Empirical Significance of Learning in a New Keynesian Model with Firm-Specific Capital

James Murray
Indiana University

July 20, 2007

Introduction

- Purpose:
  - Estimate the empirical significance of learning.
  - Determine what features of U.S. data constant gain least squares learning can explain.
- Learning suggested to deliver features:
  - Inflation scares.
  - Great moderation.
  - Time varying volatility.
  - Macroeconomic persistence.
- Estimate a NK model with learning and RE by MLE.
  - Estimate with and without endogenous capital.
  - Estimate under alternative initial conditions for beliefs of agents.
- Examine forecast errors, evolution of shocks, and evolution of agents' coefficients.

Utility function:

$$U_0 = E_0^* \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} \xi_t \left(c_t(i) - \eta c_{t-1}(i) \right)^{1-\sigma} - \frac{1}{1+\mu} n_t(i)^{1+\mu} \right]$$

- E_t^*: possibly non-rational expectations operator.
- $c_t(i)$: consumption at time t.
- $n_t(i)$: labor supply at time t.
- ξ_t: common preference shock.
- β: discount factor.
- $\sigma \in (0, \infty)$: inverse of the intertemporal elasticity of substitution.
- $\eta \in [0, 1)$: degree of habit formation.
- $\mu \in (0, \infty)$: inverse of the elasticity of labor supply.

Final good production:

$$y_t = \left[\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

- y_t output of final good, $y_t(i)$ output of intermediate good i.
- $\theta \in (1, \infty)$: elasticity of substitution in production.

Intermediate goods production:

$$\text{Model with capital: } y_t(i) = z_t k_t(i)^{\alpha} \left(\nu^t n_t(i)\right)^{1-\alpha}$$

$$\text{Model without capital: } y_t(i) = z_t n_t(i)$$

- z_t: common technology shock.
- $k_t(i)$: firm-specific capital good.
- ν: gross growth rate of labor productivity.

- Follow Calvo (1983) pricing: fraction $1-\omega$ firms re-optimize their price each period.
- Inflation indexation: Those who cannot re-optimize may adjust according to:

$$\ln(p_t(i)) = \ln(p_{t-1}(i)) + \gamma\pi_{t-1}$$

- With firm-specific capital (Woodford, 2005), leads to Phillips curve:

$$\pi_t = \frac{\gamma}{1+\beta\gamma}\pi_{t-1} + \frac{\beta}{1+\beta\gamma}E_t\pi_{t+1} + \kappa\hat{s}_t$$

- $\hat{s}_t$: average marginal cost in the economy (percentage deviation from steady state).
- κ: function of many parameters.

- Final good is converted to a firm-specific capital good.
- Investment of $I_t(i)$ leads to capital stock next period:

$$k_{t+1}(i) = (1-\delta)k_t(i) + \mu_t I_t(i) - \frac{\phi}{2}\left[\frac{k_{t+1}(i)}{k_t(i)} - 1\right]^2 k_t(i)$$

  - μ_t: common investment technology shock.
  - δ: depreciation rate.
  - ϕ: capital adjustment cost parameter.

- Profit maximizing marginal cost:

$$\hat{s}_t = \frac{\mu+\alpha}{1-\alpha}\hat{y}_t - \frac{\alpha(\mu+1)}{1-\alpha}\hat{k}_t - \hat{\lambda}_t - \frac{\mu+1}{1-\alpha}\hat{z}_t,$$

Market clearing

- Market clearing condition:

$$\text{Model with capital: } y_t = c_t + I_t + g_t$$

$$\text{Model without capital: } y_t = c_t$$

- g_t: Exogenous demand shocks such as changes in government spending and net exports.

- Model *without* capital: interest rate responds to *output gap*:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r)(\psi_\pi \pi_t + \psi_y \tilde{y}_t) + \epsilon_{r,t}$$

  - $\psi_\pi \in (0, \infty)$: feedback on inflation.
  - $\psi_y \in (0, \infty)$: feedback on output.
  - $\rho_r \in (0, 1)$: smoothing parameter.

- Model *with* capital: interest rate responds to the deviation of output from steady state:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r)(\psi_\pi \pi_t + \psi_y \hat{y}_t) + \epsilon_{r,t}$$

Exogenous shocks

- Non-policy shocks (percentage deviations from steady state) are AR(1):
  - Preference shock: $\hat{\xi}_t = \rho_\xi \hat{\xi}_{t-1} + \epsilon_{\xi,t}$
  - Technology shock: $\hat{z}_t = \rho_z \hat{z}_{t-1} + \epsilon_{z,t}$
  - Investment shock: $\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \epsilon_{\mu,t}$
  - Demand shock: $\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g,t}$
- $\epsilon_{\xi,t}$, $\epsilon_{z,t}$, $\epsilon_{\mu,t}$, $\epsilon_{g,t}$, $\epsilon_{r,t}$ are independently normally distributed with mean zero.

Model without capital

- Re-write the model without capital in terms of the output gap.
- Notation:
  - Tilde denotes (percentage) deviation from fully flexible outcome.
  - Superscript f denotes fully flexible outcome.
  - Superscript $*$ denotes steady state value.
- Natural rate of interest:

$$r_t^n = r_t^f - E_t \pi_{t+1}^f$$

$$r_t^n = (1 - \rho_{r^n})\, r^* + \rho_n r_{t-1}^n + \epsilon_t^{r^n}$$

- Consumer first order conditions + market clearing:

$$\tilde{\lambda}_t = E_t \tilde{\lambda}_{t+1} + (r_t - r_t^n - E_t \pi_{t+1})$$

$$\tilde{\lambda}_t = \frac{1}{(1-\beta\eta)(1-\eta)} \left[\beta\eta\sigma E_t \tilde{y}_{t+1} - \sigma(1+\beta\eta^2)\tilde{y}_t + \sigma\eta\tilde{y}_{t-1} \right]$$

- Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left(\mu \tilde{y}_t - \tilde{\lambda}_t \right) + u_t$$

- Cost push shock:

$$u_t = \rho_u u_{t-1} + \epsilon_t^u$$

General Form

- The New Keynesian model has the form:

$$\Omega_0 x_t = \Omega_1 x_{t-1} + \Omega_2 E_t x_{t+1} + \Psi z_t$$

$$z_t = A z_{t-1} + \epsilon_t$$

  - x_t vector of time t variables, observable to agents at following time period.
  - z_t: vector of time t shocks, observable to agents in current period.
- Rational expectations solution:

$$x_t = G x_{t-1} + M z_t$$

- Agents estimate elements of G and M by least squares.
- Agents know coefficients in A.
- X_t: vector of regressors: $X_t' = [1\ x_{t-2}'\ z_{t-1}']$.
- Y_t: vector of dependent variables: $Y_t = x_{t-1}$.
- $\hat{\phi}_t$: least squares estimate of the coefficients in G and M.
- Information at available at time t: x_{t-1}, z_t.
- Evolution of $\hat{\phi}_t$ in recursive form:

$$\hat{\phi}_t = \hat{\phi}_{t-1} + gR_t^{-1}X_{t-1}\left(Y_t - X_{t-1}'\hat{\phi}_{t-1}\right)$$

$$R_t = R_{t-1} + g(X_{t-1}X_{t-1}' - R_{t-1})$$

- where g is the constant learning gain.

- Method: Maximum likelihood estimation.
- Data: Quarterly data for 1960 through 2005.
- Model with no capital:
    - CBO measure of the output gap.
    - Annualized quarterly inflation rate of the GDP deflator.
    - Annualized quarterly Federal funds rate.
- Model with endogenous capital:
    - Real GDP per capita.
    - Real private consumption per capita.
    - Real gross private domestic investment per capita.
    - Annualized quarterly inflation rate of the GDP deflator.
    - Annualized quarterly Federal funds rate.
- Calibrated parameters:
    - Model without capital: $\beta = 0.99$, $\mu = 0$ (perfectly elastic labor supply).
    - Model with capital: $\beta = 0.99$, $\delta = 0.025$, $\alpha = 0.33$, $\nu = 1.00533$.

- $\hat{\phi}_t$ and R_t must be initialized for estimation.
- MSV solution, expected outer product of the state vector under RE.
  - Much of learning theory valid for the neighborhood of the MSV solution.
  - Initial conditions are supported by the micro-foundations.
  - Problem: Learning dynamics smallest when near MSV solution.
- Joint estimation:
  - Problems: As many as 40 additional parameters (endogenous capital estimation).
  - Motivation: illustrates how predictions depend on initial conditions.
- Alternative approach: Use a VAR(1) from pre-sample data
  - Problem: latent variables are regressors - capital stock, structural shocks.

Results Without Capital Accumulation: RE Solution for Initial Conditions

Description	Parameter	RE	Std. dev.	Learning	Std. dev.
Learning gain	g	.	.	0.0104	0.0048
Habit formation	η	0.9953	0.0105	0.9470	0.0214
Inverse elasticity sub.	σ	0.0830	0.0724	0.5057	0.4069
Phillips curve slope	κ	0.0001	0.0001	0.0001	0.0001
Price indexation	γ	0.9686	0.0458	0.9730	0.0172
Interest rate smoothing	ρ_r	0.8449	0.0269	0.8943	0.0178
Policy feedback on output	ψ_y	0.2073	0.0778	0.3346	0.0931
Policy feedback on inflation	ψ_π	1.2812	0.2230	1.5895	0.3079
Persistence nat. int. rate	ρ_n	0.9944	0.0081	0.9016	0.0273
Persistence cost push	ρ_u	1.50E-6	0.0684	0.0000	0.0224
Std. dev. nat. int. rate	s_n	0.0009	0.0005	0.1150	0.1065
Std. dev. cost push	s_u	2.45E-6	3.40E-6	2.49E-6	3.00E-7
Std. dev. policy shock	s_r	5.48E-6	3.40E-6	5.31E-6	3.30E-7
Steady state inflation	π^*	7.9795	2.8442	5.6139	1.0109

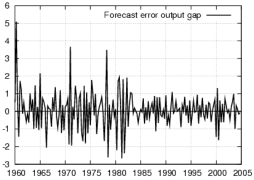
Results Without Capital Accumulation: RE Solution for Initial Conditions

- Learning gain statistically significantly different from zero.
- Habit formation and indexation remain significant sources of persistence.
- Learning model predicts lower steady state level inflation.
- Rest of the parameters are very similar.

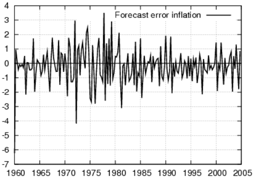
Forecast Errors:
RE and Learning with RE initial conditions

Rational Expectations

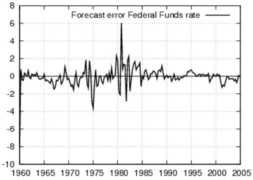
Output gap



Inflation

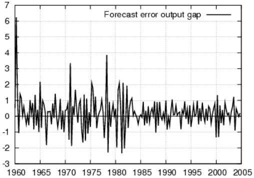


Interest Rate

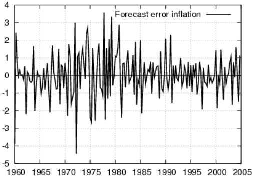


Learning (RE initial conditions)

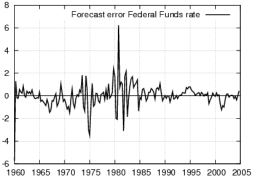
Output gap



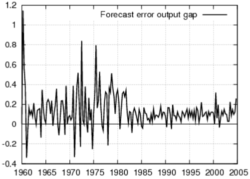
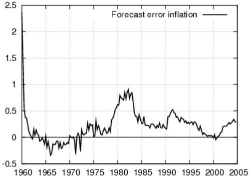
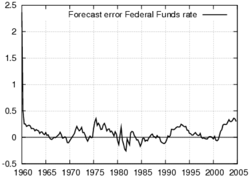
Inflation



Interest Rate



Differences in Forecast Errors

Output gap	Inflation	Interest Rate
		

Correlation of Forecast Errors

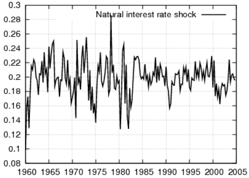
Output gap	Inflation	Interest Rate
0.9858	0.9707	0.9833

Forecast Errors: RE and Learning with RE initial conditions

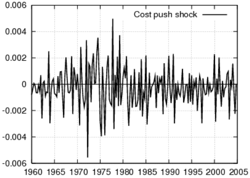
- Forecast errors essentially the same.
- Forecast errors for all variables are largest in 1970s.
    - Neither model explains monetary policy in 1979-1982 period.
    - Neither model can explain changes in volatility.
    - Learning model actually makes slightly larger forecast errors.

Rational Expectations

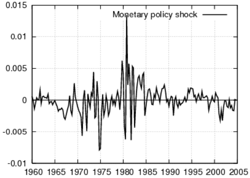
Natural Rate Shock



Cost Push Shock

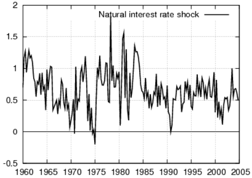


Policy Shock

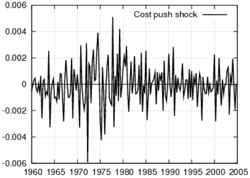


Learning (RE initial conditions)

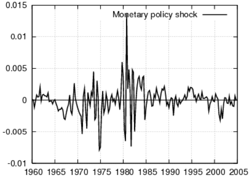
Natural Rate Shock



Cost Push Shock

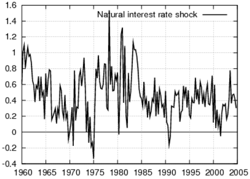


Policy Shock

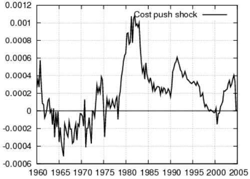


Difference in Smoothed Shocks

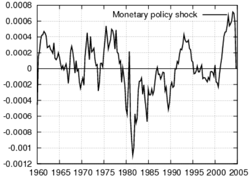
Natural Rate Shock



Cost Push Shock



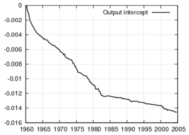
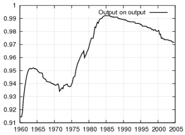
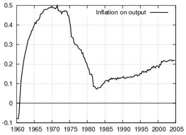
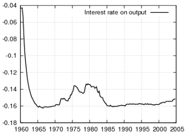
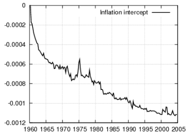
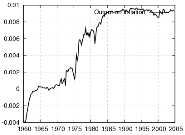
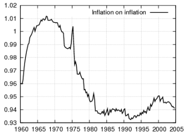
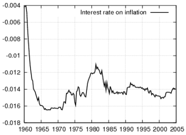
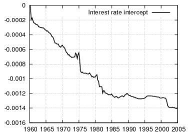
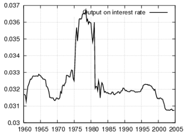
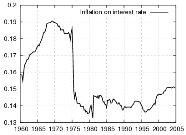
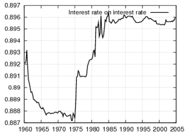
Policy Shock



Correlation of Smoothed Shocks

Natural Rate Shock	Cost Push Shock	Policy Shock
0.7150	0.9799	0.9907

Learning (RE initial conditions): Evolution of Coefficients

Output: Intercept	Output	Inflation	Fed Funds Rate
			
Inflation: Intercept	Output	Inflation	Fed Funds Rate
			
Fed Funds: Intercept	Output	Inflation	Fed Funds Rate
			

Learning (RE initial conditions): Evolution of Coefficients

- Perceived persistence in output starts low, increases over the 1970s and early 1980s.
- Perceived impact of past inflation on output opposite sign of RE solution.
- Perceived persistence of inflation falls during 1970s.
- Perceived monetary policy during late 1970s:
  - Believed to respond more to output, less to inflation.
  - Permanent increase in perceived interest rate persistence.
  - Spike in the predictive power of interest rate.

Results Without Capital Accumulation: Estimated Initial Conditions

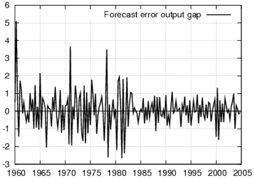
Description	Parameter	Estimate	Std. dev.
Learning gain	g	0.0116	0.0072
Habit formation	η	0.0436	0.0404
Inverse elasticity sub.	σ	2.4018	2.8262
Phillips curve slope	κ	0.0001	0.0003
Price indexation	γ	0.9940	1.1123
Interest rate smoothing	ρ_r	0.8940	0.0279
Policy feedback on output	ψ_y	0.2973	0.1329
Policy feedback on inflation	ψ_π	1.0576	0.1989
Persistence nat. int. rate	ρ_n	0.9987	1.00E-8
Persistence cost push	ρ_u	0.0002	0.0823
Std. dev. nat. int. rate	s_n	0.0007	0.0080
Std. dev. cost push	s_u	1.5E-05	6.06E-06
Std. dev. policy shock	s_r	5.4E-06	4.6E-07
Steady state inflation	π^*	1.6478	0.8418

Results Without Capital Accumulation: Estimated Initial Conditions

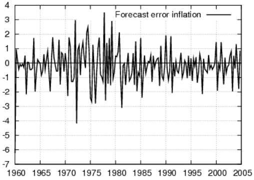
- Not shown: estimates of initial conditions.
  - Many estimates have large standard deviations.
- Learning gain statistically significantly different from zero.
- Degree of habit formation is close to zero.
- Point estimate of price indexation still high, but insignificant.
- Point estimate of steady state inflation is low.

Rational Expectations

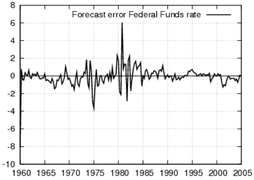
Output gap



Inflation

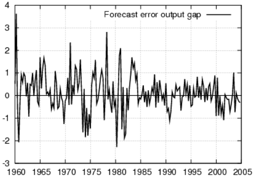


Interest Rate

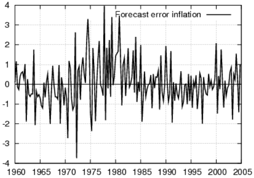


Learning (Estimated initial conditions)

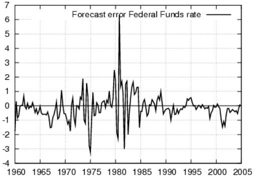
Output gap



Inflation



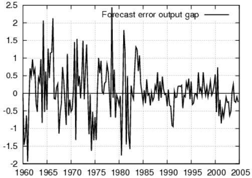
Interest Rate



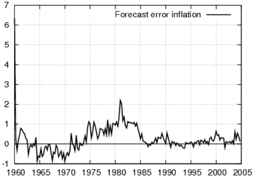
Forecast Errors: RE and Learning with Estimated Initial Conditions

Differences in Forecast Errors

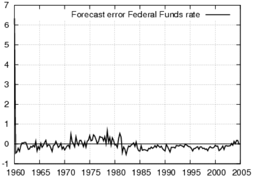
Output gap



Inflation



Interest Rate

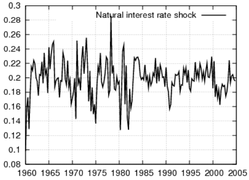


Correlation of Forecast Errors

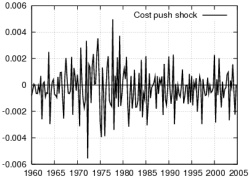
Output gap	Inflation	Interest Rate
0.7333	0.8472	0.8861

Rational Expectations

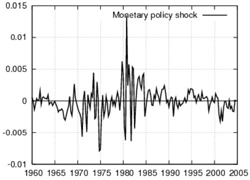
Natural Rate Shock



Cost Push Shock

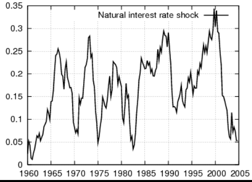


Policy Shock

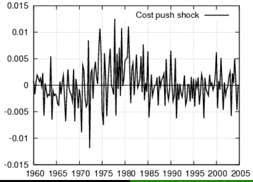


Learning (Estimated initial conditions)

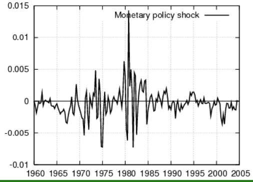
Natural Rate Shock



Cost Push Shock

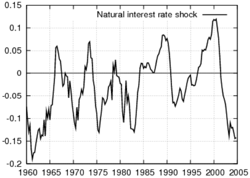


Policy Shock

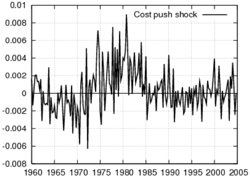


Difference in Smoothed Shocks

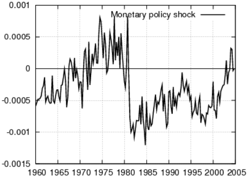
Natural Rate Shock



Cost Push Shock



Policy Shock

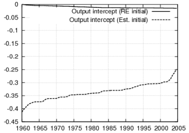


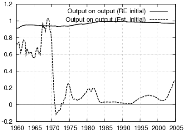
Correlation of Smoothed Shocks

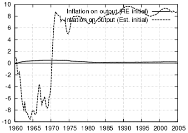
Natural Rate Shock	Cost Push Shock	Policy Shock
0.7150	0.9799	0.9907

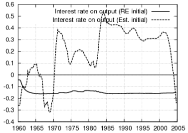
Estimated Initial Conditions: Evolution of Coefficients

Output: Intercept | Output | Inflation | Fed Funds Rate

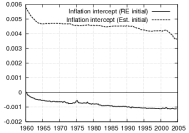


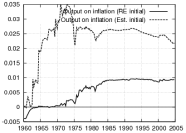


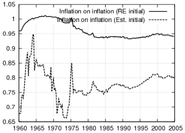


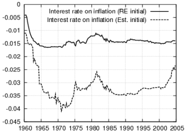


Inflation: Intercept | Output | Inflation | Fed Funds Rate

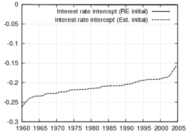


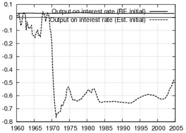


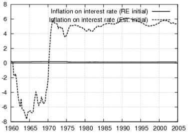


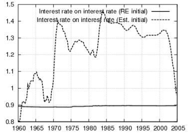


Fed Funds: Intercept | Output | Inflation | Fed Funds Rate









	RE	Learning (RE init.)	Learning (Est. init.)
MSE Output	1.0920	1.1041	0.7303
MSE Inflation	1.6613	1.5781	1.4175
MSE Fed Funds	1.2594	1.0879	0.9435
Log-likelihood	-303.8	-291.9	-241.6

Results With Capital Accumulation: RE Solution for Initial Conditions

Description	Parameter	RE	Std. dev.	Learning	Std. dev.
Learning gain	g	.	.	3.51E-6	7.01E-6
Habit formation	η	0.0855	0.0846	0.1078	0.0805
Inverse elasticity sub.	σ	16.5056	13.8189	16.8358	14.5459
Capital / output ratio	k_y	4.4799	0.2129	4.5061	0.2129
Consumption / output ratio	c_y	0.7141	0.0547	0.7234	0.0516
Inverse elasticity labor	μ	1.53E-5	0.3944	1.00E-5	0.4396
Capital adjustment cost	ϕ	27.9001	5.4801	27.4460	5.9845
Phillips curve slope	κ	0.0116	0.0034	0.0119	0.0037
Price indexation	γ	0.9999	0.0000	0.9999	0.0000
Interest rate smoothing	ρ_r	0.8147	0.0196	0.8103	0.0203
Policy feedback on output	ψ_y	0.0326	0.0116	0.0356	0.0122
Policy feedback on inflation	ψ_π	1.5401	0.1504	1.4735	0.1405
Persistence in tech. shock	ρ_z	1.40E-5	0.0888	1.00E-5	0.0902
Persistence in pref. shock	ρ_ξ	0.9793	0.0176	0.9800	0.0167
Persistence in inv. shock	ρ_μ	0.9295	0.0235	0.9282	0.0253
Persistence in AD shock	ρ_g	0.9989	0.0042	0.9993	0.0035
Std. dev. tech. shock	s_z	0.2399	0.1262	0.2326	0.1325
Std. dev. inv. shock	s_μ	0.1996	0.1219	0.1900	0.1251
Std. dev. pref. shock	s_ξ	0.1722	0.1447	0.1857	0.1626
Std. dev. policy shock	s_r	0.0025	0.0001	0.0024	0.0001
Std. dev. AD shock	s_g	0.0222	0.0091	0.0238	0.0098
Steady state inflation	π^*	8.6918	2.3997	10.3678	3.0793
Steady state output	y^*	1.2521	0.1033	1.2355	0.0984

Results With Capital Accumulation: Estimated Initial Conditions

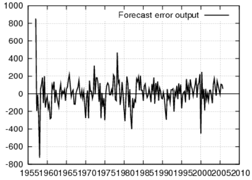
Description	Parameter	Estimate	Std. dev.
Learning gain	g	0.0288	0.0077
Habit formation	η	0.0772	0.1031
Inverse elasticity sub.	σ	16.6941	14.4002
Capital / output ratio	k_y	4.5086	1.5187
Consumption / output ratio	c_y	0.7213	0.0842
Inverse elasticity labor	μ	0.0132	1.3992
Capital adjustment cost	ϕ	28.1764	38.5551
Phillips curve slope	κ	0.0120	0.0154
Price indexation	γ	0.9999	0.0000
Interest rate smoothing	ρ_r	0.8776	0.0351
Policy feedback on output	ψ_y	0.0374	0.0661
Policy feedback on inflation	ψ_π	1.5654	0.4185
Persistence in tech. shock	ρ_z	0.0000	0.0124
Persistence in pref. shock	ρ_ξ	0.9611	0.0234
Persistence in inv. shock	ρ_μ	0.9364	0.0405
Persistence in AD shock	ρ_g	0.9999	0.0001
Std. dev. tech. shock	s_z	0.2569	0.5566
Std. dev. inv. shock	s_μ	0.1830	0.1692
Std. dev. pref. shock	s_ξ	0.1707	0.1461
Std. dev. policy shock	s_r	0.0024	0.0001
Std. dev. AD shock	s_g	0.0255	0.0154
Steady state inflation	π^*	9.5185	6.1255
Steady state output	y^*	1.2396	0.1424

- With RE initial conditions:
    - Learning gain essentially zero.
    - All parameter estimates are very similar.
    - Habit formation close to zero under *both* RE and Learning.
    - Inflation indexation very close to one.
- With Estimated initial conditions.
    - Point estimates similar as with RE.
    - Not shown: standard deviations for some initial conditions huge.
    - Learning gain greater than zero.
    - Inflation indexation remains significant and close to one.
    - Standard deviations of some structural parameters increased:
        - Capital adjustment cost.
        - Policy feedback on inflation.
        - Steady state inflation.

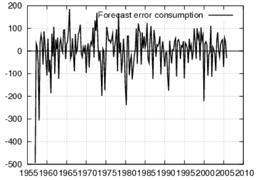
Forecast Errors: RE and Learning with RE Initial Conditions

Rational Expectations

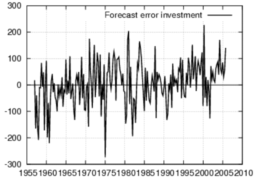
Output



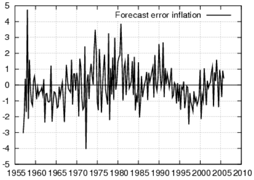
Consumption



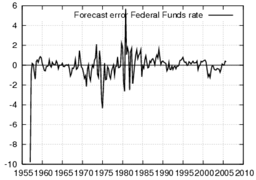
Investment



Inflation



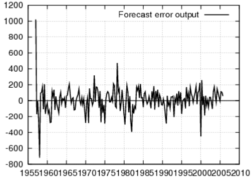
Interest Rate



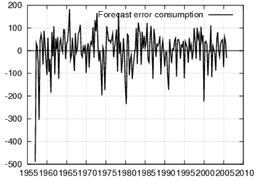
Forecast Errors:
RE and Learning with RE Initial Conditions

Learning (RE initial conditions)

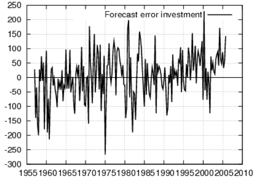
Output



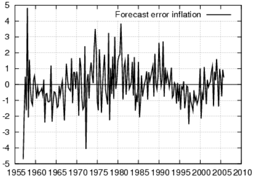
Consumption



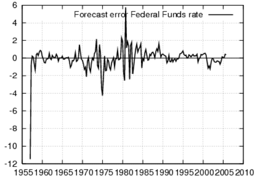
Investment



Inflation



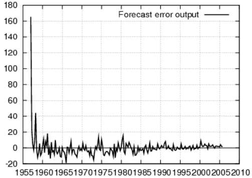
Interest Rate



Forecast Errors:
RE and Learning with RE Initial Conditions

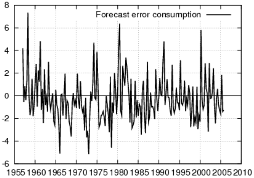
Differences in Forecast Errors (Correlations)

Output



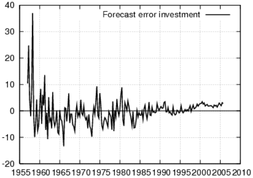
(0.9966)

Consumption



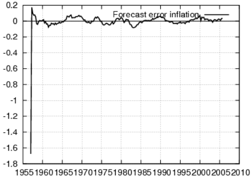
(0.9997)

Investment



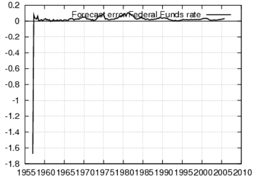
(0.9982)

Inflation



(0.9956)

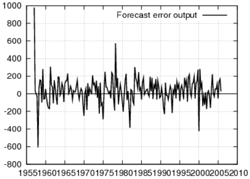
Interest Rate



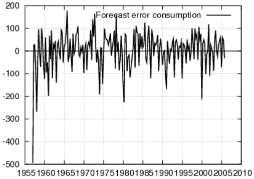
(0.9968)

Learning (Estimated initial conditions)

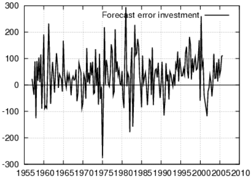
Output



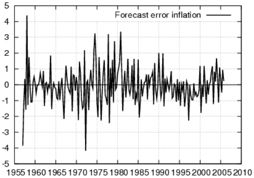
Consumption



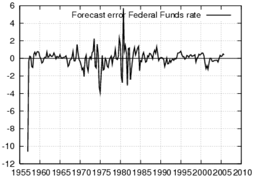
Investment



Inflation



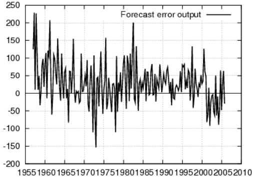
Interest Rate



Forecast Errors: RE and Learning with Estimated Initial Conditions

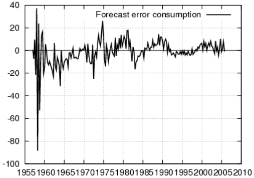
Differences in Forecast Errors (Correlations)

Output



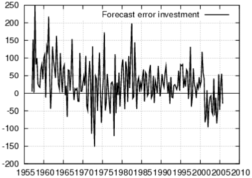
(0.9213)

Consumption



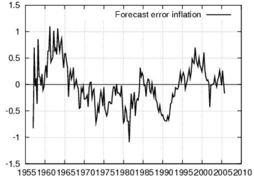
(0.9906)

Investment



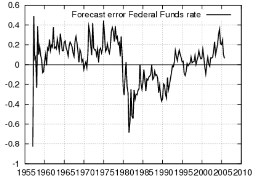
(0.7294)

Inflation



(0.9527)

Interest Rate



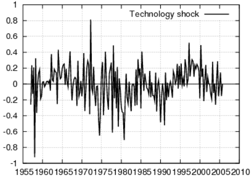
(0.9847)

- Forecast errors for inflation and federal funds rate are clustered in 1970s.
- Volatility of forecast errors is not clustered for consumption and investment.
- Learning model with estimated initial conditions:
    - Out performs the RE model for predicting inflation during 1970s.
    - Under performs the RE model for predicting output during 1980s.
- Even with estimated initial conditions: models have similar performance.

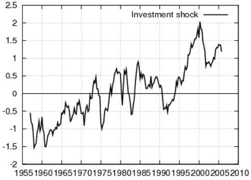
Smoothed Shocks: RE and Learning with RE Initial Conditions

Rational Expectations

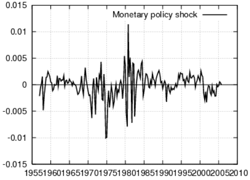
Technology shock



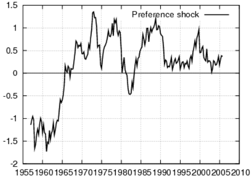
Investment shock



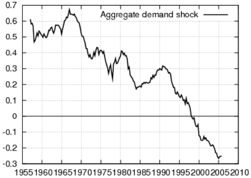
Monetary policy shock



Preference shock



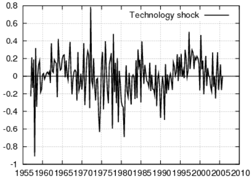
Aggregate demand shock



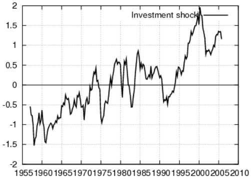
Smoothed Shocks: RE and Learning with RE Initial Conditions

Learning (RE initial conditions)

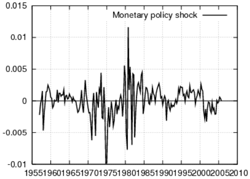
Technology shock



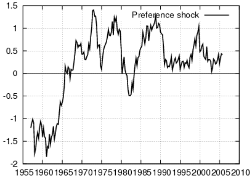
Investment shock



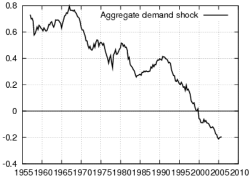
Monetary policy shock



Preference shock



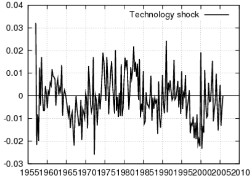
Aggregate demand shock



Smoothed Shocks: RE and Learning with RE Initial Conditions

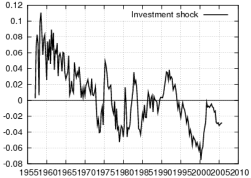
Differences in smoothed shocks (Correlations)

Technology shock



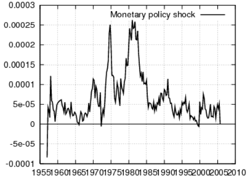
(0.9996)

Investment shock



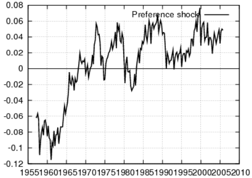
(0.9998)

Monetary policy shock



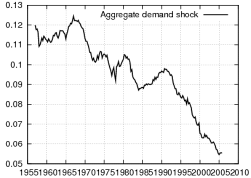
(0.9997)

Preference shock



(0.9998)

Aggregate demand shock

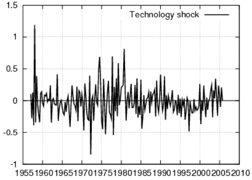


(0.9999)

Smoothed Shocks: RE and Learning with Estimated Initial Conditions

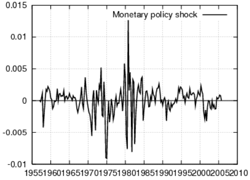
Learning (Estimated initial conditions)

Technology shock

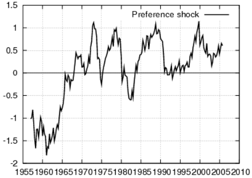


Investment shock

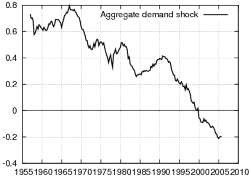
Monetary policy shock



Preference shock



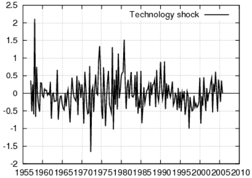
Aggregate demand shock



Smoothed Shocks:
RE and Learning with Estimated Initial Conditions

Differences in smoothed shocks (Correlations)

Technology shock

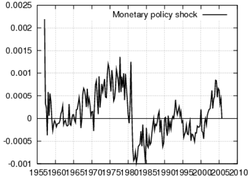


(-0.9528)

Investment shock

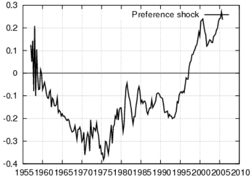
(0.5308)

Monetary policy shock



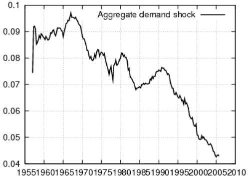
(0.9794)

Preference shock



(0.9760)

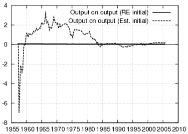
Aggregate demand shock

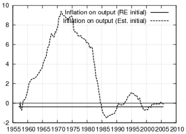


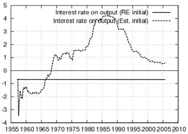
(0.9999)

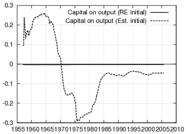
Comparison of Coefficients

Output: Output | Inflation | Fed Funds Rate | Capital

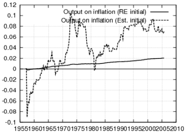


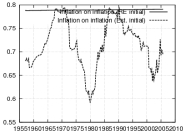


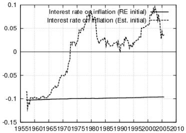


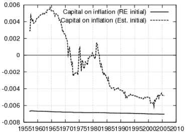


Inflation: Output | Inflation | Fed Funds Rate | Capital

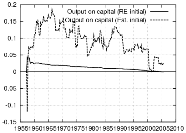


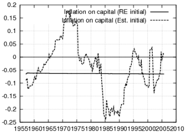


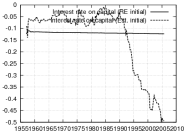


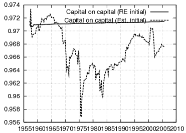


Capital: Output | Inflation | Fed Funds Rate | Capital









	RE	Learning (RE init.)	Learning (Est. init.
MSE Output	23558.2	24868.6	24652.0
MSE Consumption	7129.3	7105.1	6883.4
MSE Investment	7118.3	6930.3	7656.7
MSE Inflation	1.6727	1.7508	1.3893
MSE Fed Funds	1.4116	1.5910	1.4483
Log-likelihood	-3042.2	-3041.6	-241.6

Conclusion

- Estimated a NK model without capital under RE and Learning.
  - RE initial conditions: fails to explain persistence, volatility clustering.
  - Estimated initial conditions: some persistence explained, not volatility clustering.
- Estimated a NK model with endogenous capital under RE and learning.
  - Using RE initial conditions: No difference, capital explained some persistence and volatility.
  - Using Estimated Initial conditions:
    - Can get very different paths of shocks and coefficients.
    - Model fit not greatly improved.
    - Still does not explain volatility clustering.
- Conclusion: Expectations framework with larger deviations from RE likely needed to explain data.