Regime Switching in Fiscal Debt Targets and Fiscal Composition

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Describe fiscal policy dynamics

Government expenditures
Income tax rate
Net transfer payments
Deficits
Debt

Describe debt service

1. How do these fiscal policy variables respond to *debt / GDP*?
2. What is the implied target for *debt / GDP*?
3. Is there switching in these fiscal policy responses?
4. Is there switching in the long-run debt target?

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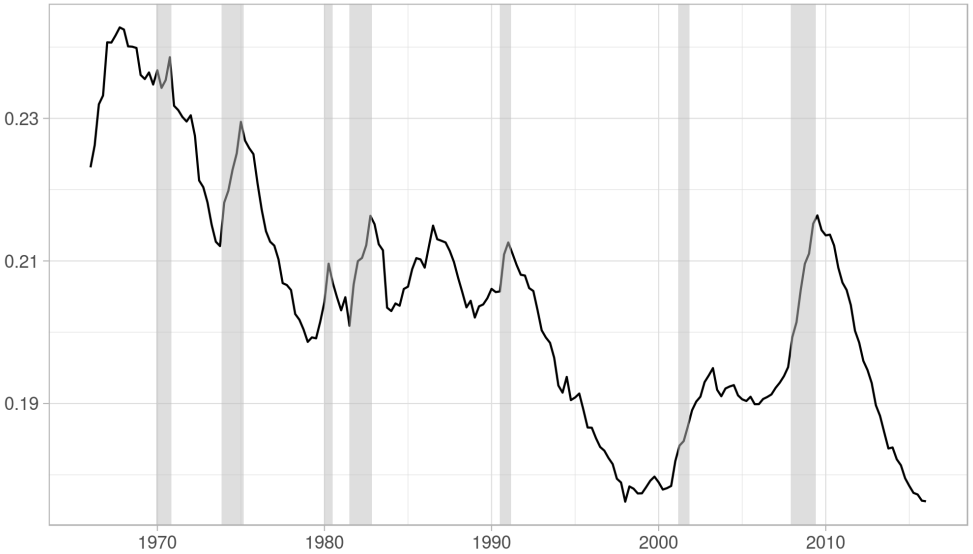
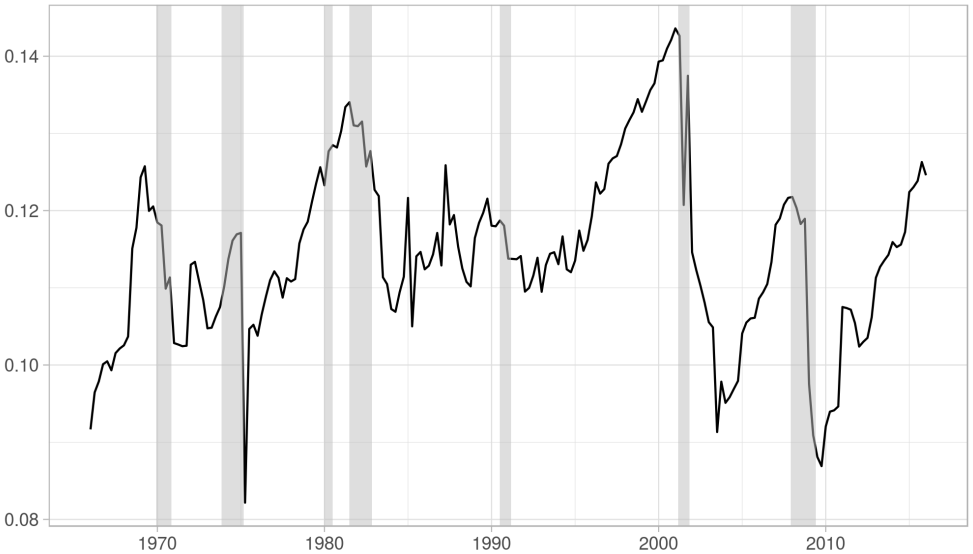
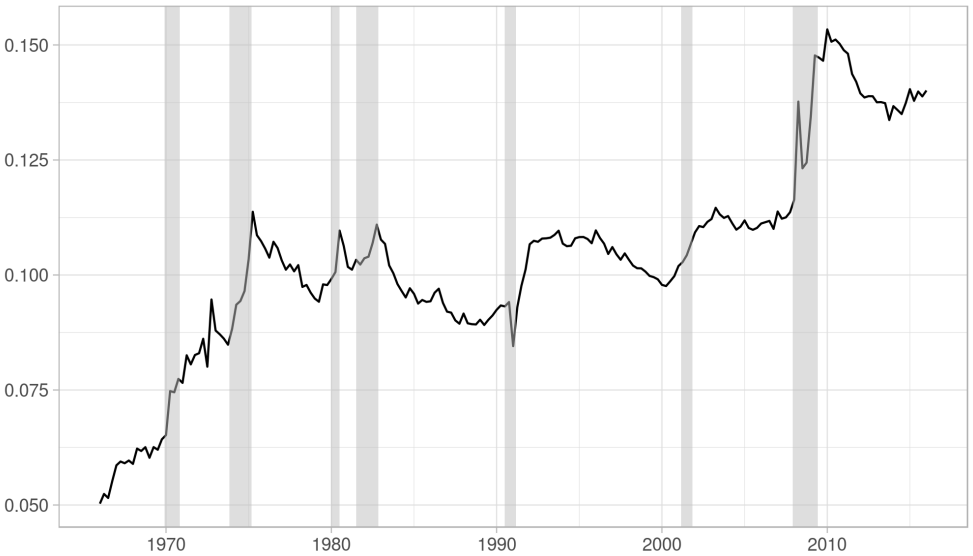


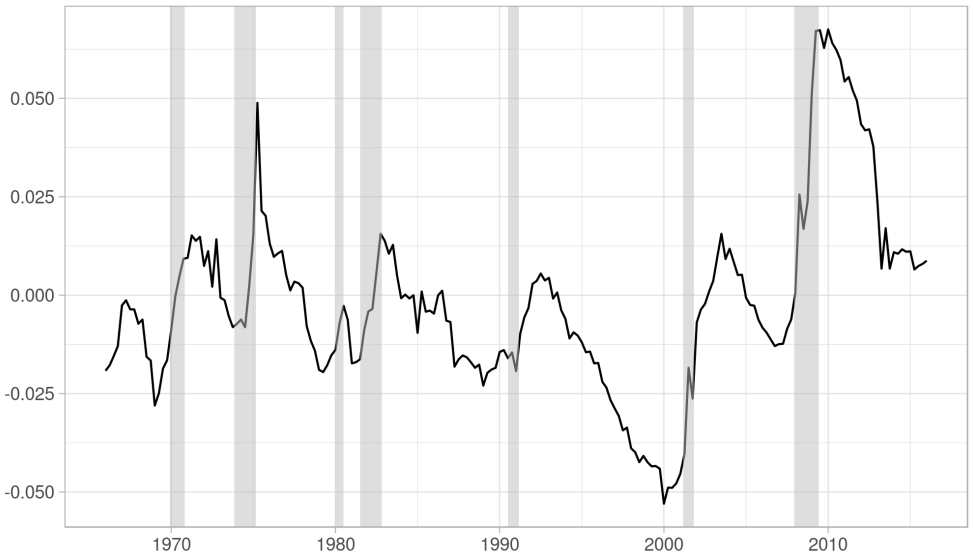
Government Spending / GDP Ratio



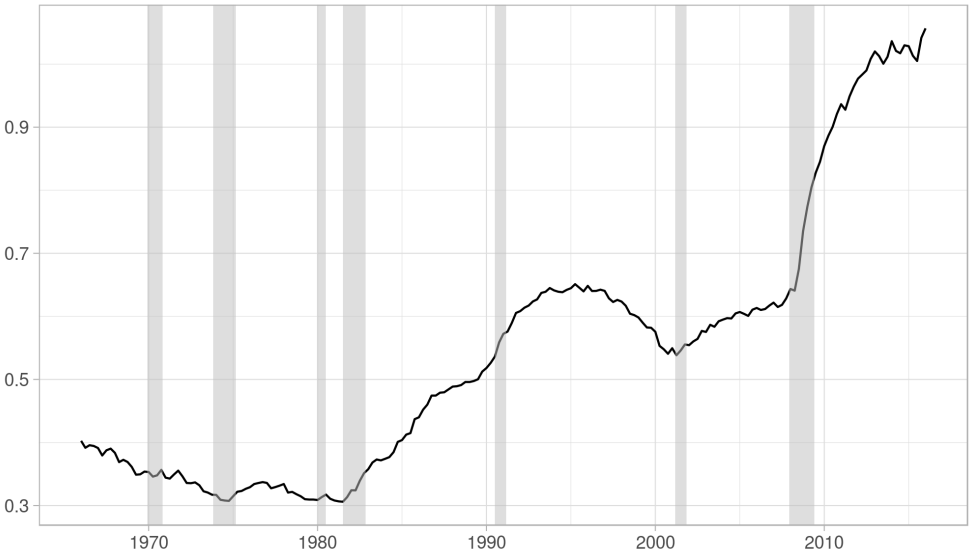
Federal Tax Revenue / GDP Ratio



Transfers / GDP Ratio



Deficit / GDP Ratio



Debt / GDP Ratio

Debt target and tax response matter

- Expected smaller debt/GDP target and/or expected larger response of taxes to debt,
  $\rightarrow$ Higher expected income taxes
  $\rightarrow$ lower consumption, investment, real GDP.
- Richter and Throckmorton (EER, 2015):
  - Unknown debt targets amplify impact of tax shocks
  - Uncertain long-run debt targets reduced impact of ARRA, extensions to Bush tax cut

Fiscal composition matters

Leeper, Plante, and Traum (JoE, 2010)

- Rich set of fiscal variables responding to debt fits data best
- Magnitude of fiscal shocks depend on composition
- Fiscal multipliers can have unexpected signs, depending on composition

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- Favero and Montecelli (2005): Deficit feedback rule with *Markov switching*
  - Switching explains data better
  - Deficits switch between *active* and *passive* regimes
- Ko and Morita (2013): Switching in government expenditures and taxes in Japan

Switching and Monetary/Fiscal Interactions

- Eg: Chung, Davig, Leeper (2007), Davig and Leeper (2011)
- Evidence for switching between active/passive fiscal and monetary policies
- Implications for fiscal multipliers and stabilizing impact of monetary policy

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Gradual movement toward target

$$G_t = \rho_g G_{t-1} + (1 - \rho_g) G_t^*,$$

- $\rho_g \in (0, 1)$ persistence parameter
- G_t: Actual nominal government expenditures
- G_t^*: Target level for government expenditures

Divide by nominal GDP (Y_t)

$$g_t = \rho_g \left(\frac{1}{y_t}\right) g_{t-1} + (1 - \rho_g) g_t^*$$

- $g_t \equiv G_t/Y_t$, $g_t^* \equiv G_t^*/Y_t$: Actual / Target government expenditures to GDP ratio
- $y_t \equiv Y_t/Y_{t-1}$: Gross growth rate of nominal GDP

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Target Policy Behavior

- Use government expenditures to stabilize business cycle
  $\rightarrow$ Decrease gov exp in response to output gap
- Decrease government expenditures in response to rising debt
- Long-run target for government expenditures / GDP ratio

Structure

$$g_t^* = \bar{g}(s_t) + \psi_g(s_t)x_t + \gamma_g(s_t)\left[b_{t-1} - \bar{b}(s_t)\right] + u_{g,t},$$

- $s_t \in \{1, .., M\}$: Fiscal regime... more later
- $\bar{g}(s_t)$: Long-run government expenditures / GDP target
- b_{t-1}: Lagged government debt / GDP ratio
- $\bar{b}(s_t)$ Long-run target debt / GDP ratio
- $\psi_g(s_t) < 0$: Response to increase in output gap
- $\gamma_g(s_t) < 0$: Response to increase in government debt

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Fiscal policy variables

$$f_t \in \left\{ \begin{array}{ll} g_t: \text{ Gov exp / GDP}, & n_t: \text{ Net transfers / GDP} \\ \tau_t: \text{ Taxes / GDP}, & d_t: \text{ Deficits / GDP} \end{array} \right\}$$

Similar evolution for all fiscal variables

$$f_t^* = \bar{f}(s_t) + \psi_f(s_t) x_t + \gamma_f(s_t) \left[b_{t-1} - \bar{b}(s_t) \right] + u_{f,t},$$

$$u_{f,t} = \alpha_f u_{f,t-1} + \epsilon_{f,t}, \ \epsilon_{f,t} \sim \mathcal{N}\left(0, \sigma_f^2(s_t)\right)$$

Notation

f_t	Fiscal variable	x_t	Output gap
f_t^*	Time t target for f_t	ρ_f	Persistence of f_t
$\bar{f}(s_t)$	*Long-run* target for f_t	$\psi_f(s_t)$	Feedback on output gap
b_t	Debt / GDP ratio	$\gamma_f(s_t)$	Feedback on debt/GDP
$\bar{b}(s_t)$	*Long-run* target for debt/GDP	$u_{f,t}$	Innovations to f_t

Intertemporal government budget constraint

$$B_t = (1 + r_{t-1})B_{t-1} + D_t - (M_t - M_{t-1}),$$

B_t: Nominal government debt
r_{t-1}: interest rate on past-issued debt
D_t: Nominal budget deficit
$M_t - M_{t-1}$: seigniorage

Empirical government budget constraint

Divide both sides by Y_t and allow for measurement error (v_t)

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$$u_{b,t} = (1 - \alpha_b)\bar{u}_b + \alpha_b u_{b,t-1} + e_{b,t}$$

Budget constraint

- Budget constraint describes relationship between long-run targets for...
  (1) Debt / GDP, $\bar{b}(s_t)$, and
  (2) deficits / GDP, $\bar{d}(s_t)$
- Evaluate budget constraint at steady state and a constant fiscal regime $s_{t-1} = s_t = s$:

$$\bar{d}(s) = \left(\frac{\bar{y} - \bar{r} - 1}{\bar{y}} \right) \bar{b}(s) - \bar{u}_b$$

Long-run deficit dependencies

Debt target
Long-run nominal interest rate
Long-run nominal GDP growth
Long-run seigniorage

Jointly estimate these long-run targets

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Regime-dependent variances for fiscal shocks

$\sigma_g^2(s_t)$: Var(shock to gov exp)
$\sigma_\tau(s_t)$: Var(shock to taxes)
σ_n^2: Var(shock to transfers)
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Correlations of fiscal shocks

- Fiscal policy decisions are dependent on one another.
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Three Sources for Regime Switching

Long-run Debt Target Regimes

Regime L: *Low* long-run target for debt/GDP (low value for $\bar{b}(s_t)$)

Regime H: *High* long-run target for debt/GDP (high value for $\bar{b}(s_t)$)

Fiscal Financing

- Targets for fiscal components: $\bar{g}(s_t)$, $\bar{\tau}(s_t)$, $\bar{n}(s_t)$, $\bar{d}(s_t)$
- Behavior toward output gap and debt: $\psi_f(s_t)$ and $\gamma_f(s_t)$, for $f \in \{g, \tau, n, d\}$

Regime 1: Fiscal behavior 1

Regime 2: Fiscal behavior 2

Fiscal Volatility

Two regimes to determine variances, $\sigma_g^2(s_t)$, $\sigma_\tau^2(s_t)$, $\sigma_n^2(s_t)$, and $\sigma_d^2(s_t)$:

Regime S: *Stable*, relatively smaller variances

Regime V: *Volatile*, relatively larger variances

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Possible Regimes

Eight regime combinations are possible!

(L) Low debt,	(1) Financing Regime 1,	(S) Low volatility
(H) High debt,	(1) Financing Regime 1,	(S) Low volatility
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Regime matters. Suppose taxes increased. Was this..

- one time shock, no change in regime?
- change in long-run target for debt/GDP?
- change in long-run target for taxes/GDP?
- change in policy using taxes more heavily to repay debt
- change in policy where taxes are more/less response to output gap
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Describe Regimes

- How large is debt/GDP ratio in each regime?
- Describe how fiscal financing regimes are different.
- How large are differences in volatility regimes.

Identify Time Periods

For each quarter over 1966-2016, identify probabilities that fiscal policy was in each regime.

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Markov regime switching

Action to remain/switch determined randomly

$$P(s_t = 1) = p_1 \ \mathbb{1}(s_{t-1} = 1) + (1 - p_2) \ \mathbb{1}(s_{t-1} = 2)$$

$$P(s_t = 2) = (1 - p_1) \ \mathbb{1}(s_{t-1} = 1) + p_2 \ \mathbb{1}(s_{t-1} = 2)$$

- $p_1 = P(s_t = 1 | s_{t-1} = 1)$ be prob policy remains in reg 1
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Independent sources of regime switching

- Same *independent* regime switching procedure for each source
- Independent regime switching allows for rich possibilities:
  - Changes in priorities for taxes, transfers, spending, *without adjusting long-run targets for debt/GDP*
  - Changes in debt-targets, *without adjusting purposes and priorities for fiscal components*
  - Changes in volatility of fiscal outcomes, *without changing goals or purposes*

Regime Switching Procedure

Markov regime switching

Action to remain/switch determined randomly

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Loose ends

- Relationship between $\bar{d}(s_t)$ and $\bar{b}(s_t)$ depends on...
  - long-run values for nominal GDP growth ($\bar{y}$)
  - long-run average interest rate ($\bar{r}$)
- Identify effects of output gap on fiscal policy behavior *from effects of fiscal policy actions on output gap*.

Next steps

- Specify monetary policy
- Specify inter-dependent behavior of macro variables: GDP growth, output gap, and inflation

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Taylor-like (1993) rule

$$r_t = (1-\rho_r)\bar{r} + \rho_r r_{t-1} + (1-\rho_r)\left[\phi_x x_t + \phi_\pi\left(\pi_t - \bar{\pi}\right)\right] + u_{r,t},$$

- $\bar{r}$: long-run nominal interest rate
- ρ_r: Monetary policy persistence
- $\phi_x > 0$: Response to output gap
- $\phi_\pi > 0$: Response to inflation
- π_t: inflation rate
- $\bar{\pi}$: target inflation rate
- x_t: output gap
- $u_{r,t}$: shock to monetary policy

Policy shock

$$u_{r,t} = \alpha_r u_{r,t-1} + e_{r,t}, \quad e_{r,t} \sim \mathcal{N}\left(0, \sigma_r^2\right)$$

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Dependent variables

Augmented vector autoregression for...

1. nominal GDP growth, y_t,
2. output gap, x_t,
3. inflation, π_t

Explanatory variables

- One lag of all dependent variables: y_{t-1}, x_{t-1}, π_{t-1}
- Fiscal policy variables: g_t, τ_t, n_t
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Estimation Outcomes

- Long-run values for $\bar{y}$ and $\bar{r}$
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Fiscal policy variables

1. Nominal government expenditures: NIPA Table 1.1.5, Line 22
2. Tax revenue: NIPA Table 3.2, Line 3
3. Net transfers: Federal current transfer pmts - receipts
   - NIPA Table 3.2, (Line 25 - Line 18)
4. Primary budget deficit:
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State equation with regime switching:

$$\xi_t = h^*(s_t) + F^*(s_t)\xi_{t-1} + M^* e_t, \quad e_t \sim \mathcal{N}(0, Q(s_t))$$

State Vector: ξ_t	Stochastic vector: e_t
g_t and g_t^*	$e_{g,t}$
τ_t and τ_t^*	$e_{\tau,t}$
n_t and n_t^*	$e_{n,t}$
d_t	$e_{d,t}$
b_t and b_t^*	$e_{b,t}$
y_t,	$e_{y,t}$
x_t,	$e_{x,t}$
pi_t,	$e_{\pi,t}$
r_t	$e_{r,t}$
AR(1) shocks	

Observation equation: Indicator matrix picking off observable values in state vector

Kalman filter

- Kalman filter: iterative procedure that approximates values in the state vector over sample period, *given a set of parameters* and given observable variables.
- *Constant coefficients, no regime changing*
- Likelihood function: Probability distribution describing likelihood of observed data, *given parameters*.

Kim filter

- Kim and Nelson (1999): Extend Kalman filter to make updates on regime switching
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Bayesian estimation

- *Beliefs* on parameters have distributions
- *Prior distribution:* Beliefs before taking the model to the data
- *Posterior distribution:* (i.e. the estimation results), updated beliefs on the parameters after taking the model to the data

Prior distributions

- Impose $(0, 1)$ intervals for a number of parameters (persistence, fiscal components ratio to GDP, et al.)
- Impose sign restrictions on some parameters, eg:
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  - Monetary policy responses to output gap
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Endogoneity problem: two-way causation

- *Ceteris paribus*, an increase in output gap leads to higher taxes (captured by parameter $\psi_\tau(s_t)$ in fiscal policy equation)
- *Ceteris paribus*, an increase in taxes leads lower aggregate demand and therefore a lower output gap (captured by coef in augmented VAR for x_t)

Sign restrictions

- Faust (1998), Canova and De Nicolo (2002), and Uhlig (2005)
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- Impulse response function examples:
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  - Response = time path of response to tax revenue
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- Require some responses be non-negative or non-positive

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Example Sign Restrictions

Impulse = single shock to output gap

- Responses = resulting time paths for output gap, tax revenue
- Output gap response should be positive
- Tax revenue response should be positive

Impulse = single shock to tax

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Time frame

Impose sign restriction over *4 quarters* of responses, including shock period

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Fiscal policy sign restrictions

	Impulse Variable				
Response	Gov Exp	Taxes	Transfers	Deficit	Output gap
Output gap	positive	negative	positive	positive	positive
Output growth	positive	negative	positive	positive	positive
Gov exp	positive	(none)	(none)	(none)	negative
Taxes	(none)	positive	(none)	(none)	positive
Transfers	(none)	(none)	positive	(none)	negative
Deficits	(none)	(none)	(none)	positive	negative

Monetary policy sign restrictions

	Impulse Variable		
Response	Interest rate	Output gap	Inflation
Output gap	negative	positive	(none)
Output growth	negative	positive	(none)
Inflation	negative	positive	(none)
Interest rate	positive	positive	positive

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Deficits	(none)	(none)	(none)	positive	negative

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Goals of this paper

- Answer: Is there evidence of differences in regimes in multiple dimensions (debt, fiscal composition, volatility)?
- Identify periods in U.S. history with different debt targets, fiscal goals, volatility
- Describe nature of changing fiscal composition (changes in priorities & goals for gov exp, taxes, transfers)
- Describe nature of regime-switching sources overlapping
- Describe and illustrate (with IRFS) inter-dependence of fiscal policies

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