Math417

Deadline: April 16, 2013
(the late submission will be subject to 50% less grade)

Programming assignment 5

Direct Methods for Solving Linear Systems

A linear system of n equations in matrix-vector form can be written as

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \tag{1}$$

where $\mathbf{A}$ is the coefficient $n \times n$-matrix, $n > 0$, $\mathbf{b}$ is given right hand side vector of size n, and $\mathbf{x}$ is the unknown vector of size n.

In this programming assignment we are interested on numerical approximation to the solution of the system (1) using direct methods.

1 Gaussian Eliminations

Write a program that solves (1) using Gaussian elimination with backward substitution. Use Algorithm 6.1 from the book, 9th edition. If you come up with the different way of implementation, explain your algorithm line by line. Unexplained solution will not be graded.

2 LU Factorization

Write a program that solves (1) using LU factorization. For this bring the matrix $\mathbf{A}$ into $\mathbf{A} = \mathbf{L}\mathbf{U}$, where $\mathbf{L}$ is lower triangular and $\mathbf{U}$ is upper triangular matrices. By introducing the new unknown vector $\mathbf{y} = \mathbf{U}\mathbf{x}$ solve first $\mathbf{L}\mathbf{y} = \mathbf{b}$ for $\mathbf{y}$. Once $\mathbf{y}$ is knonwn, solve $\mathbf{U}\mathbf{x} = \mathbf{y}$ for $\mathbf{x}$.

Use Algorithm 6.4 from the book, 9th edition. If you come up with the different way of implementation, explain your algorithm line by line. Unexplained solution will not be graded.

3 Testing your program

Consider a linear system of equations

$$\begin{cases} x_1 + x_2 - 2x_3 - x_4 = 2, \\ 2x_1 + x_2 - x_3 + x_4 = 1, \\ -x_1 + 2x_2 + 3x_3 - x_4 = 4, \\ 3x_1 - x_2 - x_3 + 2x_4 = -3. \end{cases} \tag{2}$$

3.1

Solve the system (2) with pen and paper using Gaussian eliminations. Denote your solution by $\mathbf{x}_{\text{exact}}$.

3.2

Solve the system (2) using your programs from Sections 1 and 2. Compute the Euclidean norm of the error $\mathbf{e} = \mathbf{x}_{\text{exact}} - \mathbf{x}_{\text{approx}}$

$$\|\mathbf{e}\| = (\mathbf{e}_1^2 + \mathbf{e}_2^2 + \ldots + \mathbf{e}_n^2)^{1/2}.$$

Report your numerical solutions, corresponding errors and norm of errors.

Up to this point do not use a backslash operator in Matlab to solve any above systems!

3.3

Assume we are given 100×100 linear system of equations with random coefficient matrix $\mathbf{A}$ and right hand side vector $\mathbf{b}$. Let us assume that the "exact" solution is computed by the backslash operator in Matlab $\mathbf{x}_{\text{exact}} := \mathbf{A} \backslash \mathbf{b}$. Now use your programs to compute $\mathbf{x}$ from $\mathbf{Ax} = \mathbf{b}$ by two different direct methods and compute the Euclidean norm of the error and report the norms only. What is the norm of the error for 500×500 random system?

Hints. In Matlab you can use functions *rand(100)* and *rand(100,1)* to construct random matrix and column vector.