# Math308, Quiz 4, 09/26/13

**First Name: ..................................** **Last Name: ..................................**

Grade: ..........

**Show all work!**

**Problem 1. 100%.** Solve the following non-homogeneous differential equation:

$$y'' + 2y' + y = 2\cos t. \tag{1}$$

## Solutions

**Problem 1.** a) First we find the general solution of corresponding homogeneous differential equation:

$$y'' + 2y' + y = 0. \tag{2}$$

The characteristic equation is

$$r^2 + 2r + 1 = 0, \tag{3}$$

which has two repeated real roots: $r_{1,2} = -1$. Therefore, one solution of the homogeneous equation is $y_1(t) = e^{-t}$ and the second solution is $y_2 = te^{-t}$. The general solution of Equation (2) is then written as

$$y_{\text{hom}}(t) = C_1 e^{-t} + C_2 t e^{-t}. \tag{4}$$

b) Let us now search for a particular solution of the non-homogeneous equation (1). The right hand side of (1) is a trigonometric function, thus we assume that $Y(t) = A\sin t + B\cos t$, where $A$ and $B$ are constants to be determined. By inserting $Y(t)$ to Equation (1) we obtain:

$$\begin{aligned}
&Y(t)'' + 2Y(t)' + Y(t) \\
&\qquad = (A\sin t + B\cos t)'' + 2(A\sin t + B\cos t)' + (A\sin t + B\cos t) \\
&\qquad = -A\sin t - B\cos t + 2A\cos t - 2B\sin t + A\sin t + B\cos t \\
&\qquad = (-A - 2B + A)\sin t + (-B + 2A + B)\cos t \\
&\qquad = -2B\sin t + 2A\cos t \\
&\qquad = 2\cos t.
\end{aligned}$$

Where by matching the coefficient we find that $A = 1$ and $B = 0$. Therefore,

$$Y(t) = \sin t,$$

and the general solution of the non-homogeneous equation is

$$y_{\text{nonhom}}(t) = C_1 e^{-t} + C_2 t e^{-t} + \sin t \tag{5}$$