

Empirical Significance of Learning in a New Keynesian Model with Firm-Specific Capital

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- Purpose:
 - Estimate the empirical significance of learning.
 - Determine what features of U.S. data (if any) learning can explain.
- Learning may deliver features:
 - Orphanides and Williams (2005): “Inflation scares”.
 - Milani (2005): Persistence.
 - Volatility persistence.
 - Bullard and Singh (2007), Primiceri (2005): Great Moderation.
- Estimate a NK model with learning and RE by MLE.
- Examine forecast errors, evolution of shocks, and evolution of expectations.

Utility function:

$$U_0 = E_0^* \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} \xi_t (c_t(i) - \eta c_{t-1}(i))^{1-\sigma} - \frac{1}{1+\mu} n_t(i)^{1+\mu} \right]$$

- E_t^* : possibly non-rational expectations operator.
- $c_t(i)$: consumption at time t .
- $n_t(i)$: labor supply at time t .
- ξ_t : common preference shock.
- β : discount factor.
- $\sigma \in (0, \infty)$: related to the intertemporal elasticity of substitution.
- $\eta \in [0, 1)$: degree of habit formation.

Final good production:

$$y_t = \left[\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

- y_t output of final good, $y_t(i)$ output of intermediate good i .
- $\theta \in (1, \infty)$: elasticity of substitution in production.

Intermediate goods production:

$$y_t(i) = z_t k_t(i)^\alpha n_t(i)^{1-\alpha}$$

- z_t : common technology shock.
- $k_t(i)$: firm-specific capital good.

- Follow Calvo (1983) pricing: fraction $1 - \omega$ firms re-optimize their price each period.
- Inflation indexation: Those who cannot re-optimize may adjust according to:

$$p_t(i) = p_{t-1}(i) + \gamma \pi_{t-1}$$

- Even with endogenous capital (Woodford, 2005), leads to Phillips curve:

$$\pi_t = \frac{\gamma}{1 + \beta\gamma} \pi_{t-1} + \frac{\beta}{1 + \beta\gamma} E_t^* \pi_{t+1} + \kappa \hat{S}_t$$

- \hat{S}_t : average marginal cost in the economy (percentage deviation from steady state).
- κ : function of many parameters.

- Final good is converted to a firm-specific capital good.
- Investment of $I_t(i)$ leads to capital stock next period:

$$k_{t+1}(i) = (1 - \delta)k_t(i) + \mu_t I_t(i) - \frac{\phi}{2} \left[\frac{k_{t+1}(i)}{k_t(i)} - 1 \right]^2 k_t(i)$$

- μ_t : common investment technology shock.
- δ : depreciation rate.
- ϕ : capital adjustment cost parameter.

- Market clearing condition: $y_t = c_t + I_t$
- Monetary policy: $\hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r) (\psi_\pi \pi_t + \psi_y \hat{y}_t) + \epsilon_{r,t}$
 - $\psi_\pi \in (0, \infty)$: feedback on inflation.
 - $\psi_y \in (0, \infty)$: feedback on output.
 - $\rho_r \in (0, 1)$: smoothing parameter.
- Non-policy shocks (percentage deviations from steady state) are AR(1):
 - Preference shock: $\hat{\xi}_t = \rho_\xi \hat{\xi}_{t-1} + \epsilon_{\xi,t}$
 - Technology shock: $\hat{z}_t = \rho_z \hat{z}_{t-1} + \epsilon_{z,t}$
 - Investment shock: $\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \epsilon_{\mu,t}$

Suppose a DSGE model of the form:

$$\Omega_0 x_t = \Omega_1 x_{t-1} + \Omega_2 E_t^* x_{t+1} + \Psi \epsilon_t$$

- x_t vector of time t variables, all observable to agents.
- E_t^* : possibly non-rational expectations operator.

Rational expectations solution implies:

$$E_t x_{t+1} = G x_t$$

- Agents know the form of this solution, but estimate elements of G by least squares.
- Use as explanatory variables past observations of x_t^k .
- x_t^k includes a constant and all variables in x_t where the associated column in G is non-zero.

- Let G_t^k include non-zero columns of G and a constant.
- Ordinary least squares estimate for G^k at time t :

$$(\hat{G}_t^k)' = \left(\frac{1}{t-1} \sum_{\tau=1}^{t-1} x_{\tau-1}^k x_{\tau-1}^{k'} \right)^{-1} \left(\frac{1}{t-1} \sum_{\tau=1}^{t-1} x_{\tau-1}^k x_{\tau}' \right)$$

- Least squares forecast:

$$E_t^* x_{t+1} = \hat{g}_{0,t} + \hat{G}_t E_t^* x_t = (I + \hat{G}_t) \hat{g}_{0,t} + \hat{G}_t^2 x_{t-1} \quad (1)$$

- Evolution of \hat{G}_t^k in recursive form:

$$\hat{G}_t^k = \hat{G}_{t-1}^k + g(x_{t-1} - \hat{G}_{t-1}^k x_{t-2}^k) x_{t-2}^{k'} R_t^{-1} \quad (2)$$

$$R_t = R_{t-1} + g(x_{t-2}^k x_{t-2}^{k'} - R_{t-1}) \quad (3)$$

- where $g = 1/(t-1)$ is the learning gain.

- Ordinary least squares \rightarrow learning dynamics disappear as t grows.
- Constant gain: assumes g is constant.
- With a constant gain, learning dynamics persist in the long run.
- Dynamics of expectations depend on the size of the constant learning gain.
- Appropriate initial condition + ($g = 0$) \rightarrow RE.
- Standard statistical test for $g = 0$ can conclude a rejection failure to RE.

Estimate by MLE using Kalman filter (Hamilton, 1994).

- State equation:

$$x_t = \Omega_0^{-1} \Omega_2 \left(I + \hat{G}_t \right) \hat{g}_{0,t} + \Omega_0^{-1} \left(\Omega_1 + \Omega_2 \hat{G}_t^2 \right) x_{t-1} + \Omega_0^{-1} \Psi \epsilon_t. \quad (4)$$

- Observation equations:

$$\begin{aligned} GDP_t^g &= 100 (\hat{y}_t - \hat{y}_{t-1}) \\ I_t^g &= 100 (\hat{l}_t - \hat{l}_{t-1}) \\ INF_t &= \pi^* + 400 \pi_t \\ FF_t &= r^* + 400 \hat{r}_t \end{aligned}$$

- Initialize Kalman filter and learning algorithm with the MSV solution under RE.

Description	Parameter	Learning	RE
Learning gain	g	0.017225***	—
Discount factor	β	0.994528***	0.994893***
Habit formation	η	0.269967**	0.213315
Inverse elasticity sub.	σ	16.264823	17.589033
Elasticity of sub. production	θ	11.459122	10.030577
Inverse elasticity labor supply	μ	2.509288	1.791766
Calvo parameter	ω	0.715756	0.713950
Inflation indexation	γ	0.441247***	0.958006***
MP interest rate smoothing	ρ_r	0.866560***	0.823025***
MP feedback on output	ψ_y	0.123166**	0.055319*
MP feedback on inflation	ψ_π	0.995049***	1.067211***
Pref. shock persistence	ρ_ξ	0.931082***	0.962095***
Tech. shock persistence	ρ_z	0.000010	0.000010
Steady state inflation	π^*	2.814514***	3.296221**
Std. dev. technology shock	σ_z	0.301741*	0.340167
Std. dev. preference shock	σ_ξ	0.280402*	0.254504
Std. dev. interest rate shock	σ_r	0.002283***	0.002344***

* Significantly different from zero at the 10% level.

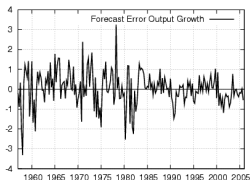
** Significantly different from zero at the 5% level.

*** Significantly different from zero at the 1% level.

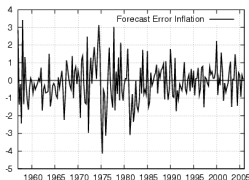
- Learning statistically significant.
- Learning leads to lower degree of inflation indexation.
- Habit formation still significant source of persistence.
- Very similar estimates for the degree of price flexibility.
- Possible reasons for differences with Milani (2005):
 - MLE vs. Bayesian methods.
 - Different initial condition for recursive learning process.
 - Different data: Growth rate of real GDP vs. CBO measure of output gap.

Learning Inflation

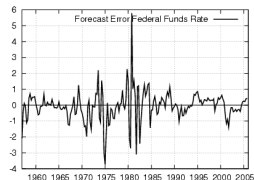
Output



Inflation

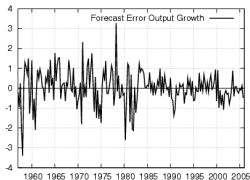


Interest Rate

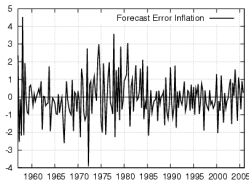


Rational Expectations

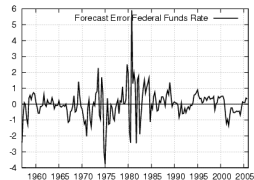
Output



Inflation



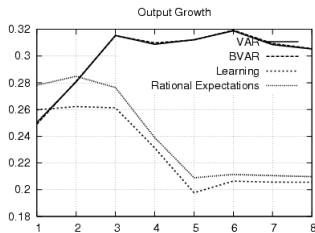
Interest Rate



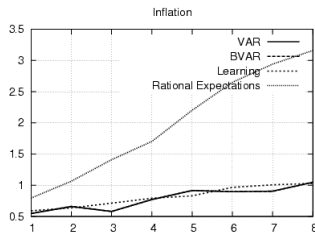
- Forecast errors are very similar for Learning and RE.
- Forecast errors are more volatile in 1970s.
- Huge forecast errors for federal funds rate in late 70s, early 80s.
- Learning appears not to be explaining U.S. dynamics better than RE.
- Both fail to explain high volatility in 70s with low volatility after mid 80s.

- Estimate the model through 1989:Q4.
- Use estimated parameters to forecast 1990:Q1 - 2005:Q4.
- For each quarter, forecast eight periods ahead.
- Given forecasts of each horizon, compute MSE.
- Do the same for a VAR(4) and Litterman (1986) BVAR(4).

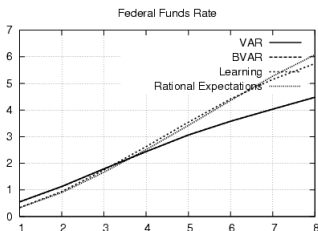
Output



Inflation

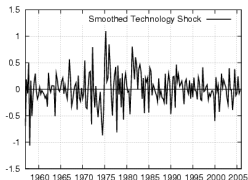


Interest Rate

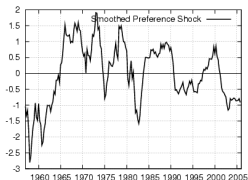


Learning

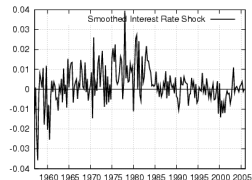
Technology Shock



Preference Shock

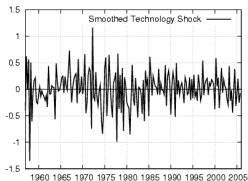


Interest Rate Shock

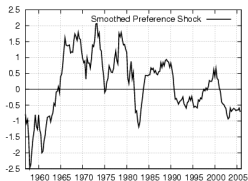


Rational Expectations

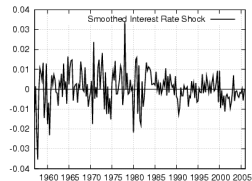
Technology Shock



Preference Shock

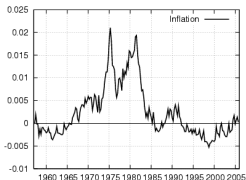
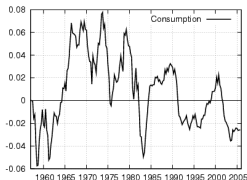


Interest Rate Shock

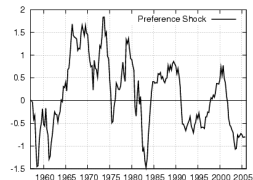


Learning Inflation

Consumption

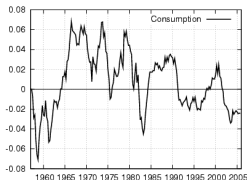


Preference shock

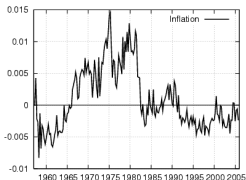


Rational Expectations

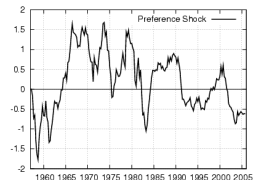
Consumption



Inflation



Preference shock

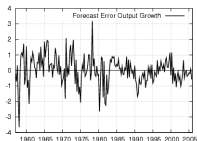


Description	Parameter	Learning	RE
Learning gain	g	0.024327**	—
Discount factor	β	0.993366***	0.992763***
Habit formation	η	0.281563**	0.286030*
Inverse elasticity sub.	σ	18.558950	16.328695
Elasticity of sub. production	θ	13.122359	7.317025
Inverse elasticity labor supply	μ	3.329474	6.219549
Capital share of income	α	0.174750	0.186636
Depreciation rate	δ	0.163489	0.299872
Cost of adjusting capital	ϕ	13.455016	11.558055
Calvo parameter	ω	0.658099	0.774919
Inflation indexation	γ	0.404221***	0.760702***
MP interest rate smoothing	ρ_r	0.869496***	0.859279***
MP feedback on output	ψ_y	0.064696*	0.128857***
MP feedback on inflation	ψ_π	0.992672***	0.967512***
Pref. shock persistence	ρ_ξ	0.984689***	0.980813***
Tech. shock persistence	ρ_z	0.012960	0.000010
Inv. shock persistence	ρ_μ	0.804935***	0.824007***
Steady state inflation	π^*	3.570552***	4.073905***
Std. dev. technology shock	σ_z	0.229175**	0.400000**
Std. dev. investment shock	σ_μ	0.060246***	0.052502***
Std. dev. preference shock	σ_ξ	0.231848*	0.214337*
Std. dev. interest rate shock	σ_r	0.002291***	0.002286***

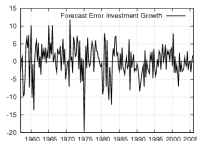
- Learning remains statistically significant.
- Including capital leads to a lower degree of inflation indexation.
- Learning further lowers the degree of inflation indexation.
- Habit formation still significant source of persistence.
- Learning leads to lower estimates for the degree of price flexibility.
- Learning leads to lower estimates for the variance of the technology shock.

Learning

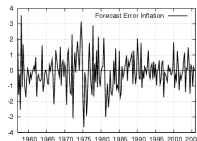
Output



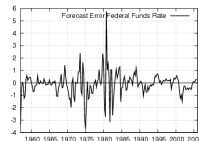
Investment



Inflation

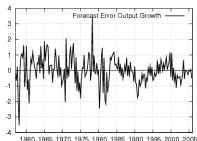


Interest Rate



Rational Expectations

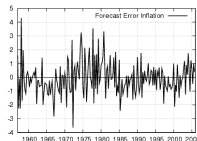
Output



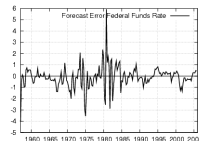
Investment



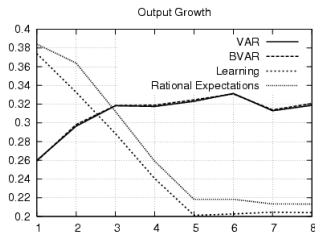
Inflation



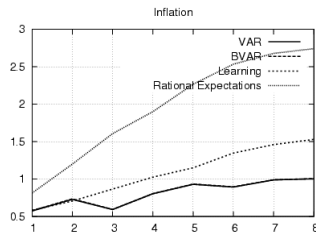
Interest Rate



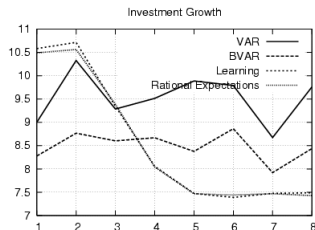
Output



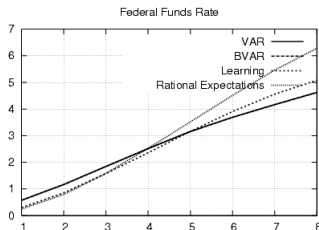
Inflation



Investment

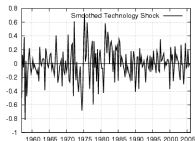


Interest Rate

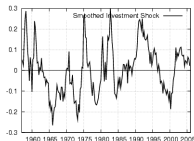


Learning

Technology Shock



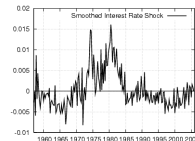
Investment Shock



Preference Shock

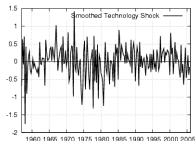


Interest Rate Shock

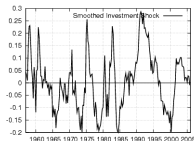


Rational Expectations

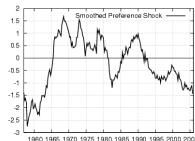
Technology Shock



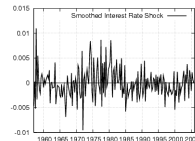
Investment Shock



Preference Shock

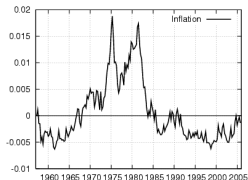
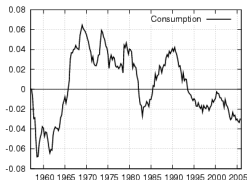


Interest Rate Shock

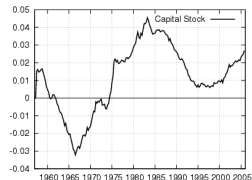


Learning Inflation

Consumption

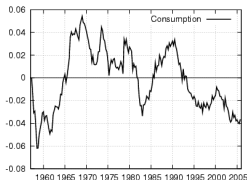


Capital stock

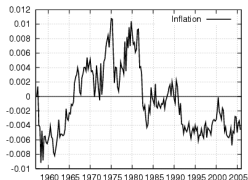


Rational Expectations

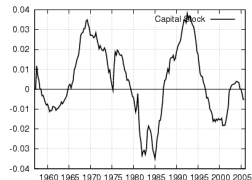
Consumption



Inflation



Capital stock



- Learning successes:
 - Lower estimates for inflation indexation.
 - Lower estimates for the Calvo parameter (using capital).
 - Lower estimates for variance of the technology shock (using capital).
 - Some evidence of inflation scares.
 - Better performance out-of-sample, especially for inflation.
- Learning failures:
 - Very similar to RE in explaining the data.
 - 1970s inflation, monetary policy.