- Purpose:
  - Estimate the empirical significance of learning.
  - Determine what features of U.S. data learning can explain.
- Learning suggested to deliver features:
  - Orphanides and Williams (2005): Inflation scares.
  - Milani (2005): Macroeconomic persistence.
  - Milani (2007): Time varying volatility.
- Estimate a NK model with learning and RE by MLE.
- Examine forecast errors, evolution of shocks, and evolution of expectations.

**Utility function:**

$$U_0 = E_0^* \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{1-\sigma} \xi_t \left( c_t(i) - \eta c_{t-1}(i) \right)^{1-\sigma} - \frac{1}{1+\mu} n_t(i)^{1+\mu} \right]$$

- $E_t^*$: possibly non-rational expectations operator.
- $c_t(i)$: consumption at time $t$.
- $n_t(i)$: labor supply at time $t$.
- $\xi_t$: common preference shock.
- $\beta$: discount factor.
- $\sigma \in (0, \infty)$: related to the intertemporal elasticity of substitution.
- $\eta \in [0, 1)$: degree of habit formation.

**Final good production:**

$$y_t = \left[ \int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

- $y_t$ output of final good, $y_t(i)$ output of intermediate good $i$.

- $\theta \in (1, \infty)$: elasticity of substitution in production.

**Intermediate goods production:**

$$\text{Model with capital: } y_t(i) = z_t k_t(i)^{\alpha} \left(\nu^t n_t(i)\right)^{1-\alpha}$$

$$\text{Model without capital: } y_t(i) = z_t n_t(i)$$

- $z_t$: common technology shock.
- $k_t(i)$: firm-specific capital good.
- $\nu$: growth rate of labor productivity (growth rate of real output per capita).

- Follow Calvo (1983) pricing: fraction $1 - \omega$ firms re-optimize their price each period.
- Inflation indexation: Those who cannot re-optimize may adjust according to:

$$\ln(p_t(i)) = \ln(p_{t-1}(i)) + \gamma \pi_{t-1}$$

- With firm-specific capital (Woodford, 2005), leads to Phillips curve:

$$\pi_t = \frac{\gamma}{1+\beta\gamma}\pi_{t-1} + \frac{\beta}{1+\beta\gamma}E_t\pi_{t+1} + \kappa \hat{s}_t$$

- $\hat{s}_t$: average marginal cost in the economy (percentage deviation from steady state).
- $\kappa$: function of many parameters.

- Final good is converted to a firm-specific capital good.
- Investment of $I_t(i)$ leads to capital stock next period:

$$k_{t+1}(i) = (1-\delta)k_t(i) + \mu_t I_t(i) - \frac{\phi}{2}\left[\frac{k_{t+1}(i)}{k_t(i)} - 1\right]^2 k_t(i)$$

  - $\mu_t$: common investment technology shock.
  - $\delta$: depreciation rate.
  - $\phi$: capital adjustment cost parameter.
- Profit maximizing marginal cost:

$$\hat{s}_t = \frac{\mu+\alpha}{1-\alpha}\hat{y}_t - \frac{\alpha(\mu+1)}{1-\alpha}\hat{k}_t - \hat{\lambda}_t - \frac{\mu+1}{1-\alpha}\hat{z}_t,$$

- Market clearing condition:

$$\text{Model with capital: } y_t = c_t + I_t + g_t$$

$$\text{Model without capital: } y_t = c_t$$

- $g_t$: Exogenous demand shocks such as changes in government spending and net exports.

- Model *without* capital: interest rate responds to *output gap*:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r)\left(\psi_\pi \pi_t + \psi_y \tilde{y}_t\right) + \epsilon_{r,t}$$

  - $\psi_\pi \in (0, \infty)$: feedback on inflation.
  - $\psi_y \in (0, \infty)$: feedback on output.
  - $\rho_r \in (0, 1)$: smoothing parameter.

- Model *with* capital: interest rate responds to the deviation of output from steady state:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r)\left(\psi_\pi \pi_t + \psi_y \hat{y}_t\right) + \epsilon_{r,t}$$

- Non-policy shocks (percentage deviations from steady state) are AR(1):

  - Preference shock: $\hat{\xi}_t = \rho_\xi \hat{\xi}_{t-1} + \epsilon_{\xi,t}$
  - Technology shock: $\hat{z}_t = \rho_z \hat{z}_{t-1} + \epsilon_{z,t}$
  - Investment shock: $\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \epsilon_{\mu,t}$
  - Demand shock: $\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g,t}$

- $\epsilon_{\xi,t}$, $\epsilon_{z,t}$, $\epsilon_{\mu,t}$, $\epsilon_{g,t}$, $\epsilon_{r,t}$ are independently normally distributed with mean zero.

- Re-write the model without capital in terms of the output gap.
- Notation:
  - Tilde denotes (percentage) deviation from fully flexible outcome.
  - Superscript $f$ denotes fully flexible outcome.
  - Superscript $*$ denotes steady state value.
- Natural rate of interest:

$$r_t^n = r_t^f - E_t \pi_{t+1}^f$$

$$r_t^n = (1 - \rho_{r^n})\, r^* + \rho_n r_{t-1}^n + \epsilon_t^{r^n}$$

- Consumer first order conditions + market clearing:

$$\tilde{\lambda}_t = E_t \tilde{\lambda}_{t+1} + (r_t - r_t^n - E_t \pi_{t+1})$$

$$\tilde{\lambda}_t = \frac{1}{(1-\beta\eta)(1-\eta)} \left[\beta\eta\sigma E_t \tilde{y}_{t+1} - \sigma(1+\beta\eta^2)\tilde{y}_t + \sigma\eta\tilde{y}_{t-1}\right]$$

- Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left(\mu \tilde{y}_t - \tilde{\lambda}_t\right) + u_t$$

- Cost push shock:

$$u_t = \rho_u u_{t-1} + \epsilon_t^u$$

- The New Keynesian model has the form:

$$\Omega_0 x_t = \Omega_1 x_{t-1} + \Omega_2 E_t x_{t+1} + \Psi z_t$$

$$z_t = A z_{t-1} + \epsilon_t$$

  - $x_t$ vector of time $t$ variables, observable to agents at following time period.
  - $z_t$: vector of time $t$ shocks, observable to agents in current period.
- Rational expectations solution:

$$x_t = G x_{t-1} + M z_t$$

- Agents estimate elements of $G$ and $M$ by least squares.
- $X_t$: vector of regressors: $X_t' = [1\ x_{t-1}'\ z_t']$.
- $Y_t$: vector of dependent variables: $Y_t = x_t$.
- $\hat{\phi}_t$: least squares estimate of the coefficients in $G$ and $M$.
- Information at available at time $t$: $x_{t-1}$, $z_t$.
- Evolution of $\hat{\phi}_t$ in recursive form:

$$\hat{\phi}_t = \hat{\phi}_{t-1} + gR_t^{-1}X_{t-1}\left(Y_t - X_{t-1}'\hat{\phi}_{t-1}\right)$$

$$R_t = R_{t-1} + g(X_{t-1}X_{t-1}' - R_{t-1})$$

- where $g$ is the constant learning gain.

- Method: Maximum likelihood estimation.
- Data: Quarterly data for 1960 through 2005.
- Model with no capital:
  - CBO measure of the output gap.
  - Annualized quarterly inflation rate of the GDP deflator.
  - Annualized quarterly Federal funds rate.
- Model with endogenous capital:
  - Real GDP per capita.
  - Real private consumption per capita.
  - Real gross private domestic investment per capita.
  - Annualized quarterly inflation rate of the GDP deflator.
  - Annualized quarterly Federal funds rate.
- Calibrated parameters:
  - Model without capital: $\beta = 0.99$, $\mu = 0$ (perfectly inelasticy labor supply).
  - Model with capital: $\beta = 0.99$, $\delta = 0.025$, $\alpha = 0.33$, $\nu = 1.00533$.

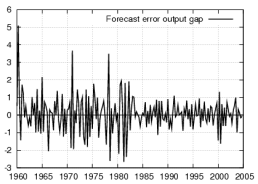
- $\hat{\phi}_t$ and $R_t$ must be initialized for estimation.
- MSV solution, expected outer product of the state vector under RE.
  - Much of learning theory valid for the neighborhood of the MSV solution.
  - Initial conditions are supported by the microfoundations.
  - Problem: Learning dynamics smallest when near MSV solution.
- Joint estimation:
  - Problems: As many as 40 additional parameters (endogenous capital estimation).
  - Motivation: illustrates how estimates for structural parameters depend on initial conditions.
- Alternative approach: Use a VAR(1) from pre-sample data
  - Problem: latent variables are regressors - capital stock, structural shocks.

| Description | Parameter | RE | Std. dev. | Learning | Std. dev. |
|---|---|---|---|---|---|
| Learning gain | $g$ | . | . | 0.0104 | 0.0048 |
| Habit formation | $\eta$ | 0.9953 | 0.0105 | 0.9470 | 0.0214 |
| Inverse elasticity sub. | $\sigma$ | 0.0830 | 0.0724 | 0.5057 | 0.4069 |
| Phillips curve slope | $\kappa$ | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| Price indexation | $\gamma$ | 0.9686 | 0.0458 | 0.9730 | 0.0172 |
| Interest rate smoothing | $\rho_r$ | 0.8449 | 0.0269 | 0.8943 | 0.0178 |
| Policy feedback on output | $\psi_y$ | 0.2073 | 0.0778 | 0.3346 | 0.0931 |
| Policy feedback on inflation | $\psi_\pi$ | 1.2812 | 0.2230 | 1.5895 | 0.3079 |
| Persistence nat. int. rate | $\rho_n$ | 0.9944 | 0.0081 | 0.9016 | 0.0273 |
| Persistence cost push | $\rho_u$ | 1.50E-6 | 0.0684 | 0.0000 | 0.0224 |
| Std. dev. nat. int. rate | $s_n$ | 0.0009 | 0.0005 | 0.1150 | 0.1065 |
| Std. dev. cost push | $s_u$ | 2.45E-6 | 3.40E-6 | 2.49E-6 | 3.00E-7 |
| Std. dev. policy shock | $s_r$ | 5.48E-6 | 3.40E-6 | 5.31E-6 | 3.30E-7 |
| Steady state inflation | $\pi^*$ | 7.9795 | 2.8442 | 5.6139 | 1.0109 |

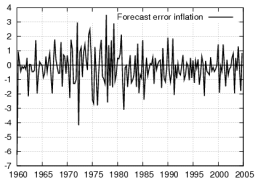
- Learning gain statistically significantly different from zero.
- Habit formation and indexation remain significant sources of persistence.
- Most parameters are very similar.

**Rational Expectations**

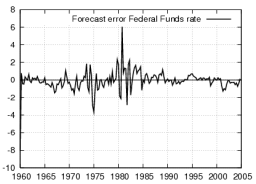
Output gap




Inflation

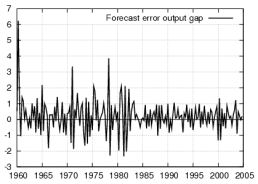



Interest Rate

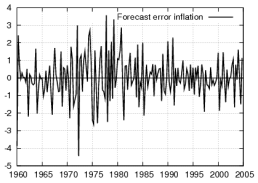



**Learning (RE initial conditions)**

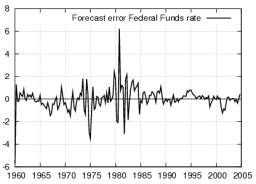
Output gap




Inflation

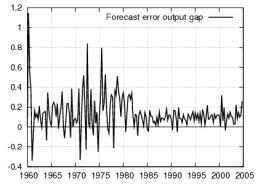



Interest Rate

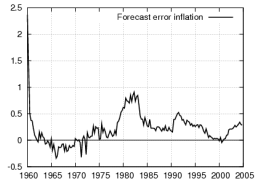



**Differences in Forecast Errors**

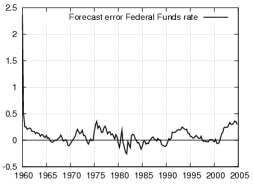
Output gap




Inflation




Interest Rate



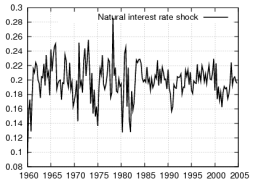

**Correlation of Forecast Errors**

| Output gap | Inflation | Interest Rate |
|---|---|---|
| 0.9858 | 0.9707 | 0.9833 |

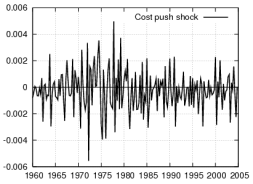
- Forecast errors essentially the same.
- Forecast errors are largest for 1970s.
  - Neither model explains monetary policy in 1979-1982 period.
  - Neither model explains changes in volatility.

## Rational Expectations

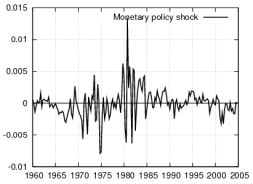
Natural Rate Shock




Cost Push Shock

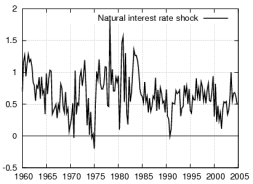



Policy Shock

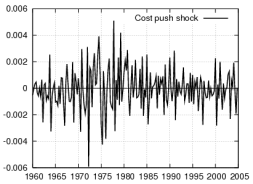



## Learning (RE initial conditions)

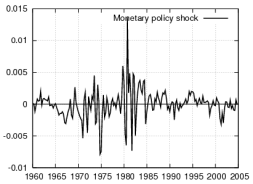
Natural Rate Shock




Cost Push Shock

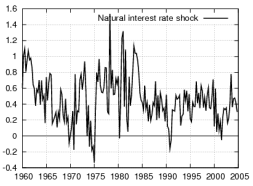



Policy Shock

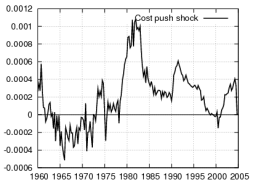



## Difference in Smoothed Shocks

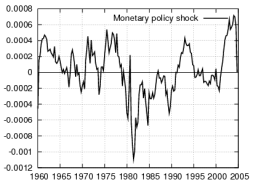
Natural Rate Shock




Cost Push Shock



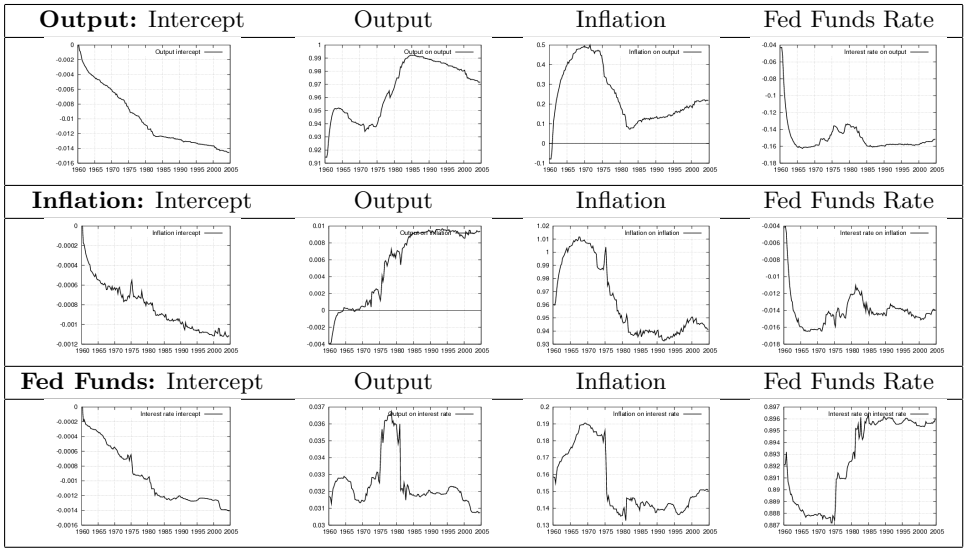

Policy Shock




## Correlation of Smoothed Shocks

| Natural Rate Shock | Cost Push Shock | Policy Shock |
|---|---|---|
| 0.7150 | 0.9799 | 0.9907 |

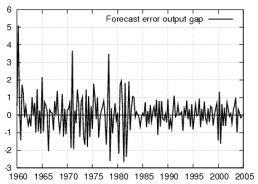
| Description | Parameter | Estimate | Std. dev. |
|---|---|---|---|
| Learning gain | $g$ | 0.0116 | 0.0072 |
| Habit formation | $\eta$ | 0.0436 | 0.0404 |
| Inverse elasticity sub. | $\sigma$ | 2.4018 | 2.8262 |
| Phillips curve slope | $\kappa$ | 0.0001 | 0.0003 |
| Price indexation | $\gamma$ | 0.9940 | 1.1123 |
| Interest rate smoothing | $\rho_r$ | 0.8940 | 0.0279 |
| Policy feedback on output | $\psi_y$ | 0.2973 | 0.1329 |
| Policy feedback on inflation | $\psi_\pi$ | 1.0576 | 0.1989 |
| Persistence nat. int. rate | $\rho_n$ | 0.9987 | 1.00E-8 |
| Persistence cost push | $\rho_u$ | 0.0002 | 0.0823 |
| Std. dev. nat. int. rate | $s_n$ | 0.0007 | 0.0080 |
| Std. dev. cost push | $s_u$ | 1.5E-05 | 6.06E-06 |
| Std. dev. policy shock | $s_r$ | 5.4E-06 | 4.6E-07 |
| Steady state inflation | $\pi^*$ | 1.6478 | 0.8418 |

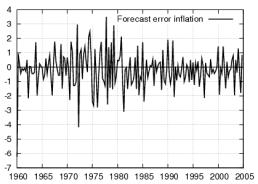


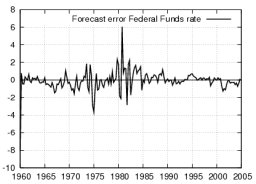

- Learning gain statistically significantly different from zero.
- Degree of habit formation is close to zero.
- Point estimate of price indexation still high, still significant.

**Rational Expectations**

Output gap | Inflation | Interest Rate

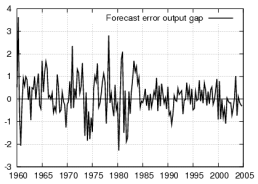


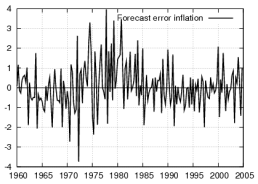


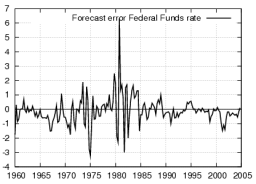



**Learning (Estimated initial conditions)**

Output gap | Inflation | Interest Rate

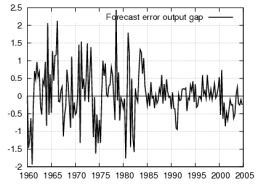


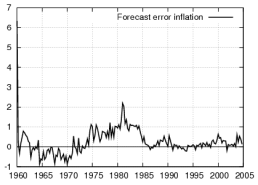


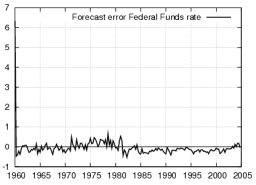



**Differences in Forecast Errors**

Output gap | Inflation | Interest Rate





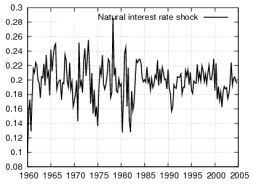



**Correlation of Forecast Errors**

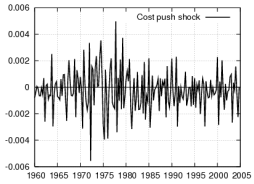
| Output gap | Inflation | Interest Rate |
|---|---|---|
| 0.7333 | 0.8472 | 0.8861 |

## Rational Expectations

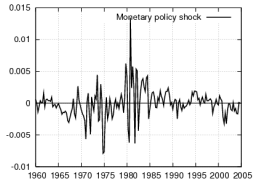
Natural Rate Shock




Cost Push Shock

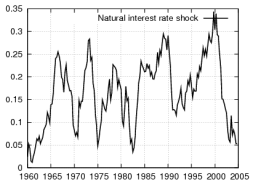



Policy Shock

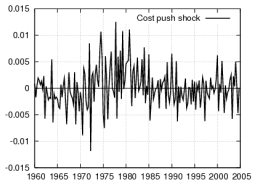



## Learning (Estimated initial conditions)

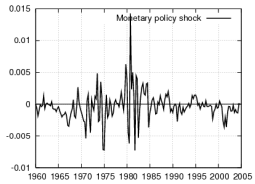
Natural Rate Shock




Cost Push Shock

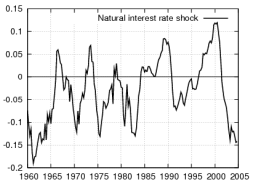



Policy Shock

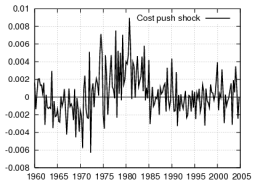



## Difference in Smoothed Shocks

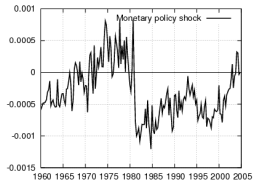
Natural Rate Shock




Cost Push Shock

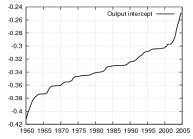
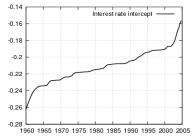



Policy Shock




## Correlation of Smoothed Shocks

| Natural Rate Shock | Cost Push Shock | Policy Shock |
|---|---|---|
| 0.7150 | 0.9799 | 0.9907 |

| **Output:** Intercept | Output | Inflation | Fed Funds Rate |
|---|---|---|---|
|  |  |  |  |
| **Inflation:** Intercept | Output | Inflation | Fed Funds Rate |
|  |  |  |  |
| **Fed Funds:** Intercept | Output | Inflation | Fed Funds Rate |
|  |  |  |  |

| | RE | Learning (RE init.) | Learning (Est. init.) |
|---|---|---|---|
| MSE Output | 1.0920 | 1.1041 | 0.7303 |
| MSE Inflation | 1.6613 | 1.5781 | 1.4175 |
| MSE Fed Funds | 1.2594 | 1.0879 | 0.9435 |
| Log-likelihood | -303.8 | -291.9 | -241.6 |

| Description | Parameter | RE | Std. dev. | Learning | Std. dev. |
|---|---|---|---|---|---|
| Learning gain | $g$ | . | . | 3.51E-6 | 7.01E-6 |
| Habit formation | $\eta$ | 0.0855 | 0.0846 | 0.1078 | 0.0805 |
| Inverse elasticity sub. | $\sigma$ | 16.5056 | 13.8189 | 16.8358 | 14.5459 |
| Capital / output ratio | $k_y$ | 4.4799 | 0.2129 | 4.5061 | 0.2129 |
| Consumption / output ratio | $c_y$ | 0.7141 | 0.0547 | 0.7234 | 0.0516 |
| Inverse elasticity labor | $\mu$ | 1.53E-5 | 0.3944 | 1.00E-5 | 0.4396 |
| Capital adjustment cost | $\phi$ | 27.9001 | 5.4801 | 27.4460 | 5.9845 |
| Phillips curve slope | $\kappa$ | 0.0116 | 0.0034 | 0.0119 | 0.0037 |
| Price indexation | $\gamma$ | 0.9999 | 0.0000 | 0.9999 | 0.0000 |
| Interest rate smoothing | $\rho_r$ | 0.8147 | 0.0196 | 0.8103 | 0.0203 |
| Policy feedback on output | $\psi_y$ | 0.0326 | 0.0116 | 0.0356 | 0.0122 |
| Policy feedback on inflation | $\psi_\pi$ | 1.5401 | 0.1504 | 1.4735 | 0.1405 |
| Persistence in tech. shock | $\rho_z$ | 1.40E-5 | 0.0888 | 1.00E-5 | 0.0902 |
| Persistence in pref. shock | $\rho_\xi$ | 0.9793 | 0.0176 | 0.9800 | 0.0167 |
| Persistence in inv. shock | $\rho_\mu$ | 0.9295 | 0.0235 | 0.9282 | 0.0253 |
| Persistence in AD shock | $\rho_g$ | 0.9989 | 0.0042 | 0.9993 | 0.0035 |
| Std. dev. tech. shock | $s_z$ | 0.2399 | 0.1262 | 0.2326 | 0.1325 |
| Std. dev. inv. shock | $s_\mu$ | 0.1996 | 0.1219 | 0.1900 | 0.1251 |
| Std. dev. pref. shock | $s_\xi$ | 0.1722 | 0.1447 | 0.1857 | 0.1626 |
| Std. dev. policy shock | $s_r$ | 0.0025 | 0.0001 | 0.0024 | 0.0001 |
| Std. dev. AD shock | $s_g$ | 0.0222 | 0.0091 | 0.0238 | 0.0098 |
| Steady state inflation | $\pi^*$ | 8.6918 | 2.3997 | 10.3678 | 3.0793 |
| Steady state output | $y^*$ | 1.2521 | 0.1033 | 1.2355 | 0.0984 |

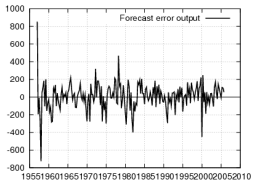
- Learning gain essentially zero.
- All parameter estimates are very similar.
- Habit formation close to zero under *both* RE and Learning.
- Inflation indexation very close to one.

| Description | Parameter | Estimate | Std. dev. |
|---|---|---|---|
| Learning gain | $g$ | 0.0288 | 0.0077 |
| Habit formation | $\eta$ | 0.0772 | 0.1031 |
| Inverse elasticity sub. | $\sigma$ | 16.6941 | 14.4002 |
| Capital / output ratio | $k_y$ | 4.5086 | 1.5187 |
| Consumption / output ratio | $c_y$ | 0.7213 | 0.0842 |
| Inverse elasticity labor | $\mu$ | 0.0132 | 1.3992 |
| Capital adjustment cost | $\phi$ | 28.1764 | 38.5551 |
| Phillips curve slope | $\kappa$ | 0.0120 | 0.0154 |
| Price indexation | $\gamma$ | 0.9999 | 0.0000 |
| Interest rate smoothing | $\rho_r$ | 0.8776 | 0.0351 |
| Policy feedback on output | $\psi_y$ | 0.0374 | 0.0661 |
| Policy feedback on inflation | $\psi_\pi$ | 1.5654 | 0.4185 |
| Persistence in tech. shock | $\rho_z$ | 0.0000 | 0.0124 |
| Persistence in pref. shock | $\rho_\xi$ | 0.9611 | 0.0234 |
| Persistence in inv. shock | $\rho_\mu$ | 0.9364 | 0.0405 |
| Persistence in AD shock | $\rho_g$ | 0.9999 | 0.0001 |
| Std. dev. tech. shock | $s_z$ | 0.2569 | 0.5566 |
| Std. dev. inv. shock | $s_\mu$ | 0.1830 | 0.1692 |
| Std. dev. pref. shock | $s_\xi$ | 0.1707 | 0.1461 |
| Std. dev. policy shock | $s_r$ | 0.0024 | 0.0001 |
| Std. dev. AD shock | $s_g$ | 0.0255 | 0.0154 |
| Steady state inflation | $\pi^*$ | 9.5185 | 6.1255 |
| Steady state output | $y^*$ | 1.2396 | 0.1424 |

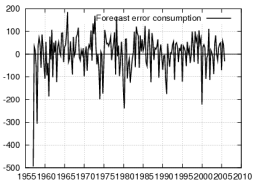
- Learning gain parameter significantly greater than zero.
- All parameter estimates are very similar.
- Habit formation close to zero under *both* RE and Learning.
- Inflation indexation very close to one.

**Rational Expectations**

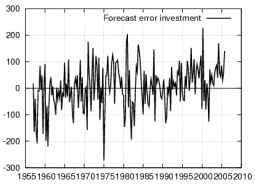
Output



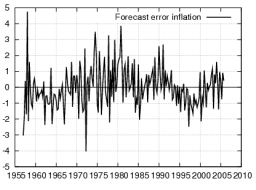

Consumption



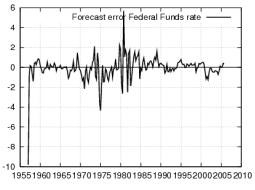

Investment




Inflation

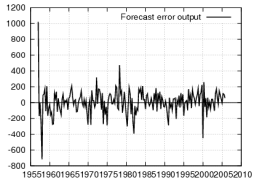



Interest Rate

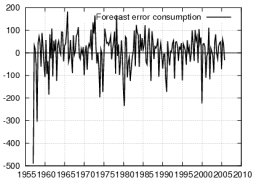


**Learning (RE initial conditions)**

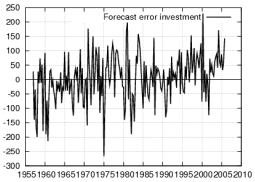
Output



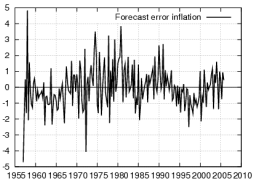

Consumption



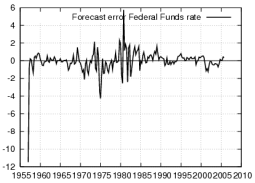

Investment




Inflation

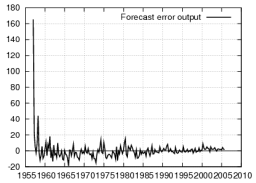



Interest Rate



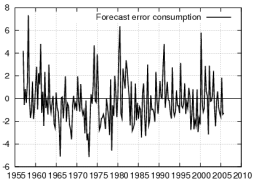

**Differences in Forecast Errors (Correlations)**

Output



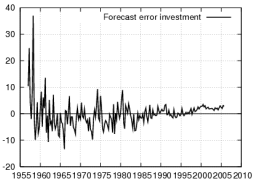

(0.9966)

Consumption



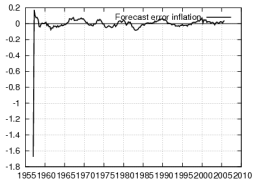

(0.9997)

Investment



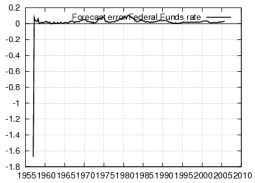

(0.9982)

Inflation




(0.9956)

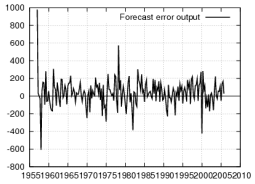
Interest Rate



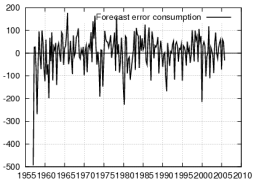

(0.9968)

**Learning (Estimated initial conditions)**

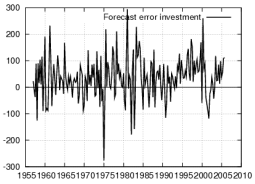
Output



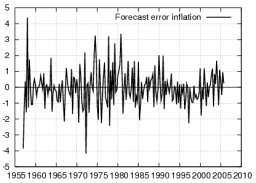

Consumption



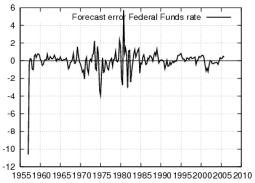

Investment




Inflation

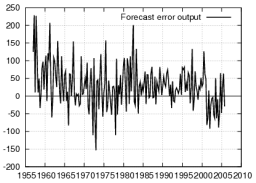



Interest Rate



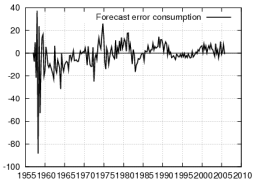

**Differences in Forecast Errors (Correlations)**

Output



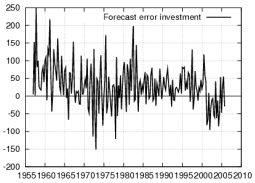

(0.9213)

Consumption



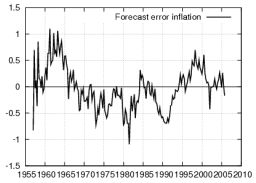

(0.9906)

Investment



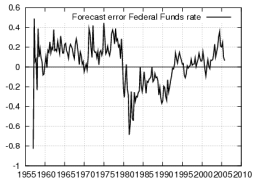

(0.7294)

Inflation




(0.9527)

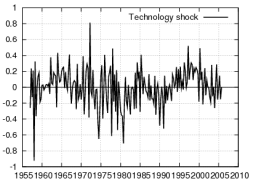
Interest Rate




(0.9847)

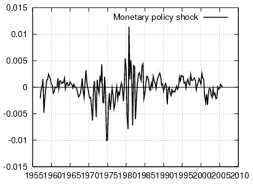
## Rational Expectations

Technology shock

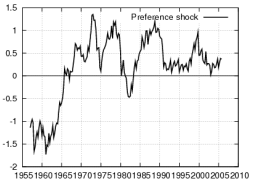



Investment shock

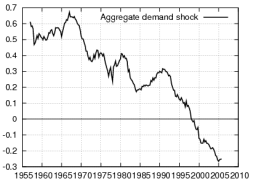

Monetary policy shock




Preference shock

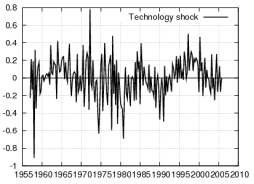



Aggregate demand shock



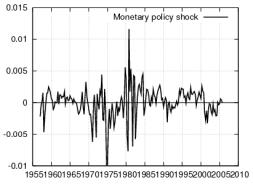

## Learning (RE initial conditions)

Technology shock

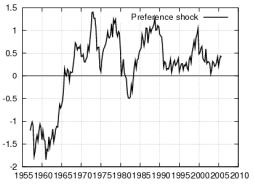



Investment shock

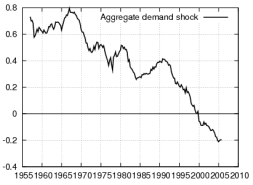

Monetary policy shock




Preference shock

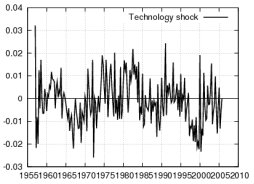



Aggregate demand shock



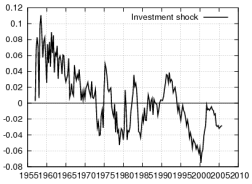
**Differences in smoothed shocks (Correlations)**

Technology shock



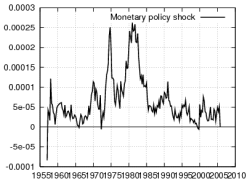

(0.9996)

Investment shock



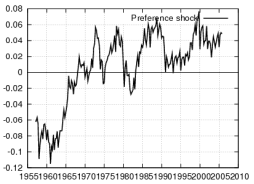

(0.9998)

Monetary policy shock




(0.9997)

Preference shock

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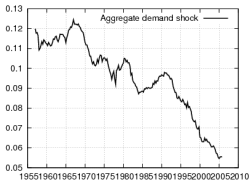
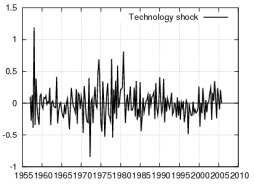
Aggregate demand shock




(0.9999)

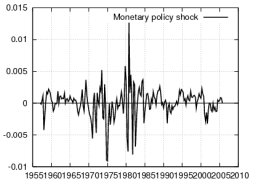
**Learning (Estimated initial conditions)**

Technology shock

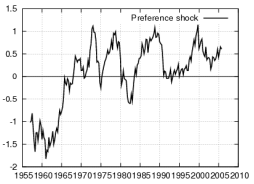



Investment shock

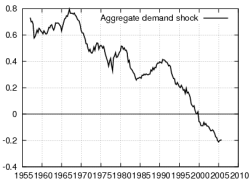

Monetary policy shock




Preference shock

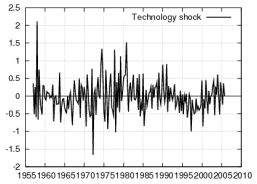



Aggregate demand shock



**Differences in smoothed shocks (Correlations)**

Technology shock

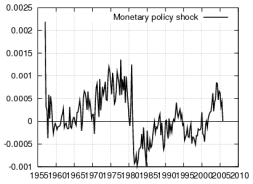



(-0.9528)

Investment shock

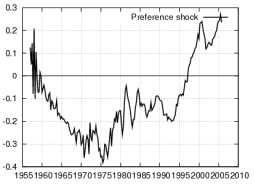

(0.5308)

Monetary policy shock



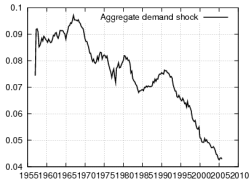

(0.9794)

Preference shock

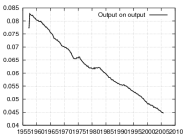
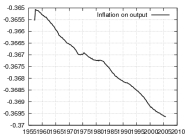
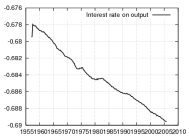
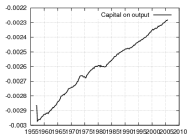
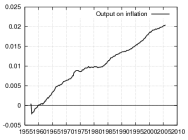
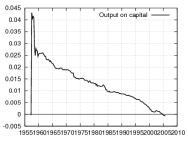
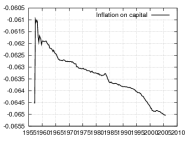
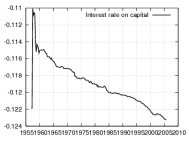
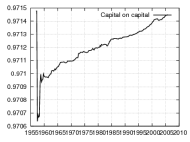



(0.9760)

Aggregate demand shock

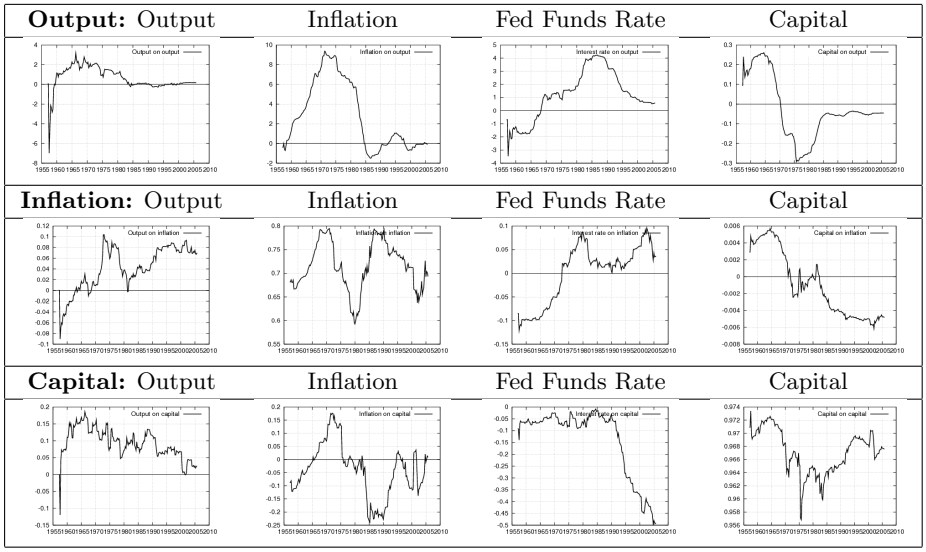



(0.9999)

| **Output:** Output | Inflation | Fed Funds Rate | Capital |
|---|---|---|---|
|  |  |  |  |
| **Inflation:** Output | Inflation | Fed Funds Rate | Capital |
|  |  |  |  |
| **Capital:** Output | Inflation | Fed Funds Rate | Capital |
|  |  |  |  |

- Recall estimated gain: $g = 3.51(10^{-6})$
- Tiny changes in coefficients.

- Very slow to adjust.




| | RE | Learning (RE init.) | Learning (Est. init.) |
|---|---|---|---|
| MSE Output | 23558.2 | 24868.6 | 24652.0 |
| MSE Consumption | 7129.3 | 7105.1 | 6883.4 |
| MSE Investment | 7118.3 | 6930.3 | 7656.7 |
| MSE Inflation | 1.6727 | 1.7508 | 1.3893 |
| MSE Fed Funds | 1.4116 | 1.5910 | 1.4483 |
| Log-likelihood | -3042.2 | -3041.6 | -241.6 |