Math308, Quiz 5, 10/03/13

First Name: .... **Last Name:** ....

Grade: ....

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Problem 1. 40%. Solve the following initial value problem:

$$\begin{aligned} &u'' + u = 0 \\ &u(0) = 0, \quad u'(0) = 1. \end{aligned} \tag{1}$$

Problem 2. 50%. Rewrite the general solution of (1) in the form

$$u = R\cos(\omega_0 t - \delta).$$

Problem 3. 10%. What happens to $u(t)$ when $t \to \infty$?

Solutions

Problem 1. The characteristic equation for problem (1) is

$$r^2 + 1 = 0,$$

where its solution $r_1 = -i$ and $r_2 = i$. The general solution of (1) is then

$$u(t) = Ae^{-it} + Be^{it} = A\cos t + B\sin t.$$

Next, we find the constants A and B using the initial data:

$$\left.\begin{array}{r} u(0) = 0 \\ u'(0) = 1 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} A + 0 = 0, \\ 0 + B = 1, \end{array}\right. \tag{2}$$

therefore the solution of the given initial value problem is

$$u(t) = \sin t.$$

Problem 2. We have

$$u = R\cos(\omega_0 t - \delta) = R\cos\delta\cos\omega_0 t + R\sin\delta\sin\omega_0 t.$$

Now, by comparing to the exact solution $u(t) = \sin t$ we get that $\omega_0 = 1$ and

$$\left\{\begin{array}{l} R\cos\delta = 0 \\ R\sin\delta = 1. \end{array}\right. \tag{3}$$

From here we get that $R = 1$. To find δ we use the second equation of (3):

$$\sin\delta = 1,$$

for which the solution is $\delta = \frac{\pi}{2}$. Therefore, $u(t) = \sin t$ can be written as

$$u = R\cos(\omega_0 t - \delta) = \cos(t - \frac{\pi}{2}).$$

Note, that you can also find δ by dividing the second equation to the first one:

$$\tan\delta = \frac{1}{0} \Rightarrow \tan(\delta) = \infty,$$

which of course gives us that $\delta = \frac{\pi}{2}$.

Problem 3. The function $|u(t)| = |\sin(t)| = |\cos(t - \frac{\pi}{2})|$ is bounded by $R = 1$ so the limit is also bounded as $t \to \infty$, and since there is no damping the solution is a simple harmonic motion for any t.