Math417

Deadline: April 2, 2013
(the late submission will be subject to 50% less grade)

Programming assignment 4

1 Initial-Value Problem for Ordinary Differential Equations

Consider the initial-value problem

$$\begin{aligned} y'(t) &= f(t, y), \quad t \in [a, b] \\ y(0) &= y_0, \end{aligned} \tag{1}$$

where $f(t, y) = y - 4t^2 + 1$, $a = 0$, $b = 1$, $y_0 = 1$.

1.1 Exact Solution

Find the exact solution of the IVP (1).

1.2 Numerical Approximation

Write four functions to compute the approximation of the solution $y(t)$ of the IVP (1) using

1. Euler's method,
2. Taylor method of order two,
3. Modified Euler method, and
4. Runge-Kutta method of order four

with 6 sub-intervals. Compare your results with the exact solution at $t = b$. Report your results in a table which should contain the points t_i, exact solution and numerical solutions from the above four methods in columns at points t_i. Also, make one figure that contains all results, put a legend to describe the lines, put the axes and title. Motivate your results. Which methods performs better?

1.3 Convergence Rates

Assume that the given interval is divided by N equal sub-intervals. We denote the length of the sub-intervals by h. Then, $h = t_i - t_{i-1} = \frac{b-a}{N}$ for any $i = 1, 2, \ldots, N$. Now, use your functions from Problem 1.2 to compute the approximate solution of the IVP (1) for $N = 5$, 10, 20, 40, 80, 160, 320, 640, 1280, 2560. Plot h versus h^α and h versus the error $|y_{exact}(t_N) - y_{approx}(t_N)|$ in four different *loglog*-plots in Matlab. Motivate your results. You can take the appropriate convergence rate α for each method from the book.

1.4 Testing your program

Now, use your program and repeat the steps 1.1-1.3 to solve the above IVP problem with the following data:

$$f(t, y) = 2\sin(t), \quad a = 0,\, b = \pi, \quad y_0 = 0.$$

Some useful advice for your report. Please, be clear when you write your report, motivate your answer, do not submit a plain code with no explanation, put axis, title and legend to all figures, explain what is plotted, try to minimize your computational data in your presented table, do not submit a table if it contains more than one page, try to make it shorter.