Empirical Significance of Learning in a New Keynesian Model with Firm-Specific Capital

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February 2, 2007

- Least squares learning: no rational expectations, instead agents form all expectations using least squares estimates.
- Orphanides and Williams (2005): inflation scares.
- Primiceri (2005): High inflation (1970s) followed by dis-inflation (1980s).
- Milani (2005): Learning, not habit formation or inflation indexation, explains persistence.
- Mark (2005): Dollar / DM real exchange rate depreciation (1970s), appreciation (early 1980s), depreciation (late 1980s).

Motivation for Estimation

- All these results depend on the size of the learning gain.
- Milani (2005) finds a statistically significant learning gain with a 3 equation NKPC model.
- Learning provides a better fit to the data:
  - Allows for greater persistence.
  - Allows for time varying volatility.
- Is learning significant because the model is too simple?
- Contribution: estimate with endogenous firm-specific capital (Woodford, 2005).

Motivation for Endogenous Capital

- Introduces an additional source of persistence.
- Affects how agents make expectations.
- Evolution of output and inflation depend on investment decisions.
- Additional data for estimation: investment.

- Goals:
  - Find out whether adding capital is important.
  - Find out if learning is statistically significant with more general model.
  - Find out what dynamics of U.S. data learning helps explain.
- Method:
  - Examine impulse response functions with/without capital.
  - Estimate NK model with capital and learning by MLE.
  - Examine the forecast errors.
- Findings:
  - Capital makes a difference.
  - Learning remains statistically significant.
  - Learning helps explain volatility.
  - Learning + capital best able to explain U.S. experience.

1. Overview Woodford (2005) model.
2. Overview of learning in DSGE model.
3. Present impulse responses.
4. Present estimation results.
5. Conclude.

- Continuum of consumer types, each type supplies a unique type of labor.
- Consumers exhibit habit formation.
- One final good produced in a perfectly competitive market.
- Final good produced with a continuum of intermediate goods.
- Intermediate goods each hire specific labor, require a firm-specific capital good.
- Intermediate goods firms are monopolistically competitive.
- Calvo (1983) pricing for intermediate goods.

Utility function:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} \xi_t \left(c_t - \eta c_{t-1} \right)^{1-\sigma} - \frac{1}{1+\mu} n_t(i)^{1+\mu} \right]$$

- c_t: consumption at time t.
- $n_t(i)$: labor supply at time t.
- ξ_t: common preference shock.
- β: discount factor.
- $\sigma \in (0, \infty)$: related to the intertemporal elasticity of substitution.
- $\eta \in [0, 1)$: degree of habit formation.

Utility maximization + log-linearization:

$$\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + \hat{r}_t - E_t \pi_{t+1}$$

$$\hat{\lambda}_t = \frac{1}{(1-\beta\eta)(1-\eta)} \left[\beta\eta\sigma E_t \hat{c}_{t+1} - \sigma(1+\beta\eta^2)\hat{c}_t + \sigma\eta\hat{c}_{t-1} \right] + \hat{\xi}_t$$

- Hats denote percentage deviations from steady state.
- λ_t is Lagrange multiplier = marginal utility of income.
- Note: habit formation adds a "mechanical" source of persistence.

Final good production:

$$y_t = \left[\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

- y_t output of final good, $y_t(i)$ output of intermediate good i.
- $\theta \in (1, \infty)$: elasticity of substitution in production.

Intermediate goods production:

$$y_t(i) = z_t k_t(i)^{\alpha} n_t(i)^{1-\alpha}$$

- z_t: common technology shock.
- $k_t(i)$: firm-specific capital good.

- Follow Calvo (1983) pricing.
- Even with endogenous capital (Woodford, 2005), leads to standard looking Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{s}_t$$

- s_t: average marginal cost in the economy.
- κ: function of many parameters.
  - No closed form expression when there is endogenous capital.
  - Increasing with degree of price flexibility.

- Final good is converted to a firm-specific capital good.
- Investment of $I_t(i)$ leads to capital stock next period:

$$k_{t+1}(i) = (1-\delta)k_t(i) + \mu_t I_t(i) - \frac{\phi}{2}\left[\frac{k_{t+1}(i)}{k_t(i)} - 1\right]^2 k_t(i)$$

- μ_t: common investment technology shock.
- δ: depreciation rate.
- ϕ: capital adjustment cost parameter.

- Optimal demand for labor and capital:

$$\hat{s}_t = \frac{\mu+\alpha}{1-\alpha}\hat{y}_t - \frac{\alpha(\mu+1)}{1-\alpha}\hat{k}_t - \hat{\lambda}_t - \frac{\mu+1}{1-\alpha}\hat{z}_t$$

- Optimal investment:

$$\hat{\lambda}_t + \phi\left(\hat{k}_{t+1} - \hat{k}_t\right) = \beta(1-\delta)E_t\hat{\lambda}_{t+1}$$

$$+\left(\frac{1-\beta\left(1-\delta\right)}{1-\alpha}\right)\left[(\mu+1)E_t\hat{y}_{t+1} - (1+\alpha\mu)\hat{k}_{t+1}\right]$$

$$+\beta\phi\left(E_t\hat{k}_{t+2} - \hat{k}_{t+1}\right) + \hat{\mu}_t$$

Log-linear market clearing condition:

$$\hat{y}_t = c_y \hat{c}_t + \delta k_y \hat{I}_t$$

- c_y: steady state consumption / output ratio.
- k_y: steady state capital / output ratio.

Monetary policy:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r)\left(\psi_\pi \pi_t + \psi_y \hat{y}_t\right) + \epsilon_{r,t}$$

- $\psi_\pi \in (0, \infty)$: feedback on inflation.
- $\psi_y \in (0, \infty)$: feedback on output.
- $\rho_r \in (0, 1)$: smoothing parameter.

- There are four structural shocks in the model:
  - $\hat{\xi}_t$: preference shock
  - $\hat{z}_t$: technology shock
  - $\hat{\mu}_t$: investment shock
  - $\hat{\epsilon}_{r,t}$: monetary policy shock
- Assume shocks are all iid normally distributed with no serial correlation.
  - Serial correlation would definitely fit data better.
  - Purpose: explain which *economic explanations* explain U.S. data.
  - Look at forecast errors.

- Agents do not know any parameters of the model.
- Expectations are all replaced by least squares forecasts.
- Timing:
  1. Beginning of period t: data is collected through period $t+1$.
  2. Agents compute least squares forecasts.
  3. Based on expectations, make consumption, production, investment, pricing decisions.
  4. End of time t: time t observations are realized.
- Timing is both realistic, incredibly convenient.

Suppose a DSGE model of the form:

$$\Omega_0 x_t = \Omega_1 x_{t-1} + \Omega_2 E_t^* x_{t+1} + \Psi \epsilon_t$$

- x_t vector of time t variables, all observable to agents.
- E_t^*: possibly non-rational expectations operator.

Rational expectations solution implies:

$$E_t x_{t+1} = G x_t$$

- Agents know the form of this solution, but estimate elements of G by least squares.
- Use as explanatory variables past observations of x_t^k.
- x_t^k includes all variables in x_t where the associated column in G is non-zero.

- Let G_t^k denote non-zero columns of G.
- Ordinary least squares estimate for G^k at time t:

$$\left(\hat{G}_t^k\right)' = \left(\frac{1}{t-1}\sum_{\tau=1}^{t-1} x_{\tau-1}^k {x_{\tau-1}^k}'\right)^{-1}\left(\frac{1}{t-1}\sum_{\tau=1}^{t-1} x_{\tau-1}^k x_\tau'\right)$$

- Least squares forecast:

$$E_t^* x_{t+1} = \hat{G}_t E_t^* x_t = \hat{G}_t^2 x_{t-1}$$

- Evolution of $\hat{G}_t$ in recursive form:

$$\hat{G}_t^k = \hat{G}_{t-1}^k + g(x_{t-1} - \hat{G}_{t-1} x_{t-2}) {x_{t-2}^k}' R_t^{-1},$$

$$R_t = R_{t-1} + g(x_{t-2}^k {x_{t-2}^k}' - R_{t-1})$$

- where $g = 1/(t-1)$ is the learning gain.

- Ordinary least squares $\rightarrow$ learning dynamics disappear as t grows.
- Constant gain: assumes g is constant.
- With a constant gain, learning dynamics persist in the long run.
- Dynamics of expectations depend on the size of the constant learning gain.
- Appropriate initial condition $+$ $(g = 0) \rightarrow$ RE.
- Standard statistical test for $g = 0$ can conclude a rejection failure to RE.

- Substitution of the forecast $E_t^* x_{t+1}$ into the structural form:

$$x_t = \Omega_0^{-1}\left(\Omega_1 + \Omega_2 \hat{G}_t^2\right) x_{t-1} + \Omega_0^{-1}\Psi\epsilon_t$$

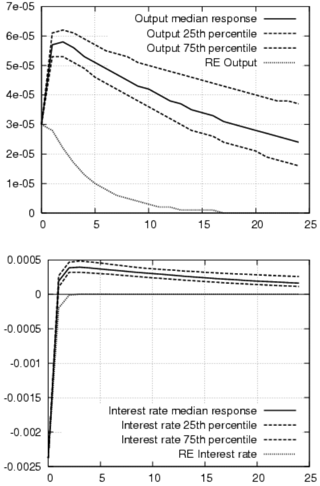
- This solution is used to generate IRFs, estimate model by MLE.
- The impact of x_{t-1} on x_t is time varying.
- Depends on expectations, and therefore past data.
- Therefore, learning delivers persistence, time varying volatility.

- Technology shock (z_t) and a preference shock (ξ_t).
- Look at case with fixed capital and endogenous capital.
- Look at $g = 0.05$, and $g = 0$.
- Response will depend on initial condition for $\hat{G}_t$.
- Simulate 1000 IRFs with different initial conditions, report quartiles.

Table: Parameter values used for Impulse Response Functions

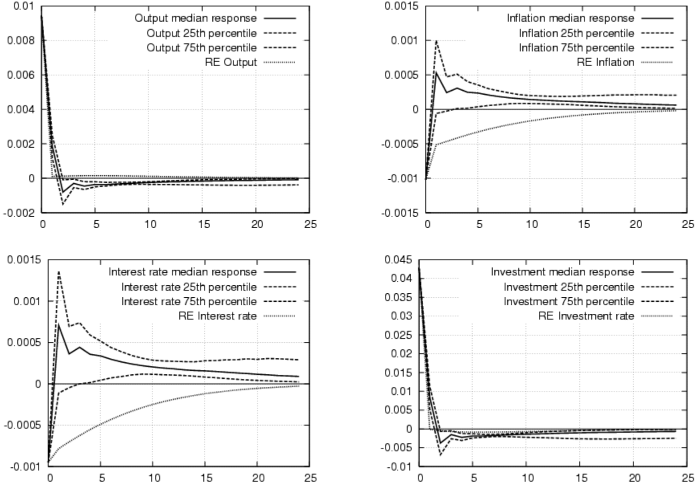
Description	Parameter	Value
Discount rate	β	0.99
Learning gain	g	0.05
Habit formation	η	0.8
Depreciation rate	δ	0.025
Inverse elasticity sub.	σ	2
Inverse elasticity labor supply	μ	1
Cost of adjusting capital	ϕ	0.1
Elasticity sub. in production	θ	4
Price flexibility	κ	0.1
MP interest rate smoothing	ρ_r	0.1
MP feedback on output	ψ_y	0.5
MP feedback on inflation	ψ_π	1.5
Std. dev. technology shock	σ_z	0.02
Std. dev. investment shock	σ_μ	0.02
Std. dev. preference shock	σ_ξ	0.02
Std. dev. interest rate shock	σ_r	0.02

Figure: Technology shock IRF (fixed capital stock)



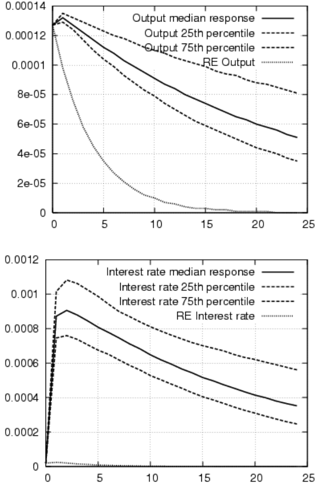
- Habit formation causes some persistence.
- Increased expectations of future consumption.
- Causes an increase in demand for current consumption.
    - Causes prolonged increases in output.
    - Increase in demand (caused by expectations) causes an *increase* in prices.
- Expectations take time to return to normal.

Figure: Technology shock IRF (endogenous capital stock)



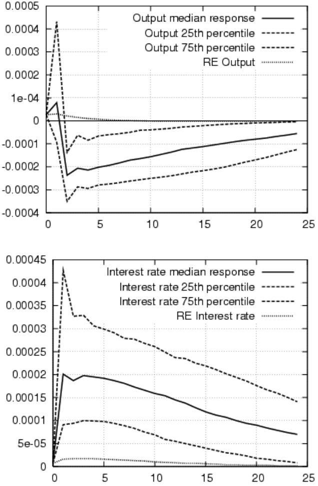
- Output response completely different.
- Demand side expectations are same.
- Supply side expectations: producers expect higher productivity in the future.
- Firms increase investment in period of shock AND period following the shock.
- Over-investment decreases marginal product of capital, expectations reverse.
- Period 3 onward: expectations decrease supply and increase demand.
  - Investment decreases.
  - Small changes in output.
  - Prolonged inflation.

Figure: Preference shock IRF (fixed capital stock)



- Habit formation causes some persistence.
- Increased expectations of future consumption.
- Causes an increase in demand for current consumption.
  - Causes prolonged increases in output.
  - Increase in demand causes further *increase* in prices.
- Expectations take time to return to normal.

Figure: Preference shock IRF (endogenous capital stock)



- Output response completely different.
- Demand side expectations are same.
- Supply side expectations: producers expect higher demand in the future.
- Firms over-invest.
- Investment decreases.
- Small changes in output.
- Prolonged inflation.

Data: quarterly U.S. data from 1957:Q1 through 2005:Q4.

- Growth rate of real GDP (de-meaned).
- Growth rate of real gross private domestic investment (de-meaned).
- Annualized inflation rate of CPI.
- Annualized federal funds rate.

Estimate by MLE using Kalman filter (Hamilton, 1994).

- State equation:

$$x_t = \Omega_0^{-1}\left(\Omega_1 + \Omega_2 \hat{G}_t^2\right) x_{t-1} + \Omega_0^{-1}\Psi\epsilon_t$$

- Observation equations:

$$\begin{aligned} GDP_t^g &= 100\left(\hat{y}_t - \hat{y}_{t-1}\right) \\ I_t^g &= 100\left(\hat{I}_t - \hat{I}_{t-1}\right) \\ INF_t^{CPI} &= \pi^* + 400\pi_t \\ FF_t &= r^* + 400\hat{r}_t \end{aligned}$$

- π^*: steady state inflation. NK model assumes is zero.
- r^*: steady state nominal interest rate.

- Fix $\beta = 0.99$, $\delta = 0.025$, $\alpha = 0.33$.
- Maximize likelihood with respect to:

$$\Theta_1 = [g \ \eta \ \sigma \ \mu \ \phi \ \theta \ \kappa \ \rho_r \ \psi_y \ \psi_\pi \ \pi^* \ \sigma_z \ \sigma_\mu \ \sigma_\xi \ \sigma_r]'$$

- When estimating the model without capital, μ, ϕ, θ, and σ_μ, become unidentifiable.

$$\Theta_2 = [g \ \eta \ \sigma \ \kappa \ \rho_r \ \psi_y \ \psi_\pi \ \pi^* \ \sigma_z \ \sigma_\xi \ \sigma_r]'$$

- Estimate four cases:
  1. Fixed capital + learning.
  2. Fixed capital + ($g = 0$).
  3. Endogenous capital + learning.
  4. Endogenous capital + ($g = 0$).

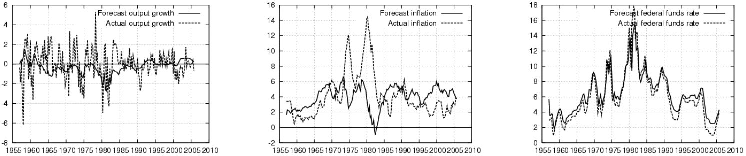
Table: MLE Results with Learning and Fixed Capital

Parameter		Estimate	Std. dev.	P-value.
Learning gain	g	0.008503	0.004215	0.021818
Habit formation	η	0.823211	0.070769	0.000000
Inverse elasticity sub.	σ	1.064657	2.170468	0.311883
Price flexibility	κ	0.011064	0.003194	0.000267
MP interest rate smoothing	ρ_r	0.814581	0.021260	0.000000
MP feedback on output	ψ_y	0.022568	0.012843	0.039440
MP feedback on inflation	ψ_π	1.003403	0.097723	0.000000
Steady state inflation	π^*	1.656158	0.579859	0.002144
Std. dev. technology shock	σ_z	0.340227	0.095211	0.000176
Std. dev. preference shock	σ_ξ	0.790604	1.041036	0.223795
Std. dev. interest rate shock	σ_r	0.002440	0.000084	0.000000
MSE Output growth		2.609730		
MSE Inflation		10.482353		
MSE Interest rate		1.400302		

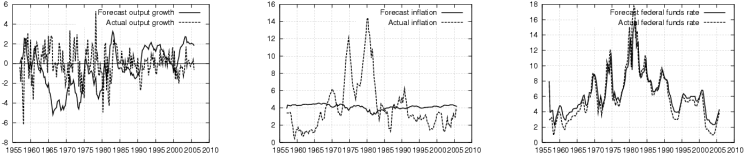
- Results similar to Milani (2005).
- Learning significant.
- Habit formation still significant source of persistence.

Figure: Forecast Errors with Fixed Capital

Learning



Rational Expectations



- Learning forecast errors for output and inflation are less correlated.
- Learning generates more volatility for inflation.
- Both estimates still miss inflation episodes in the 1970s.

Table: MLE Results with Learning and Endogenous Capital

Parameter		Estimate	Std. Dev.	P-value
Learning gain	g	0.025106	0.002111	0.000000
Habit formation	η	0.716807	0.000066	0.000000
Inverse elasticity sub.	σ	6.424134	0.002669	0.000000
Inverse elasticity labor supply	μ	0.911733	0.002737	0.000000
Cost of adjusting capital	ϕ	8.821684	0.000867	0.000000
Elasticity sub. in production	θ	3.650960	0.000395	0.000000
Price flexibility	κ	0.001962	0.000021	0.000000
MP interest rate smoothing	ρ_r	0.877350	0.000137	0.000000
MP feedback on output	ψ_y	0.190299	0.000015	0.000000
MP feedback on inflation	ψ_π	0.514582	0.000134	0.000000
Steady state inflation	π^*	0.929303	0.002806	0.000000
Std. dev. technology shock	σ_z	0.947077	0.000021	0.000000
Std. dev. investment shock	σ_μ	0.322885	0.000024	0.000000
Std. dev. preference shock	σ_ξ	1.402110	0.002658	0.000000
Std. dev. interest rate shock	σ_r	0.011280	0.000075	0.000000
MSE Output growth		5.324316		
MSE Investment growth		147.418957		
MSE Inflation		5.038744		
MSE Interest rate		1.929975		

- Even higher statistically significant estimate for the learning gain.
- Habit formation is still a significant source of persistence.
- Taylor rule parameters look very different.

Figure: Forecast Errors with Learning and Endogenous Capital

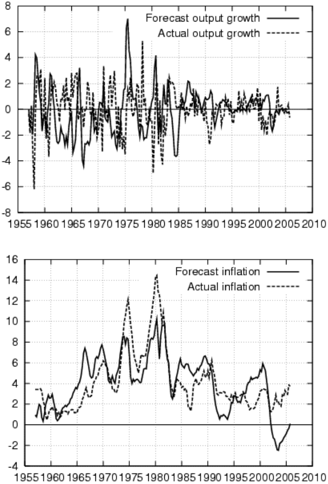
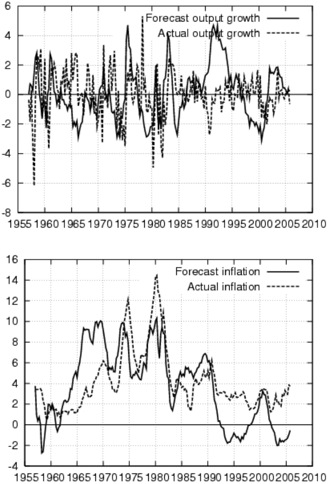


Figure: Forecast Errors with No Learning and Endogenous Capital



Comparison

- Without learning: predicts persistent volatility throughout sample in output, investment, and inflation.
- With Learning:
  - Correctly predicts volatility in output, investment and inflation in 1970s.
  - Correctly predicts low volatility in output and investment from mid-1980s onward.
  - Correctly predicts low volatility in inflation during late 1950s, early 1960s and during 1980s and 1990s.

Conclusion

- Capital in the New Keynesian model has non-trivial implications for output and inflation dynamics, especially with learning.
- Including capital still leads to statistically significant learning gain.
- Capital + learning is best able to predict the changing volatility of output, investment, and inflation in the post-war period.

Work yet to do

- Generate impulse response functions for expectations or expectation errors.
- Produce plots of agents' forecast errors through the sample.
- Results are currently sensitive to initial guess for MLE solution.
  - Maximize likelihood using simulated annealing: its been running for days.
- Estimate subsamples, look for structural changes especially in monetary policy.