Math308, Quiz 9, 11/07/13

First Name: ....   **Last Name:** ....

Grade: ....

Show all work!

Given the system of differential equations:

$$\begin{cases} x_1' = 2x_2, & x_1(0) = 2 \\ x_2' = 2x_1, & x_2(0) = 0. \end{cases}$$

Problem 1. 30%. Transform the given system into a single equation of second order.

Problem 2. 70%. Find x_1 and x_2 that satisfies the above initial value problem.

Solutions

Problem 1. We find x_2 from the first equation and insert it into the second one:

$$\begin{cases} x_2 = \dfrac{1}{2}x_1', \\ \dfrac{1}{2}x_1'' = 2x_1. \end{cases}$$

Now, we get the second order equation for x_1:

$$x_1'' - 4x_1 = 0.$$

Problem 2. The solution of the last equation is easy to find using the characteristic equation:

$$r^2 - 4 = 0,$$

where we find that

$$x_1 = C_1 e^{-2t} + C_2 e^{2t}.$$

Using the fact from Problem 1 we easily find x_2:

$$x_2 = \frac{1}{2}x_1' = -C_1 e^{-2t} + C_2 e^{2t}.$$

Apply the above initial conditions to get:

$$\begin{cases} x_1(0) = 2 & \Rightarrow \quad C_1 + C_2 = 2, \\ x_2(0) = 0 & \Rightarrow \quad -C_1 + C_2 = 0, \end{cases}$$

which gives us $C_2 = 1$ and $C_1 = 1$. Therefore, the solution of the given system of differential equations is

$$x_1(t) = e^{-2t} + e^{2t}, \quad x_2(t) = -e^{-2t} + e^{2t}.$$