Contents

Week 8: Concrete Formulation I

In this week, we focus on the first two steps of optimization modeling: the English description and the concrete formulation. By the end of this week, you will be able to formulate linear optimization models to tackle a wide range problems. The key transferable skills are mathematical modeling and creative problem solving.

Session 15: Appropriately Phrasing the English Description

Recap of the Four Steps of Optimization Modeling

1. English description: write a succinct verbal description of the decision, objective and constraints.

2. Concrete formulation: translate the above into a linear optimization formulation, illustrating with made-up numbers from a toy example.

3. Abstract formulation: identify patterns in the above and rewrite the formulation into one that can be scaled up to arbitrary data, by defining data variables and using index and summation notations.

4. Reusable software: write Python code to take in any input data of a certain format and output the optimal decision.

Example from Week 7: Assortment Planning

Amazon.com is expanding its business by launching a physical store in West LA. As the manager, you need to select which bestsellers to carry at the store's grand opening. The following table provides the list of Top 10 Bestsellers in Literature & Fiction, along with their genres. Note that some bestsellers belong to more than one genre.

Rank \ Genre	Literary	Sci-Fi	Romance	Thriller
1	✓			
2		✓		✓
3			✓	✓
4	✓		✓	
5	✓			
6			✓	
7		✓		
8				✓
9	✓	✓		
10			✓	

You wish to carry the minimum number of bestsellers, while ensuring that there are at least two bestsellers in each genre. Formulate this as a linear optimization problem.

Sample English Description

Decision: Which bestsellers to carry.

Objective: Minimize the total number of bestsellers carried.

Constraints: For each of the four genres, we need to carry at least two books of that genre. In other words, for each genre,

$$\#\text{ of books carried of this genre} \geq 2$$

Sample Concrete Formulation

Decision variables:

- x_i: whether to carry book i. (Binary)

Objective:

$$\text{Minimize:} \quad x_1 + x_2 + \cdots + x_{10}$$

Constraints:

$$\begin{aligned} \text{(Literary)} \quad & x_1 + x_4 + x_5 + x_9 \geq 2 \\ \text{(Sci-Fi)} \quad & x_2 + x_7 + x_9 \geq 2 \\ \text{(Romance)} \quad & x_3 + x_4 + x_6 + x_{10} \geq 2 \\ \text{(Thriller)} \quad & x_2 + x_3 + x_8 \geq 2 \end{aligned}$$

Note on Latex: to type x_{10}, you need `$x_{10}$`, otherwise it will look like x_10.

Exercise 8.1: Project Sub-Contracting

Tom Burke, owner of Burke Construction, has promised to complete five projects this winter. Burke Construction has 10 workers that will work 40 hours a week for 12 weeks this winter. Since this is a limited workforce, Tom knows that he will not be able to complete all of his construction projects without subcontracting some of them. In the table below, he has estimated the amount of labor hours required by each project, and the profit to his company. (All profits are in thousands of dollars.)

Project	1	2	3	4	5
Labor hours required	1300	950	1000	1400	1600
Profit (if done by own company)	30	10	26	18	20
Profit (if subcontracted)	6	2	8	9	4

To maximize profit, which jobs should Tom schedule for his company, and which should be subcontracted? (Assume that projects cannot be partially subcontracted; that is, a project will be completed entirely by either Burke Construction or the subcontractor.)

a) Describe in English the decision, objective and constraints in the above problem. Try your best to use **helpful keywords** and ensure that the descriptions are **precise**, **complete** and **succinct**.

Decision:

Objective:

Constraints:

b) Formulate the above as a linear optimization model and typeset it using LaTex.

Decision variable:

Objective:

Constraints:

Exercise 8.2: Supply Chain Planning

The following table provides the shipping cost for a certain item, from three of Amazon's fulfillment centers (FC) to four regions (A, B, C and D).

Region FC	1	2	3
A. Kings County, NY	20	8	25
B. Los Angeles County, CA	18	23	8
C. King County, WA	21	24	8
D. Harris County, TX	8	8	19

The following table summarizes the weekly demand for the item from each region.

Region A	Region B	Region C	Region D
30	50	10	20

Suppose that each FC is able to ship up to 40 units each week in total. Formulate a linear program to determine the minimum transportation cost needed to satisfy all demand while respecting FC capacities, as well as the optimal shipment plan.

a) Describe in English the decision, objective and constraints in the above problem. Try your best to use **helpful keywords** and ensure that the descriptions are **precise**, **complete** and **succinct**.

Decision:

Objective:

Constraints:

b) Write a linear optimization formulation of the above and typeset it using Latex.

Decision variables:

Objective:

Constraints:

Exercise 8.3: Assignment of Consultants to Projects

There are two projects and four consultants: Alice, Bob, Charles, and Daphne. Each consultant can be assigned to at most one project, and each project requires at least two consultants. As a manager, you evaluated the relative fitness of the four consultants for each project on a scale of 1 to 5, with 5 being the best fit and 1 being the worst.

	Project 1	Project 2
Alice	5	2
Bob	3	2
Charles	4	5
Daphne	3	1

Furthermore, Alice, Bob and Daphne are senior consultants and each project requires at least one senior on the team.

Formulate a linear optimization problem to maximize the total fitness of the consultants to their assigned project, subject to all the business constraints.

English Description

Decision:

Objective:

Constraints:

Concrete Formulation

Decision variables:

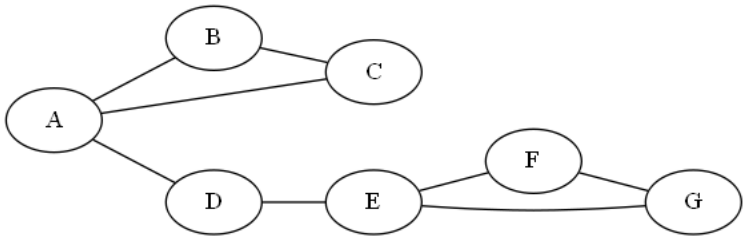
Objective:

Constraints:

Session 16: Encoding Logical Constraints using Linear Expressions

Sample Problem: Project Selection

Ebony is an ambitious master's student who would like to maximize the number of extra-curricular business analytics projects she takes part of this year. However, projects may conflict with one another. The following graph summarizes the conflicts. (For example, project A conflicts with B, C and D, but projects B and D can be done together.)



Beside the conflict above,

- Project A is a prerequisite to project F (meaning that pursuing F requires also pursuing A.)
- Project B is a prerequisite to project G.

Formulate a linear optimization problem to help her decide which projects to pursue.

English Description

Decision:

Objective:

Constraints:

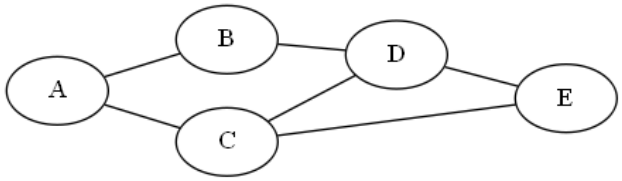
Concrete Formulation

Decision Variables:

Objective and Constraints:

Exercise 8.4: Optimal Sensor Placement

To reduce traffic congestion around USC, the university plans to install sensors at various streets in order to monitor every segment of every street around the university. To be precise, a segment is defined to be the line between two adjacent intersections. In order to monitor a segment, it suffices to have a sensor installed at one of the two ends. For example, consider the following map, which has 5 intersections (A, B, C, D, E) and 6 segment (A-B, A-C, C-D, B-D, C-E, and D-E). Installing a sensor at intersection A would monitor segments A-B and A-C, but no other segments.



The following table summarizes the cost of installing a sensor at each intersection

	A	B	C	D	E
Cost ($)	100	150	180	160	130

Moreoer, in order to install sensors at intersections B or C, the university would have to apply for a special permit from the city, which costs $50, and this cost is in addition to the installation costs above. Once the university has this permit, then it can install sensors at both B and C under the same permit.

Formulate a linear optimiztaion problem to decide which intersections to install sensors in order to minimize cost, subject to monitoring every segment.

English Description

Decision:

Objective:

Constraints:

Concrete Formulation

Decision Variables:

Objective:

Constraints:

Exercise 8.5: Warehouse Planning

The following table provides the shipping cost for one-pound packages, from 7 of Amazon's fulfillment centers (FC) to 4 regions.

Region	1	2	3	4	5	6	7
A. Kings County, NY	20.25	7.70	24.59	23.26	7.69	7.70	7.69
B. Los Angeles County, CA	18.43	23.30	7.69	7.69	24.16	22.12	24.91
C. King County, WA	21.28	24.18	7.70	17.67	23.91	22.98	24.57
D. Harris County, TX	7.69	7.70	18.73	7.71	18.79	7.70	19.47

A shipping cost of $10 or less indicates that the package will be transported via ground shipping; otherwise, it will be transported via air shipping.

For a certain item that weights a pound, Amazon would like to stock it in as few FCs as possible, while guaranteeing that it can fulfill demand in all 4 regions via ground shipping. Moreover,

- the item must be stocked in at least one of FCs 5 or 7;
- the item cannot be stocked in FC 4 unless it is also stocked in FC 1;
- if the item is stocked in FC 2, then it cannot also be stocked in FC 3.

Formulate a linear optimization problem to find the minimum number of FCs needed.

English Description

Decision:

Objective:

Constraints:

Concrete Formulation

Decision Variables:

Objective and constraints:

Exercise 8.6: Nurse Scheduling

Hospital administrators must schedule nurses so that the hospital's patients are provided with adequate care. At the same time, in the face of tighter competition in the health care industry, they must pay careful attention to keeping costs down.

From historical records, administrators estimated the minimum number of nurses to have on hand for the various times of the day, as shown in the following table.

Shift	Time	Minimum number of nurses needed
1	Midnight-4am	5
2	4am-8am	12
3	8am-noon	14
4	noon-4pm	8
5	4pm-8pm	14
6	8pm-Midnight	10

Nurses work 8 hours a day in two consecutive shifts. As a result, in each shift, there are two types of nurses: those that started in the previous shift (and are now working their second shift), and those that just started in this shift (and will be working in the next shift as well). Note that if a nurse who starts at the 8pm-Midnight shift would finish work on the next day's Midnight-4am shift.

Formulate a linear optimization problem to minimize the total number of nurses subject to being able to fulfill all business constraints.

English Description

Decision:

Objective:

Constraints:

Concrete Formulation

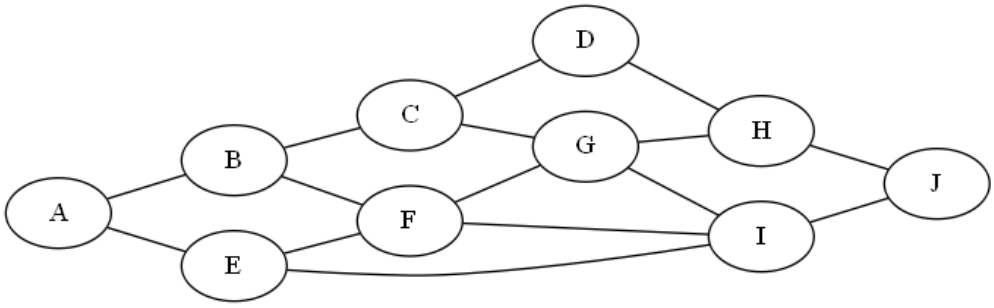
Decision variable:

Objective

Constraints:

Exercise 8.7: Off-Campus Security

USC would like to protect every intersection around USC by stationing security staff, so that every intersection either has a staff stationed, or is connected directly to another intersection that has a staff stationed. For example, in the following sample map with 10 intersections, a staff stationed at intersection A is able to protect the intersections A, B and E; a staff stationed at intersection E is able to protect intersections A, E, F and I.



For the above map, formulate a linear optimization problem to minimize the total number of staff needed, subject to protecting all 10 intersections, as well as satisfying the following constraints:

- Staff cannot be stationed at both intersections A and B.
- At least three of E, F, G and H must have a staff directly stationed.
- No one can be stationed at intersection C.
- If someone is stationed at D, then at least one of H or J must be unstationed.

a) What is the decision (in words)?

b) What is the objective (in words)?

c) Other than the constraints listed in bullets above, describe another constraint implied by the problem text (in words).

d) Write a concrete formulation using the correct mathematical notation, specifying the decision variables, the objective function and all constraints.

Decision Variables:

Objective:

Constraints: