Assignment Two

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To prove or (match x xs) = elem x xs, we must prove the base case and induction step, which are as follows:

Base Case. P([ ]) = or (match x [ ]) = elem x [ ])
Induction Step. P(xs) ⇒ P(x:xs)
or (match x xs) = elem x xs ⇒ or (match x (y:ys)) = elem x (y:ys)

<u>Base Case</u>
or (match x [ ]) = elem x [ ])
or [ ] = elem x [ ] *by match.1*
False = elem x [ ] *by or.1*
False = False *by elem.1*
The base case holds.

We assume that or (match x xs) = elem x xs holds, and try to prove that or (match x (y:ys)) = elem x (y:ys) holds.

<u>RHS:</u>
elem x (y:ys)
(x == y) || (elem x ys) *by elem.2*

<u>LHS:</u>
or (match x (y:ys))
or ((x == y):(match x ys)) *by match.2*

(x == y) || (or (match x ys))   *by or.2*

Since in both the LHS and RHS, (x == y) is one of the disjuncts, we need only focus on the second parts,(or (match x ys)) and (elem x ys). However, by our hypothesis, we assumed that or (match x xs) = elem x xs holds. In other words, by the hypothesis, we can change either the LHS to match the RHS, or the RHS to match the LHS. The proof is complete.

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