Contents

Solutions for Week 10: Using the Gurobi Solver

Grading Scheme:

Important: This Jupyter notebook needs to be completed and submitted via Blackboard before the due date to receive a non-zero grade.

- 5: Every question is completed and the solution is essentially correct. This means that the math formulation correctly models the given problem and the code outputs the correct results. However, it is okay if the code returns another optimal solution that is equally good.
- 4: Almost complete, but certain questions are blank, the code there does not run, or the output is clearly incorrect. For math formulations, this would be if the formulation is seriously flawed.
- 3: This score will not be assigned, as everyone should strive to get 4 or 5.
- 2: Not close to complete, but at least 50% complete.
- 1: At least 10% complete, but less than 50% complete
- 0: Less than 10% complete, or response is identical to someone else's, indicating plagiarism.

A perfect score is 5. Note that your code does not need to be absolutely perfect to receive a 5, but you need to complete every question and ensure that the outputs are correct on all of the sample runs included here. Furthermore, you should double check that your math formulations make sense, by plugging in concrete numbers and verifying that the constraints are satisfied.

These exercises are intended to be completed in 6-8 hours, including the individual-work time in class. You should budget at least this much time before the due date.

Name: XXX

Exercise 10.1 Numerical Solution for Project Sub-Contracting

Following the assortment planning example, incrementally produce a version of the Gurobi code that does not hard-code in numbers but obtain them from appropriate data structures.

Decision variable:

- Let x_i denote whether to schedule job i for own company. (Binary)
- Let y_i denote whether to subcontract job i. (Binary)

Objective:

$$\text{Maximize:} \qquad 30x_1 + 10x_2 + 26x_3 + 18x_4 + 20x_5 + 6y_1 + 2y_2 + 8y_3 + 9y_4 + 4y_5$$

Constraints:

$$\begin{aligned}
\text{(Labor)} \quad & 1300x_1 + 950x_2 + 1000x_3 + 1400x_4 + 1600x_5 \leq 4800 \\
\text{(Doing every project)} \quad & x_1 + y_1 = 1 \\
& x_2 + y_2 = 1 \\
& x_3 + y_3 = 1 \\
& x_4 + y_4 = 1 \\
& x_5 + y_5 = 1
\end{aligned}$$

Version 1: Hard-coding in everything

For comparison purposes, write a version of the code that hard-codes in everything, similar to version 1 of the previous example. Remember to set the sense in the objective to GRB.MAXIMIZE.

```
[ ]: # Write your code here
```

```
[12]: # Correct output
```

```
Optimal objective: 88.0
Optimal solution: x1=1.0, x2=1.0, x3=1.0, x4=1.0, x5=0.0
```

Version 2: Using addVars to create multiple variables at once

Using addVars, generate all of the x's using one command, and all of the y's using one command. Also, make the optimal solution easier to read, as in the output below.

```
[ ]: # Write your code here (you must use addVars instead of addVar)
```

```
[13]: # Correct output
```

```
Optimal objective: 88.0
Optimal solution: do projects 1 2 3 4 yourself
```

Version 3 and 4: Using list comprehension and for loops

Instead of hard-coding in the numbers, obtain them from the following data structures. Moreover, use list comprehension to generate the large sums, and for loops to generate the repetitive constraints. Build up the formulation part by part and double in the end check by making Gurobi display the entire concrete formulation.

```
[14]: import pandas as pd
      projects=range(1,6)
      ownLabor=4800
      profit=pd.DataFrame([[30,10,26,18,20],[6,2,8,9,4]], \
                          index=['Yourself','Subcontract'], columns=projects)
      profit
```

```
              1   2   3   4   5
Yourself     30  10  26  18  20
Subcontract   6   2   8   9   4
```

```
[15]: laborRequired=pd.Series([1300,950,1000,1400,1600],index=projects)
      laborRequired
```

```
1    1300
2     950
3    1000
4    1400
5    1600
dtype: int64
```

```
[ ]: # Objective function
```

```
[16]: # Correct output for the objective function
```

```
<gurobi.LinExpr: 30.0 x[1] + 6.0 y[1] + 10.0 x[2] + 2.0 y[2] + 26.0 x[3] + 8.0 y[3] + 18.
↪0 x[4] + 9.0 y[4] + 20.0 x[5] + 4.0 y[5]>
```

```
[ ]: # Labor constraint
```

```
[17]: # Correct output for the labor constraint
```

```
<gurobi.TempConstr: 1300.0 x[1] + 950.0 x[2] + 1000.0 x[3] + 1400.0 x[4] + 1600.0 x[5]␣
↪<= 4800>
```

```
[ ]: # Entire formulation
```

```
[18]: # Correct output after using %cat (Mac/linux) or !type (Windows)
```

```
\ LP format - for model browsing. Use MPS format to capture full model detail.
Maximize
  30 x[1] + 10 x[2] + 26 x[3] + 18 x[4] + 20 x[5] + 6 y[1] + 2 y[2] + 8 y[3]
   + 9 y[4] + 4 y[5]
Subject To
 Labor: 1300 x[1] + 950 x[2] + 1000 x[3] + 1400 x[4] + 1600 x[5] <= 4800
 Project_1: x[1] + y[1] = 1
 Project_2: x[2] + y[2] = 1
 Project_3: x[3] + y[3] = 1
 Project_4: x[4] + y[4] = 1
 Project_5: x[5] + y[5] = 1
Bounds
Binaries
 x[1] x[2] x[3] x[4] x[5] y[1] y[2] y[3] y[4] y[5]
End
```

Final code

Use the final version of your formulation to produce the following output.

```
[ ]: # Write your final code here. Make sure that it is self-contained.
```

```
[19]: # Correct output
```

```
Optimal objective: 88.0
Optimal solution: do projects 1 2 3 4 yourself
```

Exercise 10.2: Numerical Solution for Warehouse Planning

The concrete formulation of Exercise 8.5 is reproduced below:

Decision Variables: Let $X_1, \cdots, X_7$ denote whether to use each FC. (Binary)

Objective and constraints:

$$\begin{aligned}
\text{Minimize}\quad & X_1 + X_2 + \cdots + X_7 \\
\text{s.t.}\quad & X_2 + X_5 + X_6 + X_7 \geq 1 \\
& X_3 + X_4 \geq 1 \\
& X_3 \geq 1 \\
& X_1 + X_2 + X_4 + X_6 \geq 1 \\
& X_5 + X_7 \geq 1 \\
& X_4 \leq X_1 \\
& X_2 + X_3 \leq 1
\end{aligned}$$

a) Implement the above using Gurobi, while using for loops and list comprehensions as much as possible to automate recurring patterns.

After you are done, use `mod.write`, and `%cat` in Mac or `!type` in Windows to output what the linear optimization formulation looks like according to Gurobi. You can use this to verify that you have indeed implemented the above.

```
[ ]: # Write your code here
```

```
[20]: # Correct output after using %cat (Mac/linux) or !type (windows)
```

```
\ LP format - for model browsing. Use MPS format to capture full model detail.
Minimize
  X[1] + X[2] + X[3] + X[4] + X[5] + X[6] + X[7]
Subject To
 R0: X[2] + X[5] + X[6] + X[7] >= 1
 R1: X[3] + X[4] >= 1
 R2: X[3] >= 1
 R3: X[1] + X[2] + X[4] + X[6] >= 1
 R4: X[5] + X[7] >= 1
 R5: - X[1] + X[4] <= 0
 R6: X[2] + X[3] <= 1
Bounds
Binaries
 X[1] X[2] X[3] X[4] X[5] X[6] X[7]
End
```

b) Solve the MIP and print the minimum number of FCs needed, as well as where to stock the items. The output format should match the sample output below.

```
[ ]: # Write your code here
```

```
[21]: # Correct output
```

```
Minimum # of FCs needed: 3
Stock item in the following:
        FC1
        FC3
        FC7
```

Exercise 10.3: Numerical Solution for Assignment of Consultants to Projects

Incrementally create Gurobi code to solve Exercise 8.3, following the example given in the lecture.

Recap of Exercise 8.3: There are two projects and four consultants: Alice, Bob, Charles, and Daphne. Each consultant can be assigned to at most one project, and each project requires at least two consultants.

As a manager, you evaluated the relative fitness of the four consultants for each project on a scale of 1 to 5, with 5 being the best fit and 1 being the worst.

	Project 1	Project 2
Alice	5	2
Bob	3	2
Charles	4	5
Daphne	3	1

Furthermore, Alice, Bob and Daphne are senior consultants and each project requires at least one senior on the team.

Formulate a linear optimization problem to maximize the total fitness of the consultants to their assigned project, subject to all the business constraints.

Concrete Formulation:

Decision variables:

- x_{ij}: whether to assign consultant i to project j. (Binary)

Objective:

$$\text{Maximize: } 5x_{A1} + 2x_{A2} + 3x_{B1} + 2x_{B2} + 4x_{C1} + 5x_{C2} + 3x_{D1} + x_{D2}$$

Constraints:

$$\begin{aligned}
\text{(Alice)} \quad & x_{A1} + x_{A2} \leq 1 \\
\text{(Bob)} \quad & x_{B1} + x_{B2} \leq 1 \\
\text{(Charles)} \quad & x_{C1} + x_{C2} \leq 1 \\
\text{(Daphne)} \quad & x_{D1} + x_{D2} \leq 1 \\
\text{(Project 1 Total)} \quad & x_{A1} + x_{B1} + x_{C1} + x_{D1} \geq 2 \\
\text{(Project 2 Total)} \quad & x_{A2} + x_{B2} + x_{C2} + x_{D2} \geq 2 \\
\text{(Project 1 Senior)} \quad & x_{A1} + x_{B1} + x_{D1} \geq 1 \\
\text{(Project 2 Senior)} \quad & x_{A2} + x_{B2} + x_{D2} \geq 1
\end{aligned}$$

Implement the above in Gurobi while obtaining all numbers from the below data structures. See the desired intermediate outputs for every step below. You should write your code in such a way such that if the input data is changed, the code will still work.

```
[34]: # Input Data
      import pandas as pd
      consultants=['Alice', 'Bob', 'Charles', 'Daphne']
      projects=[1,2]
      fitness=pd.DataFrame([[5,2],[3,2],[4,5],[3,1]],index=consultants,columns=projects)
      senior=['Alice','Bob','Daphne']
      capacity=pd.Series([1,1,1,1],index=consultants)
      demand=pd.Series([2,2],index=projects)
      seniorDemand=pd.Series([1,1],index=projects)
```

```
[35]: # Creating variables
      from gurobipy import Model, GRB
      mod=Model()
      x=mod.addVars(consultants,projects,vtype=GRB.BINARY,name='x')
```

```
    mod.update()
    x
```

```
{('Alice', 1): <gurobi.Var x[Alice,1]>,
 ('Alice', 2): <gurobi.Var x[Alice,2]>,
 ('Bob', 1): <gurobi.Var x[Bob,1]>,
 ('Bob', 2): <gurobi.Var x[Bob,2]>,
 ('Charles', 1): <gurobi.Var x[Charles,1]>,
 ('Charles', 2): <gurobi.Var x[Charles,2]>,
 ('Daphne', 1): <gurobi.Var x[Daphne,1]>,
 ('Daphne', 2): <gurobi.Var x[Daphne,2]>}
```

```
[ ]: # Objective
```

```
[36]: # Correct output for the objective
```

```
<gurobi.LinExpr: 5.0 x[Alice,1] + 2.0 x[Alice,2] + 3.0 x[Bob,1] + 2.0 x[Bob,2] + 4.0␣
↪x[Charles,1] + 5.0 x[Charles,2] + 3.0 x[Daphne,1] + x[Daphne,2]>
```

```
[ ]: # (Alice) constraint
```

```
[37]: # Correct output for the (Alice) constraint
```

```
<gurobi.TempConstr: x[Alice,1] + x[Alice,2] <= 1>
```

```
[ ]: # (Project 1 Total) constraint
```

```
[38]: # Correct output for the (Project 1 Total) constraint
```

```
<gurobi.TempConstr: x[Alice,1] + x[Bob,1] + x[Charles,1] + x[Daphne,1] >= 2>
```

```
[ ]: # (Project 1 Senior) constraint
```

```
[39]: # Correct output for the (Project 1 Senior) constraint
```

```
<gurobi.TempConstr: x[Alice,1] + x[Bob,1] + x[Daphne,1] >= 1>
```

```
[ ]: # Full formulation
```

```
[40]: # Correct output after running %cat (Mac/linux) or !type (Windows)
```

```
\ LP format - for model browsing. Use MPS format to capture full model detail.
Maximize
  5 x[Alice,1] + 2 x[Alice,2] + 3 x[Bob,1] + 2 x[Bob,2] + 4 x[Charles,1]
   + 5 x[Charles,2] + 3 x[Daphne,1] + x[Daphne,2]
Subject To
 Alice: x[Alice,1] + x[Alice,2] <= 1
 Bob: x[Bob,1] + x[Bob,2] <= 1
 Charles: x[Charles,1] + x[Charles,2] <= 1
 Daphne: x[Daphne,1] + x[Daphne,2] <= 1
 Project_1: x[Alice,1] + x[Bob,1] + x[Charles,1] + x[Daphne,1] >= 2
 Project_2: x[Alice,2] + x[Bob,2] + x[Charles,2] + x[Daphne,2] >= 2
 Project_1_Senior: x[Alice,1] + x[Bob,1] + x[Daphne,1] >= 1
 Project_2_Senior: x[Alice,2] + x[Bob,2] + x[Daphne,2] >= 1
Bounds
Binaries
 x[Alice,1] x[Alice,2] x[Bob,1] x[Bob,2] x[Charles,1] x[Charles,2]
 x[Daphne,1] x[Daphne,2]
End
```

```
[ ]: # Numerical Solution
```

```
[41]: # Correct output
```

```
Maximum total fitness: 15.0
```

```
[ ]: # Creating the output table
```

```
[42]: # Expected output
```

```
           1    2
Alice    NaN  NaN
Bob      NaN  NaN
Charles  NaN  NaN
Daphne   NaN  NaN
```

```
[ ]: # Filling in the output table
```

```
[43]: # Correct output
```

```
         1  2
Alice    1  0
Bob      0  1
Charles  0  1
Daphne   1  0
```

```
[1]: # Final code
```

```
[44]: # Correct output
```

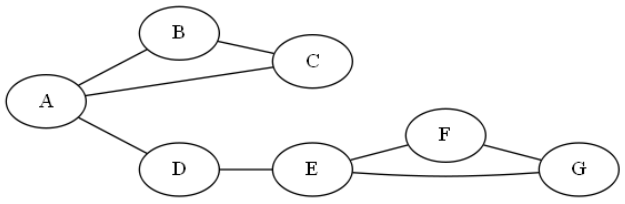
```
Maximum total fitness: 15.0
```

```
         1  2
Alice    1  0
Bob      0  1
Charles  0  1
Daphne   1  0
```

Exercise 10.4: Numerical Solution for Project Selection

Implement the linear optimization model for the "Project Selection" problem from last week.

Recap of the problem: Ebony is an ambitious master's student who would like to maximize the number of extra-curricular business analytics projects she takes part of this year. However, projects may conflict with one another. The following graph summarizes the conflicts. (For example, project A conflicts with B, C and D, but projects B and D can be done together.)



Beside the conflict above,

- Project A is a prerequisite to project F (meaning that pursuing F requires also pursuing A.)
- Project B is a prerequisite to project G.

Formulate a linear optimization problem to help her decide which projects to pursue.

Concrete Formulation

Decision Variables:

X_i: whether to pursue project i. (Binary)

Objective and Constraints:

$$\begin{aligned} \text{Maximize} \quad & X_A + X_B + \cdots + X_G \\ \text{s.t.} \quad & \\ & X_A + X_B \leq 1 \\ & X_B + X_C \leq 1 \\ & X_A + X_C \leq 1 \\ & X_A + X_D \leq 1 \\ & X_D + X_E \leq 1 \\ & X_E + X_F \leq 1 \\ & X_F + X_G \leq 1 \\ & X_E + X_G \leq 1 \\ & X_A \geq X_F \\ & X_B \geq X_G \end{aligned}$$

Input data

```
[45]: projects=['A','B','C','D','E','F','G']
      conflicts=[['A','B'],['B','C'],['A','C'],['A','D'],\
                 ['D','E'],['E','F'],['F','G'],['E','G']]
      prereqs=[['A','F'],['B','G']]
```

```
[46]: # Examples of looping through the above data structures
      # Standard way
      for pair in conflicts:
          print(f'{pair[0]} and {pair[1]} are in conflict.')
      # Shortcut
      for p1,p2 in prereqs:
          print(f'{p1} is a pre-requisite to {p2}')
```

```
A and B are in conflict.
B and C are in conflict.
A and C are in conflict.
A and D are in conflict.
D and E are in conflict.
E and F are in conflict.
F and G are in conflict.
E and G are in conflict.
A is a pre-requisite to F
B is a pre-requisite to G
```

Python Code

Write Python code to implement the above using Gurobi. Your code must obtain all data from the above input data structures, such that if new projects are added or the list of conflicts and pre-reqs change, the code will continue to work.

```
[ ]: # Write your code here
```

```
[47]: # Correct output
```

```
Optimal objective: 3.0
Optimal projects to pursue: B D G
```

Exercise 10.5: Numerical Solution for Food Production

Solve the following concrete formulation, while loading input data from the given data structures.

Decision Variables:

- $X_1, X_2, \cdots, X_6$: amount of oil to buy in each month. (continuous)
- $Y_1, Y_2, \cdots, Y_6$: amount of oil stored at the end of each month. (continuous)

Objective:

$$\text{Min. } 150X_1 + 160X_2 + 180X_3 + 170X_4 + 180X_5 + 160X_6$$

Constraints:

$$\begin{aligned}
Y_1 &= X_1 - 2000 \\
Y_2 &= X_2 + Y_1 - 2000 \\
Y_3 &= X_3 + Y_2 - 2000 \\
Y_4 &= X_4 + Y_3 - 2000 \\
Y_5 &= X_5 + Y_4 - 2000 \\
Y_6 &= X_6 + Y_5 - 2000 \\
Y_t &\leq 1000 && \text{for each month } t \in \{1, 2, \cdots, 6\}. \\
X_t, Y_t &\geq 0 && \text{for each month } t.
\end{aligned}$$

```
[48]: # Input data
      import pandas as pd
      months=range(1,7)
      price=pd.Series([150,160,180,170,180,160],index=months)
      usage=2000
      storage_capacity=1000
```

```
[ ]: # Write your code here
```

```
[49]: # Correct output
```

```
Minimum purchase cost: 1960000.0
Month   Buy     Store
1       3000.0  1000.0
2       2000.0  1000.0
3       1000.0  0.0
4       3000.0  1000.0
5       1000.0  0.0
6       2000.0  0.0
```

Exercise 10.6: Optimal Advertising Plan

In this exercise, you need to complete the English description, concrete formulation, and Gurobi code. It is not enough to have correct Python code; the correctness of the concrete formulation also counts.

SALS Marketing Inc. is developing an advertising campaign for a large consumer goods corporation. An advertising plan specifies how many units of each kind of advertisement to purchase. SALS has promised a plan that will yield the highest possible "exposure rating," which is a measure of the ability to reach the appropriate demographic group and generate demand. The options for advertisements with their respective costs (per unit of advertising) and per-unit exposure ratings are given in the table below (K stands for thousands).

Category	Subcategory	Cost/Unit	Exposure/Unit
Magazines	Literary	\$7.5 K	15 K
	News	\$10 K	22.5 K
	Topical	\$15 K	24 K
Newspapers	Morning	\$2 K	37.5 K
	Evening	\$3 K	75 K
Television	Morning	\$20 K	275 K
	Midday	\$10 K	180 K
	Evening	\$60 K	810 K
Radio	Morning	\$15 K	180 K
	Midday	\$15 K	17 K
	Evening	\$10 K	16 K

Of course, certain restrictions exist for the advertising campaign. The client corporation has budgeted 800K for the campaign, but to restrict overexposure to any particular audience it wants no more than 300K put into any one category (Magazine, Newspaper, etc.). Also, to ensure a broad range of exposure, at least 100K must be spent in each category. Finally, one has to purchase an integer number of units of each kind of advertisement, as no fractional units are allowed. Formulate and solve a linear optimization model to determine the optimal advertising plan.

English Description

Decision:

Objective:

Constraints:

Concrete Formulation

Decision variables:

Objective and constraints:

Python Code

Write Gurobi code to implement the above formulation. Your code should read in the data from the following data structures rather than hard code in the numbers. For convenience, all numerical values are in the units of K (thousands).

The outputs should be in the same format as the sample outputs below. Note: Gurobi might output strangely formatted numbers like `-0`, and you can make it `0` by converting it to `int`.

```
[50]: # Constructing the subcategories
      subcat={}
      subcat['Magazines']=['Literary Mag.','News Mag.','Topical Mag.']
      subcat['Newspapers']=['Morning News','Evening News']
      subcat['Television']=['Morning TV','Midday TV','Evening TV']
      subcat['Radio']=['Morning Radio','Midday Radio','Evening Radio']
      subcat
```

```
{'Magazines': ['Literary Mag.', 'News Mag.', 'Topical Mag.'],
 'Newspapers': ['Morning News', 'Evening News'],
 'Television': ['Morning TV', 'Midday TV', 'Evening TV'],
 'Radio': ['Morning Radio', 'Midday Radio', 'Evening Radio']}
```

```
[51]: # Input table
      import pandas as pd
     ␣
 ↪allSubCat=subcat['Magazines']+subcat['Newspapers']+subcat['Television']+subcat['Radio']
      data=pd.DataFrame([[7.5,15],[10,22.5],[15,24],\
                         [2,37.5],[3,75],
                         [20,275],[10,180],[60,810],\
                         [15,180],[15,17],[10,16]],\
                         index=allSubCat,columns=['Cost','Exposure'])
      data
```

```
               Cost  Exposure
Literary Mag.   7.5      15.0
News Mag.      10.0      22.5
Topical Mag.   15.0      24.0
Morning News    2.0      37.5
Evening News    3.0      75.0
Morning TV     20.0     275.0
Midday TV      10.0     180.0
Evening TV     60.0     810.0
Morning Radio  15.0     180.0
Midday Radio   15.0      17.0
Evening Radio  10.0      16.0
```

```
[ ]: # Write your code here
```

```
[52]: # Correct output (it's okay if you have the same maximum total exposure but a␣
 ↪different allocation of funds)
```

```
Maximum total exposure (in thousands): 14235.0
# of units to purchase:
        Literary Mag.: 0
        News Mag.: 10
        Topical Mag.: 0
        Morning News: 0
        Evening News: 98
        Morning TV: 0
        Midday TV: 30
        Evening TV: 0
        Morning Radio: 7
        Midday Radio: 0
        Evening Radio: 0
```