Math417

Deadline: February 12, 2013

Programming assignment 2

1 Iterative Methods

We are interested to find the roots of the following equation

$$f(x) = 0, \quad x \in [a, b], \tag{1}$$

where a and b are some real numbers.

1.1 Problem 1

Write four functions corresponding to the following iterative methods: **(a)** Bisection method, **(b)** Fixed-Point iteration, **(c)** Newton-Raphson and **(d)** the Secant methods. The input values are an inital guess p_0, tolerance TOL, maximum number of iterations N_0, and the output is a vector consisting of the sequence of approximated values $\{p_n\}_{n=0}^{\infty}$.

1.2 Problem 2

Assume $f(x) = 2 - e^x + x^2 - 3x$. Determine an interval $[a, b]$ on which fixed-point iteration will converge. Then, use your functions from Problem 1 to find the roots of $f(x) = 0$ with two different initial data and $TOL = 10^{-10}$. Make a table and print your outputs from the different methods and discuss the result.

1.3 Problem 3

Now, let us take $f(x) = x^3 + 4x^2 - 10$. This function has a unique root in $[1, 2]$, $p = 1.365230013$. This problem can be rewritten as the fixed-point form $x = \frac{1}{2}(10 - x^3)^{1/2}$.

The convergence rate α can be determined as follows:

$$\alpha \approx \ln\left(\frac{p_{n+2} - p}{p_{n+1} - p}\right) / \ln\left(\frac{p_{n+1} - p}{p_n - p}\right) \tag{2}$$

Use the functions from Problem 1 to find approximate solution with $TOL = 10^{-10}$. Compute the convergence rate α for four implemented iterative methods.

2 Interpolations

For given $n+1$ distint points $x_0, x_1, \ldots, x_n$ and f is a function whose values are given at these points, the Lagrange polynomial interpolation $P(x)$ is defined as follows:

$$P(x) = \sum_{k=0}^{n} f(x_k) L_{n,k}(x), \tag{3}$$

where $L_{n,k}$ is the Lagrange coefficient polynomials and they are calculated as

$$L_{n,k} = \prod_{i=0, i\neq k}^{n} \frac{x - x_i}{x_k - x_i}. \tag{4}$$

2.1 Problem 1

The average monthly temperature of College Station is given in the following table (in °F)

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
61	66	73	79	85	92	95	96	91	82	71	63

Write a program which computes the Lagrange coefficient polynomial $L_{n,k}(x)$ for $n = 12$, $k = 0, \ldots, n-1$, and compute the polynomial approximation $P(x)$ that interpolates the average temperature given in the above table. What is the temperature in February 12? Plot the given data from the table together with the function $P(x)$.

2.2 Problem 2

Modify your program from the previous problem, that computes $L_{n,k}(x)$ and $P(x)$ for $n = 4, 8, 16$ for the following functions:

1. $f(x) = \sin(2\pi x)$ for $x \in [-1, 1]$,
2. $f(x) = |x - 0.5|$ for $x \in [0, 1]$.

Use at least 1000 values of x while plotting the results.