Learning with Expectational Shocks in the New Keynesian Model

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Purpose

1. How much of macroeconomic volatility in post-war U.S. is explained by...
   - traditional structural shocks (natural rate shock, cost shock, monetary policy shock),
   - and shocks to judgment (aka expectational shocks)?
2. How much of judgment is explained by...
   - actual events (i.e. current structural shocks),
   - and expectational shocks?

Framework

- Examine predictions using an estimated three equation New Keynesian model (Woodford [2003]).
- Replace rational expectations with least-squares learning.
- Actual expectations = Least-squares forecast + Judgment.

Learning

- Oraphanides and Williams (JEDC, 2005): Monetary authority was optimizing, but misinformed.
- Primiceri (QJE, 2006): Monetary authority misinformed, expectations improved with time.
- Milani (JME, 2007): Agents learn, little evidence of difference in policy parameters.
- Milani (2008): Time varying expectations explains time-varying volatility.
- Bullard and Singh (2007): bad luck + Bayesian learning.

Judgment

- Svensson (2005), Reifschneider, Stockton, and Wilcox (1997): judgment essential for central banking policy.
- Bullard, Evans, Honkapohja (2008), (2010): exuberance equilibria.
- Missing: empirical evaluation.

New Keynesian Model

- Consumer behavior:
  - Choose consumption and labor to maximize utility.
  - Habit formation: utility on consumption depends on past consumption.
- Producer behavior:
  - Intermediate goods are produced with labor in monopolistically competitive markets.
  - Intermediate goods subject to Calvo (1983) price friction.
  - Price indexation: when not re-optimizing prices, price can adjust according to past inflation.
- Taylor (1993) type monetary policy:
  - Nominal interest rate responds to inflation, expected future output gap, and past interest rate.

Optimal Consumer Behavior

$$\tilde{\lambda}_t = E_t\tilde{\lambda}_{t+1} + \hat{r}_t - E_t\pi_{t+1} - r_t^n,$$

$$\tilde{\lambda}_t = \frac{1}{(1-\beta\eta)(1-\eta)}\left[\beta\eta E_t\tilde{y}_{t+1} - (1+\beta\eta^2)\tilde{y}_t + \eta\tilde{y}_{t-1}\right]$$

- Variables:
  - $\tilde{\lambda}_t$: marginal utility of income.
  - $\tilde{y}_t$: output gap.
  - $\hat{r}_t$: nominal interest rate.
  - π_t: inflation.
  - r_t^n: "natural rate" shock, deviation of interest rate from flexible price outcome.
- Parameters:
  - $\eta \in [0, 1)$: degree of habit formation.
  - $\beta \in (0, 1)$: discount rate.

Producer Behavior

- Phillips curve:

$$\pi_t = \frac{1}{1+\beta\gamma}\left[\gamma\pi_{t-1} + \beta E_t\pi_{t+1} + \kappa(\tilde{y}_t - \mu\tilde{\lambda}_t) + u_t\right]$$

- Cost push shock: u_t.
- Parameters:
  - $\gamma \in [0, 1)$: degree of price indexation.
  - $\kappa \in (0, \infty)$: reduced form parameter inversely related to degree of price flexibility.

Monetary policy

- Nominal interest rate responds to expected output gap and inflation:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r)\left(\psi_\pi E_t \pi_{t+1} + \psi_y E_t \tilde{y}_{t+1}\right) + \epsilon_{r,t}$$

- Monetary policy shock: $\epsilon_{r,t}$.
  - $\psi_\pi \in (0, \infty)$: feedback on inflation.
  - $\psi_y \in (0, \infty)$: feedback on output.
  - $\rho_r \in (0, 1)$: smoothing parameter.

Structural Shocks

- Natural interest rate shock:

$$r_t^n = \rho_n r_{t-1}^n + \epsilon_{n,t}$$

- Cost push shock:

$$u_t = \rho_u u_{t-1} + \epsilon_{u,t}$$

- Monetary policy shock, $\epsilon_{r,t}$ is not autoregressive.

Learning

- Log-linearized New Keynesian model has the structural form:

$$\Omega_0 x_t = \Omega_1 x_{t-1} + \Omega_2 x^e_{t+1} + \Omega_3 x^e_{t+2} + \Psi z_t$$

- All observable by the agents: $x_t = [\tilde{y}_t \ \pi_t \ \hat{r}_t]$
- Shocks not observable to agents that learn: $z_t = [r^n_t \ u_t \ \epsilon_{r,t}]'$
- Rational expectations solution:

$$E_t x_{t+1} = G x_t + H z_t$$

- Agents estimate G by constant gain least squares.
  - Regressors: constant, first lag of x_t.
  - Constant learning gain, g, measures degree to which expectations are adaptive.

Judgment

Judgment, η_t, is possibly informed by current structural shocks, and subject to is own shock:

$$\eta_t = \phi_0 + \Phi z_t + \zeta_t,$$

$$\zeta_{y,t} = \rho_{\zeta,y}\zeta_{y,t-1} + \xi_{y,t},$$

$$\zeta_{\pi,t} = \rho_{\zeta,\pi}\zeta_{\pi,t-1} + \xi_{\pi,t},$$

Notation:

- η_t is 2x1 vector, includes judgment on $\tilde{y}^e_{t+1}$ and π^e_{t+1}.
- ϕ_0: degree/direction of a consistent bias in judgment.
- Φ: dependence of judgment on actual structural shocks.
- ζ_t: persistent expectational shocks.

Expectations

- Expectations are the sum of least squares forecasts ($E_t^* x_{t+1}$) and judgment (η_t).

$$\begin{aligned} x_{t+1}^e &= E_t^* x_{t+1} + \eta_t \\ &= E_t^* x_{t+1} + \phi_0 + \Phi z_t + \zeta_t \end{aligned}$$

- Special cases:
  - $\Phi = H$, structural shocks are perfectly observable, expectations rational.
  - $\Phi = 0$, structural shocks are completely unobservable.
  - $Var(\zeta) = 0$, there are no expectational shocks.
- Elements of ϕ, Φ, and $Var(\zeta)$ are estimated jointly with all other parameters.

Estimation

- Bayesian Estimation - Metropolis Hastings Simulation Procedure.
- Quarterly data from 1968:Q3 through 2007:Q1 on
  - Output gap: measured by Congressional Budget Office.
  - GDP deflator inflation rate.
  - Federal funds rate.
  - Survey of Professional Forecasters One-Quarter ahead forecast on real GDP.
  - Survey of Professional Forecasters One-Quarter ahead forecast on GDP deflator.
- Pre-sample (1954:Q3 - 1968:Q2) data on first three variables initialize VAR(1) learning forecasts.

Judgment Results

		Posterior Distribution	
Parameter	Median	5th Percentile	95th Percentile
$\rho_{\zeta,y}$	0.9430	0.8872	0.9871
$\rho_{\zeta,\pi}$	0.8922	0.7918	0.9639
$\phi_{y,0}$	-0.0333	-0.0606	-0.0020
$\phi_{y,n}$	-0.0187	-0.0357	-0.0089
$\phi_{y,u}$	0.1377	-0.0965	0.3448
$\phi_{y,r}$	0.0715	0.0481	0.0938
$\phi_{\pi,0}$	-0.0024	-0.0037	-0.0005
$\phi_{\pi,n}$	0.0005	-0.0032	0.0052
$\phi_{\pi,u}$	-0.6204	-0.7316	-0.5201
$\phi_{\pi,r}$	-0.0064	-0.1257	0.1232

Judgment Results

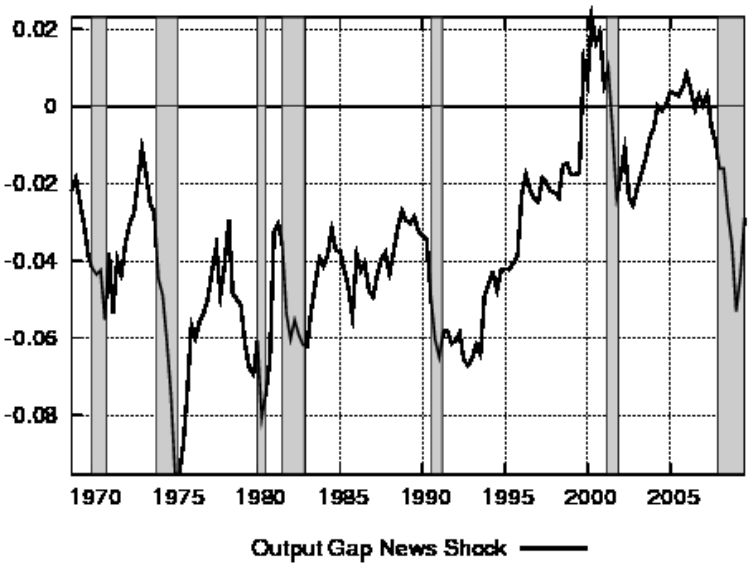
- Judgment is very persistent.
- Output and inflation is consistent underestimated.
- Elements of Φ indicate judgments are at least partially based on structural shocks.

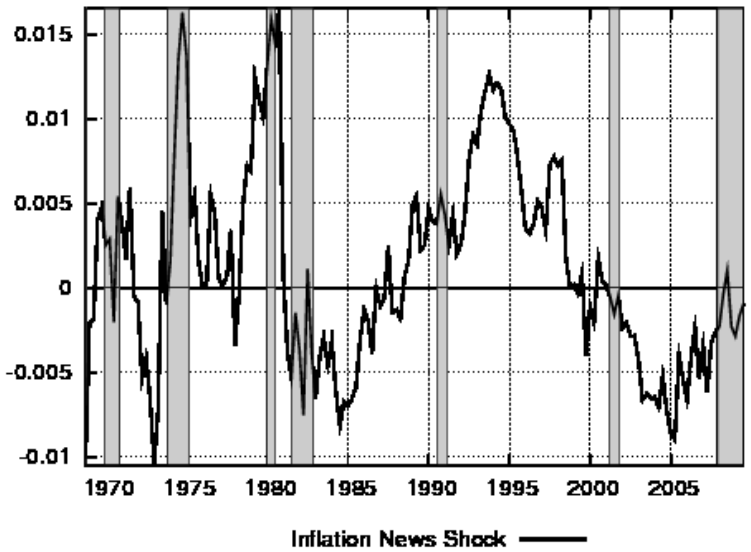
Variance Decompositions

Stochastic Shock	Output Judg.	Inflation Judg.
Natural Rate Shock	14.93%	0.08%
Cost Shock	0.25%	38.34%
Monetary Policy Shock	0.00%	0.00%
Output Judgment Shock	84.82%	–
Inflation Judgment Shock	–	61.58%
Total	100.00%	100.00%

- Judgment strongly influenced by uninformed stochastic shocks.
- Natural rate shock only informs output judgment.
- Cost shock only informs inflation judgment.
- Monetary policy shocks do not influence judgment.

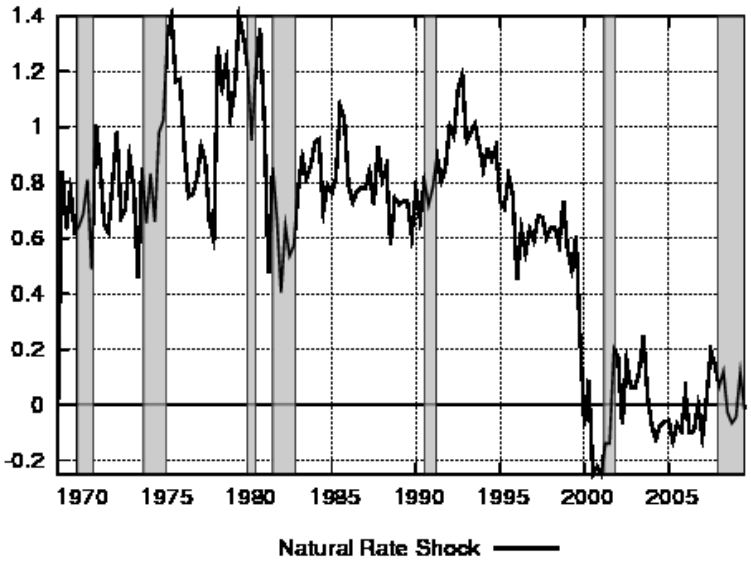
Expectational Shocks

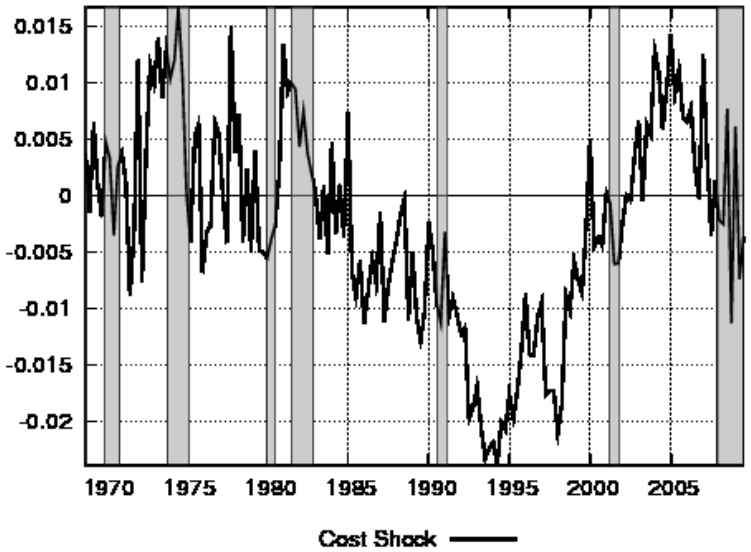




- Negative shocks to output expectations precede each recession since 1970.
- Positive shocks to inflation expectations are associated with recessions in middle and late 1970s.

Structural Shocks





- Natural rate shock positive during most period.
- Fall in natural rate shock precedes the 2001 recession.
- Little change in volatility of shocks during "Great Moderation".

Conclusions

- Findings:
  - Judgment is primarily determined by expectational shocks.
  - Preliminary evidence suggests expectational shocks can be causing recessions.
  - Preliminary evidence suggests expectational shocks Changes in macroeconomic volatility.
- Next steps:
  - Impulse response functions to expectational shocks.
  - Decomposition of forecast errors.