Math308, Quiz 3, 09/19/13

First Name: ....   **Last Name:** ....

Grade: ....

Show all work!

Consider the following initial value problem:

$$\begin{aligned} & y'' - 6y' + 5y = 0, \\ & y(0) = 2,\ y'(0) = 6. \end{aligned} \tag{1}$$

Problem 1. 90%. Solve the problem.

Problem 2. 10%. Find $\lim_{t\to\infty} y(t)$.

Solutions

Problem 1. First we write the characteristic equation that is obtained by assuming that the solution of (1) has the form of $y(t) = e^{rt}$:

$$r^2 - 6r + 5 = 0. \tag{2}$$

We find that $r_1 = 1$ and $r_2 = 5$ are the roots of the characteristic equation. Therefore, the general solution of (1) is

$$y(t) = C_1 e^t + C_2 e^{5t}. \tag{3}$$

We now use the initial condition to find the constants in (3).

$$\left.\begin{aligned} y(0) &= 2 \\ y'(0) &= 6 \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} C_1 + C_2 &= 2, \\ C_1 + 5C_2 &= 6, \end{aligned}\right. \tag{4}$$

which is a linear system for C_1 and C_2, that can be solved easily: $C_1 = 1$, $C_2 = 1$. Therefore, the solution of the initial value problem (1) is

$$y(t) = e^t + e^{5t}. \tag{5}$$

Problem 2. We have:

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty} (e^t + e^{5t}) = +\infty. \tag{6}$$

So, the solution goes to infinity as t growth.