Math417

Deadline: February 26, 2013

Programming assignment 3

1 Numerical Integration

Consider the function $f(x) = \sin(x)$ on the interval $x \in [0, \pi]$.

1.1 Integration Rules

Write three functions to compute the approximation of the integral

$$I = \int_0^{\pi} f(x)dx \tag{1}$$

using **(a)** Midpoint, **(b)** Trapezoidal and **(c)** Simpson's Rules with 8 sub-intervals. Compare your results with the exact value of the integral.

1.2 Convergence Rates

Assume that the given interval is divided by n equal sub-intervals. We denote the length of the sub-intervals by h. Then, $h = x_i - x_{i-1} = \frac{\pi}{n}$ for any $i = 1, 2, \ldots, n$. Now, use your functions from Problem 1 to compute the above integral for $n = 5, 10, 20, 40, 80, 160, 320$. Report your results in a table which should contain h, the approximate integrals I_{approx}, and the convergence rate of the numerical integration α. Plot h versus h^{α} and h versus the error $|I_{exact} - I_{approx}|$ in one *loglog*-plot in Matlab. Motivate your results.

Hint. You can either use the formula defined in Problem 3, Programming Assignment 2 or the relation (2) defined in the next question to compute the order of convergence α.

1.3 Bonus Question (5 points)

Assume that $I_{approx}(h)$ and $I_{approx}(h/2)$ are numerical approximations of the integral I_{exact}, corresponding to sub-interval lengths h and $h/2$. Prove the following relation:

$$\log_2 \left| \frac{I_{exact} - I_{approx}(h)}{I_{exact} - I_{approx}(h/2)} \right| = \alpha + \mathcal{O}(h). \tag{2}$$

1.4 Gaussian Quadrature Rules

Use Gaussian Quadrature with 3, 4 and 5 points to approximate the above integral.