

# Three Essays in Adaptive Expectations in New Keynesian Monetary Economies

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## Three Essays

1. Initial Expectations in New Keynesian Models with Learning
2. Empirical Significance of Learning in a New Keynesian Model with Firm-Specific Capital
3. Regime Switching, Learning, and the Great Moderation

## Purpose

How does least squares learning affect our understanding of post-war U.S. data?

- Does it explain prolonged inflation?
  - Orphanides and Williams (2005a)
- Does it explain time-varying volatility? Great Moderation?
  - Primiceri (QJE, 2005)
  - Orphanides and Williams (JEDC, 2005b)
  - Milani (2007)
- How does learning influence the impact of structural shocks?
- How do initial expectations and agents' information sets influence these results?
- Can dynamic gain learning explain time-varying volatility?

## Essay 1 Outline

### Initial Expectations in New Keynesian Models with Learning

- Examine constant gain learning in a standard New Keynesian model.
  - No capital, output used for consumption.
  - Habit formation.
  - Calvo (JME 1983) sticky prices.
  - Inflation indexation.
  - Taylor rule responds to expectations.
- How do initial expectations and assumptions for information sets influence results on...
  - parameter estimates?
  - impact of structural shocks?
  - forecast errors?

## Four Expectations Frameworks

- Case 1: Rational Expectations.
  - Agents observe structural shocks (natural rate shock, cost push shock).
- Case 2: Constant gain least squares learning.
  - Agents observe structural shocks.
  - Expectations initialized to rational expectations solution.
- Case 3: Constant gain learning with a limited information set.
  - Shocks are not observable, not used as explanatory variables.
  - Expectations on remaining variables set equal to rational expectations solution.
- Case 4: Constant gain learning with pre-sample initial expectations.
  - Shocks are not observable, not used as explanatory variables.
  - Initial conditions set to pre-sample (1954:Q3-1959:Q4) WLS results.

## Estimation Procedure

- Maximum Likelihood
- Calibrated parameters:
  - Discount factor:  $\beta = 0.9925$
  - Steady state inflation:  $\pi^* = 3.52$
  - Phillips coefficient:  $\kappa = 0.1$

## Parameter Estimates

	Parameter	Case 1	Case 2	Case 3	Case 4
$\eta$	Habit Persistence	0.9929 (0.0892)	0.6515 (0.0174)	0.9577 (0.4132)	0.7065 (0.2465)
$\sigma^{-1}$	Inverse IES	0.0015 (0.0281)	0.4162 (0.0536)	0.0308 (0.5686)	0.2457 (0.4541)
$\gamma$	Price Indexation	0.0000 (0.0377)	0.7126 (0.0238)	0.9994 (0.0754)	0.6322 (0.1325)
$\rho_r$	MP Persistence	0.8857 (0.0195)	0.7843 (0.0030)	0.8558 (0.0208)	0.7043 (0.0391)
$\psi_y$	MP Output Gap	0.3864 (0.1228)	0.0758 (0.0163)	0.1434 (0.0320)	0.2265 (0.0451)
$\psi_\pi$	MP Inflation	3.6813 (0.6479)	1.7419 (0.0343)	2.2153 (0.2974)	1.5009 (0.0942)
$\rho_n$	Natural Rate Pers.	0.3636 (0.0381)	0.7699 (0.0045)	0.3060 (0.0406)	0.5102 (0.0434)
$\rho_u$	Cost Push Pers.	0.8568 (0.0155)	0.2398 (0.0366)	0.0000 (0.0438)	0.2880 (0.0684)
$\sigma_n$	Natural Rate Std. Dev.	0.0635 (0.0128)	0.0055 (0.0000)	0.2173 (0.0584)	0.0328 (0.0112)
$\sigma_u$	Cost Push Std. Dev.	0.0021 (0.0001)	0.0066 (0.0003)	0.0122 (0.0005)	0.0100 (0.0008)
$\sigma_r$	Policy Shock Std. Dev.	0.0032 (0.0001)	0.0031 (0.0001)	0.0030 (0.0000)	0.0031 (0.0001)
$\pi^*$	Steady State Inflation	3.5304 (0.2017)	3.2369 (0.3026)	4.5971 (0.4958)	4.0443 (0.3728)
$g$	Learning Gain	—	0.0119 (0.0015)	0.0202 (0.0020)	0.0175 (0.0027)

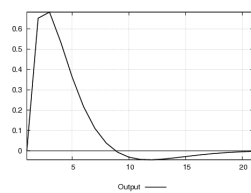
Learning gain is statistically significant. Learning predicts consumption and monetary policy decisions are less responsive to expectations Learning models predict significant inflation persistence.

## Impulse Response Functions

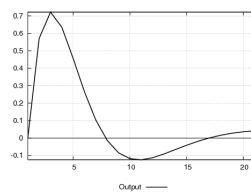
- Does learning influence the impact a structural shock has on the economy?
- Minimal impact from learning occurs when expectations are at the RE steady state.
- Absent of shocks, state variables still evolve when expectations are away from their steady state.
- Methodology:
  - Take expectations at time 2008:Q4.
  - Simulate data with a one standard deviation shock to natural interest rate.
  - Simulate data absent of any shocks.
  - Take the difference.

## IRF: Natural Rate Shock on Output

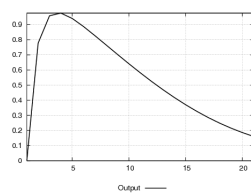
Case 1



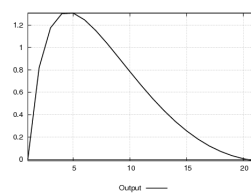
Case 2



Case 3

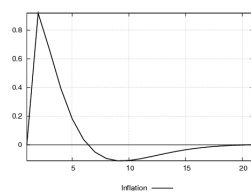


Case 4

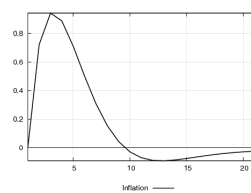


## IRF: Natural Rate Shock on Inflation

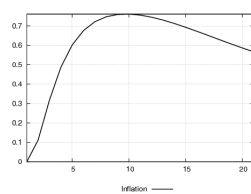
Case 1



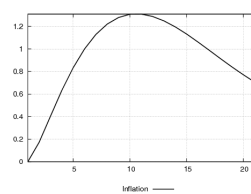
Case 2



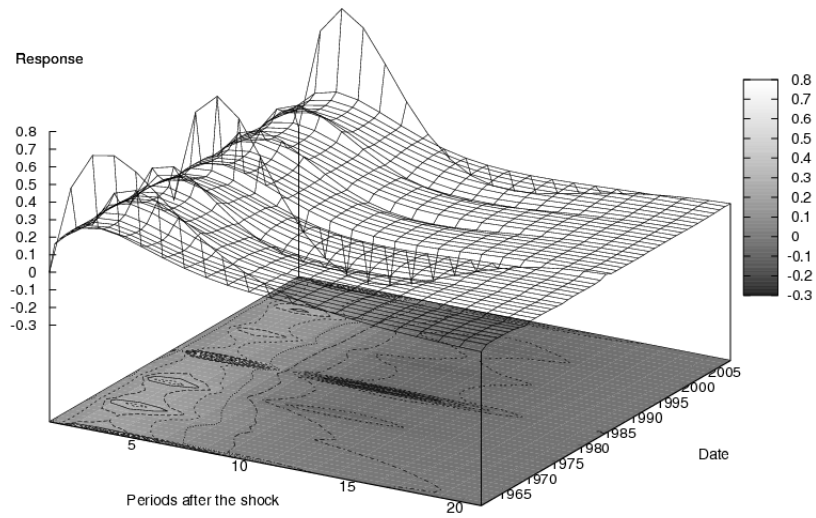
Case 3



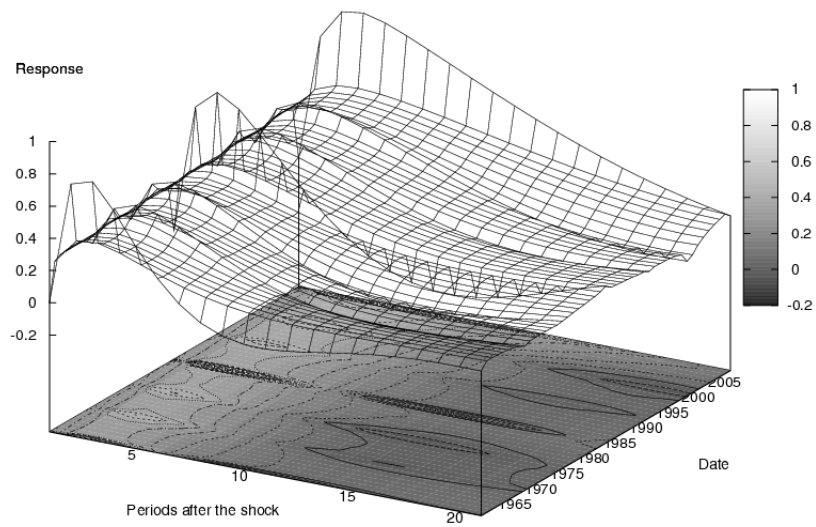
Case 4



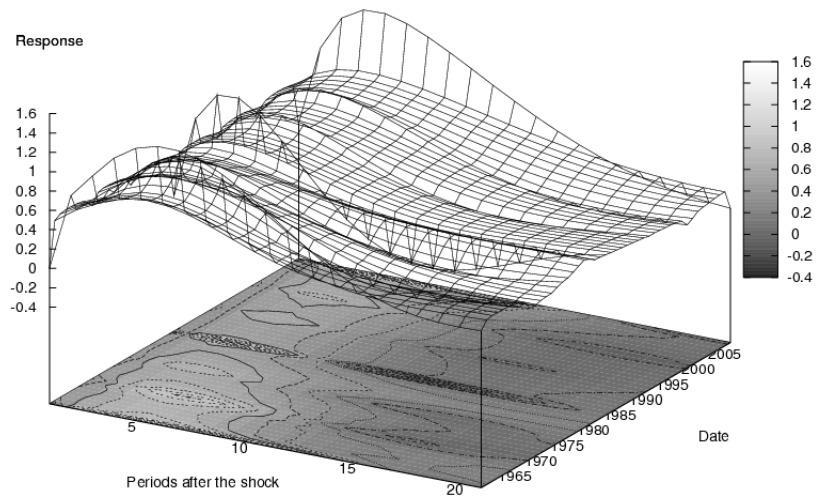
## Case 2: Natural Rate Shock on Output



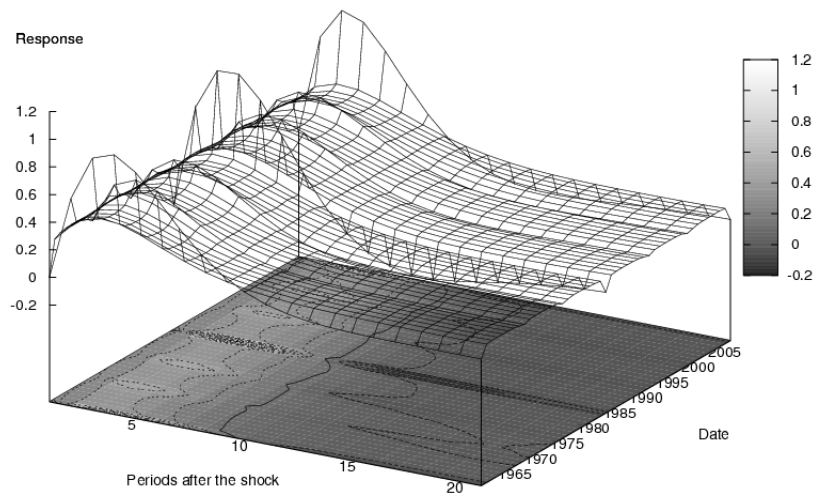
### Case 3: Natural Rate Shock on Output



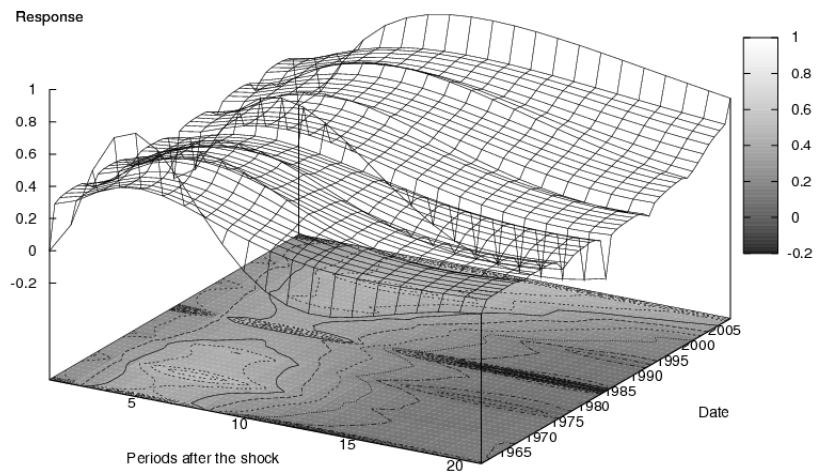
### Case 4: Natural Rate Shock on Output



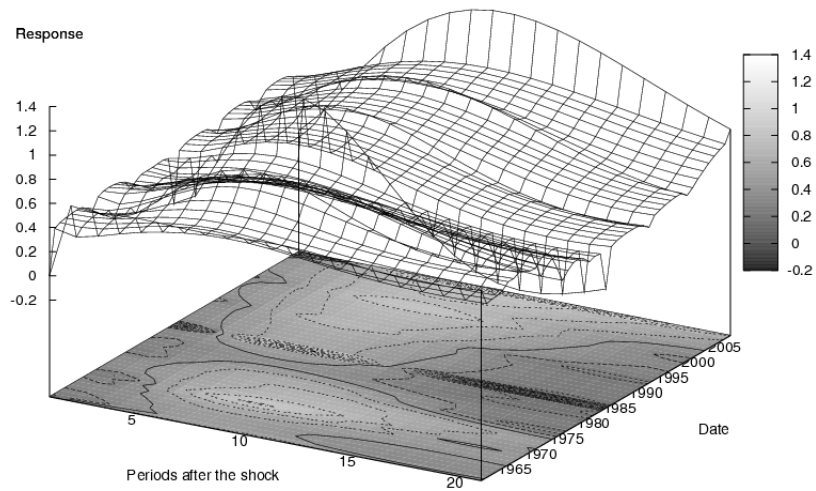
### Case 2: Natural Rate Shock on Inflation



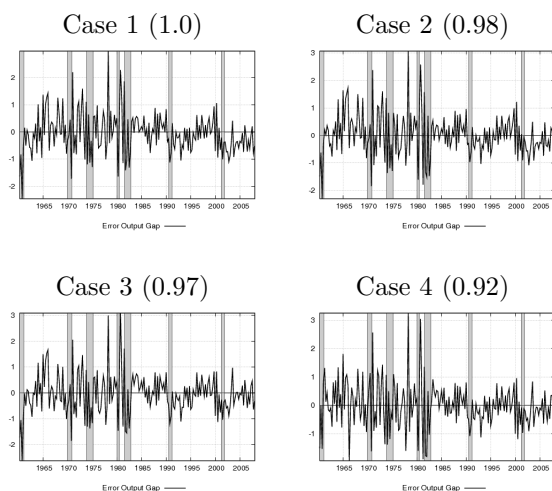
### Case 3: Natural Rate Shock on Inflation



#### Case 4: Natural Rate Shock on Inflation

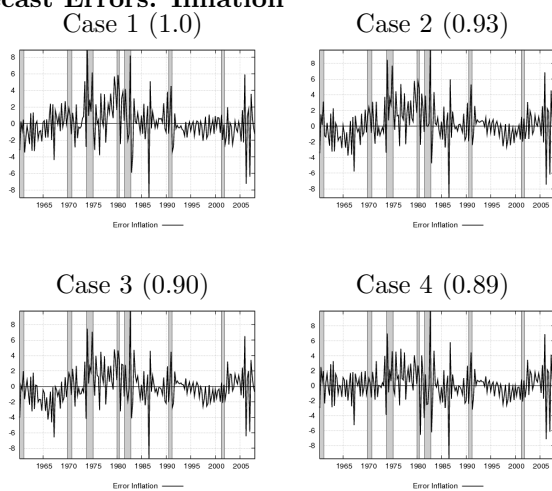


#### Forecast Errors: Output Gap



- (Correlation with Rational Expectations)
- All models made similar errors
- Most volatile during recessions in 1970s, early 1980s

### Forecast Errors: Inflation

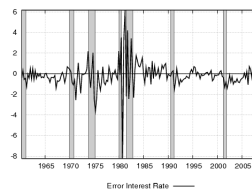


- (Correlation with Rational Expectations)
- All models made similar errors
- Largest errors during recessions in 1970s, early 1980s

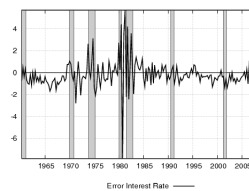


## Forecast Errors: Federal Funds Rate

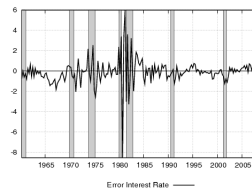
Case 1 (1.0)



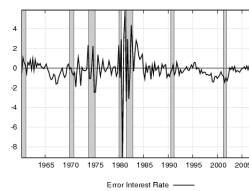
Case 2 (0.95)



Case 3 (0.95)



Case 4 (0.91)



- (Correlation with Rational Expectations)
- All models made similar errors
- Learning *does not explain* change in policy beginning 1979/1980.

## Conclusions

- Learning gain is statistically significant.
- Incorporating learning leads to parameter estimates that imply less sensitivity to expectations.
- Largest errors for every specification still occur during 1970s and early 1980s.
- Learning + Limited information sets leads to prolonged and oscillatory impulse responses.
- 3D Impulse Responses show the United States was more sensitive to shocks following recessions in 1970s, early 1980s, and now.

## Essay 2 Outline

Empirical Significance of Learning in a New Keynesian Model with Firm-Specific Capital

- Examine constant gain learning in a New Keynesian model with Firm-Specific capital.
  - Output used for consumption and investment in capital.
  - Habit formation.

- Calvo (1983) sticky prices.
- Inflation indexation.
- Taylor rule responds to expectations.
- How do initial expectations and assumptions for information sets influence results on...
  - parameter estimates?
  - impact of structural shocks?
  - forecast errors?
- Look at the same four cases for initial conditions and agents' information sets.

## Data

- Pre-sample period: 1953:Q3 - 1969:Q4, Sample period: 1970:Q1-2008:Q1
- Data:
  - Real consumption per capita
  - Real gross private domestic investment per capita
  - CPI inflation
  - Federal Funds rate
- Consumption and Investment is de-trended:

$$CONS_t^* = \frac{CONS_t}{(1 + g_y)^t}, \quad INV_t^* = \frac{INV_t}{(1 + g_y)^t}$$

- $g_y \equiv$  average quarterly growth rate of  $CONS_t + INV_t$ .

## Estimation Procedure

- Maximum Likelihood
- Calibrate parameters:
  - Discount factor  $\beta = 0.9925$ .
  - Steady state inflation  $\pi^* = 3.52$ .
  - Depreciation rate  $\delta = 0.025$ .
  - Steady state level of (de-trended) output set equal to average.
  - Phillips curve slope  $\kappa = 0.1$ .
  - Capital share of income  $\alpha = 0.24$ .

## Parameter Estimates

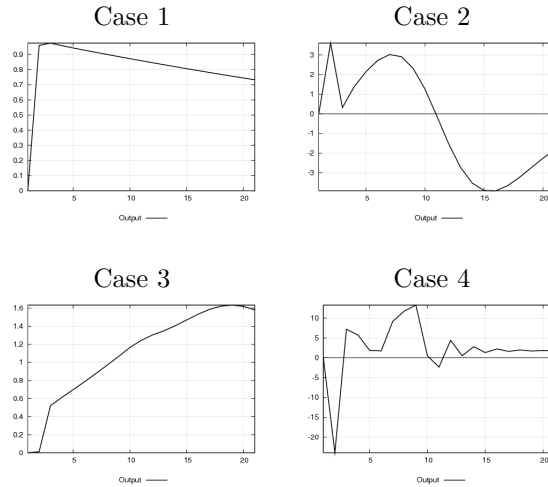
	Parameter	Case 1	Case 2	Case 3	Case 4
$\eta$	Habit Persistence	0.1060 (0.0272)	0.1289 (0.0399)	0.1224 (0.0264)	0.2728 (0.0232)
$\sigma$	IES	0.1603 (0.0342)	0.0513 (0.0072)	0.0157 (0.0001)	0.1220 (0.0175)
$\mu$	Inverse Elas. Labor	30.6713 (8.9559)	0.0499 (0.0771)	2.0877 (0.4286)	0.3324 (0.2817)
$c/y$	C/Y Ratio	0.8339 (0.0038)	0.8672 (0.0024)	0.8361 (0.0000)	0.8289 (0.0028)
$\phi$	Capital Adj. Cost	7.6832 (1.4003)	24.8826 (1.4036)	26.8332 (4.1201)	26.9755 (1.7912)
$\gamma$	Price Indexation	0.3624 (0.1929)	0.0000 (0.0349)	0.5236 (0.0852)	0.6090 (0.1214)
$\rho_r$	MP Persistence	0.1945 (0.0497)	0.6592 (0.0141)	0.7250 (0.0280)	0.0956 (0.1061)
$\psi_y$	MP Output	0.0000 (0.0172)	0.0576 (0.0090)	0.0458 (0.0092)	0.0041 (0.0131)
$\psi_\pi$	MP Inflation	2.1212 (0.1893)	1.4448 (0.0658)	1.7735 (0.1278)	1.8491 (0.0974)
$\rho_\xi$	Pref. Shock Pers.	0.9826 (0.0069)	0.9925 (0.0040)	0.9636 (0.0070)	1.0000 (0.0000)
$\rho_z$	Tech. Shock Pers.	0.9668 (0.0058)	0.6741 (0.0254)	0.9638 (0.0193)	0.9506 (0.0121)
$\rho_\iota$	Inv. Shock Pers.	0.9060 (0.0151)	0.9301 (0.0084)	0.9297 (0.0103)	0.9234 (0.0173)
$\sigma_\xi$	Pref. Shock Std. Dev.	0.0926 (0.0252)	0.1647 (0.0211)	0.6165 (0.0511)	0.6879 (0.0446)
$\sigma_z$	Tech. Shock Std. Dev.	0.0104 (0.0003)	0.0199 (0.0018)	0.0294 (0.0037)	0.0730 (0.0191)
$\sigma_\iota$	Inv. Shock Std. Dev.	0.0206 (0.0021)	0.1130 (0.0149)	0.0387 (0.0016)	0.0683 (0.0069)
$\sigma_r$	MP Shock Std. Dev.	0.0027 (0.0003)	0.0032 (0.0001)	0.0036 (0.0001)	0.0053 (0.0003)
$g$	Learning Gain	—	0.0240 (0.0043)	0.0236 (0.0026)	0.0381 (0.0038)

Learning gain is statistically significant. Learning predicts that consumption de-

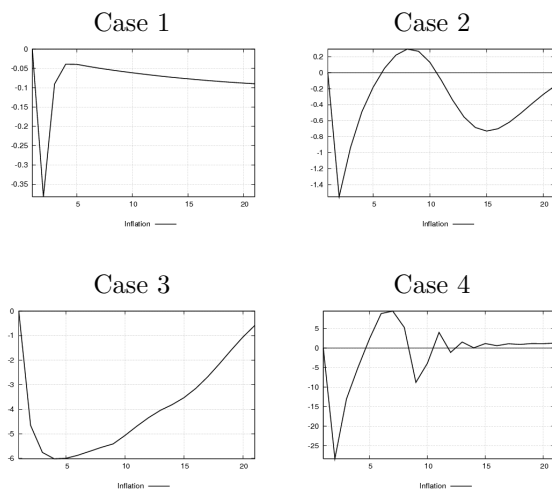
cisions are less responsive to changes in expectations. Learning predicts much

more elastic labor supply, indicating investment decisions less responsive to expectations of future output and capital. Learning predicts larger capital adjustment cost, indicating investment decisions less responsive to expectations of future output and capital.

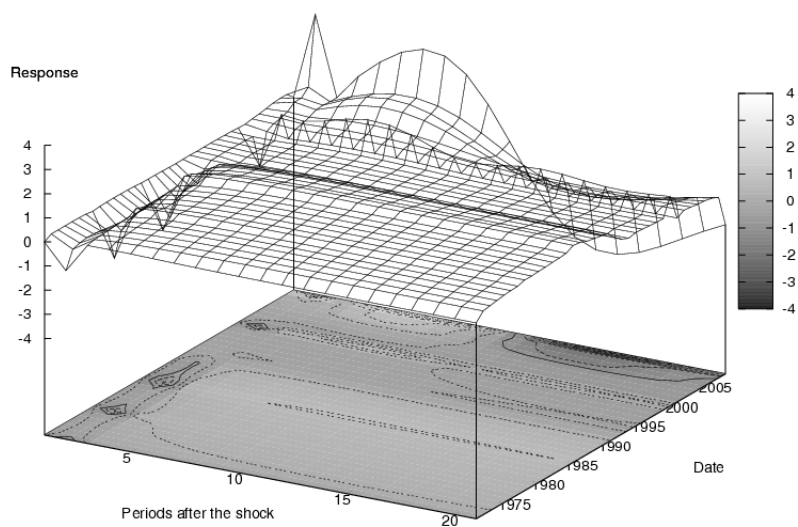
## IRF: Technology Shock on Output



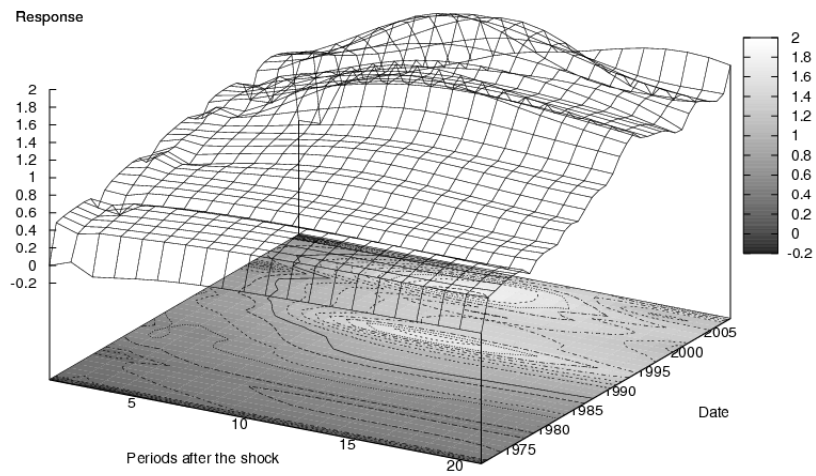
## IRF: Technology Shock on Inflation



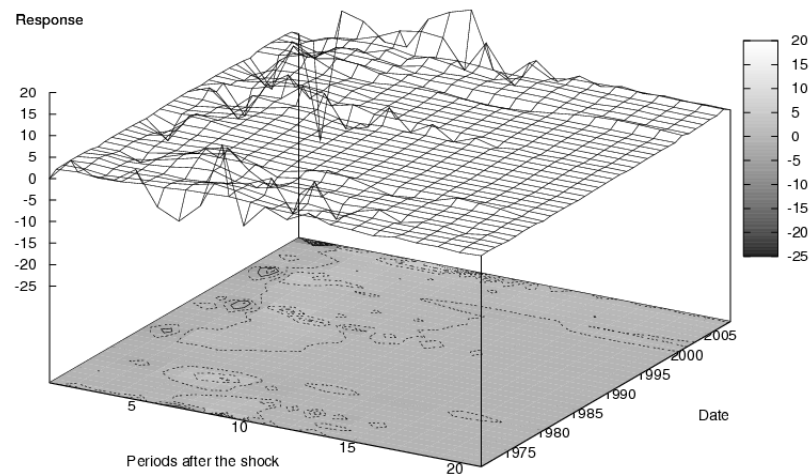
### Case 2: Technology Shock on Output



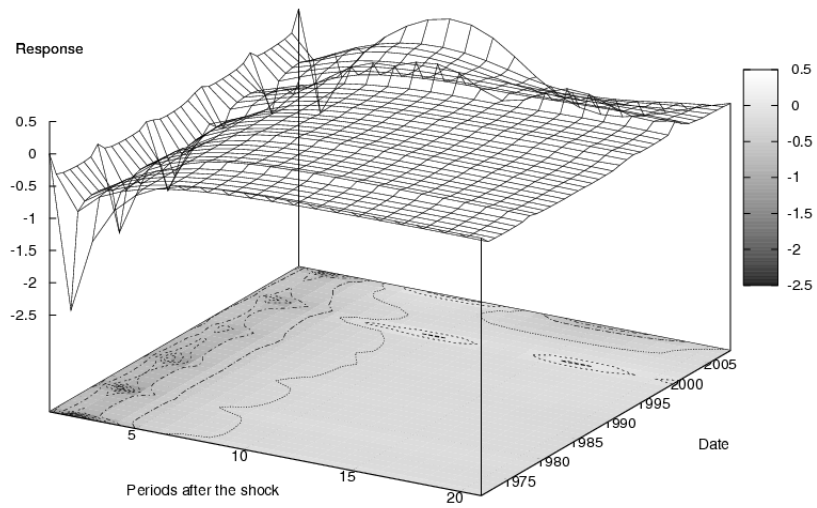
### Case 3: Technology Shock on Output



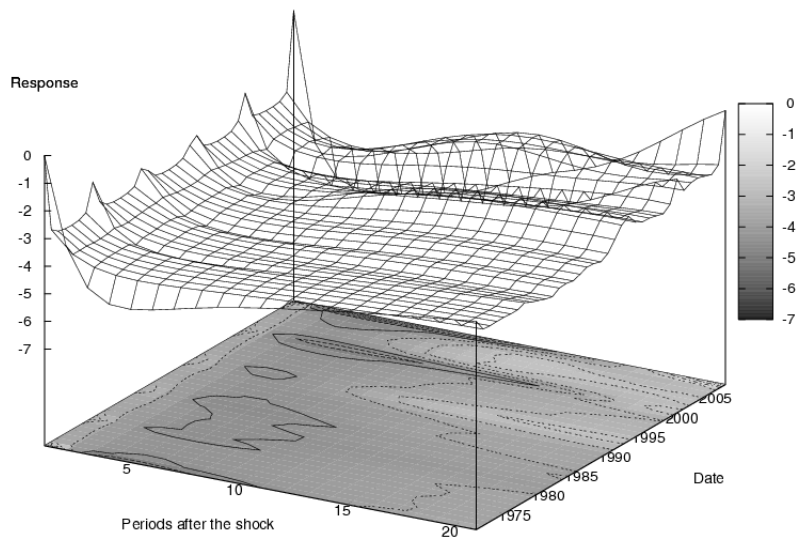
#### Case 4: Technology Shock on Output



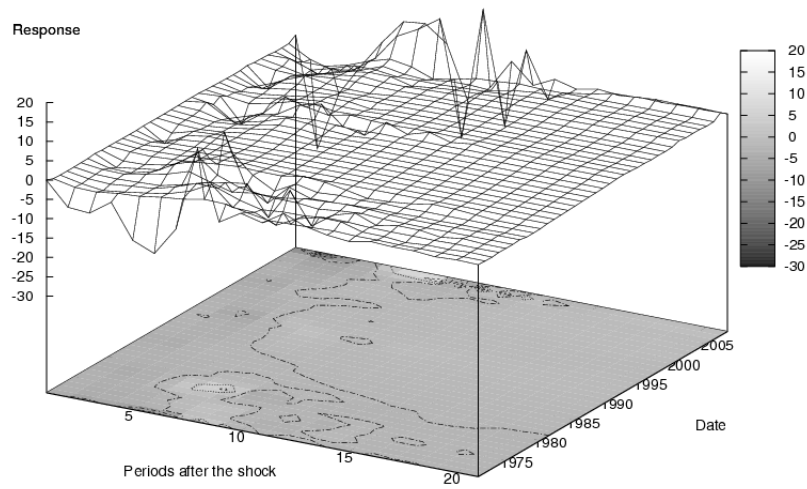
#### Case 2: Technology Shock on Inflation



### Case 3: Technology Shock on Inflation

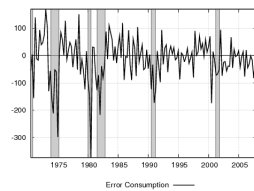


### Case 4: Technology Shock on Inflation

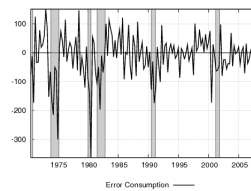


### Forecast Errors: Consumption

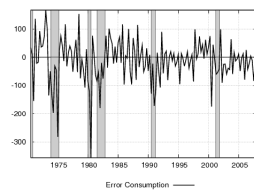
Case 1 (1.0)



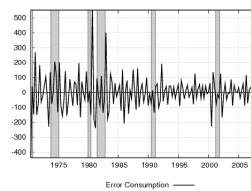
Case 2 (0.95)



Case 3 (0.99)

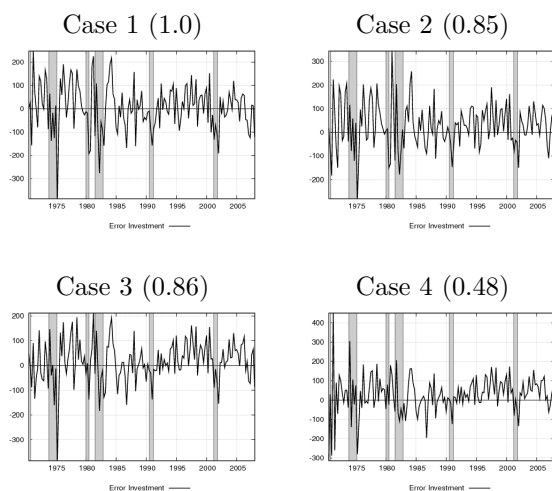


Case 4 (0.70)



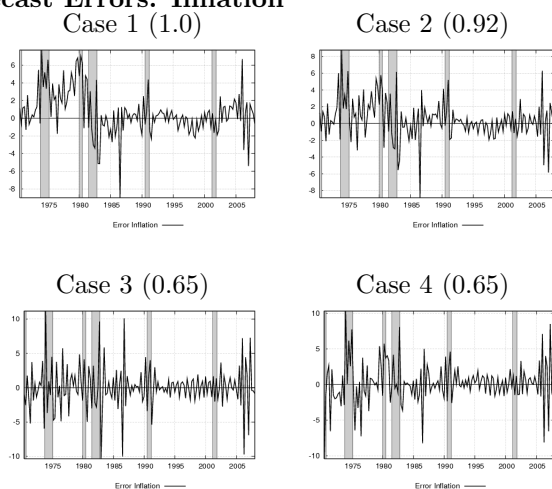
- (Correlation with Rational Expectations)
- All models made similar errors
- Largest errors during recessions in 1970s, early 1980s

### Forecast Errors: Investment



- (Correlation with Rational Expectations)
- Models with RE initial expectations make similar errors
- Large errors before 1985.

### Forecast Errors: Inflation

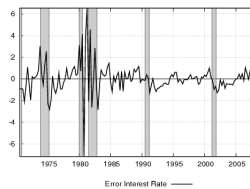


- (Correlation with Rational Expectations)
- Models with limited information somewhat less correlated with RE.
- Largest errors during mid-1970s and early 1980s recessions, and today.

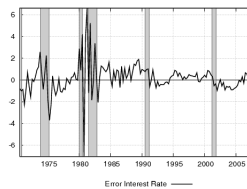


## Forecast Errors: Federal Funds Rate

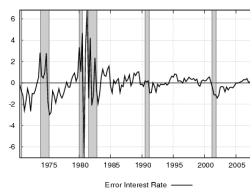
Case 1 (1.0)



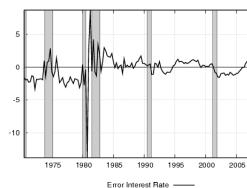
Case 2 (0.95)



Case 3 (0.95)



Case 4 (0.69)



- (Correlation with Rational Expectations)
- All models fail to account for change in policy at beginning of Paul Volcker period.

## Conclusions

- Learning gain is statistically significant.
- Incorporating learning leads to parameter estimates that imply less sensitivity to expectations.
- Not presented: learning models do not significantly out-perform rational expectations in in-sample and out-of-sample forecast error measures.
- Initial conditions and agents' information sets have significant impacts on predicted impulse responses.
- 3D Impulse Responses show the United States was more sensitive to shocks during mid-1980s, early 1990s, and especially today.

## Essay 3

Regime Switching, Learning, and Great Moderation

- Three equation New Keynesian model (same as Essay 1)
- Examine whether dynamic gain learning [Marcet and Nicolini (AER 2003)] can explain time-varying volatility.
  - Agents start using decreasing learning gain consistent with OLS.
  - Agents switch to a high learning if recent forecast errors become larger than historical average.

– Milani (2007) finds evidence this creates ARCH effects.

- Regime-switching volatility.

### Dynamic gain learning

- Learning process:

$$\hat{G}_t^* = \hat{G}_{t-1}^* + g_t(x_{t-1} - \hat{G}_{t-1}^* x_{t-2}^*) x_{t-2}^{*'} R_t^{-1}$$

$$R_t = R_{t-1} + g_t(x_{t-2}^* x_{t-2}^{*'} - R_{t-1})$$

- Learning gain process:

$$g_t^{-1} = \begin{cases} g_{t-1}^{-1} + 1 & \text{if } \frac{1}{J} \sum_{j=1}^J \frac{1}{n} \sum_{v=1}^n |x_{t-j}(v) - \hat{G}_{t-j}^*(v) x_{t-j-1}^*| < \nu_t \\ g^{-1} & \text{otherwise} \end{cases}$$

$$\nu_t = \frac{1}{t-1} \sum_{j=1}^{t-1} \frac{1}{n} \sum_{v=1}^n |x_{t-j}(v) - \hat{G}_{t-j}^*(v) x_{t-j-1}^*|$$

### Volatility Switching

- Variances of natural rate, cost push, and monetary policy shocks switching according to a Markov chain.

$$Var[\epsilon_t(s_t)] = \begin{cases} \begin{bmatrix} \sigma_{n,L}^2 & 0 & 0 \\ 0 & \sigma_{u,L}^2 & 0 \\ 0 & 0 & \sigma_{r,L}^2 \end{bmatrix}, & \text{if } s_t = L \\ \begin{bmatrix} \sigma_{n,H}^2 & 0 & 0 \\ 0 & \sigma_{u,H}^2 & 0 \\ 0 & 0 & \sigma_{r,H}^2 \end{bmatrix}, & \text{if } s_t = H \end{cases}$$

- $\sigma_{n,H}^2 \geq \sigma_{n,L}^2$ ,  $\sigma_{u,H}^2 \geq \sigma_{u,L}^2$ ,  $\sigma_{r,H}^2 \geq \sigma_{r,L}^2$
- Transition probabilities:  $P(s_t = H | s_{t-1} = H) = p_H$ ,  $P(s_t = L | s_{t-1} = L) = p_L$

### Estimation Procedure

- Maximum Likelihood
- Data: output gap, CPI inflation rate, federal funds rate.
- Pre-sample period: 1954:Q3 - 1959:Q4. Sample period: 1960:Q1 - 2008:Q1.
- Expectations are initialized to pre-sample VAR(1) results.
- Calibrate: discount factor  $\beta = 0.9925$ .

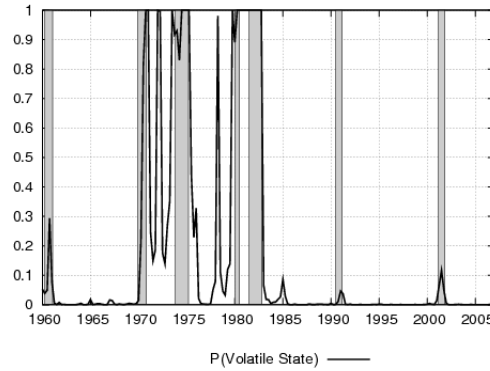
## Parameter Estimates

	Parameter	Rational Expectations	Dynamic Gain	Constant Gain
$\eta$	Habit Formation	0.3643 (0.0478)	0.2580 (0.0308)	0.3659 (0.0288)
$\sigma$	IES	0.0073 (0.0154)	0.2560 (0.1171)	0.1824 (0.1140)
$\mu$	Elas. Labor	0.0000 (40.9507)	0.3219 (2.2075)	0.0001 (5.0920)
$\kappa$	Phillips Coefficient	0.0011 (0.0186)	0.0237 (0.0256)	0.0054 (0.0146)
$\gamma$	Price Indexation	0.8945 (0.0330)	0.9849 (0.1926)	0.9990 (0.0004)
$\rho_r$	MP Persistence	0.9355 (0.0289)	0.9234 (0.0084)	0.9196 (0.0092)
$\psi_y$	MP Output	0.2507 (0.0498)	0.1878 (0.0367)	0.2758 (0.0425)
$\psi_\pi$	MP Inflation	1.9577 (0.2591)	1.7363 (0.1687)	1.6354 (0.1189)
$\rho_n$	Nat. Rate Pers.	0.8705 (0.0353)	0.7484 (0.0267)	0.6936 (0.0272)
$\rho_u$	Cost Push Pers.	0.0000 (0.0000)	0.0062 (0.0376)	0.0031 (0.0085)
$\pi_*$	SS Inflation	3.5446 (0.2808)	4.4419 (0.2220)	5.3272 (0.2825)
$\sigma_{n,L}$	Nat. Rate (Low)	0.1768 (0.3720)	0.0454 (0.0217)	0.0931 (0.0572)
$\sigma_{u,L}$	Cost Push (Low)	0.0023 (0.0001)	0.0045 (0.0004)	0.0042 (0.0001)
$\sigma_{r,L}$	MP Shock (Low)	0.0013 (0.0001)	0.0012 (0.0000)	0.0012 (0.0000)
$\sigma_{n,H}$	Nat. Rate (High)	0.4295 (0.9056)	0.0966 (0.0485)	0.1794 (0.1144)
$\sigma_{u,H}$	Cost Push (High)	0.0044 (0.0004)	0.0092 (0.0010)	0.0085 (0.0005)
$\sigma_{r,H}$	MP Shock (High)	0.0070 (0.0005)	0.0064 (0.0003)	0.0056 (0.0002)
$p_L$	P(Remain Low)	0.9609 (0.0224)	0.9724 (0.0097)	0.9780 (0.0109)
$p_H$	P(Remain High)	0.8099 (0.0578)	0.8924 (0.0264)	0.9412 (0.0159)
$g$	Learning Gain	—	0.0045 (0.0007)	0.0000 (0.0018)

Expectations are not adaptive. Regimes are highly persistent. Learning predicts smaller variances of the natural rate shock. Variances of cost push and monetary shock are similar.

## Regime-Switching Volatility

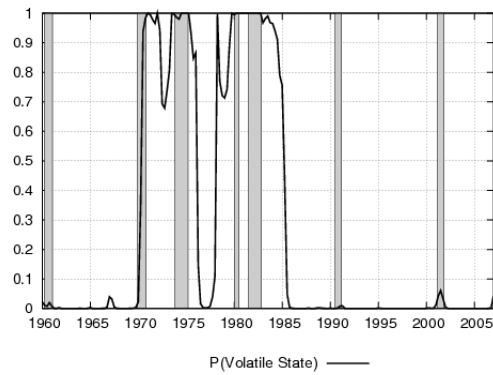
Rational Expectations Probability Economy is in the Volatile Regime



Expected 7.77 volatile years

## Regime-Switching Volatility

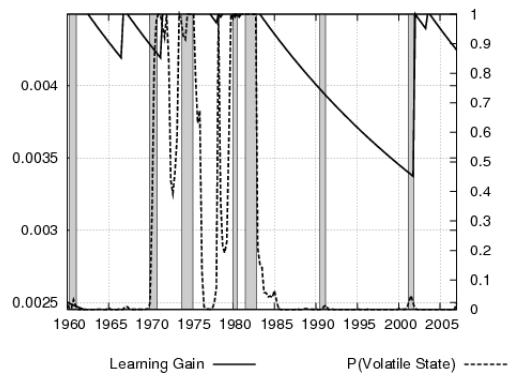
Constant Gain Learning Probability Economy is in the Volatile Regime



Expected 12.26 volatile years

### Regime-Switching Volatility

Dynamic Gain Learning Probability Economy is in the Volatile Regime and



Evolution of the Learning Gain

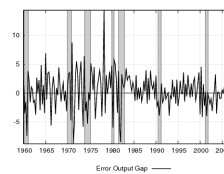
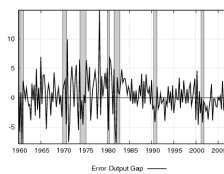
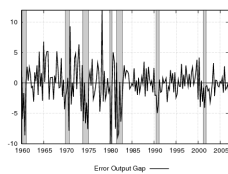
Expected 9.17 volatile years

### Forecast Errors: Output Gap

Rational Exp. (1.0)

Constant Gain (0.86)

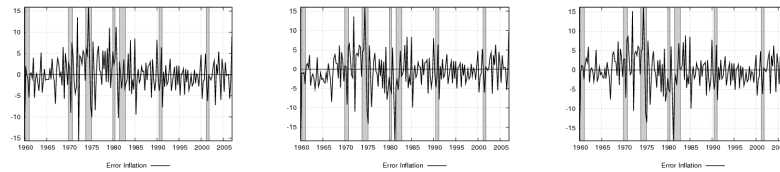
Dynamic Gain (0.82)



- (Correlation with Rational Expectations)
- All models made similar errors
- Most volatile during recessions in 1970s, early 1980s

### Forecast Errors: Inflation

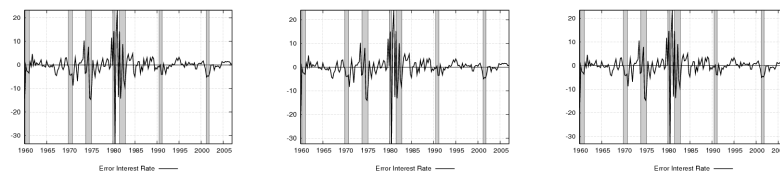
Rational Exp. (1.0)    Constant Gain (0.85)    Dynamic Gain (0.80)



- (Correlation with Rational Expectations)
- All models made similar errors.
- Most volatile during recessions in 1970s, early 1980s.

### Forecast Errors: Federal Funds Rate

Rational Exp. (1.0)    Constant Gain (0.99)    Dynamic Gain (0.99)



- (Correlation with Rational Expectations)
- Essentially identical errors.
- Do not explain change in policy in early 1980s.

### Conclusions

- When allowing for regime-switching volatility, there is little evidence of adaptive expectations.
- Constant gain learning and dynamic gain learning both produce less volatility for the natural rate shock.
- Learning frameworks actually deliver a higher prediction for the time spent in volatile regime.
- All models make similar forecast errors at similar points in sample.
- Not presented: the rational expectations model actually yields smallest in-sample MSE.