Math308, Quiz 1, 09/05/13

First Name: .... **Last Name: ....**

Grade: ....

SHOW ALL WORK!

Problem 1. Determine **1. the order**, **2. the kind** (i.e. PDE or ODE), **3. linear or nonlinearity** of the given differential equation:

$$\frac{d^2y}{dt^2} + \sin(t+y) = \sin t.$$

Problem 2. Solve the following initial value problem:

$$\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^{y}},$$
$$y(0) = 1.$$

Solutions

Problem 1.

1. 2nd order, since the highest derivative is $\frac{d^2y}{dx^2}$;
2. ODE, since only one ordinary derivative is used;
3. nonlinear, since $\sin(t+y)$ is a nonlinear function.

Problem 2.

First of all note that $y + e^y \neq 0$. Then, by multipling the equation by $\dfrac{dx}{y+e^y}$ we obtain that

$$(y + e^y)dy = (x - e^{-x})dx.$$

This equation is separable, so by method of direct integration we get

$$\frac{y^2}{2} + e^y = \frac{x^2}{2} + e^{-x} + C,$$

where C is some arbitrary constant. To find the constant corresponding to our initial data we use $y(0) = 1$:

$$\frac{1^2}{2} + e^1 = 0 + e^0 + C$$

$$\Rightarrow \frac{1}{2} + e = 0 + 1 + C$$

$$\Rightarrow C = e - \frac{1}{2}.$$

Finally we get the exact solution to our initial value problem:

$$\frac{y^2}{2} + e^y = \frac{x^2}{2} + e^{-x} + e - \frac{1}{2}.$$