Learning with Judgment Shocks in the New Keynesian Model

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Abstract

This paper examines the role of judgment shocks in combination with other structural shocks in explaining post-war economic volatility within the context of a New Keynesian model. Agents form expectations using constant gain learning then augment these forecasts with judgment. These judgments may be interpreted as a reaction to current news stories or policy announcements that would influence people's expectations. I allow for the possibility that these judgments be informatively based on information about structural shocks, but judgment itself may also be subject to its own stochastic shocks. I estimate a standard New Keynesian model that includes these shocks using Bayesian simulation methods. To aid in identifying expectational shocks from other structural shocks I include data on professional forecasts along with data on output gap, inflation, and interest rates.

Keywords: Learning, add-factors, New Keynesian model, Metropolis-Hastings.
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1 Introduction

Rational expectations with full information about quantities of relevant state variables and stochastic shocks is the most common assumption in research that uses dynamic models of the macroeconomy. The assumption makes solving, evaluating, and estimating these models possible with standard tools (albeit, still rather sophisticated), but the informational requirements and information processing requirements behind the assumption are rather extreme. Least squares learning is a type of non-rational, adaptive expectations that attempts to use more realistic forecasting methods within the context of macroeconomic models to understand macroeconomic dynamics. In such a framework, economic agents gather past data and use simple least squares time series techniques to form their expectations for future outcomes before making forward looking decisions (a feasible and rather simple statistical exercise).

One drawback of least squares learning is that expectations are based only on collections of past data that are passed through some statistical procedure. Forward looking decisions might also be well guided by relevant current events that have not yet made themselves evident in historical data. Examples of such events may include a change in political landscape, the passing of a new law, a natural disaster, news of a recent technological development, or changes in the economic or political landscape of major trading partners, just to name a few. As soon as such events are made known through the media, optimizing economic agents would do well to immediately change their expectations and decisions accordingly. One could argue that rational expectations captures this realistic component of expectations formation that learning does not. Typically in dynamic stochastic general equilibrium (DSGE) models with rational expectations, exact values for current stochastic shocks are known before expectations are made. We might view the values of these shocks as precisely quantified impacts of the current events above.

It might also happen that current events are misinterpreted, the quantified impact is misjudged, or that judgment is otherwise misinformed and not based on actual events. One example of an exaggerated news story in recent U.S. history might be the Y2K computer bug widely discussed in the late 1990s. The September 11, 2001, terrorist attacks may be an example of a very real event, but whose implications to economic activity may have been overestimated. Rational expectations cannot account for such misinterpretations of news of this type.

Judgment based on information in the news that is impractical or impossible to quantify may therefore benefit optimal decision making, be detrimental, or more probably a combination of both. I examine within the context of a standard stochastic New Keynesian model expectations that are formed by least squares learning forecasts and are then augmented by judgment. The least squares forecasts incorporates historical data on the output gap, inflation rate, and the federal funds rate, data that can be readily obtained and which are informative for expectations within the context of the New Keynesian model. If expectations were rational and agents had full information, they would also use current realizations of structural shocks in forming expectations. In the learning environment with judgment, values for structural shocks cannot be obtained or estimated, but judgments based on news and current events may incorporate some of this information. Judgment may also be subject to its own stochastic shocks that are independent to all other shocks and state variables. This stochastic component of judgment can be viewed as the detrimental component to using judgment; shocks that are unrelated to economic fundamentals affect agents expectations and forward looking decisions. One of the contributions of this paper is to provide an estimate for the degree to which judgment has influenced expectations in the post-war U.S. monetary economy, and how much of this judgment is informative (that is, related to current realizations of structural shocks) and how much is disruptive (independent of structural shocks). I further illustrate the influence judgment shocks have had on the dynamics of inflation, output, and interest rates in U.S. history, along with traditional supply, demand, and monetary policy shocks.

Before moving forward, it is prudent to define the following terms used in this paper that I give precise meanings to, perhaps using these somewhat differently than other papers in the literature:

Expectation: the value agents actually expect a variable to take in the future. This will be the sum of agents' econometric forecast and their judgment. When taking the model to the data, *expectations* are matched to median responses from the Survey of Professional Forecasters.

Econometric forecast: a forecast for a future variable computed using least squares methods and past data on the output gap, inflation rate, and the federal funds rate.

Judgment: sometimes referred to as "add-factors" in the literature. A value that is added to agents' econometric forecasts to reflect their actual expectations of what is to come. Judgment is a linear combination of structural shocks and judgment shocks.

Structural shocks or *fundamental shocks*: traditional stochastic shocks in the New Keynesian model: a natural rate shock, a cost shock, and a monetary policy shock. Current values of structural shocks affect macroeconomic dynamics but they have no influence on agents' econometric forecasts. They may, however, influence judgment.

Judgment shocks: stochastic shocks to judgment that are independent of the structural shocks.

2 Related Literature

The literature on learning specific to monetary economics can be broadly put into two categories: 1) theoretical work that examines stability of equilibria under learning versus rational expectations, and 2) empirical and descriptive research that examines the difference in macroeconomic dynamics between learning and rational expectations. The first branch explores the conditions for expectational stability, or E-stability, on monetary policy parameters. A model with learning that is E-stable will have expectations that converge to the rational expectations equilibrium, within the neighborhood of the rational expectations solution. Examples papers of this type are numerous, but include Bullard and Mitra (2002) and (2007), Evans and Honkapohja (2003a) and (2003b), and Preston (2005), just to name a few. These papers demonstrate that conditions on monetary policy for E-stability can be different and more restrictive than conditions for determinacy (the relevant condition when expectations are rational); the implication is that the economy can become unstable and volatile if monetary policy strays from these restrictions.

Such concerns have motivated the second branch of literature which investigates whether learning can explain macroeconomic dynamics we see in the data that is not well explained by traditional rational expectations models. Orphanides and Williams (2005b) use a calibrated model with learning to demonstrate that transient inflation shocks can lead to “inflation scares”, prolonged periods of high inflation. Findings like these suggests that learning can explain macroeconomic persistence. Milani (2007) finds evidence for this with an estimated New Keynesian model with learning. He finds learning can explain persistence in inflation and output without the need for common “mechanical” sources of persistence, such as habit formation and inflation indexation which are typically augmented to rational expectations models. Learning has also been used to explain characteristics of the “Great Inflation” and “Great Moderation”, the large run-up of inflation and macroeconomic volatility in the 1970s followed by a long period of relatively moderate volatility and low inflation since 1984. Examples of such papers include Orphanides and Williams (2005a), Primiceri (2006), Bullard and Eusepi (2005), and Bullard and Singh (2007).

Preceding this paper, relatively little work has investigated the importance of judgment on expectations. Reifschneider, Stockton, and Wilcox (1997) and Svensson (2005) demonstrate the usefulness of judgment for central bankers when making monetary policy decisions. Bullard, Evans, and Honkapohja (2008) and (2010) incorporate judgment of the kind that is purely disruptive (judgments depend exclusively on stochastic shocks that are independent of economic fundamentals) into simple monetary models and demonstrate that judgment can create “exuberance equilibria”, a condition that is susceptible to self-fulfilling judgments even when an equilibrium is otherwise locally determinant and/or E-stable. They go on to suggest appropriate monetary policy to prevent such unstable outcomes.

These papers by Bullard, Evans, and Honkapohja fall into the first branch of learning literature mentioned above: they provide theoretical evidence that expectations formed by judgment and learning can lead to economic instability. The present work is the first attempt to bring the issue to the second branch: to determine whether judgment with learning can explain characteristics of business cycle fluctuations seen in data for the post-war United States.

3 Model

Learning and judgment are examined within the context of a standard New Keynesian model, a model that has been estimated at great length with rational expectations and learning to investigate the roles stochastic shocks play in explaining macroeconomic dynamics. Notable examples using rational expectations include Ireland (2004a) and (2004b), Nason and Smith (2005), and Smets and Wouters (2003), (2005), and (2007), just to name a few. Examples of estimated DSGE models that examine the role for learning include Milani (2007) and Slobodyan and Wouters (2007) and (2008).

The New Keynesian models used in this literature vary in their extentions and notation, but all have the same basic setup. There are three sectors that describe consumer behavior, producer behavior in markets characterized by monopolistic competition and imperfectly flexible prices, and monetary policy. Optimal consumer behavior is described by one or more equations that determine current consumption based on past consumption, interest rates, and expectations of future consumption and future inflation. Producer behavior is modeled with a "Phillips equation" which predicts the inflation rate that arises from firm's optimal pricing strategies when subject to a pricing friction. The final sector is monetary policy follows a Taylor (1993) type rule where the central bank sets a nominal interest rate which responds to expectations of future output and inflation. These equations jointly determine the dynamics of the output gap (the percentage difference between real GDP and potential GDP) and/or output growth, the inflation rate, and the nominal interest rate.

The previously cited papers each derive a New Keynesian model from its utility maximizing and profit maximizing microfoundations. For the purposes of this paper it is not necessary to recreate this step so I simply adopt the model used by Ireland (2004b), present its linearized form here using the same notation, and refer interested readers to this source for full details. This paper was chosen for its simplicicity in modeling strategy and exposition and because it addresses a question similar to the present paper. The model allows for exogenous economic growth (caused by a random walk technology shock) and is linearized around its steady state growth path. It focuses only on the dynamics of output, inflation, and interest rates and does not attempt to use highly stylized labor or capital markets to match data on investment or employment. Ireland (2004b) addresses similar issues as this paper when he examines the role various structural shocks have played in post-war business cycle fluctuations.

Consumer behavior is governed by the following equation,

$$\hat{x}_t = \alpha_x \hat{x}_{t-1} + (1-\alpha_x)\hat{x}^e_{t+1} - (\hat{r}_t - \hat{\pi}^e_{t+1}) + (1-\omega)(1-\rho_a)\hat{a}_t, \tag{1}$$

which is often times refered to as the "IS equation". Here, $\hat{x}_t$ is the output gap (percentage deviation of real GDP from potential GDP), $\hat{r}_t$ is the deviation of the nominal interest rate from its steady state, $\hat{\pi}_t$ is the difference in the inflation rate from its steady state, and $\hat{a}_t$ is a preference shock that increases the marginal utility of consumption, essentially a positive consumption demand shock today that comes at the expense of consumption demand tomorrow. The parameter ω is positively related to the elasticity of labor supply and ρ_a is the persistence of the preference shock. Ireland adds the final parameter α_x after log-linearization to allow for persistence in consumption demand without taking a stance on microfounded causes such as habit formation. The only piece of notation that differs from Ireland (2004b) is that expectations at time t for time $t+1$ variables are denoted with an e superscript instead of the rational expectations operator, E_t.

Production behavior is described by the Phillips curve,

$$\hat{pi}_t = \beta\left[\alpha_\pi \hat{\pi}_{t-1} + (1-\alpha_\pi)\hat{\pi}^e_{t+1}\right] + \psi\hat{x}_t - e_t. \tag{2}$$

Here, e_t is a cost-push shock which is directly related to the time-varying elasticity of substitution among the intermediate goods for production of the final good. Intermediate goods are produced in monopolistically competitive markets so if the elasticity of substitution increases, the markup of price over marginal cost decreases. This causes a decrease in the cost of production of the final good, e_t increases, and there is downward pressure on inflation.

The parameter α_π is added to the model after log-linearization to allow for persistence in inflation without taking a stance on microfounded causes such as price indexation to past inflation. The Phillips curve coefficient ψ is directly related to the degree of price flexibility. Microfoundations for ψ typically rely on imperfectly flexible pricing schemes such as quadratic price adjustments costs as suggested by Rotemberg (1982), or infrequent re-optimization of prices as suggested by Calvo (1983).

The central bank adjusts the nominal interest rate to target steady state levels of inflation, output gap, and GDP growth according to the modified Taylor (1993) rule,

$$r_t - r_{t-1} = \rho_\pi \hat{\pi}_t + \rho_g \hat{g}_t + \rho_x \hat{x}_t + \epsilon_{r,t}, \tag{3}$$

where $\hat{g}_t$ is the growth rate real GDP, the coefficients ρ_π, ρ_g, and ρ_x measure the reponsiveness of interest rate changes to inflation, output growth, and the output gap; and $\epsilon_{r,t}$ is an independently and identically distributed monetary policy shock.

Ireland (2004b) shows that output growth and the output gap are related according by the following equation,[1]

$$\hat{g}_t = \hat{x}_t - \hat{x}_{t-1} + \omega(\hat{a}_t - \hat{a}_{t-1}) + z_t, \tag{4}$$

where z_t is the stochastic component of a random walk technology shock and is independently and identically distributed.

There are a total of four structrual shocks. The technology shock and monetary policy shock are independently normally distributed with mean zero and variances σ_z^2 and σ_r^2, respectively. The preference shock and cost shock evolve according to the following AR(1)

[1]Equation (4) is not explicitly expressed in Ireland (2004b) but is found by combining equations (20) and (21) in that paper.

processes, respectively,

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_{a,t}, \quad \epsilon_{a,t} \sim \mathcal{N}(0, \sigma_a^2) \tag{5}$$

$$\hat{e}_t = \rho_e \hat{e}_{t-1} + \epsilon_{e,t}, \quad \epsilon_{e,t} \sim \mathcal{N}(0, \sigma_e^2) \tag{6}$$

4 Expectations

4.1 Learning

The log-linearized model in the previous section can be expressed in the general form:

$$\Omega_0 x_t = \Omega_1 x_{t-1} + \Omega_2 x_{t+1}^e + \Omega_2 x_{t+2}^e + \Psi z_t, \tag{7}$$

$$z_t = A z_{t-1} + \epsilon_t \tag{8}$$

where the notation x_{t+1}^e has replaced $E_t x_{t+1}$ to denote possibly non-rational expectations; x_t is a vector of minimum state variables, given by $x_t = [\tilde{y}_t \ \pi_t \ \hat{r}_t]'$, and z_t is a vector of structural shocks, given by $z_t = [r_t^n \ u_t \ \epsilon_{r,t}]'$. The variable $\tilde{\lambda}_t$ can be eliminated by substituting equation (**??**) into equations (**??**) and (**??**), which leads to the inclusion of the two-period ahead expectation for the output gap, $E_t \tilde{y}_{t+2}$, in the IS equation. The minimum state variable (MSV) solution under rational expectations is given by,

$$E_t x_{t+1} = G x_t + H E_t z_{t+1}, \tag{9}$$

where the elements of the matrices G and H are a function of the parameters of the model and may be determined by the method of undetermined coefficients. Agents that learn do not know the the parameters that govern the economy, but do use this reduced form as their forecasting model. Agents' information sets are restricted only to past data on x_t, so they are unable to collect data on past structural shocks to estimate matrix H.

In period t agents are able to assemble data sets only through period $t-1$. At this point the agents estimate G using least squares and use the model to make econometric forecasts for future output and inflation. There is no constant term in the general form of the model, equation (7), or in the rational expectation, given in equation (9), since all variables are expressed in terms of percentage deviations from the steady state or flexible price outcome. Since agents are not endowed with information about the parameters of the model to determine steady state values, it is realistic to suppose that agents also estimate a constant term in equation (9). Let $\hat{G}_t^*$ denote agents' time t estimate for the columns of matrix G and a column for a constant term so that $\hat{G}_t^* = [\hat{g}_t\ \hat{G}_t]$, where $\hat{g}_t$ is the time t estimate of the constant term.

If agents use ordinary least squares (OLS), then,

$$\left(\hat{G}_t^*\right)' = \left(\frac{1}{t-1}\sum_{\tau=1}^{t-1} x_{\tau-1}^* x_{\tau-1}^{*\,\prime}\right)^{-1} \left(\frac{1}{t-1}\sum_{\tau=1}^{t-1} x_{\tau-1}^* x_\tau'\right), \tag{10}$$

where $x_t^{*\prime} = [1\ x_t']$ is the vector of explanatory variables including the constant. This equation can be conveniently rewritten in the following recursive form:

$$\hat{G}_t^* = \hat{G}_{t-1}^* + g_t(x_{t-1} - \hat{G}_{t-1}^* x_{t-2}^*) x_{t-2}^{*\,\prime} R_t^{-1}, \tag{11}$$

$$R_t = R_{t-1} + g_t(x_{t-2}^* x_{t-2}^{*\,\prime} - R_{t-1}), \tag{12}$$

where $g_t = 1/(t-1)$ is the learning gain.[2] The recursive form shows precisely how expectations are adaptive. The term enclosed in parentheses in equation (11) is the realized forecast error using the previous estimate $\hat{G}_{t-1}^*$. The degree to which agents adjust their expectations depends on the size of this forecast error, the variance of the estimated coefficients, captured by the inverse of matrix R_t, and the size of the learning gain, g_t. The larger is the learning gain, the more expectations respond to the latest forecast error. When agents use OLS, g_t approaches zero as time approaches infinity. Under constant gain learning, g_t remains at some constant level, g, so the degree to which new observations can affect expectations is always the same.

[2]To show this, let $R_t = \frac{1}{t-1}\sum_{\tau=1}^{t-1} x_{\tau-1}^* x_{\tau-1}^{*\prime}$ and $\left(\hat{G}_t^*\right)' = R_t^{-1}\left(\frac{1}{t-1}\sum_{\tau=1}^{t-1} x_{\tau-2}^* x_{\tau-1}'\right)$.

Agents use the least squares estimate of the coefficients in G to form the econometric forecasts,

$$E_t^* x_{t+1} = \hat{g}_t + \hat{G}_t E_t^* x_t = (I + \hat{G}_t)\hat{g}_t + \hat{G}_t^2 x_{t-1},$$
$$E_t^* x_{t+2} = \hat{g}_t + \hat{G}_t E_t^* x_{t+1} = \left[I + \hat{G}_t(I + \hat{G}_t)\right]\hat{g}_t + \hat{G}_t^3 x_{t-1}, \tag{13}$$

where E_t^* denotes expectation that is equal to the econometric forecast.

4.2 Judgment

Agents are not able to collect realizations of stochastic shocks, z_t, in their forecasts.[3] However, current events may reveal some noisy information about structural shocks, which becomes part of judgment when forming expectations. Examples of such events may be the announcement of technological innovations, natural disasters, onset of war or political instability among trading partners, changes in weather effecting agricultural production, etc. These events cannot be instantly mapped to data to make econometric forecasts, but are nonetheless valuable information when forming expectations. Let agents' final expectations be the following sum of the econometric forecast given in equation (13) and judgment,

$$x_{t+1}^e = E_t^* x_{t+1} + \eta_t, \tag{14}$$

where η_t is a 2x1 vector that includes judgment concerning the future output gap ($\eta_{y,t}$) and future inflation rate ($\eta_{\pi,t}$). The judgment vector depends on current events that includes some information about z_t, but it also includes expectational shocks, its own stochastic component that is independent of economic fundamentals. Let judgment evolve according

[3]Central banks do use a number of such sophisticated models that incorporate the presence of latent structural shocks when forming forecasts. Reifschneider, Stockton, and Wilcox (1997) and Svensson (2005) point out that judgment is nonetheless an important component of central bank expectations and decisions.

to,

$$\eta_t = \Phi z_t + \zeta_t,$$

$$\zeta_{y,t} = \rho_{\zeta,y}\zeta_{y,t-1} + \xi_{y,t}, \tag{15}$$

$$\zeta_{\pi,t} = \rho_{\zeta,\pi}\zeta_{\pi,t-1} + \xi_{\pi,t},$$

where matrix Φ captures the degree to which judgment successfully picks up information about structural shocks and ζ_t is a vector of autocorrelated disturbances to the judgment variables. The second and third equations allow these disturbances to be autocorrelated so that agents' ill-informed judgment about a particular variable may persist for multiple quarters depending on the parameters $\rho_{\zeta,y}$ and $\rho_{\zeta,y}$. The judgment shocks $\xi_{y,t}$ and $\xi_{\pi,t}$ are independently and normally distributed with mean zero and standard deviation given by $\sigma_{\xi,y}$ and $\sigma_{\xi,\pi}$, respectively.

The structural form for evolution of judgment in equation (15) is quite general and allows as special cases common specifications for expectations in DSGE models. If $\rho_{zeta,y} = \rho_{\zeta,y} = 0$ and $Var(\xi_{y,t}) = Var(\xi_{\pi,t}) = 0$ then expectations are not subject to judgment shocks. If $\Phi = 0$, then stochastic shocks are always unobservable when forming expectations, which is a common assumption among empirical learning papers. If $\Phi = HA$, where H is the coefficient on expected shocks in the MSV solution in equation (9) and A is the degree of persistence of structural shocks given in (8), then agents are capable of observing quantities for structural shocks and the influence these shocks have on expectations is equal to the rational expectations solution. In fact, the entire model encompasses rational expectations as a special case when these conditions are met, the learning gain is equal to zero ($g = 0$), and the initial condition for learning coefficients, G_t^* in equation (11), is consistent with the MSV solution, G in equation (9). This initial condition is estimated using pre-sample data as described in the next section, and all other parameters mentioned here are estimated jointly with the New Keynesian structural parameters, so this framework can be viewed as quite unrestricted.

5 Estimation

The model is estimated using U.S. quarterly data from 1968:Q3 through 2007:Q1 on the output gap (percentage difference between real GDP reported by the Bureau of Economic Analysis and the measure of potential real GDP from the Congressional Budget Office), the inflation rate of the GDP deflator, and the Federal Funds Rate. Quarterly data from the same period on one quarter ahead expectations from the Survey of Professional Forecasters is also used to help identify the parameters of the learning and judgment process. The median responses were obtained for the one quarter ahead forecast for real GDP and the GDP deflator. The expectation for the output gap is found by computing the percentage difference between the forecast for real GDP and the CBO estimate for potential GDP in the next quarter. The expectation for inflation is found by computing the percentage difference between the forecast for the GDP deflator next period and the current GDP deflator. The base year used for the forecasts from the Survey of Professional Forecasters changes throughout the sample, so the data was first appropriately rescaled.

5.1 State Space Representation

Equations (7), (8), (11), (12), (13), and (15) can be combined into following single state equation convenient for evaluating a Kalman filter,[4]

$$s_t = f_t + F_t s_{t-1} + v_t \tag{16}$$

where $s_t = [\tilde{y}_t \ \pi_t \ \hat{r}_t \ \tilde{y}^e_{t+1} \ \tilde{y}^e_{t+2} \ \pi^e_{t+1} \ \eta_{y,t} \ \eta_{\pi,t} \ r^n_t \ u_t \ \zeta_{y,t} \ \zeta_{\pi,t}]'$ is a vector of state variables, and $v_t = [\epsilon_{n,t} \ \epsilon_{u,t} \ \epsilon_{r,t} \ \xi_{y,t} \ \xi_{\pi,t}]$ is a vector of all the independently and identically distributed

[4]Habit formation causes the two period ahead expectation, $\tilde{y}^e_{t+2}$ to appear in the model, which in turn requires an evaluating a time t expectation for judgment $\eta_{y,t+1}$. For simplicity, I suppose this judgment is formed using the mathematical expectation operator on the equations in (15), advanced to period $t+1$. This implies that when using judgment for expectations two periods ahead, agents already discount this judgment depending on the degree of persistence, $\rho_{\zeta,y}$; and the degree to which stochastic shocks z_t impact judgment two periods ahead is determined by the actual degree of persistence dictated persistence of the natural rate and cost-push shocks (given by parameters ρ_n and ρ_u).

stochastic shocks. The time-varying component of vector f_t and matrix F_t comes from the coefficients in $\hat{g}_t$ and $\hat{G}_t$ determined by the learning process in (13). Since f_t and F_t depend only on lagged realizations of some of the state variables, they can be treated as predetermined when evaluating the Kalman filter.

Let GAP_t denote the data on the output gap, INF_t denote data on inflation, FF_t denote data on the Federal Funds Rate, and SPF_GAP_t and SPF_INF_t denote data on expected one-quarter ahead output gap and inflation rate, respectively, implied by the Survey of Professional Forecasters. The observation equations are given by,

$$
\begin{aligned}
GAP_t &= 100\tilde{y}_t, \\
INF_t &= \pi^* + 400\pi_t, \\
FF_t &= r^* + \pi^* + 400\hat{r}_t. \\
SPF\_GAP_t &= 100\tilde{y}^e_{t+1}, \\
SPF\_INF_t &= \pi^* + 400\pi^e_{t+1}.
\end{aligned}
$$

The state variables are multiplied by 100 to convert the decimals percentages, and the inflation rate, expected inflation rate, and federal funds rate are multiplied by 4 to convert quarterly rates to annualized rates. The New Keynesian model assumes that the steady state inflation rate is equal to zero, but since this is not likely the case in the data, the annualized steady state inflation rate, given by π^*, is included in the observation equations above. The steady state gross real interest rate is set equal to the inverse of the discount factor; therefore $r^* = 400(1 - 1/\beta)$. These steady state parameters are calibrated to $\pi^* = 3.4$ and $\beta = 0.9956$ so the steady state values match the average inflation rate and nominal interest rate in the sample.

5.2 Initial Conditions

Aside from standard initial conditions for the Kalman filter, it is necessary to specify initial conditions for $\hat{G}^*_0$ and R_0, the initial values for learning process given in equations (11) and (12). I use pre-sample data from 1954:Q2 through 1968:Q2 on the output gap, inflation rate, and federal funds rate and and transform these into the same terms as the state vector, x_t, according to,

$$\tilde{y}_t = \frac{1}{100} GAP_t,$$

$$\pi_t = \frac{1}{400}(INF_t - \pi^*),$$

$$\hat{r}_t = \frac{1}{400}(FF_t - r^* - \pi^*).$$

I estimate a VAR(1) (the same reduced form as used in the least squared learning process described in Section 4) on this data using ordinary least squares to set initial values for the learning matrices. The coefficients from the regression are used to initialize $\hat{G}_0^*$, and the elements from the sum of squares component of the estimate of the variance of the coefficients is used to initialize R_0.

5.3 Bayesian Estimation

Table 1 lists the parameters to be estimated, along with the prior distribution imposed for Bayesian estimation. The parameters include the learning gain, the parameters of the New Keynesian Model described in Section 3, the coefficients Φ in equation (15) governing how stochastic shocks informatively impact judgment, the persistence of stochastic components to judgment, also in equation (15), and the standard deviation of the structural shocks and judgment shocks.

The model is estimated with Bayesian methods using the Metropolis-Hastings algorithm. The vector of parameters were drawn from the posterior distribution 400,000 times and the first 100,000 draws were discarded for a burn-in period. Table 1 shows the prior distributions used for the estimation. The prior distributions for the New Keynesian parameters are similar to others used in the literature. The prior mean for the learning gain is set to 0.02, with a rather large standard deviation of 0.03 which allows for a wide range for learning dynamics. The value of the learning gain two standard deviations above the mean is 0.08, which implies the sample size agents to develop their econometric only $0.08^{-1} = 12.5$, or just over only 3 years of data. The large standard deviation for the prior on the learning gain also allows for a relatively large probability that the learning gain is close to zero, implying agents use a very large number of observations and agents adjust their econometric estimates for the coefficients only very slowly. The prior distributions for the coefficients in judgment process are intentionally made very wide in recognition that no previous literature has attempted to measure or even discuss such coefficients.

6 Results

6.1 Parameters

The prior and posterior distributions for the parameters are listed in Table 2. The last three columns present the median, 5th percentile and 95th percentile of the posterior distributions for the parameters. The estimate for the learning gain is found to be 0.0322 with a relatively tight posterior distribution relative to the prior. This implies that agents use approximately $0.0322^{-1} = 31.06$ observations for forming least squares forecasts, which corresponds to about 7.75 years. This is a magnitude similar to that found by Milani (2007), and Slobodyan and Wouters (2007) and (2008). Habit formation is found to be a significant source of persistence, with an estimate $\eta = 0.6871$. Price indexation is found to be less important in explaining persistence with an estimate $\gamma = 0.1407$. Other significant sources of persistence come from the natural rate shock ($\rho_n = 0.95$), cost shock ($\rho_u = 0.78$), and the persistence of disturbances to judgment on output and inflation, with $\rho_{\zeta,y} = 0.94$ and $\rho_{\zeta,\pi} = 0.89$, respectively. The most volatile shock driving business cycles is the natural rate shock, where the cost shock, monetary policy shock, and judgment shocks have standard deviations with similar magnitudes significantly below the standard deviation of the natural rate shock. Only the preference parameters σ and μ are poorly identified by the data; these posterior distributions largely mirror the priors.

6.2 Judgment

The posterior distributions for the coefficients in Φ for the judgment process are very significantly informed by the data; these posterior distributions are considerably tight given the very wide prior distributions. Recall the parameters in Φ determine how much judgment depends on actual stochastic shocks. Using equation (15), the variance of judgment can be decomposed into variance caused by structural parameters (informed judgment) and variance from the independent stochastic component (judgment shocks) as follows,

$$Var(\eta_t) = \Phi Var(z_t)\Phi' + Var(\zeta_t). \tag{17}$$

Both z_t and ζ_t are autoregressive stochastic processes whose variances depend on the variances for the shocks. To illustrate, the variance for z_t can be derived from the variance of the underlying independently and identically distributed shocks using equation (8) as follows,

$$Var(z_t) = AVar(z_{t-1})A' + Var(\epsilon_t)$$

Since the variance of z_t does not depend on time, we can solve for this variance using the $vec(\dot{})$ operator on both sides of this equation,

$$vec(Var(z_t)) = A \otimes Avec(Var(z_t)) + vec(Var(\epsilon_t)).$$

Solving leads to the expression,

$$vec(Var(z_t)) = (I - A \otimes A)^{-1} vec(Var(\epsilon_t)). \tag{18}$$

The off-diagonal elements of $Var(\epsilon_t)$ are zero, and the diagonal elements are given by squares of σ_n, σ_u, and σ_r, whose estimates are reported in Table 2. The output $vec(Var(z_t))$ is then appropriately re-sized to yield $Var(z_t)$ to substitute into equation (17). A symmetric exercise performed on the autoregressive equations in (15) yields $Var(\zeta_t)$.

Given the estimates for these variances, equation (**??**) can be used to determine what percentage of the variability in judgment depends on structural shocks and judgment shocks. Table 3 reports these results. About 85% of the variability judgment in output is explained by the variance of the shock to judgment and the remaining 15% of variability is explained primarily by the variance of the natural rate shock. This implies judgment on output is primarily ill-informed: only a small amount of judgment is based on information related to fundamentals in the economy. The second column of Table 3 shows the result is very similar for judgment regarding inflation. About 62% of the judgment in inflation is ill-informed, and the remaining 38% depends on information from the cost shock. The impact of monetary policy shocks on judgment of both variables was essentially equal to zero.

Its interesting that both the natural rate shock and cost shock help inform judgment, but strangely, the natural rate shock is not used for judgments regarding inflation, and the cost shock is not used for judgments regarding output. Both of these shocks influence output and inflation in equilibrium - yet agents mistakenly believe that cost shocks only drive inflation, and natural rate shocks only drive output.

7 Conclusion

Rational expectations is a prominent assumption used in evaluating economic issues analyzed with DSGE models, but in reality people consider statistical forecasts then use judgment when forming their actual expectations. I estimate a standard New Keynesian model using data on the output gap, inflation rate, and interest rates along with data on expectations from the Survey of Professional Forecasters. Fundamental structural shocks in the model include the natural rate shock, cost shock, and monetary policy shock. I allow judgment to be based on these shocks, indicating it can be informed by current fundamental shocks, but it can also be subject to its own stochastic disturbances that are orthogonal to current structural shocks and past state variables. Stochastic shocks to judgment is found to be a significant source of economic persistence and economic volatility in U.S. history. Furthermore, judgment is found to be determined primarily by its own stochastic disturbances; very little of the variability in judgment is shown to depend on fundamental shocks.

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Table 1: Parameters and Prior Distributions

			Prior Distribution		
Parameter	Description	Domain	Distribution	Mean	Std.Dev.
g	Learning gain	R^+	Gamma	0.02	0.03
η	Habit Formation	$(0,1)$	Beta	0.70	0.10
σ	Elasticity of Substitution	R^+	Normal	1.50	0.25
μ	Labor Elasticity	R^+	Normal	2.00	0.50
κ	Phillips Curve Slope	R^+	Gamma	0.10	0.05
γ	Price Indexation	$(0,1)$	Beta	0.50	0.15
ρ_r	Policy Persistence	$(0,1)$	Beta	0.75	0.10
ψ_y	Policy Feedback Output	R	Normal	0.50	0.25
ψ_π	Policy Feedback Inflation	R	Normal	1.50	0.25
ρ_n	Natural Rate Persistence	$(0,1)$	Beta	0.50	0.20
ρ_u	Cost Shock Persistence	$(0,1)$	Beta	0.50	0.20
$\rho_{\zeta,y}$	Output Judgment Persistence	$(0,1)$	Beta	0.75	0.20
$\rho_{\zeta,\pi}$	Inflation Judgment Persistence	$(0,1)$	Beta	0.75	0.20
σ_n	Std.Dev. Natural Rate	R^+	InvGamma	0.10	0.10
σ_u	Std.Dev. Cost Shock	R^+	InvGamma	0.10	0.10
σ_r	Std.Dev. Policy Shock	R^+	InvGamma	0.10	0.10
$\sigma_{\zeta,y}$	Std.Dev. Output Judgment	R^+	InvGamma	0.10	0.10
$\sigma_{\zeta,\pi}$	Std.Dev. Inflation Judgment	R^+	InvGamma	0.10	0.10
$\phi_{y,0}$	Output Judgment Constant	R	Normal	0.00	4.00
$\phi_{y,n}$	Nat.Rate Impact on Output Judgment	R	Normal	0.00	4.00
$\phi_{y,u}$	Cost Shock Impact on Output Judgment	R	Normal	0.00	4.00
$\phi_{y,r}$	Policy Shock Impact on Output Judgment	R	Normal	0.00	4.00
$\phi_{\pi,0}$	Inflation Judgment Constant	R	Normal	0.00	4.00
$\phi_{\pi,n}$	Nat.Rate Impact Inflation Judgment	R	Normal	0.00	4.00
$\phi_{\pi,u}$	Cost Shock Impact Inflation Judgment	R	Normal	0.00	4.00
$\phi_{\pi,r}$	Policy Shock Impact Inflation Judgment	R	Normal	0.00	4.00

Table 2: Estimation Results

		Prior Distribution			Posterior Distribution		
Parameter	Domain	Distribution	Mean	Std.Dev.	Median	5th Percentile	95th Percentile
g	R^+	Gamma	0.02	0.03	0.0322	0.0224	0.0418
η	$(0,1)$	Beta	0.70	0.10	0.6871	0.6088	0.7482
σ	R^+	Normal	1.50	0.25	1.6858	1.3274	2.0728
μ	R^+	Normal	2.00	0.50	2.3458	1.6232	3.0770
κ	R^+	Gamma	0.10	0.05	0.0233	0.0114	0.0489
γ	$(0,1)$	Beta	0.50	0.15	0.2462	0.1407	0.3600
ρ_r	$(0,1)$	Beta	0.75	0.10	0.7528	0.6674	0.8236
ψ_y	R	Normal	0.50	0.25	0.1499	-0.0140	0.3201
ψ_π	R	Normal	1.50	0.25	1.8801	1.5013	2.2551
ρ_n	$(0,1)$	Beta	0.50	0.20	0.9532	0.9233	0.9796
ρ_u	$(0,1)$	Beta	0.50	0.20	0.7831	0.6936	0.8677
$\rho_{\zeta,y}$	$(0,1)$	Beta	0.75	0.20	0.9430	0.8872	0.9871
$\rho_{\zeta,\pi}$	$(0,1)$	Beta	0.75	0.20	0.8922	0.7918	0.9639
σ_n	R^+	InvGamma	0.10	0.10	0.1861	0.1180	0.2905
σ_u	R^+	InvGamma	0.10	0.10	0.0067	0.0059	0.0075
σ_r	R^+	InvGamma	0.10	0.10	0.0039	0.0036	0.0044
$\sigma_{\zeta,y}$	R^+	InvGamma	0.10	0.10	0.0091	0.0083	0.0101
$\sigma_{\zeta,\pi}$	R^+	InvGamma	0.10	0.10	0.0038	0.0034	0.0043
$\phi_{y,0}$	R	Normal	0.00	4.00	-0.0333	-0.0606	-0.0020
$\phi_{y,n}$	R	Normal	0.00	4.00	-0.0187	-0.0357	-0.0089
$\phi_{y,u}$	R	Normal	0.00	4.00	0.1377	-0.0965	0.3448
$\phi_{y,r}$	R	Normal	0.00	4.00	0.0715	0.0481	0.0938
$\phi_{\pi,0}$	R	Normal	0.00	4.00	-0.0024	-0.0037	-0.0005
$\phi_{\pi,n}$	R	Normal	0.00	4.00	0.0005	-0.0032	0.0052
$\phi_{\pi,u}$	R	Normal	0.00	4.00	-0.6204	-0.7316	-0.5201
$\phi_{\pi,r}$	R	Normal	0.00	4.00	-0.0064	-0.1257	0.1232

Table 3: Judgment Variance Decomposition

Stochastic Shock	Judgment Output (%)	Judgment Inflation (%)
Natural Rate Shock	14.93	0.08
Cost Shock	0.25	38.34
Monetary Policy Shock	0.00	0.00
Output Judgment Shock	84.82	–
Inflation Judgment Shock	–	61.58
Total	100.00	100.00

Figure 1: Impulse Response Functions: Judgment Shocks

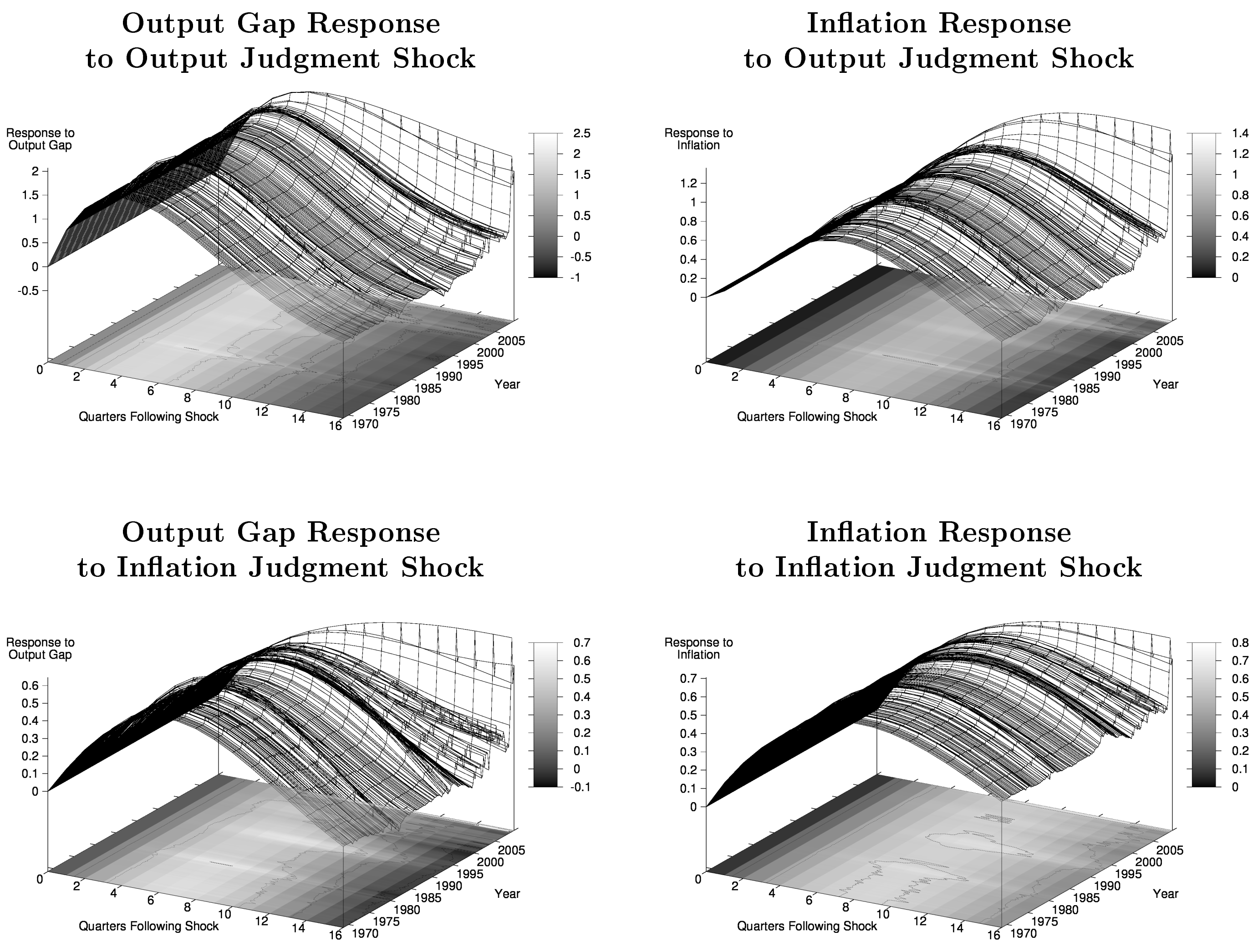

Figure 2: Root Mean Squared Response to Judgment Shocks

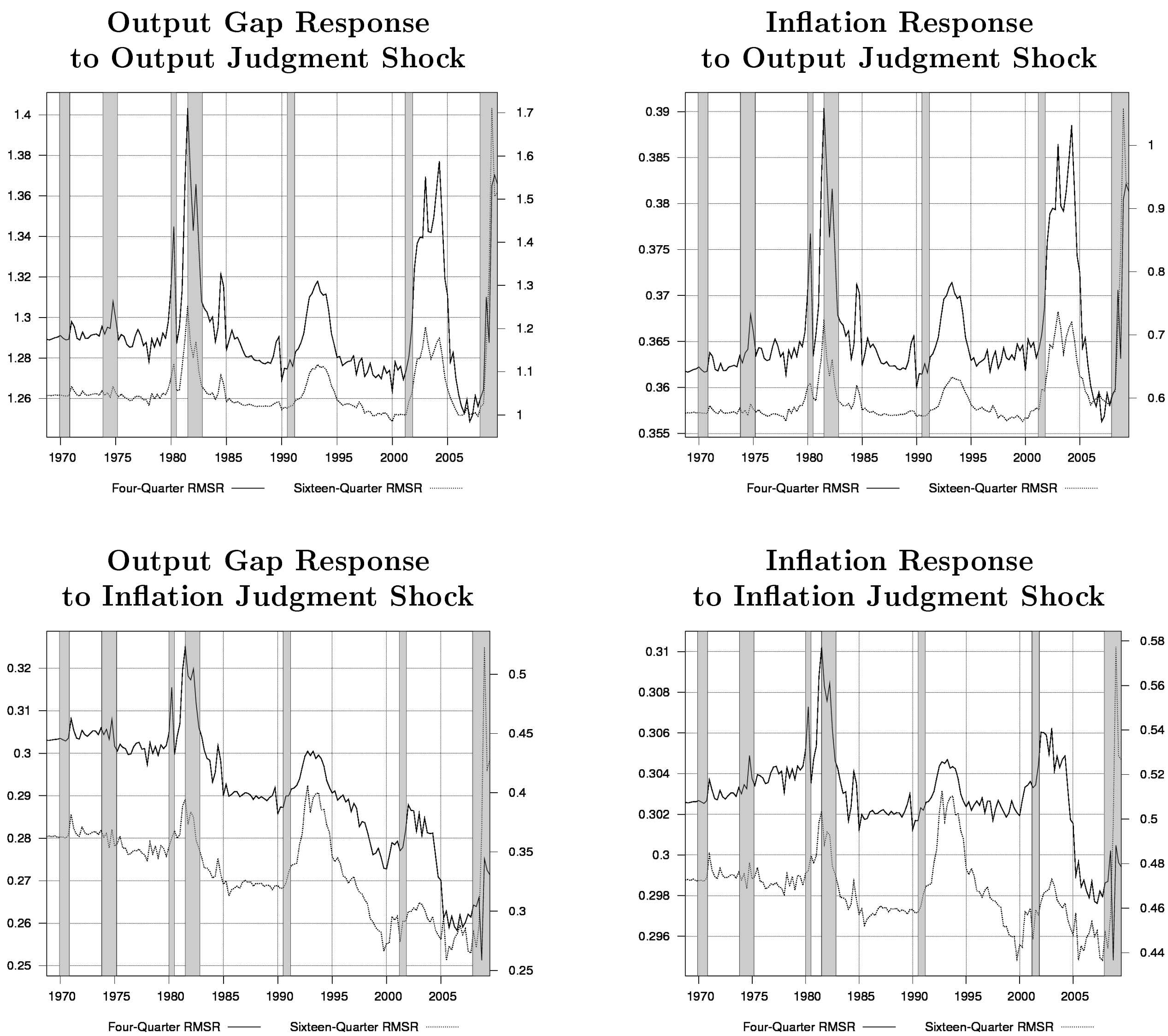

Figure 3: Impulse Response Functions: Structural Shocks

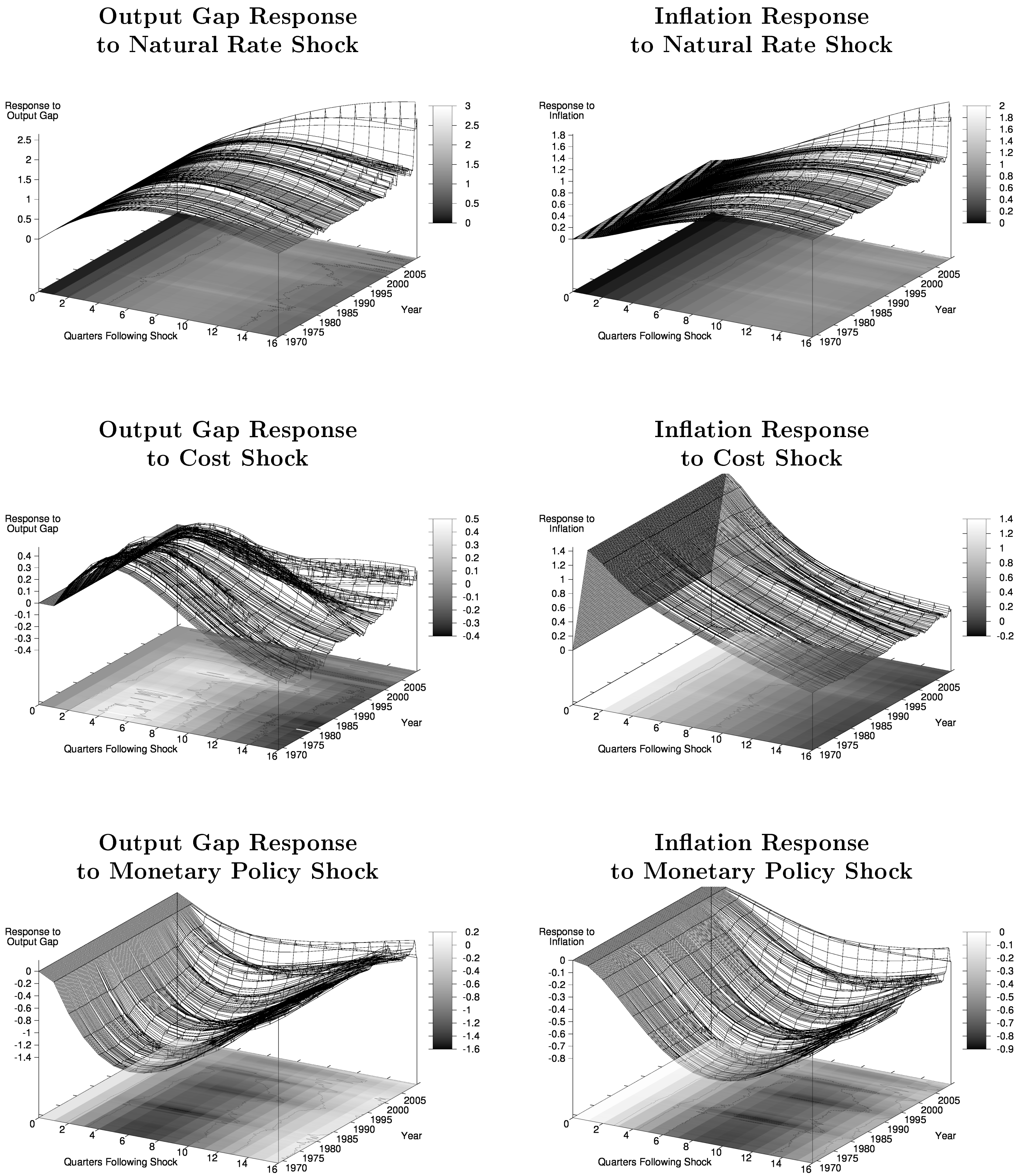

Figure 4: Root Mean Squared Response Structural Shocks