# Empirical Significance of Learning in a New Keynesian Model with Firm-Specific Capital

James Murray Indiana University

October 5, 2007



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- Learning: Agents form expectations with least squares forecasts.
- Popular assumption: agents use the correct specification.
- Misspecifications
  - Some variables in economy are unobservable to agents...
  - Agents ignore multi-variate structure, use univariate methods.
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- Prolonged inflation following a shock:
  - Orphanides and Williams (RED 2005).
- Bad monetary policy prescriptions:
  - Orphanides and Williams (JEDC 2005), Primiceri (QJE 2006)
- Output and inflation persistence
  - Milani (2005)

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- Initial beliefs can play a major role.
- Orphanides and Williams (JEDC 2005)
  - Central Bank began under-estimating natural rate of unemployment.
- Primiceri:
  - Central Bank began under-estimating unemployment and inflation persistence.
- Milani:
  - Many initial beliefs set to pre-sample VAR(1).
  - Assumes lower inflation persistence, sensitivity of output to inflation.
  - Assumes shocks are observable, sets initial impacts to zero
- Missing from empirical literature:
  - Systematic way for specifying initial conditions.
  - Estimate initial conditions
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- Use the rational expectations solution.
  - Benefit: Initial conditions are consistent with model.
  - Draw back: Learning dynamics are small near the RE equilibrium. (Williams 2003).
- Assume limited information set.
  - Agents cannot observe realizations of stochastic shocks
  - Initialize beliefs of remaining coefficients equal to RE solution
  - Benefit: more realistic.
- Using limited information, set initial beliefs to pre-sample least squares estimates.
  - Benefit: Most likely to mirror actual beliefs.
  - Draw back: sometimes so far from RE the learning model is unstable (Slobodyan and Wouters 2007).
- Jointly estimate initial conditions.
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• Linear Dynamic Stochastic General Equilibrium models:

$$\Omega_0 x_t = \Omega_1 x_{t-1} + \Omega_2 E_t^* x_{t+1} + \Psi v_t$$

$$v_t = Av_{t-1} + \epsilon_t$$

- x<sub>t</sub> vector of time t macroeconomic variables, observable to agents at following time period.
- $z_t$ : vector of time t shocks, possibly observable to agents in current period.
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$$x_t = Gx_{t-1} + Mv_t$$

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- Agents estimate elements of G and M by least squares.
- Information at available at time t:  $x_{t-1}$ ,  $v_t$ :
  - $X_t$ : vector of regressors:  $X'_t = [1 \ x'_{t-2} \ v'_{t-1}]$ .
  - $Y_t$ : vector of dependent variables:  $Y_t = x_{t-1}$ .
- When  $v_t$  is observable, I suppose agents know A.
- Let  $\Phi_t$  vector of least squares estimates of the coefficients in matrices G and M.
- Ordinary Least Squares estimate of  $\phi_t$ :

$$\Phi_t' = \left(\frac{1}{t-1} \sum_{\tau=2}^t X_\tau X_\tau'\right)^{-1} \left(\frac{1}{t-1} \sum_{\tau=2}^t X_\tau Y_\tau'\right)$$

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#### • Recursive form:

$$\Phi_t = \Phi_{t-1} + g_t(Y_t - \Phi_{t-1}X_t)X_t'R_t^{-1}$$

$$R_t = R_{t-1} + g_t(X_t X_t' - R_{t-1})$$

- Where  $g_t = 1/(t-1)$ .
- OLS learning: dynamics disappear in the long run.
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# Motivation for Constant Gain Learning

- Learning dynamics persist in the long run.
  - Reasonable approximation to a rolling window estimation procedure.
    - Typically 30-50 year windows are used in forecasting quarterly macroeconomic variables.
    - Johnston Williamson (2007): data available for U.S. output and price level back to 1790.
- Swanson and White (RES 1997):
  - Adaptive estimation procedures out-perform non-adaptive procedures.
  - Multivariate procedures out-perform univariate procedures.
  - Linear models out-perform non-linear models.
  - Adaptive, multivariate, linear models outperform professional forecast surveys.

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#### Examine two popular specifications

- Standard three equation model: IS equation, Phillips curve, monetary policy. Woodford (2003).
- ② Add endogenous capital. Woodford (IJCB 2005).
- Details behind IS equation:
  - Utility maximizing consumers
  - Habit formation in utility function (source of persistence)
  - Intertemporal substitution
  - Goods market clearing
- Details behind the Phillips curve:
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- Monetary Policy (Taylor rule):
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- Specification 1: Three equation model (without capital)
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  - Cost-push shock.
  - Monetary policy shock.
- Specification 2: Model with capital
  - Preference shock
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- Estimate two NK specifications by maximum likelihood.
- For each specification, estimate five expectations frameworks:
  - Rational Expectations
  - ② Learning with RE solution for initial  $\Phi_t$ ,  $R_t$ .
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- Elimited information set with pre-sample coefficients
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  - **2** Learning with RE solution for initial  $\Phi_t$ ,  $R_t$ .
  - Learning with limited information set.
    - Shocks are unobservable.
    - Initial coefficients on remaining variables set equal to RE solution.
  - 4 Limited information set with pre-sample coefficients.
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Learning algorithm for constant gain least squares:

$$\Phi_t = \left(\sum_{\tau=0}^{t-1} (1-g)^t X_{t-\tau} X_{t-\tau}'\right)^{-1} \left(\sum_{\tau=0}^{t-1} (1-g)^\tau X_{t-\tau} Y_{t-\tau}'\right)$$

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#### Specification 1:

- $\bullet$   $Y_t$  vector includes output gap, inflation rate, federal funds rate.
- X<sub>t</sub> vector includes constant, previous period's output gap, inflation rate, federal funds rate..

#### • Specification 2:

- Y<sub>t</sub> vector includes consumption, capital stock, inflation rate federal funds rate.
- X<sub>t</sub> vector includes constant, previous period's consumption capital stock, inflation rate, federal funds rate.
- All these variables expressed as percentage deviation from the steady state.
- Data on capital stock constructed using data on investment.

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- Quarterly data for 1960:Q1 through 2005:Q4.
- Pre-sample data: Quarterly data for 1954:Q1 through 1959:Q4.
- Specification 1: Model with no capital:
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- Specification 2: Endogenous capital:
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  - Real gross private domestic investment per capita.
  - Output is defined as the sum of consumption and investment
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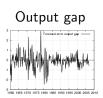
#### No Capital: RE vs. Learning (RE Init.)

|                     |                  | Ca       | se 1      | Ca       | se 2      |
|---------------------|------------------|----------|-----------|----------|-----------|
| Description         | Parameter        | Estimate | Std. Dev. | Estimate | Std. Dev. |
| Habit Formation     | η                | 0.4221   | 0.1062    | 0.4241   | 0.1216    |
| Inverse IES         | $\sigma$         | 0.5152   | 0.4401    | 0.5236   | 0.4865    |
| Phillips Slope      | $\kappa$         | 0.0001   | 0.0002    | 0.0001   | 0.0002    |
| Price Indexation    | $\gamma$         | 0.9900   | 0.0634    | 0.9901   | 0.0907    |
| MP Persistence      | $ ho_r$          | 0.9207   | 0.0207    | 0.9207   | 0.0214    |
| MP Output           | $\psi_{y}$       | 0.4946   | 0.1901    | 0.4949   | 0.1967    |
| MP Inflation        | $\psi_{\pi}$     | 1.9994   | 0.0000    | 1.9995   | 0.0000    |
| Nat. Rate Pers.     | $ ho_n$          | 0.8488   | 0.0684    | 0.8489   | 0.0645    |
| Cost Push Pers.     | $ ho_u$          | 0.0000   | 0.0692    | 0.0000   | 0.0608    |
| Nat. Rate Std. Dev. | $\sigma_n$       | 0.0751   | 0.0706    | 0.0736   | 0.0741    |
| Cost Push Std. Dev. | $\sigma_{\it u}$ | 0.0029   | 0.0002    | 0.0029   | 0.0003    |
| MP Std. Dev.        | $\sigma_r$       | 0.0030   | 0.0001    | 0.0030   | 0.0001    |
| SS Inflation        | $\pi^*$          | 5.9904   | 1.2374    | 5.9905   | 1.2739    |
| Learning Gain       | g                | _        | _         | 0.0067   | 0.0070    |
| Log-likelihood      |                  |          | -459.9390 |          | -459.5154 |
| MSE Output Gap      |                  |          | 0.6087    |          | 0.6061    |
| MSE Inflation       |                  |          | 1.3313    |          | 1.3269    |
| MSE Fed. Funds Rate |                  |          | 1.6480    |          | 1.6519    |

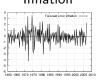
|                     |                  | Ca       | se 1      | Ca        | se 2      |
|---------------------|------------------|----------|-----------|-----------|-----------|
| Description         | Parameter        | Estimate | Std. Dev. | Estimate  | Std. Dev. |
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#### No Capital: Forecast Errors



Rational Expectations Inflation



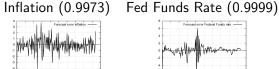
Fed Funds Rate



Learning with RE Initial Conditions Output gap (0.9993)



1965 1970 1975 1980 1985 1990 1995 2000 2005 2010





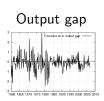
|                     |                  | Ca       | se 1      | Ca       | se 3      |
|---------------------|------------------|----------|-----------|----------|-----------|
| Description         | Parameter        | Estimate | Std. Dev. | Estimate | Std. Dev. |
| Habit Formation     | η                | 0.4221   | 0.1062    | 0.3027   | 0.1216    |
| Inverse IES         | $\sigma$         | 0.5152   | 0.4401    | 0.2251   | 0.4865    |
| Phillips Slope      | $\kappa$         | 0.0001   | 0.0002    | 0.0004   | 0.0002    |
| Price Indexation    | $\gamma$         | 0.9900   | 0.0634    | 0.9999   | 0.0907    |
| MP Persistence      | $ ho_r$          | 0.9207   | 0.0207    | 0.9131   | 0.0214    |
| MP Output           | $\psi_{y}$       | 0.4946   | 0.1901    | 0.4762   | 0.1967    |
| MP Inflation        | $\psi_\pi$       | 1.9994   | 0.0000    | 1.9865   | 0.0000    |
| Nat. Rate Pers.     | $ ho_n$          | 0.8488   | 0.0684    | 0.8413   | 0.0645    |
| Cost Push Pers.     | $ ho_u$          | 0.0000   | 0.0692    | 0.0002   | 0.0608    |
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| Cost Push Std. Dev. | $\sigma_{\it u}$ | 0.0029   | 0.0002    | 0.0054   | 0.0003    |
| MP Std. Dev.        | $\sigma_r$       | 0.0030   | 0.0001    | 0.0030   | 0.0001    |
| SS Inflation        | $\pi^*$          | 5.9904   | 1.2374    | 5.9539   | 1.2739    |
| Learning Gain       | g                | _        | _         | 0.0042   | 0.0070    |
| Log-likelihood      |                  |          | -459.9390 |          | -458.8326 |
| MSE Output Gap      |                  |          | 0.6087    |          | 0.6032    |
| MSE Inflation       |                  |          | 1.3313    |          | 1.3371    |
| MSE Fed. Funds Rate |                  |          | 1.6480    |          | 1.6378    |

|                     |                  | Ca       | se 1      | Ca       | se 3      |
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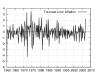
|                     |                  | Ca       | se 1      | Ca       | se 3      |
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#### No Capital: Forecast Errors

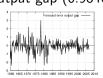


Rational Expectations Inflation

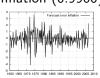


Fed Funds Rate

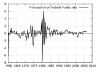




Learning Without Observable Shocks



Output gap (0.9846) Inflation (0.9960) Fed Funds Rate (0.9996)



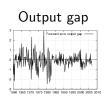
|                     |                                     | Ca       | se 1      | Ca       | se 4      |
|---------------------|-------------------------------------|----------|-----------|----------|-----------|
| Description         | Parameter                           | Estimate | Std. Dev. | Estimate | Std. Dev. |
| Habit Formation     | η                                   | 0.4221   | 0.1062    | 0.5293   | 0.1216    |
| Inverse IES         | $\sigma$                            | 0.5152   | 0.4401    | 0.2502   | 0.4865    |
| Phillips Slope      | $\kappa$                            | 0.0001   | 0.0002    | 0.0064   | 0.0002    |
| Price Indexation    | $\gamma$                            | 0.9900   | 0.0634    | 0.9989   | 0.0907    |
| MP Persistence      | $ ho_r$                             | 0.9207   | 0.0207    | 0.8454   | 0.0214    |
| MP Output           | $\psi_{y}$                          | 0.4946   | 0.1901    | 0.3200   | 0.1967    |
| MP Inflation        | $\psi_\pi$                          | 1.9994   | 0.0000    | 1.5109   | 0.0000    |
| Nat. Rate Pers.     | $ ho_n$                             | 0.8488   | 0.0684    | 0.6810   | 0.0645    |
| Cost Push Pers.     | $ ho_u$                             | 0.0000   | 0.0692    | 0.4419   | 0.0608    |
| Nat. Rate Std. Dev. | $\sigma_n$                          | 0.0751   | 0.0706    | 0.5835   | 0.0741    |
| Cost Push Std. Dev. | $\sigma_{\scriptscriptstyle \it U}$ | 0.0029   | 0.0002    | 0.0086   | 0.0003    |
| MP Std. Dev.        | $\sigma_r$                          | 0.0030   | 0.0001    | 0.0030   | 0.0001    |
| SS Inflation        | $\pi^*$                             | 5.9904   | 1.2374    | 5.8862   | 1.2739    |
| Learning Gain       | g                                   | _        | _         | 0.0828   | 0.0070    |
| Log-likelihood      |                                     |          | -459.9390 |          | -573.3274 |
| MSE Output Gap      |                                     |          | 0.6087    |          | 0.7989    |
| MSE Inflation       |                                     |          | 1.3313    |          | 2.7104    |
| MSE Fed. Funds Rate |                                     |          | 1.6480    |          | _ 1.7396  |

|                  | Ca   | se 1   | Ca  | se 4  |
|------------------|--|--|---|---|
| Parameter        | Estimate   | Std. Dev.  | Estimate  | Std. Dev.   |
| η                | 0.4221   | 0.1062   | 0.5293  | 0.1216  |
| $\sigma$         | 0.5152   | 0.4401   | 0.2502  | 0.4865  |
| $\kappa$         | 0.0001   | 0.0002   | 0.0064  | 0.0002  |
| $\gamma$         | 0.9900   | 0.0634   | 0.9989  | 0.0907  |
| $ ho_r$          | 0.9207   | 0.0207   | 0.8454  | 0.0214  |
| $\psi_{y}$       | 0.4946   | 0.1901   | 0.3200  | 0.1967  |
| $\psi_\pi$       | 1.9994   | 0.0000   | 1.5109  | 0.0000  |
| $ ho_n$          | 0.8488   | 0.0684   | 0.6810  | 0.0645  |
| $ ho_{u}$        | 0.0000   | 0.0692   | 0.4419  | 0.0608  |
| $\sigma_n$       | 0.0751   | 0.0706   | 0.5835  | 0.0741  |
| $\sigma_{\it u}$ | 0.0029   | 0.0002   | 0.0086  | 0.0003  |
| $\sigma_r$       | 0.0030   | 0.0001   | 0.0030  | 0.0001  |
| $\pi^*$          | 5.9904   | 1.2374   | 5.8862  | 1.2739  |
| g                | _  | _  | 0.0828  | 0.0070  |
|                  |  | -459.9390  |   | -573.3274   |
|                  |  | 0.6087   |   | 0.7989  |
|                  |  | 1.3313   |   | 2.7104  |
|                  |  | 1.6480   |   | 1.7396  |
|                  | $\eta$ $\sigma$ $\kappa$ $\gamma$ $\rho_r$ $\psi_y$ $\psi_\pi$ $\rho_n$ $\sigma_n$ $\sigma_u$ $\sigma_r$ $\pi^*$ | $\begin{array}{c c} \text{Parameter} & \text{Estimate} \\ \hline \eta & 0.4221 \\ \sigma & 0.5152 \\ \kappa & 0.0001 \\ \gamma & 0.9900 \\ \rho_r & 0.9207 \\ \psi_y & 0.4946 \\ \psi_\pi & 1.9994 \\ \rho_n & 0.8488 \\ \rho_u & 0.0000 \\ \sigma_n & 0.0751 \\ \sigma_u & 0.0029 \\ \sigma_r & 0.0030 \\ \pi^* & 5.9904 \\ \hline \end{array}$ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ |

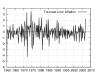
|                     |                                     | Ca       | se 1      | Ca       | se 4      |
|---------------------|-------------------------------------|----------|-----------|----------|-----------|
| Description         | Parameter                           | Estimate | Std. Dev. | Estimate | Std. Dev. |
| Habit Formation     | η                                   | 0.4221   | 0.1062    | 0.5293   | 0.1216    |
| Inverse IES         | $\sigma$                            | 0.5152   | 0.4401    | 0.2502   | 0.4865    |
| Phillips Slope      | $\kappa$                            | 0.0001   | 0.0002    | 0.0064   | 0.0002    |
| Price Indexation    | $\gamma$                            | 0.9900   | 0.0634    | 0.9989   | 0.0907    |
| MP Persistence      | $ ho_r$                             | 0.9207   | 0.0207    | 0.8454   | 0.0214    |
| MP Output           | $\psi_{y}$                          | 0.4946   | 0.1901    | 0.3200   | 0.1967    |
| MP Inflation        | $\psi_\pi$                          | 1.9994   | 0.0000    | 1.5109   | 0.0000    |
| Nat. Rate Pers.     | $ ho_n$                             | 0.8488   | 0.0684    | 0.6810   | 0.0645    |
| Cost Push Pers.     | $ ho_{u}$                           | 0.0000   | 0.0692    | 0.4419   | 0.0608    |
| Nat. Rate Std. Dev. | $\sigma_n$                          | 0.0751   | 0.0706    | 0.5835   | 0.0741    |
| Cost Push Std. Dev. | $\sigma_{\scriptscriptstyle \it U}$ | 0.0029   | 0.0002    | 0.0086   | 0.0003    |
| MP Std. Dev.        | $\sigma_r$                          | 0.0030   | 0.0001    | 0.0030   | 0.0001    |
| SS Inflation        | $\pi^*$                             | 5.9904   | 1.2374    | 5.8862   | 1.2739    |
| Learning Gain       | g                                   | _        | _         | 0.0828   | 0.0070    |
| Log-likelihood      |                                     |          | -459.9390 |          | -573.3274 |
| MSE Output Gap      |                                     |          | 0.6087    |          | 0.7989    |
| MSE Inflation       |                                     |          | 1.3313    |          | 2.7104    |
| MSE Fed. Funds Rate |                                     |          | 1.6480    | 10000    | 1.7396    |

|                     |                                 | Ca       | se 1      | Ca       | se 4      |
|---------------------|---------------------------------|----------|-----------|----------|-----------|
| Description         | Parameter                       | Estimate | Std. Dev. | Estimate | Std. Dev. |
| Habit Formation     | η                               | 0.4221   | 0.1062    | 0.5293   | 0.1216    |
| Inverse IES         | $\sigma$                        | 0.5152   | 0.4401    | 0.2502   | 0.4865    |
| Phillips Slope      | $\kappa$                        | 0.0001   | 0.0002    | 0.0064   | 0.0002    |
| Price Indexation    | $\gamma$                        | 0.9900   | 0.0634    | 0.9989   | 0.0907    |
| MP Persistence      | $\rho_r$                        | 0.9207   | 0.0207    | 0.8454   | 0.0214    |
| MP Output           | $\psi_{y}$                      | 0.4946   | 0.1901    | 0.3200   | 0.1967    |
| MP Inflation        | $\psi_{\pi}$                    | 1.9994   | 0.0000    | 1.5109   | 0.0000    |
| Nat. Rate Pers.     | $\rho_n$                        | 0.8488   | 0.0684    | 0.6810   | 0.0645    |
| Cost Push Pers.     | $ ho_u$                         | 0.0000   | 0.0692    | 0.4419   | 0.0608    |
| Nat. Rate Std. Dev. | $\sigma_n$                      | 0.0751   | 0.0706    | 0.5835   | 0.0741    |
| Cost Push Std. Dev. | $\sigma_{\scriptscriptstyle U}$ | 0.0029   | 0.0002    | 0.0086   | 0.0003    |
| MP Std. Dev.        | $\sigma_r$                      | 0.0030   | 0.0001    | 0.0030   | 0.0001    |
| SS Inflation        | $\pi^*$                         | 5.9904   | 1.2374    | 5.8862   | 1.2739    |
| Learning Gain       | g                               | _        | _         | 0.0828   | 0.0070    |
| Log-likelihood      |                                 |          | -459.9390 |          | -573.3274 |
| MSE Output Gap      |                                 |          | 0.6087    |          | 0.7989    |
| MSE Inflation       |                                 |          | 1.3313    |          | 2.7104    |
| MSE Fed. Funds Rate |                                 |          | 1.6480    | 4 DF     | 1.7396    |

#### No Capital: Forecast Errors

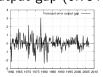


Rational Expectations Inflation

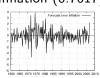


Fed Funds Rate





Learning with Pre-sample Initial Conditions







|                     |                  | Ca       | se 1      | Ca       | se 5      |
|---------------------|------------------|----------|-----------|----------|-----------|
| Description         | Parameter        | Estimate | Std. Dev. | Estimate | Std. Dev. |
| Habit Formation     | $\eta$           | 0.4221   | 0.1062    | 0.3052   | 0.1216    |
| Inverse IES         | $\sigma$         | 0.5152   | 0.4401    | 0.1960   | 0.4865    |
| Phillips Slope      | $\kappa$         | 0.0001   | 0.0002    | 0.0001   | 0.0002    |
| Price Indexation    | $\gamma$         | 0.9900   | 0.0634    | 0.9893   | 0.0907    |
| MP Persistence      | $ ho_r$          | 0.9207   | 0.0207    | 0.9193   | 0.0214    |
| MP Output           | $\psi_{y}$       | 0.4946   | 0.1901    | 0.4944   | 0.1967    |
| MP Inflation        | $\psi_\pi$       | 1.9994   | 0.0000    | 1.9992   | 0.0000    |
| Nat. Rate Pers.     | $ ho_{n}$        | 0.8488   | 0.0684    | 0.8488   | 0.0645    |
| Cost Push Pers.     | $ ho_{\it u}$    | 0.0000   | 0.0692    | 0.0000   | 0.0608    |
| Nat. Rate Std. Dev. | $\sigma_n$       | 0.0751   | 0.0706    | 0.2310   | 0.0741    |
| Cost Push Std. Dev. | $\sigma_{\it u}$ | 0.0029   | 0.0002    | 0.0054   | 0.0003    |
| MP Std. Dev.        | $\sigma_r$       | 0.0030   | 0.0001    | 0.0030   | 0.0001    |
| SS Inflation        | $\pi^*$          | 5.9904   | 1.2374    | 5.9894   | 1.2739    |
| Learning Gain       | g                | _        | -         | 0.0000   | 0.0070    |
| Log-likelihood      |                  |          | -459.9390 |          | -449.3276 |
| MSE Output Gap      |                  |          | 0.6087    |          | 0.5679    |
| MSE Inflation       |                  |          | 1.3313    |          | 1.2922    |
| MSE Fed. Funds Rate |                  |          | 1.6480    |          | _ 1.6486  |

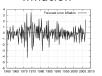
|                     |                  | Case 1   |           | Case 5   |           |
|---------------------|------------------|----------|-----------|----------|-----------|
| Description         | Parameter        | Estimate | Std. Dev. | Estimate | Std. Dev. |
| Habit Formation     | η                | 0.4221   | 0.1062    | 0.3052   | 0.1216    |
| Inverse IES         | $\sigma$         | 0.5152   | 0.4401    | 0.1960   | 0.4865    |
| Phillips Slope      | $\kappa$         | 0.0001   | 0.0002    | 0.0001   | 0.0002    |
| Price Indexation    | $\gamma$         | 0.9900   | 0.0634    | 0.9893   | 0.0907    |
| MP Persistence      | $ ho_r$          | 0.9207   | 0.0207    | 0.9193   | 0.0214    |
| MP Output           | $\psi_{y}$       | 0.4946   | 0.1901    | 0.4944   | 0.1967    |
| MP Inflation        | $\psi_{\pi}$     | 1.9994   | 0.0000    | 1.9992   | 0.0000    |
| Nat. Rate Pers.     | $ ho_n$          | 0.8488   | 0.0684    | 0.8488   | 0.0645    |
| Cost Push Pers.     | $ ho_u$          | 0.0000   | 0.0692    | 0.0000   | 0.0608    |
| Nat. Rate Std. Dev. | $\sigma_n$       | 0.0751   | 0.0706    | 0.2310   | 0.0741    |
| Cost Push Std. Dev. | $\sigma_{\it u}$ | 0.0029   | 0.0002    | 0.0054   | 0.0003    |
| MP Std. Dev.        | $\sigma_r$       | 0.0030   | 0.0001    | 0.0030   | 0.0001    |
| SS Inflation        | $\pi^*$          | 5.9904   | 1.2374    | 5.9894   | 1.2739    |
| Learning Gain       | g                | _        | _         | 0.0000   | 0.0070    |
| Log-likelihood      |                  |          | -459.9390 |          | -449.3276 |
| MSE Output Gap      |                  |          | 0.6087    |          | 0.5679    |
| MSE Inflation       |                  |          | 1.3313    |          | 1.2922    |
| MSE Fed. Funds Rate |                  |          | 1.6480    |          | 1.6486    |

|                     |                  | Case 1   |           | Case 5   |           |
|---------------------|------------------|----------|-----------|----------|-----------|
| Description         | Parameter        | Estimate | Std. Dev. | Estimate | Std. Dev. |
| Habit Formation     | $\eta$           | 0.4221   | 0.1062    | 0.3052   | 0.1216    |
| Inverse IES         | $\sigma$         | 0.5152   | 0.4401    | 0.1960   | 0.4865    |
| Phillips Slope      | $\kappa$         | 0.0001   | 0.0002    | 0.0001   | 0.0002    |
| Price Indexation    | $\gamma$         | 0.9900   | 0.0634    | 0.9893   | 0.0907    |
| MP Persistence      | $ ho_r$          | 0.9207   | 0.0207    | 0.9193   | 0.0214    |
| MP Output           | $\psi_{y}$       | 0.4946   | 0.1901    | 0.4944   | 0.1967    |
| MP Inflation        | $\psi_\pi$       | 1.9994   | 0.0000    | 1.9992   | 0.0000    |
| Nat. Rate Pers.     | $ ho_n$          | 0.8488   | 0.0684    | 0.8488   | 0.0645    |
| Cost Push Pers.     | $ ho_{\it u}$    | 0.0000   | 0.0692    | 0.0000   | 0.0608    |
| Nat. Rate Std. Dev. | $\sigma_n$       | 0.0751   | 0.0706    | 0.2310   | 0.0741    |
| Cost Push Std. Dev. | $\sigma_{\it u}$ | 0.0029   | 0.0002    | 0.0054   | 0.0003    |
| MP Std. Dev.        | $\sigma_r$       | 0.0030   | 0.0001    | 0.0030   | 0.0001    |
| SS Inflation        | $\pi^*$          | 5.9904   | 1.2374    | 5.9894   | 1.2739    |
| Learning Gain       | g                | _        | _         | 0.0000   | 0.0070    |
| Log-likelihood      |                  |          | -459.9390 |          | -449.3276 |
| MSE Output Gap      |                  |          | 0.6087    |          | 0.5679    |
| MSE Inflation       |                  |          | 1.3313    |          | 1.2922    |
| MSE Fed. Funds Rate |                  |          | 1.6480    |          | 1.6486    |





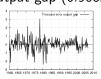
#### Rational Expectations Inflation



Fed Funds Rate



# Output gap (0.9682)



# Learning with Estimated Initial Conditions

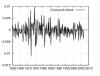


#### Inflation (0.9896) Fed Funds Rate (0.9997)





Rational Expectations Cost Push

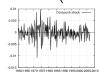


Policy Shock



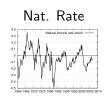


Learning with RE Initial Conditions Nat. Rate (0.9996) Cost Push (0.9965)

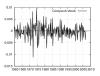


Policy Shock (1.0000)





Rational Expectations Cost Push



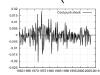
Policy Shock



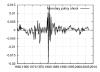
Learning Without Observable Shocks



Nat. Rate (0.9731) Cost Push (0.9932)



Policy Shock (0.9995)





Rational Expectations Cost Push



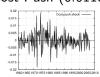
Policy Shock

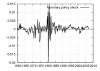


Nat. Rate (0.3652)



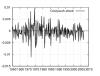
Learning with Pre-sample Initial Conditions Cost Push (0.6110) Policy Shock (0.9619)







Rational Expectations Cost Push



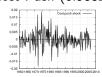
Policy Shock



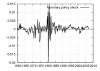
Learning with Estimated Initial Conditions Nat. Rate (0.9769)



Cost Push (0.9832)



Policy Shock (0.9998)



#### No Capital: Evolution of Expectations



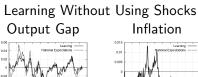
# Learning with RE Initial Conditions Output Gap Inflation

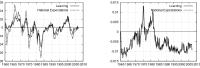




#### No Capital: Evolution of Expectations







-0.04

#### Rational Expectations

#### Output Gap



#### Inflation



# Learning with Pre-sample Initial Conditions Output Gap Inflation





#### Output Gap



#### Inflation



#### Learning with Estimated Initial Conditions Output Gap Inflation

Rational Expectations





#### Endogenous Capital: RE vs. Learning (RE Init.)

|                       |                | Case 1   |            | Case 2   |            |
|-----------------------|----------------|----------|------------|----------|------------|
| Description           | Parameter      | Estimate | Std. Dev.  | Estimate | Std. Dev.  |
| Habit Formation       | η              | 0.9181   | 0.1007     | 0.9181   | 0.1017     |
| Inverse IES           | $\sigma$       | 0.3432   | 0.7774     | 0.3432   | 0.7967     |
| Capital Share         | $\alpha$       | 0.3584   | 0.1189     | 0.3584   | 0.1219     |
| Cons / Output         | $c_y$          | 0.8753   | 0.0044     | 0.8753   | 0.0044     |
| Cost Capital Adj.     | $\phi$         | 6.9883   | 1.4836     | 6.9883   | 2.8850     |
| Phillips Slope        | $\kappa$       | 0.0090   | 0.0036     | 0.0090   | 0.0039     |
| Price Indexation      | $\gamma$       | 0.0001   | 0.0768     | 0.0001   | 0.0769     |
| MP Persistence        | $\rho_r$       | 0.7481   | 0.0472     | 0.7481   | 0.0570     |
| MP Output             | $\psi_y$       | 0.1003   | 0.0379     | 0.1003   | 0.0379     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014   | 0.1195     | 1.0014   | 0.1219     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716   | 0.0133     | 0.9716   | 0.0134     |
| Pref. Shock Pers.     | ρξ             | 0.5647   | 0.1159     | 0.5647   | 0.1160     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050   | 0.0426     | 0.9050   | 0.0435     |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094   | 0.0041     | 0.0094   | 0.0044     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306   | 0.0070     | 0.0306   | 0.0099     |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587   | 0.2699     | 0.3587   | 0.2741     |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033   | 0.0002     | 0.0033   | 0.0003     |
| SS Inflation          | π*             | 0.2209   | 4.2127     | 0.2209   | 4.2253     |
| SS Output (\$10,000)  | y*             | 1.4085   | 0.0212     | 1.4085   | 0.0213     |
| Learning gain         | g              | _        | _          | 0.0000   | 0.0147     |
| Log-likelihood        |                |          | -2391.5472 |          | -2391.5472 |
| MSE Consumption       |                | ĺ        | 7285.1049  | ĺ        | 7285.1049  |
| MSE Investment        |                |          | 14454.2922 |          | 14454.2922 |
| MSE Inflation         |                |          | 1.2633     |          | 1.2633     |
| MSE Fed. Funds Rate   |                |          | 1.7499     |          | 1.7499     |



#### Endogenous Capital: RE vs. Learning (RE Init.)

|                       |                | Case 1   |            | Case 2     |            |
|-----------------------|----------------|----------|------------|------------|------------|
| Description           | Parameter      | Estimate | Std. Dev.  | Estimate   | Std. Dev.  |
| Habit Formation       | η              | 0.9181   | 0.1007     | 0.9181     | 0.1017     |
| Inverse IES           | $\sigma$       | 0.3432   | 0.7774     | 0.3432     | 0.7967     |
| Capital Share         | $\alpha$       | 0.3584   | 0.1189     | 0.3584     | 0.1219     |
| Cons / Output         | c <sub>y</sub> | 0.8753   | 0.0044     | 0.8753     | 0.0044     |
| Cost Capital Adj.     | $\phi$         | 6.9883   | 1.4836     | 6.9883     | 2.8850     |
| Phillips Slope        | $\kappa$       | 0.0090   | 0.0036     | 0.0090     | 0.0039     |
| Price Indexation      | $\gamma$       | 0.0001   | 0.0768     | 0.0001     | 0.0769     |
| MP Persistence        | $\rho_r$       | 0.7481   | 0.0472     | 0.7481     | 0.0570     |
| MP Output             | $\psi_y$       | 0.1003   | 0.0379     | 0.1003     | 0.0379     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014   | 0.1195     | 1.0014     | 0.1219     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716   | 0.0133     | 0.9716     | 0.0134     |
| Pref. Shock Pers.     | ρξ             | 0.5647   | 0.1159     | 0.5647     | 0.1160     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050   | 0.0426     | 0.9050     | 0.0435     |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094   | 0.0041     | 0.0094     | 0.0044     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306   | 0.0070     | 0.0306     | 0.0099     |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587   | 0.2699     | 0.3587     | 0.2741     |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033   | 0.0002     | 0.0033     | 0.0003     |
| SS Inflation          | π*             | 0.2209   | 4.2127     | 0.2209     | 4.2253     |
| SS Output (\$10,000)  | y*             | 1.4085   | 0.0212     | 1.4085     | 0.0213     |
| Learning gain         | g              | -        | -          | 0.0000     | 0.0147     |
| Log-likelihood        |                |          | -2391.5472 |            | -2391.5472 |
| MSE Consumption       |                | 1        | 7285.1049  | 7285.1049  |            |
| MSE Investment        |                |          | 14454.2922 | 14454.2922 |            |
| MSE Inflation         |                |          | 1.2633     | 1.2633     |            |
| MSE Fed. Funds Rate   |                |          | 1.7499     |            | 1.7499     |



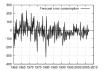
|                       |                | Case 1   |            | Case 2   |            |
|-----------------------|----------------|----------|------------|----------|------------|
| Description           | Parameter      | Estimate | Std. Dev.  | Estimate | Std. Dev.  |
| Habit Formation       | η              | 0.9181   | 0.1007     | 0.9181   | 0.1017     |
| Inverse IES           | $\sigma$       | 0.3432   | 0.7774     | 0.3432   | 0.7967     |
| Capital Share         | $\alpha$       | 0.3584   | 0.1189     | 0.3584   | 0.1219     |
| Cons / Output         | $c_y$          | 0.8753   | 0.0044     | 0.8753   | 0.0044     |
| Cost Capital Adj.     | $\dot{\phi}$   | 6.9883   | 1.4836     | 6.9883   | 2.8850     |
| Phillips Slope        | κ              | 0.0090   | 0.0036     | 0.0090   | 0.0039     |
| Price Indexation      | $\gamma$       | 0.0001   | 0.0768     | 0.0001   | 0.0769     |
| MP Persistence        | $\rho_r$       | 0.7481   | 0.0472     | 0.7481   | 0.0570     |
| MP Output             | $\psi_{V}$     | 0.1003   | 0.0379     | 0.1003   | 0.0379     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014   | 0.1195     | 1.0014   | 0.1219     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716   | 0.0133     | 0.9716   | 0.0134     |
| Pref. Shock Pers.     | $\rho_{\xi}$   | 0.5647   | 0.1159     | 0.5647   | 0.1160     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050   | 0.0426     | 0.9050   | 0.0435     |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094   | 0.0041     | 0.0094   | 0.0044     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306   | 0.0070     | 0.0306   | 0.0099     |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587   | 0.2699     | 0.3587   | 0.2741     |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033   | 0.0002     | 0.0033   | 0.0003     |
| SS Inflation          | π*             | 0.2209   | 4.2127     | 0.2209   | 4.2253     |
| SS Output (\$10,000)  | у*             | 1.4085   | 0.0212     | 1.4085   | 0.0213     |
| Learning gain         | g              | _        | -          | 0.0000   | 0.0147     |
| Log-likelihood        |                |          | -2391.5472 |          | -2391.5472 |
| MSE Consumption       |                | 1        | 7285.1049  | l        | 7285.1049  |
| MSE Investment        |                |          | 14454.2922 |          | 14454.2922 |
| MSE Inflation         |                | 1        | 1.2633     | l        | 1.2633     |
| MSE Fed. Funds Rate   |                |          | 1.7499     |          | 1.7499     |



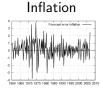
#### **Endogenous Capital: Forecast Errors**

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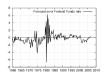
#### Consumption



#### Rational Expectations



Fed. Funds



Investment



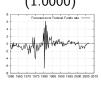
Consumption (1.0000)



Learning with RE Initial Conditions Inflation (1.0000)



Fed. Funds (1.0000)



Investment (1.0000)



|                       |                | Case 1    |            | Case 3    |            |
|-----------------------|----------------|-----------|------------|-----------|------------|
| Description           | Parameter      | Estimate  | Std. Dev.  | Estimate  | Std. Dev.  |
| Habit Formation       | η              | 0.9181    | 0.1007     | 0.8393    | 0.1888     |
| Inverse IES           | $\sigma$       | 0.3432    | 0.7774     | 0.3771    | 0.8493     |
| Capital Share         | $\alpha$       | 0.3584    | 0.1189     | 0.3870    | 0.2697     |
| Cons / Output         | $c_y$          | 0.8753    | 0.0044     | 0.8987    | 0.0000     |
| Cost Capital Adj.     | $\dot{\phi}$   | 6.9883    | 1.4836     | 6.9747    | 2.1739     |
| Phillips Slope        | κ              | 0.0090    | 0.0036     | 0.0158    | 0.0065     |
| Price Indexation      | $\gamma$       | 0.0001    | 0.0768     | 0.0007    | 0.0774     |
| MP Persistence        | $\rho_r$       | 0.7481    | 0.0472     | 0.8031    | 0.0365     |
| MP Output             | $\psi_{V}$     | 0.1003    | 0.0379     | 0.1005    | 0.0478     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014    | 0.1195     | 1.0285    | 0.1656     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716    | 0.0133     | 0.9689    | 0.0461     |
| Pref. Shock Pers.     | $\rho_{\xi}$   | 0.5647    | 0.1159     | 0.6644    | 0.1550     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050    | 0.0426     | 0.9182    | 0.0050     |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094    | 0.0041     | 0.1133    | 0.0708     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306    | 0.0070     | 0.1026    | 0.0189     |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587    | 0.2699     | 0.3532    | 0.1867     |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033    | 0.0002     | 0.0031    | 0.0001     |
| SS Inflation          | π*             | 0.2209    | 4.2127     | 3.8982    | 1.4266     |
| SS Output (\$10,000)  | у*             | 1.4085    | 0.0212     | 1.4200    | 0.0181     |
| Learning gain         | g              | -         | _          | 0.0052    | 0.0019     |
| Log-likelihood        |                |           | -2391.5472 |           | -2320.0228 |
| MSE Consumption       |                | 7285.1049 |            | 9532.5647 |            |
| MSE Investment        |                | l         | 14454.2922 | l         | 11044.5295 |
| MSE Inflation         |                | 1.2633    |            | 1.2455    |            |
| MSE Fed. Funds Rate   |                | l         | 1.7499     | l         | 1.6766     |

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|                       |                | Ca         | se 1       | Case 3   |            |
|-----------------------|----------------|------------|------------|----------|------------|
| Description           | Parameter      | Estimate   | Std. Dev.  | Estimate | Std. Dev.  |
| Habit Formation       | η              | 0.9181     | 0.1007     | 0.8393   | 0.1888     |
| Inverse IES           | $\sigma$       | 0.3432     | 0.7774     | 0.3771   | 0.8493     |
| Capital Share         | $\alpha$       | 0.3584     | 0.1189     | 0.3870   | 0.2697     |
| Cons / Output         | $c_{\nu}$      | 0.8753     | 0.0044     | 0.8987   | 0.0000     |
| Cost Capital Adj.     | $\phi$         | 6.9883     | 1.4836     | 6.9747   | 2.1739     |
| Phillips Slope        | κ              | 0.0090     | 0.0036     | 0.0158   | 0.0065     |
| Price Indexation      | $\gamma$       | 0.0001     | 0.0768     | 0.0007   | 0.0774     |
| MP Persistence        | $\rho_r$       | 0.7481     | 0.0472     | 0.8031   | 0.0365     |
| MP Output             | $\psi_{V}$     | 0.1003     | 0.0379     | 0.1005   | 0.0478     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014     | 0.1195     | 1.0285   | 0.1656     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716     | 0.0133     | 0.9689   | 0.0461     |
| Pref. Shock Pers.     | ρε             | 0.5647     | 0.1159     | 0.6644   | 0.1550     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050     | 0.0426     | 0.9182   | 0.0050     |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094     | 0.0041     | 0.1133   | 0.0708     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306     | 0.0070     | 0.1026   | 0.0189     |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587     | 0.2699     | 0.3532   | 0.1867     |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033     | 0.0002     | 0.0031   | 0.0001     |
| SS Inflation          | π*             | 0.2209     | 4.2127     | 3.8982   | 1.4266     |
| SS Output (\$10,000)  | y*             | 1.4085     | 0.0212     | 1.4200   | 0.0181     |
| Learning gain         | g              | _          | -          | 0.0052   | 0.0019     |
| Log-likelihood        |                |            | -2391.5472 |          | -2320.0228 |
| MSE Consumption       |                | 7285.1049  |            | l        | 9532.5647  |
| MSE Investment        |                | 14454.2922 |            |          | 11044.5295 |
| MSE Inflation         |                | l          | 1.2633     | l        | 1.2455     |
| MSE Fed. Funds Rate   |                | l          | 1.7499     | l        | 1.6766     |



|                       |                        | Case 1   |            | Case 3   |            |
|-----------------------|------------------------|----------|------------|----------|------------|
| Description           | Parameter              | Estimate | Std. Dev.  | Estimate | Std. Dev.  |
| Habit Formation       | η                      | 0.9181   | 0.1007     | 0.8393   | 0.1888     |
| Inverse IES           | $\sigma$               | 0.3432   | 0.7774     | 0.3771   | 0.8493     |
| Capital Share         | α                      | 0.3584   | 0.1189     | 0.3870   | 0.2697     |
| Cons / Output         | c <sub>y</sub>         | 0.8753   | 0.0044     | 0.8987   | 0.0000     |
| Cost Capital Adj.     | φ                      | 6.9883   | 1.4836     | 6.9747   | 2.1739     |
| Phillips Slope        | κ                      | 0.0090   | 0.0036     | 0.0158   | 0.0065     |
| Price Indexation      | $\gamma$               | 0.0001   | 0.0768     | 0.0007   | 0.0774     |
| MP Persistence        | $\rho_r$               | 0.7481   | 0.0472     | 0.8031   | 0.0365     |
| MP Output             | $\psi_{V}$             | 0.1003   | 0.0379     | 0.1005   | 0.0478     |
| MP Inflation          | $\psi_{\pi}$           | 1.0014   | 0.1195     | 1.0285   | 0.1656     |
| Tech. Shock Pers.     | $\rho_z$               | 0.9716   | 0.0133     | 0.9689   | 0.0461     |
| Pref. Shock Pers.     | $\rho_{\xi}$           | 0.5647   | 0.1159     | 0.6644   | 0.1550     |
| Inv. Shock Pers.      | $\rho_{\mu}$           | 0.9050   | 0.0426     | 0.9182   | 0.0050     |
| Tech. Shock Std. Dev. | $\sigma_z$             | 0.0094   | 0.0041     | 0.1133   | 0.0708     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$         | 0.0306   | 0.0070     | 0.1026   | 0.0189     |
| Pref. Shock Std. Dev. | $\sigma_{\mathcal{E}}$ | 0.3587   | 0.2699     | 0.3532   | 0.1867     |
| MP Shock Std. Dev.    | $\sigma_r$             | 0.0033   | 0.0002     | 0.0031   | 0.0001     |
| SS Inflation          | π*                     | 0.2209   | 4.2127     | 3.8982   | 1.4266     |
| SS Output (\$10,000)  | у*                     | 1.4085   | 0.0212     | 1.4200   | 0.0181     |
| Learning gain         | g                      | -        | -          | 0.0052   | 0.0019     |
| Log-likelihood        |                        |          | -2391.5472 |          | -2320.0228 |
| MSE Consumption       |                        | l        | 7285.1049  |          | 9532.5647  |
| MSE Investment        |                        | l        | 14454.2922 |          | 11044.5295 |
| MSE Inflation         |                        | l        | 1.2633     |          | 1.2455     |
| MSE Fed. Funds Rate   |                        | l        | 1.7499     |          | 1.6766     |

|                       |                | Ca         | se 1       | Case 3   |            |
|-----------------------|----------------|------------|------------|----------|------------|
| Description           | Parameter      | Estimate   | Std. Dev.  | Estimate | Std. Dev.  |
| Habit Formation       | η              | 0.9181     | 0.1007     | 0.8393   | 0.1888     |
| Inverse IES           | $\sigma$       | 0.3432     | 0.7774     | 0.3771   | 0.8493     |
| Capital Share         | $\alpha$       | 0.3584     | 0.1189     | 0.3870   | 0.2697     |
| Cons / Output         | $c_y$          | 0.8753     | 0.0044     | 0.8987   | 0.0000     |
| Cost Capital Adj.     | $\phi$         | 6.9883     | 1.4836     | 6.9747   | 2.1739     |
| Phillips Slope        | κ              | 0.0090     | 0.0036     | 0.0158   | 0.0065     |
| Price Indexation      | $\gamma$       | 0.0001     | 0.0768     | 0.0007   | 0.0774     |
| MP Persistence        | $\rho_r$       | 0.7481     | 0.0472     | 0.8031   | 0.0365     |
| MP Output             | $\psi_{V}$     | 0.1003     | 0.0379     | 0.1005   | 0.0478     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014     | 0.1195     | 1.0285   | 0.1656     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716     | 0.0133     | 0.9689   | 0.0461     |
| Pref. Shock Pers.     | $\rho_{\xi}$   | 0.5647     | 0.1159     | 0.6644   | 0.1550     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050     | 0.0426     | 0.9182   | 0.0050     |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094     | 0.0041     | 0.1133   | 0.0708     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306     | 0.0070     | 0.1026   | 0.0189     |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587     | 0.2699     | 0.3532   | 0.1867     |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033     | 0.0002     | 0.0031   | 0.0001     |
| SS Inflation          | $\pi^*$        | 0.2209     | 4.2127     | 3.8982   | 1.4266     |
| SS Output (\$10,000)  | у*             | 1.4085     | 0.0212     | 1.4200   | 0.0181     |
| Learning gain         | g              | -          | _          | 0.0052   | 0.0019     |
| Log-likelihood        | Log-likelihood |            | -2391.5472 |          | -2320.0228 |
| MSE Consumption       |                |            | 7285.1049  |          | 9532.5647  |
| MSE Investment        |                | 14454.2922 |            | l        | 11044.5295 |
| MSE Inflation         |                | 1.2633     |            |          | 1.2455     |
| MSE Fed. Funds Rate   |                | 1.7499     |            | 1.6766   |            |

|                       |                | Ca         | se 1      | Case 3   |            |
|-----------------------|----------------|------------|-----------|----------|------------|
| Description           | Parameter      | Estimate   | Std. Dev. | Estimate | Std. Dev.  |
| Habit Formation       | η              | 0.9181     | 0.1007    | 0.8393   | 0.1888     |
| Inverse IES           | $\sigma$       | 0.3432     | 0.7774    | 0.3771   | 0.8493     |
| Capital Share         | $\alpha$       | 0.3584     | 0.1189    | 0.3870   | 0.2697     |
| Cons / Output         | $c_y$          | 0.8753     | 0.0044    | 0.8987   | 0.0000     |
| Cost Capital Adj.     | $\dot{\phi}$   | 6.9883     | 1.4836    | 6.9747   | 2.1739     |
| Phillips Slope        | κ              | 0.0090     | 0.0036    | 0.0158   | 0.0065     |
| Price Indexation      | $\gamma$       | 0.0001     | 0.0768    | 0.0007   | 0.0774     |
| MP Persistence        | $\rho_r$       | 0.7481     | 0.0472    | 0.8031   | 0.0365     |
| MP Output             | $\psi_{V}$     | 0.1003     | 0.0379    | 0.1005   | 0.0478     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014     | 0.1195    | 1.0285   | 0.1656     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716     | 0.0133    | 0.9689   | 0.0461     |
| Pref. Shock Pers.     | $\rho_{\xi}$   | 0.5647     | 0.1159    | 0.6644   | 0.1550     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050     | 0.0426    | 0.9182   | 0.0050     |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094     | 0.0041    | 0.1133   | 0.0708     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306     | 0.0070    | 0.1026   | 0.0189     |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587     | 0.2699    | 0.3532   | 0.1867     |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033     | 0.0002    | 0.0031   | 0.0001     |
| SS Inflation          | π*             | 0.2209     | 4.2127    | 3.8982   | 1.4266     |
| SS Output (\$10,000)  | y*             | 1.4085     | 0.0212    | 1.4200   | 0.0181     |
| Learning gain         | g              | -          | _         | 0.0052   | 0.0019     |
| Log-likelihood        |                | -2391.5472 |           |          | -2320.0228 |
| MSE Consumption       |                |            | 7285.1049 |          | 9532.5647  |
| MSE Investment        |                | 14454.2922 |           |          | 11044.5295 |
| MSE Inflation         |                | 1.2633     |           | l        | 1.2455     |
| MSE Fed. Funds Rate   |                | 1.7499     |           | l        | 1.6766     |

|                       |                | Ca       | Case 1     |          | Case 3     |  |
|-----------------------|----------------|----------|------------|----------|------------|--|
| Description           | Parameter      | Estimate | Std. Dev.  | Estimate | Std. Dev.  |  |
| Habit Formation       | η              | 0.9181   | 0.1007     | 0.8393   | 0.1888     |  |
| Inverse IES           | $\sigma$       | 0.3432   | 0.7774     | 0.3771   | 0.8493     |  |
| Capital Share         | α              | 0.3584   | 0.1189     | 0.3870   | 0.2697     |  |
| Cons / Output         | c <sub>y</sub> | 0.8753   | 0.0044     | 0.8987   | 0.0000     |  |
| Cost Capital Adj.     | $\dot{\phi}$   | 6.9883   | 1.4836     | 6.9747   | 2.1739     |  |
| Phillips Slope        | κ              | 0.0090   | 0.0036     | 0.0158   | 0.0065     |  |
| Price Indexation      | $\gamma$       | 0.0001   | 0.0768     | 0.0007   | 0.0774     |  |
| MP Persistence        | $\rho_r$       | 0.7481   | 0.0472     | 0.8031   | 0.0365     |  |
| MP Output             | $\psi_{V}$     | 0.1003   | 0.0379     | 0.1005   | 0.0478     |  |
| MP Inflation          | $\psi_{\pi}$   | 1.0014   | 0.1195     | 1.0285   | 0.1656     |  |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716   | 0.0133     | 0.9689   | 0.0461     |  |
| Pref. Shock Pers.     | $\rho_{\xi}$   | 0.5647   | 0.1159     | 0.6644   | 0.1550     |  |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050   | 0.0426     | 0.9182   | 0.0050     |  |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094   | 0.0041     | 0.1133   | 0.0708     |  |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306   | 0.0070     | 0.1026   | 0.0189     |  |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587   | 0.2699     | 0.3532   | 0.1867     |  |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033   | 0.0002     | 0.0031   | 0.0001     |  |
| SS Inflation          | π*             | 0.2209   | 4.2127     | 3.8982   | 1.4266     |  |
| SS Output (\$10,000)  | у*             | 1.4085   | 0.0212     | 1.4200   | 0.0181     |  |
| Learning gain         | g              | -        | _          | 0.0052   | 0.0019     |  |
| Log-likelihood        |                |          | -2391.5472 |          | -2320.0228 |  |
| MSE Consumption       |                | l        | 7285.1049  | ĺ        | 9532.5647  |  |
| MSE Investment        | ivestment      |          | 14454.2922 |          | 11044.5295 |  |
| MSE Inflation         |                | 1.2633   |            |          | 1.2455     |  |
| MSE Fed. Funds Rate   |                | l        | 1.7499     | I        | 1.6766     |  |

|                       | Ca             | Case 1   |            | Case 3   |            |
|-----------------------|----------------|----------|------------|----------|------------|
| Description           | Parameter      | Estimate | Std. Dev.  | Estimate | Std. Dev.  |
| Habit Formation       | η              | 0.9181   | 0.1007     | 0.8393   | 0.1888     |
| Inverse IES           | $\sigma$       | 0.3432   | 0.7774     | 0.3771   | 0.8493     |
| Capital Share         | α              | 0.3584   | 0.1189     | 0.3870   | 0.2697     |
| Cons / Output         | c <sub>y</sub> | 0.8753   | 0.0044     | 0.8987   | 0.0000     |
| Cost Capital Adj.     | $\phi$         | 6.9883   | 1.4836     | 6.9747   | 2.1739     |
| Phillips Slope        | κ              | 0.0090   | 0.0036     | 0.0158   | 0.0065     |
| Price Indexation      | $\gamma$       | 0.0001   | 0.0768     | 0.0007   | 0.0774     |
| MP Persistence        | $\rho_r$       | 0.7481   | 0.0472     | 0.8031   | 0.0365     |
| MP Output             | $\psi_{V}$     | 0.1003   | 0.0379     | 0.1005   | 0.0478     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014   | 0.1195     | 1.0285   | 0.1656     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716   | 0.0133     | 0.9689   | 0.0461     |
| Pref. Shock Pers.     | $\rho_{\xi}$   | 0.5647   | 0.1159     | 0.6644   | 0.1550     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050   | 0.0426     | 0.9182   | 0.0050     |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094   | 0.0041     | 0.1133   | 0.0708     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306   | 0.0070     | 0.1026   | 0.0189     |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587   | 0.2699     | 0.3532   | 0.1867     |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033   | 0.0002     | 0.0031   | 0.0001     |
| SS Inflation          | π*             | 0.2209   | 4.2127     | 3.8982   | 1.4266     |
| SS Output (\$10,000)  | у*             | 1.4085   | 0.0212     | 1.4200   | 0.0181     |
| Learning gain         | g              | -        | -          | 0.0052   | 0.0019     |
| Log-likelihood        |                |          | -2391.5472 |          | -2320.0228 |
| MSE Consumption       |                | l        | 7285.1049  |          | 9532.5647  |
| MSE Investment        |                | l        | 14454.2922 |          | 11044.5295 |
| MSE Inflation         |                |          | 1.2633     |          | 1.2455     |
| MSE Fed. Funds Rate   |                |          | 1.7499     |          | 1.6766     |

#### **Endogenous Capital: Forecast Errors**

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# Consumption

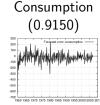
1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 201

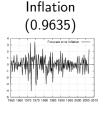
Rational Expectations Inflation

1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 2010













Learning Without Observable Shocks

|                       |                | Ca       | se 1       | Case 4   |            |
|-----------------------|----------------|----------|------------|----------|------------|
| Description           | Parameter      | Estimate | Std. Dev.  | Estimate | Std. Dev.  |
| Habit Formation       | η              | 0.9181   | 0.1007     | 0.7685   | 0.2555     |
| Inverse IES           | $\sigma$       | 0.3432   | 0.7774     | 0.6433   | 1.7938     |
| Capital Share         | $\alpha$       | 0.3584   | 0.1189     | 0.3177   | 0.3785     |
| Cons / Output         | $c_y$          | 0.8753   | 0.0044     | 0.8944   | 0.0036     |
| Cost Capital Adj.     | $\dot{\phi}$   | 6.9883   | 1.4836     | 6.8234   | 2.6605     |
| Phillips Slope        | κ              | 0.0090   | 0.0036     | 0.0113   | 0.0035     |
| Price Indexation      | $\gamma$       | 0.0001   | 0.0768     | 0.0877   | 0.1071     |
| MP Persistence        | $\rho_r$       | 0.7481   | 0.0472     | 0.8975   | 0.0382     |
| MP Output             | $\psi_{V}$     | 0.1003   | 0.0379     | 0.1565   | 0.1132     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014   | 0.1195     | 1.0462   | 0.1866     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716   | 0.0133     | 0.8205   | 0.0357     |
| Pref. Shock Pers.     | $\rho_{\xi}$   | 0.5647   | 0.1159     | 0.9007   | 0.0286     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050   | 0.0426     | 0.9999   | 0.0000     |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094   | 0.0041     | 0.3079   | 0.1910     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306   | 0.0070     | 0.1064   | 0.0133     |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587   | 0.2699     | 0.4362   | 0.4727     |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033   | 0.0002     | 0.0032   | 0.0001     |
| SS Inflation          | π*             | 0.2209   | 4.2127     | 0.0035   | 0.9250     |
| SS Output (\$10,000)  | у*             | 1.4085   | 0.0212     | 1.4074   | 0.0089     |
| Learning gain         | g              | _        | _          | 0.0060   | 0.0012     |
| Log-likelihood        |                |          | -2391.5472 |          | -2506.8255 |
| MSE Consumption       |                | l        | 7285.1049  |          | 9584.1202  |
| MSE Investment        |                | l        | 14454.2922 |          | 44510.7805 |
| MSE Inflation         |                | l        | 1.2633     |          | 3.5212     |
| MSE Fed. Funds Rate   |                | l        | 1.7499     |          | 1.5378     |



|                       |                | Ca         | se 1       | Case 4   |            |
|-----------------------|----------------|------------|------------|----------|------------|
| Description           | Parameter      | Estimate   | Std. Dev.  | Estimate | Std. Dev.  |
| Habit Formation       | η              | 0.9181     | 0.1007     | 0.7685   | 0.2555     |
| Inverse IES           | $\sigma$       | 0.3432     | 0.7774     | 0.6433   | 1.7938     |
| Capital Share         | $\alpha$       | 0.3584     | 0.1189     | 0.3177   | 0.3785     |
| Cons / Output         | c <sub>y</sub> | 0.8753     | 0.0044     | 0.8944   | 0.0036     |
| Cost Capital Adj.     | $\phi$         | 6.9883     | 1.4836     | 6.8234   | 2.6605     |
| Phillips Slope        | $\kappa$       | 0.0090     | 0.0036     | 0.0113   | 0.0035     |
| Price Indexation      | $\gamma$       | 0.0001     | 0.0768     | 0.0877   | 0.1071     |
| MP Persistence        | $\rho_r$       | 0.7481     | 0.0472     | 0.8975   | 0.0382     |
| MP Output             | $\psi_{V}$     | 0.1003     | 0.0379     | 0.1565   | 0.1132     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014     | 0.1195     | 1.0462   | 0.1866     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716     | 0.0133     | 0.8205   | 0.0357     |
| Pref. Shock Pers.     | ρξ             | 0.5647     | 0.1159     | 0.9007   | 0.0286     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050     | 0.0426     | 0.9999   | 0.0000     |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094     | 0.0041     | 0.3079   | 0.1910     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306     | 0.0070     | 0.1064   | 0.0133     |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587     | 0.2699     | 0.4362   | 0.4727     |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033     | 0.0002     | 0.0032   | 0.0001     |
| SS Inflation          | π*             | 0.2209     | 4.2127     | 0.0035   | 0.9250     |
| SS Output (\$10,000)  | y*             | 1.4085     | 0.0212     | 1.4074   | 0.0089     |
| Learning gain         | g              | _          | -          | 0.0060   | 0.0012     |
| Log-likelihood        |                |            | -2391.5472 |          | -2506.8255 |
| MSE Consumption       |                | 7285.1049  |            |          | 9584.1202  |
| MSE Investment        |                | 14454.2922 |            |          | 44510.7805 |
| MSE Inflation         |                | 1          | 1.2633     | l        | 3.5212     |
| MSE Fed. Funds Rate   |                | 1.7499     |            | 1.5378   |            |



|                       |                | Ca         | se 1       | Case 4   |            |
|-----------------------|----------------|------------|------------|----------|------------|
| Description           | Parameter      | Estimate   | Std. Dev.  | Estimate | Std. Dev.  |
| Habit Formation       | η              | 0.9181     | 0.1007     | 0.7685   | 0.2555     |
| Inverse IES           | $\sigma$       | 0.3432     | 0.7774     | 0.6433   | 1.7938     |
| Capital Share         | $\alpha$       | 0.3584     | 0.1189     | 0.3177   | 0.3785     |
| Cons / Output         | c <sub>y</sub> | 0.8753     | 0.0044     | 0.8944   | 0.0036     |
| Cost Capital Adj.     | $\phi$         | 6.9883     | 1.4836     | 6.8234   | 2.6605     |
| Phillips Slope        | $\kappa$       | 0.0090     | 0.0036     | 0.0113   | 0.0035     |
| Price Indexation      | $\gamma$       | 0.0001     | 0.0768     | 0.0877   | 0.1071     |
| MP Persistence        | $\rho_r$       | 0.7481     | 0.0472     | 0.8975   | 0.0382     |
| MP Output             | $\psi_{V}$     | 0.1003     | 0.0379     | 0.1565   | 0.1132     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014     | 0.1195     | 1.0462   | 0.1866     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716     | 0.0133     | 0.8205   | 0.0357     |
| Pref. Shock Pers.     | $\rho_{\xi}$   | 0.5647     | 0.1159     | 0.9007   | 0.0286     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050     | 0.0426     | 0.9999   | 0.0000     |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094     | 0.0041     | 0.3079   | 0.1910     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306     | 0.0070     | 0.1064   | 0.0133     |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587     | 0.2699     | 0.4362   | 0.4727     |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033     | 0.0002     | 0.0032   | 0.0001     |
| SS Inflation          | $\pi^{*}$      | 0.2209     | 4.2127     | 0.0035   | 0.9250     |
| SS Output (\$10,000)  | y*             | 1.4085     | 0.0212     | 1.4074   | 0.0089     |
| Learning gain         | g              | -          | -          | 0.0060   | 0.0012     |
| Log-likelihood        |                |            | -2391.5472 |          | -2506.8255 |
| MSE Consumption       |                | l          | 7285.1049  |          | 9584.1202  |
| MSE Investment        |                | 14454.2922 |            |          | 44510.7805 |
| MSE Inflation         |                | l          | 1.2633     |          | 3.5212     |
| MSE Fed. Funds Rate   |                | l          | 1.7499     |          | 1.5378     |



|                       |                | Case 1   |            | Case 4   |            |
|-----------------------|----------------|----------|------------|----------|------------|
| Description           | Parameter      | Estimate | Std. Dev.  | Estimate | Std. Dev.  |
| Habit Formation       | η              | 0.9181   | 0.1007     | 0.7685   | 0.2555     |
| Inverse IES           | $\sigma$       | 0.3432   | 0.7774     | 0.6433   | 1.7938     |
| Capital Share         | α              | 0.3584   | 0.1189     | 0.3177   | 0.3785     |
| Cons / Output         | $c_y$          | 0.8753   | 0.0044     | 0.8944   | 0.0036     |
| Cost Capital Adj.     | φ              | 6.9883   | 1.4836     | 6.8234   | 2.6605     |
| Phillips Slope        | $\kappa$       | 0.0090   | 0.0036     | 0.0113   | 0.0035     |
| Price Indexation      | $  \gamma  $   | 0.0001   | 0.0768     | 0.0877   | 0.1071     |
| MP Persistence        | $\rho_r$       | 0.7481   | 0.0472     | 0.8975   | 0.0382     |
| MP Output             | $\psi_y$       | 0.1003   | 0.0379     | 0.1565   | 0.1132     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014   | 0.1195     | 1.0462   | 0.1866     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716   | 0.0133     | 0.8205   | 0.0357     |
| Pref. Shock Pers.     | $\rho_{\xi}$   | 0.5647   | 0.1159     | 0.9007   | 0.0286     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050   | 0.0426     | 0.9999   | 0.0000     |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094   | 0.0041     | 0.3079   | 0.1910     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306   | 0.0070     | 0.1064   | 0.0133     |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587   | 0.2699     | 0.4362   | 0.4727     |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033   | 0.0002     | 0.0032   | 0.0001     |
| SS Inflation          | π*             | 0.2209   | 4.2127     | 0.0035   | 0.9250     |
| SS Output (\$10,000)  | у*             | 1.4085   | 0.0212     | 1.4074   | 0.0089     |
| Learning gain         | g              | -        | -          | 0.0060   | 0.0012     |
| Log-likelihood        | Log-likelihood |          | -2391.5472 |          | -2506.8255 |
| MSE Consumption       |                |          | 7285.1049  |          | 9584.1202  |
| MSE Investment        |                |          | 14454.2922 |          | 44510.7805 |
| MSE Inflation         |                |          | 1.2633     |          | 3.5212     |
| MSE Fed. Funds Rate   |                |          | 1.7499     |          | 1.5378     |

#### **Endogenous Capital: Forecast Errors**

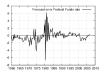
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### Consumption 1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 201

Rational Expectations Inflation



Fed. Funds



Investment



#### Learning with Pre-sample Initial Conditions

Consumption (0.8813)









-1500

|                       |                | Case 1     |            | Case 5   |            |
|-----------------------|----------------|------------|------------|----------|------------|
| Description           | Parameter      | Estimate   | Std. Dev.  | Estimate | Std. Dev.  |
| Habit Formation       | η              | 0.9181     | 0.1007     | 0.8564   | 6.7771     |
| Inverse IES           | $\sigma$       | 0.3432     | 0.7774     | 0.1667   | 16.3052    |
| Capital Share         | $\alpha$       | 0.3584     | 0.1189     | 0.3662   | 0.6600     |
| Cons / Output         | c <sub>y</sub> | 0.8753     | 0.0044     | 0.8851   | 0.0122     |
| Cost Capital Adj.     | $\phi$         | 6.9883     | 1.4836     | 6.9999   | 0.0001     |
| Phillips Slope        | $\kappa$       | 0.0090     | 0.0036     | 0.0287   | 0.0201     |
| Price Indexation      | $\gamma$       | 0.0001     | 0.0768     | 0.0002   | 0.3186     |
| MP Persistence        | $\rho_r$       | 0.7481     | 0.0472     | 0.9136   | 0.0641     |
| MP Output             | $\psi_{V}$     | 0.1003     | 0.0379     | 0.1296   | 0.2032     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014     | 0.1195     | 1.0089   | 0.4514     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716     | 0.0133     | 0.9582   | 0.0411     |
| Pref. Shock Pers.     | $\rho_{\xi}$   | 0.5647     | 0.1159     | 0.1614   | 0.1110     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050     | 0.0426     | 0.9075   | 0.0902     |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094     | 0.0041     | 0.0609   | 0.0920     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306     | 0.0070     | 0.0709   | 0.0220     |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587     | 0.2699     | 0.0918   | 0.2118     |
| MP Shock Std. Dev.    | $\sigma_r^2$   | 0.0033     | 0.0002     | 0.0030   | 0.0001     |
| SS Inflation          | $\pi^*$        | 0.2209     | 4.2127     | 0.2202   | 4.7265     |
| SS Output (\$10,000)  | y*             | 1.4085     | 0.0212     | 1.4177   | 0.0783     |
| Learning gain         | g              | -          | -          | 0.0005   | 0.0018     |
| Log-likelihood        |                |            | -2391.5472 |          | -2237.0404 |
| MSE Consumption       |                | l          | 7285.1049  | ĺ        | 6702.3679  |
| MSE Investment        |                | 14454.2922 |            | l        | 6304.4836  |
| MSE Inflation         |                | 1.2633     |            | ĺ        | 1.1815     |
| MSE Fed. Funds Rate   |                | l          | 1.7499     | l        | 1.4896     |

#### Endogenous Capital: RE vs. Learning (Estimated)

|                       |                | Ca       | se 1       | Case 5    |            |  |
|-----------------------|----------------|----------|------------|-----------|------------|--|
| Description           | Parameter      | Estimate | Std. Dev.  | Estimate  | Std. Dev.  |  |
| Habit Formation       | η              | 0.9181   | 0.1007     | 0.8564    | 6.7771     |  |
| Inverse IES           | $\sigma$       | 0.3432   | 0.7774     | 0.1667    | 16.3052    |  |
| Capital Share         | $\alpha$       | 0.3584   | 0.1189     | 0.3662    | 0.6600     |  |
| Cons / Output         | c <sub>y</sub> | 0.8753   | 0.0044     | 0.8851    | 0.0122     |  |
| Cost Capital Adj.     | $\phi$         | 6.9883   | 1.4836     | 6.9999    | 0.0001     |  |
| Phillips Slope        | κ              | 0.0090   | 0.0036     | 0.0287    | 0.0201     |  |
| Price Indexation      | $\gamma$       | 0.0001   | 0.0768     | 0.0002    | 0.3186     |  |
| MP Persistence        | $\rho_r$       | 0.7481   | 0.0472     | 0.9136    | 0.0641     |  |
| MP Output             | $\psi_{V}$     | 0.1003   | 0.0379     | 0.1296    | 0.2032     |  |
| MP Inflation          | $\psi_{\pi}$   | 1.0014   | 0.1195     | 1.0089    | 0.4514     |  |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716   | 0.0133     | 0.9582    | 0.0411     |  |
| Pref. Shock Pers.     | $\rho_{\xi}$   | 0.5647   | 0.1159     | 0.1614    | 0.1110     |  |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050   | 0.0426     | 0.9075    | 0.0902     |  |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094   | 0.0041     | 0.0609    | 0.0920     |  |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306   | 0.0070     | 0.0709    | 0.0220     |  |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587   | 0.2699     | 0.0918    | 0.2118     |  |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033   | 0.0002     | 0.0030    | 0.0001     |  |
| SS Inflation          | π*             | 0.2209   | 4.2127     | 0.2202    | 4.7265     |  |
| SS Output (\$10,000)  | у*             | 1.4085   | 0.0212     | 1.4177    | 0.0783     |  |
| Learning gain         | g              | -        | -          | 0.0005    | 0.0018     |  |
| Log-likelihood        |                |          | -2391.5472 |           | -2237.0404 |  |
| MSE Consumption       |                |          | 7285.1049  | 6702.3679 |            |  |
| MSE Investment        |                | l        | 14454.2922 |           | 6304.4836  |  |
| MSE Inflation         |                | l        | 1.2633     |           | 1.1815     |  |
| MSE Fed. Funds Rate   |                | I        | 1.7499     | 1.4896    |            |  |

#### Endogenous Capital: RE vs. Learning (Estimated)

|                       |                | Ca       | se 1       | Case 5    |            |
|-----------------------|----------------|----------|------------|-----------|------------|
| Description           | Parameter      | Estimate | Std. Dev.  | Estimate  | Std. Dev.  |
| Habit Formation       | η              | 0.9181   | 0.1007     | 0.8564    | 6.7771     |
| Inverse IES           | $\sigma$       | 0.3432   | 0.7774     | 0.1667    | 16.3052    |
| Capital Share         | $\alpha$       | 0.3584   | 0.1189     | 0.3662    | 0.6600     |
| Cons / Output         | $c_y$          | 0.8753   | 0.0044     | 0.8851    | 0.0122     |
| Cost Capital Adj.     | $\dot{\phi}$   | 6.9883   | 1.4836     | 6.9999    | 0.0001     |
| Phillips Slope        | $\kappa$       | 0.0090   | 0.0036     | 0.0287    | 0.0201     |
| Price Indexation      | $\gamma$       | 0.0001   | 0.0768     | 0.0002    | 0.3186     |
| MP Persistence        | $\rho_r$       | 0.7481   | 0.0472     | 0.9136    | 0.0641     |
| MP Output             | $\psi_y$       | 0.1003   | 0.0379     | 0.1296    | 0.2032     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014   | 0.1195     | 1.0089    | 0.4514     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716   | 0.0133     | 0.9582    | 0.0411     |
| Pref. Shock Pers.     | $\rho_{\xi}$   | 0.5647   | 0.1159     | 0.1614    | 0.1110     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050   | 0.0426     | 0.9075    | 0.0902     |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094   | 0.0041     | 0.0609    | 0.0920     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306   | 0.0070     | 0.0709    | 0.0220     |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587   | 0.2699     | 0.0918    | 0.2118     |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033   | 0.0002     | 0.0030    | 0.0001     |
| SS Inflation          | π*             | 0.2209   | 4.2127     | 0.2202    | 4.7265     |
| SS Output (\$10,000)  | у*             | 1.4085   | 0.0212     | 1.4177    | 0.0783     |
| Learning gain         | g              | -        | _          | 0.0005    | 0.0018     |
| Log-likelihood        |                |          | -2391.5472 |           | -2237.0404 |
| MSE Consumption       |                | l        | 7285.1049  | 6702.367  |            |
| MSE Investment        |                | l        | 14454.2922 | 6304.4836 |            |
| MSE Inflation         |                | l        | 1.2633     |           | 1.1815     |
| MSE Fed. Funds Rate   |                | l        | 1.7499     | l         | 1.4896     |

#### Endogenous Capital: RE vs. Learning (Estimated)

|                       |                | Case 1   |            | Case 5    |            |
|-----------------------|----------------|----------|------------|-----------|------------|
| Description           | Parameter      | Estimate | Std. Dev.  | Estimate  | Std. Dev.  |
| Habit Formation       | η              | 0.9181   | 0.1007     | 0.8564    | 6.7771     |
| Inverse IES           | $\sigma$       | 0.3432   | 0.7774     | 0.1667    | 16.3052    |
| Capital Share         | $\alpha$       | 0.3584   | 0.1189     | 0.3662    | 0.6600     |
| Cons / Output         | $c_y$          | 0.8753   | 0.0044     | 0.8851    | 0.0122     |
| Cost Capital Adj.     | $\dot{\phi}$   | 6.9883   | 1.4836     | 6.9999    | 0.0001     |
| Phillips Slope        | κ              | 0.0090   | 0.0036     | 0.0287    | 0.0201     |
| Price Indexation      | $\gamma$       | 0.0001   | 0.0768     | 0.0002    | 0.3186     |
| MP Persistence        | $\rho_r$       | 0.7481   | 0.0472     | 0.9136    | 0.0641     |
| MP Output             | $\psi_{V}$     | 0.1003   | 0.0379     | 0.1296    | 0.2032     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014   | 0.1195     | 1.0089    | 0.4514     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716   | 0.0133     | 0.9582    | 0.0411     |
| Pref. Shock Pers.     | $\rho_{\xi}$   | 0.5647   | 0.1159     | 0.1614    | 0.1110     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050   | 0.0426     | 0.9075    | 0.0902     |
| Tech. Shock Std. Dev. | $\sigma_z$     | 0.0094   | 0.0041     | 0.0609    | 0.0920     |
| Inv. Shock Std. Dev.  | $\sigma_{\mu}$ | 0.0306   | 0.0070     | 0.0709    | 0.0220     |
| Pref. Shock Std. Dev. | $\sigma_{\xi}$ | 0.3587   | 0.2699     | 0.0918    | 0.2118     |
| MP Shock Std. Dev.    | $\sigma_r$     | 0.0033   | 0.0002     | 0.0030    | 0.0001     |
| SS Inflation          | π*             | 0.2209   | 4.2127     | 0.2202    | 4.7265     |
| SS Output (\$10,000)  | у*             | 1.4085   | 0.0212     | 1.4177    | 0.0783     |
| Learning gain         | g              | _        | _          | 0.0005    | 0.0018     |
| Log-likelihood        |                |          | -2391.5472 |           | -2237.0404 |
| MSE Consumption       |                |          | 7285.1049  |           | 6702.3679  |
| MSE Investment        |                |          | 14454.2922 | 6304.4836 |            |
| MSE Inflation         |                |          | 1.2633     |           | 1.1815     |
| MSE Fed. Funds Rate   |                |          | 1.7499     |           | 1.4896     |

|                       |                | Case 1   |            | Case 5   |            |
|-----------------------|----------------|----------|------------|----------|------------|
| Description           | Parameter      | Estimate | Std. Dev.  | Estimate | Std. Dev.  |
| Habit Formation       | η              | 0.9181   | 0.1007     | 0.8564   | 6.7771     |
| Inverse IES           | $\sigma$       | 0.3432   | 0.7774     | 0.1667   | 16.3052    |
| Capital Share         | $\alpha$       | 0.3584   | 0.1189     | 0.3662   | 0.6600     |
| Cons / Output         | $c_{\nu}$      | 0.8753   | 0.0044     | 0.8851   | 0.0122     |
| Cost Capital Adj.     | $\dot{\phi}$   | 6.9883   | 1.4836     | 6.9999   | 0.0001     |
| Phillips Slope        | κ              | 0.0090   | 0.0036     | 0.0287   | 0.0201     |
| Price Indexation      | $\gamma$       | 0.0001   | 0.0768     | 0.0002   | 0.3186     |
| MP Persistence        | $\rho_r$       | 0.7481   | 0.0472     | 0.9136   | 0.0641     |
| MP Output             | $\psi_{V}$     | 0.1003   | 0.0379     | 0.1296   | 0.2032     |
| MP Inflation          | $\psi_{\pi}$   | 1.0014   | 0.1195     | 1.0089   | 0.4514     |
| Tech. Shock Pers.     | $\rho_z$       | 0.9716   | 0.0133     | 0.9582   | 0.0411     |
| Pref. Shock Pers.     | $\rho_{\xi}$   | 0.5647   | 0.1159     | 0.1614   | 0.1110     |
| Inv. Shock Pers.      | $\rho_{\mu}$   | 0.9050   | 0.0426     | 0.9075   | 0.0902     |
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| SS Output (\$10,000)  | у*             | 1.4085   | 0.0212     | 1.4177   | 0.0783     |
| Learning gain         | g              | _        | _          | 0.0005   | 0.0018     |
| Log-likelihood        |                |          | -2391.5472 |          | -2237.0404 |
| MSE Consumption       |                | l        | 7285.1049  | l        | 6702.3679  |
| MSE Investment        |                |          | 14454.2922 |          | 6304.4836  |
| MSE Inflation         |                |          | 1.2633     |          | 1.1815     |
| MSE Fed. Funds Rate   |                | 1.7499   |            | 1.4896   |            |

#### Endogenous Capital: Forecast Errors

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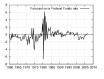
### Consumption

1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 201

#### Rational Expectations



Fed. Funds



Investment



#### Learning with Estimated Initial Conditions

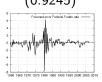
Consumption (0.8977)



Inflation (0.9843)



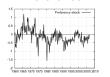
Fed. Funds (0.9245)



Investment (0.6910)



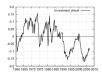
Preference



#### Rational Expectations



#### Investment



Policy Shock



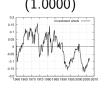
Preference (1.0000)



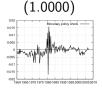
Learning with RE Initial Conditions Technology (1.0000)



Investment (1.0000)



Policy Shock



#### Endogenous Capital: Evolution of Shocks

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Preference



Rational Expectations Technology



Investment

1965 1970 1975 1980 1985 1990 1995 2000 2005 2010



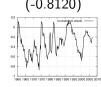
Preference (0.8697)



Learning Without Observable Shocks Technology (0.6534)



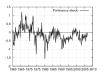
Investment (-0.8120)



Policy Shock (0.9867)



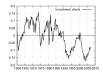
Preference



#### Rational Expectations



#### Investment



Policy Shock



#### Learning with Pre-sample Initial Conditions

Preference (0.7522)



Technology (0.5440)



Investment (-0.6584)



Policy Shock (0.8697)



Rational Expectations

#### **Endogenous Capital: Evolution of Shocks**

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Preference

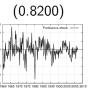
1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 2010

Technology





Preference (0.8200)

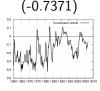


Learning with Estimated Initial Conditions Technology (0.8365)

1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 2010



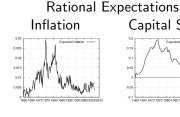
Investment (-0.7371)



Policy Shock (0.8917)



1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 2010











Learning with RE Initial Conditions







#### Rational Expectations



### Capital Stock



Output



#### Consumption



### Inflation



#### Learning Without Using Shocks Capital Stock



Output





1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 2010







Consumption





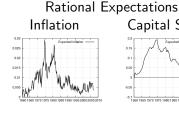
Learning with Pre-sample Initial Conditions







1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 2010









1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 2010





Learning with Estimated Initial Conditions



#### Summary of findings:

- Learning does not better explain data.
- No capital: negligible differences in forecast errors, shocks, and expectations.
- Making shocks unobservable causes MLE to predict more volatile shocks.
- Best fitting models: Learning without observable shocks.
- Worst fitting models: Pre-sample initial conditions.
- Unobservable shocks creates less volatile expectations.
- Learning expectations about capital stock causes opposite predictions for investment shocks.

#### Learning failures:

- Fails to explain persistence.
- Fails to explain Great Inflation / Great Moderation.



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