix}, \text{ if } s_t = H \end{array} \right\}$$

- $s_t = L \to$ low volatility regime, $s_t = H \to$ high volatility regime.

- $\sigma_{i,H}^2 > \sigma_{i,H}^2$.

Markov Switching

- Regimes evolve according to a Markov Chain with transition matrix:

$$P = \begin{bmatrix} p_L & 1 - p_H \\ 1 - p_L & p_H \end{bmatrix}$$

- Probabilities:

  - p_L: probability of remaining in low volatility regime.
  - p_H: probability of remaining in high volatility regime.

- State evolves according to: $S_{t+1} = PS_t$

  - Where $S_t' = [P(s_t = L)\ P(s_t = H)]$.

2.3   Dynamic Gain Learning

Learning

- Log-linearized New Keynesian model has the structural form:

$$\Omega_0 x_t = \Omega_1 x_{t-1} + \Omega_2 E_t^* x_{t+1} + \Omega_3 E_t^* x_{t+2} + \Psi z_t$$

- All observable by the agents: $x_t = [\tilde{y}_t \ \pi_t \ \hat{r}_t]$

- Shocks not observable to agents that learn: $z_t = [r_t^n \ u_t \ \epsilon_{r,t}]'$

- Rational expectations solution:

$$E_t x_{t+1} = G x_t + H z_t$$

- Agents estimate G by least squares, data on z_t is not available.

  - Regressors: constant, first lag of x_t.

Recursive algorithm

- It can be shown that least squares implies recursion:

$$\hat{G}_t^* = \hat{G}_{t-1}^* + g_t(x_{t-1} - \hat{G}_{t-1}^* x_{t-2}^*)x_{t-2}^{*\,\prime} R_t^{-1}$$

$$R_t = R_{t-1} + g_t(x_{t-2}^* x_{t-2}^{*\,\prime} - R_{t-1})$$

- OLS: $g_t = 1/(t-1)$

  - Learning dynamics disappear over time.
  - No suspicion of structural change.
  - Extremely slow to adjust to structural changes.

- Constant gain learning: $g_t = g$ for all t.

  - Time varying volatility is likely small.

Dynamic Learning Gain

- Agents use decreasing gain unless previous $J = 8$ periods forecast errors exceed a threshold.

- Threshold $= \nu_t =$ historical average absolute value forecast error.

- Agents only suspect structural change when forecast errors are exceptionally high.

$$g_t^{-1} = \left\{ \begin{array}{ll} g_{t-1}^{-1} + 1 & \text{if } \dfrac{1}{J}\displaystyle\sum_{j=1}^{J} \dfrac{1}{n} \sum_{v=1}^{n} \left| x_{t-j}(v) - \hat{G}_{t-j}^*(v) x_{t-j-1}^* \right| < \nu_t \\[2ex] g^{-1} & \text{otherwise} \end{array} \right\}$$

- Milani (2008): generates ARCH time-varying volatility, Marcet and Nicolini (2003): recurrent hyperinflations.

3 Estimation

3.1 Approach

Approach

- Use a standard, commonly estimated monetary model: New Keynesian Model.

  - Three equation model with optimizing consumers, sticky prices, monetary policy.

  - Stochastic shocks: demand shock, cost-push shock, monetary policy shock.

- Augment New Keynesian model with dynamic gain learning.

- Also estimate model under RE and constant gain learning.

- Markov switching process for variances of structural shocks.

Questions

- Does the dynamic gain model predict a lower likelihood economy was in volatile regime?

  - Spoiler: No.

- When is the economy in the volatile regime?

  - Spoiler: All models predict dates surrounding NBER recessions of 1970s.

- Does the dynamic gain model predict lower variances for volatile regime shocks?

  - Spoiler: Yes.

- When are agents using larger learning gain?

  - Spoiler: During most of the 1970s.

- What expectations mechanism fits the data best?

  - Spoiler: Rational expectations, constant gain learning, decreasing learning have nearly identical fit.

Estimation Procedure

- Maximum Likelihood

  - Use Kim and Nelson (1999) method.

- Quarterly data from 1960:Q1 through 2008:Q1

  - Output gap: measured by Congressional Budget Office.
  - CPI inflation rate.
  - Federal funds rate.

- Pre-sample period to initialize expectations: 1954:Q3 - 1959:Q4.

- Expectations are initialized to pre-sample VAR(1) results.

## 3.2   Parameter Estimates

Parameter Estimates

	Parameter	Rational Expectations	Dynamic Gain	Constant Gain
$\sigma_{n,L}$	Nat. Rate (Low)	0.1768 (0.3720)	0.0454 (0.0217)	0.0931 (0.0572)
$\sigma_{u,L}$	Cost Push (Low)	0.0023 (0.0001)	0.0045 (0.0004)	0.0042 (0.0001)
$\sigma_{r,L}$	MP Shock (Low)	0.0013 (0.0001)	0.0012 (0.0000)	0.0012 (0.0000)
$\sigma_{n,H}$	Nat. Rate (High)	0.4295 (0.9056)	0.0966 (0.0485)	0.1794 (0.1144)
$\sigma_{u,H}$	Cost Push (High)	0.0044 (0.0004)	0.0092 (0.0010)	0.0085 (0.0005)
$\sigma_{r,H}$	MP Shock (High)	0.0070 (0.0005)	0.0064 (0.0003)	0.0056 (0.0002)
p_L	P(Remain Low)	0.9609 (0.0224)	0.9724 (0.0097)	0.9780 (0.0109)
p_H	P(Remain High)	0.8099 (0.0578)	0.8924 (0.0264)	0.9412 (0.0159)
g	Learning Gain	–	0.0045 (0.0007)	0.0000 (0.0018)

Expectations are not adaptive. Regimes are highly persistent. Learning predicts smaller variances of the natural rate shock. Variances of cost push and monetary shock are similar.

3.3   Regime-Switching Volatility
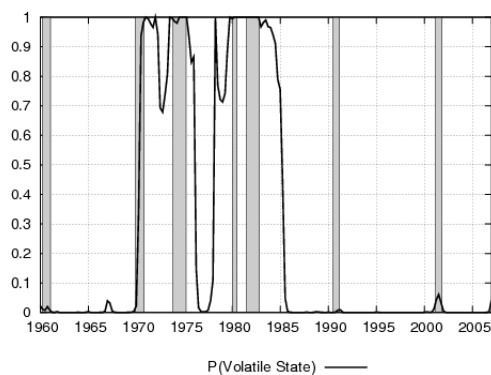
Regime-Switching Volatility

Rational Expectations Probability Economy is in the Volatile Regime

P(Volatile State) ——
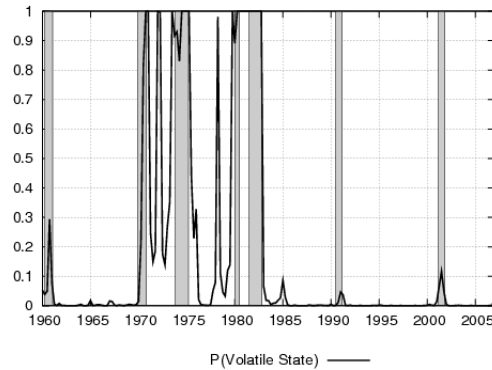
Expected 7.77 volatile years

Regime-Switching Volatility

Constant Gain Learning Probability Economy is in the Volatile Regime

P(Volatile State) ——

Expected 12.26 volatile years

Regime-Switching Volatility

Dynamic Gain Learning Probability Economy is in the Volatile Regime and
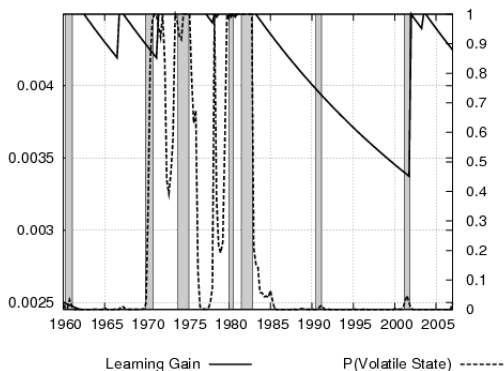
Evolution of the Learning Gain

Learning Gain ——   P(Volatile State) - - - - - -

Expected 9.17 volatile years

3.4   Forecast Errors

Forecast Errors Comparison

	Rational Expectations	Dynamic Gain	Constant Gain
RMSE Output Gap	3.12	3.13	3.18
RMSE Inflation	4.41	4.69	4.69
RMSE Federal Funds Rate	5.01	5.05	5.09
AR(1) Output Variance	0.0904 (0.0730)	0.1715 (0.0722)	0.1240 (0.0728)
AR(1) Inflation Variance	0.1760 (0.0716)	0.1364 (0.0699)	0.1073 (0.0653)
AR(1) Fed Funds Variance	0.3851 (0.0670)	0.3798 (0.0659)	0.3798 (0.0636)

- Rational Expectations actually (very slightly) fits data better than learning models.

- All models show some persistence in volatility of forecast errors.

- Models especially fail to explain changing volatility of the federal funds rate.
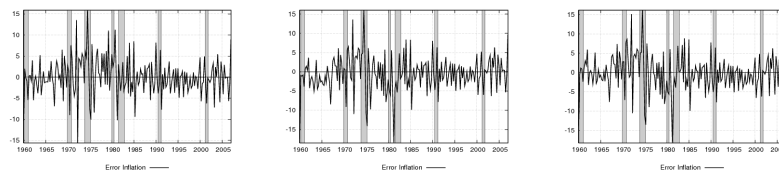
Forecast Errors: Output Gap

Rational Exp. (1.0)   Constant Gain (0.86)   Dynamic Gain (0.82)
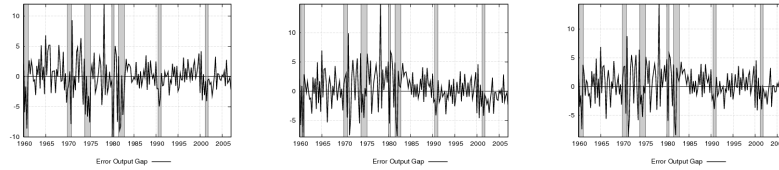
- (Correlation with Rational Expectations)

- All models made similar errors

- Most volatile during recessions in 1970s, early 1980s
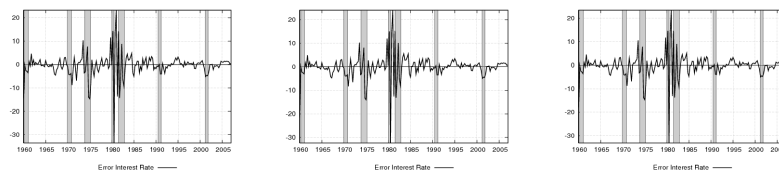
Forecast Errors: Inflation

Rational Exp. (1.0)   Constant Gain (0.85)   Dynamic Gain (0.80)

- (Correlation with Rational Expectations)

- All models made similar errors.

- Most volatile during recessions in 1970s, early 1980s.

Forecast Errors: Federal Funds Rate

Rational Exp. (1.0)   Constant Gain (0.99)   Dynamic Gain (0.99)

- (Correlation with Rational Expectations)

- Essentially identical errors.

- Do not explain change in policy in early 1980s.

4 Conclusion

Conclusions

- When allowing for regime-switching volatility, there is little evidence of adaptive expectations.

- Constant gain learning and dynamic gain learning both produce less volatility for the natural rate shock.

- Learning frameworks actually deliver a higher prediction for the time spent in volatile regime.

- All models make similar forecast errors at similar points in sample.

- Rational expectations model actually yields smallest in-sample MSE.