Math308, Quiz 6, 10/24/13

First Name: .... **Last Name: ....**

Grade: ....

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Problem 1. 100%. Use the Laplace transform to solve the following initial value problem:

$$\begin{aligned} &y'' - 2y' + 2y = 0 \\ &y(0) = 0, \quad y'(0) = 1. \end{aligned} \tag{1}$$

Solutions

Theorem. Suppose that the functions $f, f', \dots, f^{(n-1)}$ are continuous and that $f^{(n)}$ is piecewise continuous on any interval $0 \le t \le A$. Suppose that there exist constants K, a and M such that $|f(t)| \le Ke^{at}, |f'(t)| \le Ke^{at}, \dots, |f^{(n-1)}(t)| \le Ke^{at}$ for $t \ge M$. Then $\mathcal{L}[f^{(n)}(t)]$ exists for $s > a$ and given by

$$\mathcal{L}[f^{(n)}(t)] = s^n \mathcal{L}[f(t)] - s^{n-1} f(0) - \ldots - s f^{(n-2)}(0) - f^{(n-1)}(0).$$

Now, By taking the Laplace transform of the equation we obtain

$$\mathcal{L}[y''] - 2\mathcal{L}[y'] + 2\mathcal{L}[y] = 0. \tag{2}$$

We use the above theorem to express $\mathcal{L}[y'']$ and $\mathcal{L}[y']$ in terms of $\mathcal{L}[y]$:

$$\left(s^2\mathcal{L}[y] - sy(0) - y'(0)\right) - 2\left(s\mathcal{L}[y] - y(0)\right) + 2\mathcal{L}[y] = 0, \tag{3}$$

or we can simplify it as

$$\left(s^2 - 2s + 2\right)\mathcal{L}[y] - (s-2)y(0) - y'(0) = 0.$$

And we now apply the initial conditions and find $\mathcal{L}[y]$:

$$L[y] = \frac{1}{s^2 - 2s + 2},$$

or

$$L[y] = \frac{1}{(s-1)^2 + 1}.$$

Finally, the inverse Laplace transform of the right hand side is

$$y = e^t \sin t.$$