Speech signal processing using MATLAB

Basics and applications

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Slides and MATLAB scripts at https://github.com/murtex/spl

Outline

- Digital signals
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  - Time domain
  - Frequency domain
  - Filters
  - Noise
- Acoustic signals
  - Short-time analysis
  - Spectrograms
  - Activity detection
  - Landmarks detection
  - Formants detection

Digital signals/Sampling

- **continuous signal** (normalized magnitude, length L in seconds)

$$x(t) \in [-1, 1] \quad \text{with} \quad t \in [0, L]$$

```
>> x = @( t ) sin( 2*pi*f * t ); % continuous sine with frequency f
```

- **sampling rate** f_S, quantization of time

$$t \to t_i = \frac{i-1}{f_\mathrm{S}} \quad \text{with} \quad i \in \{1, \ldots, N\} \quad \text{and} \quad N = \lfloor L f_\mathrm{S} \rfloor$$

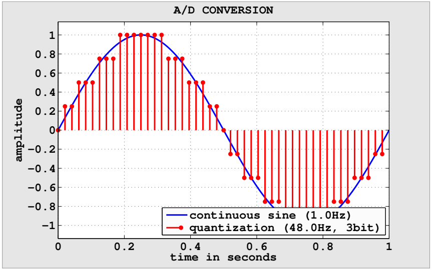
```
>> N = floor( L * fS ); % number of samples
>> ti = (0:N-1) / fS; % quantized time values
```

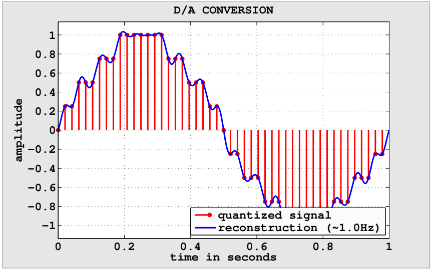
- **bits per sample** n_S, quantization of amplitude

$$x(t) \to x_i = \frac{\lfloor 2^{n_\mathrm{S}-1} x(t_i) \rceil}{2^{n_\mathrm{S}-1}}$$

```
>> xi = round( 2^(nS-1) * x( ti ) ) / 2^(nS-1); % quantized amplitudes
```

- **example**: `matlab/sampling.m`





- **exercise:**
  - verify from reconstruction that **Nyquist frequency** holds

$$f_{\mathrm{Ny}} = \frac{f_{\mathrm{S}}}{2}$$

  - compare commonly used **sampling standards** (telephony, Audio-CD, professional audio equipment, …)

Digital signals/Time domain

- **total energy**, **average power** and **root mean square**

$$E = \sum_{i=1}^{N} x_i^2, \quad P = \frac{1}{N}\sum_{i=1}^{N} x_i^2 \quad \text{and} \quad RMS = \sqrt{\frac{1}{N}\sum_{i=1}^{N} x_i^2}$$

```
>> E = sum( xi .* xi ); % total energy
>> P = mean( xi .* xi ); % average power
>> RMS = sqrt( mean( xi .* xi ) ); % root mean square
```

- **decibel full scale**, different for **power-** and **magnitude-like** quantities, e. g.

$$P_{\mathrm{dB}} = 10\log_{10}(P) \quad \text{and} \quad RMS_{\mathrm{dB}} = 20\log_{10}(RMS)$$

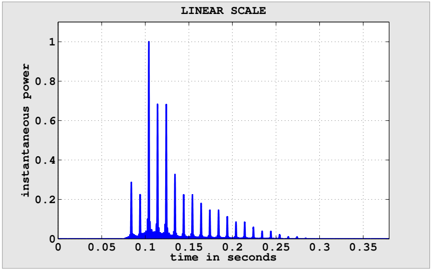
```
>> PdB = 10 * log10( P ); % power-like
>> RMSdB = 20 * log10( RMS ); % magnitude-like
```

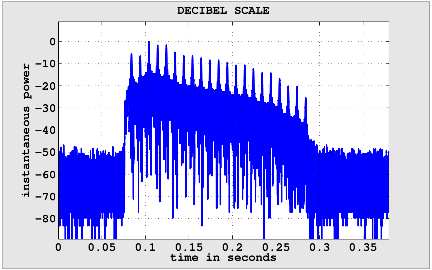
- **zero-crossings rate**

```
>> fZ = sum( abs( diff( xi >= 0 ) ) ) / N * fS;
```

- **example (playback):** `matlab/decibel.m`

$$P_i = x_i^2 \quad \text{and} \quad P_{i,\mathrm{dB}} = 10\log_{10}(x_i^2)$$





- **exercise:**
  - TODO!

Digital signals/Frequency domain

Frequency domain

- **discrete Fourier transform**, time domain → frequency domain

$$X_k = \sum_{i=1}^{N} x_i \, \mathrm{e}^{-2\pi\mathrm{i}\frac{(i-1)(k-1)}{N}} \in \mathbb{C} \quad \text{with} \quad k \in \{1, \ldots, N\}$$

```
>> Xk = fft( xi ) / N; % complex Fourier coefficients
```

- k is a **frequency index** (as i was a time index)

$$k \to f_k = \frac{k-1}{N} f_{\mathrm{S}}$$

```
>> fk = (0:N-1) / N * fS; % frequency values
```

- frequencies beyond Nyquist frequency are **negative frequencies**

$$f_k \to f_k = \begin{cases} f_k - f_{\mathrm{S}} & \text{if } f_k > f_{\mathrm{Ny}} \\ f_k & \text{otherwise} \end{cases}$$

```
>> fk(fk > fNy) = fk(fk > fNy) - fS; % imply negative frequencies
```

- **power spectral density** (also known as **power spectrum**)

$$P_k = |X_k|^2 \in \mathbb{R} \quad \Leftarrow \quad \sum_{k=1}^{N} P_k = P$$

```
>> Pk = abs( Xk ) .^ 2; % power spectral density
```

- **real valued signals** ($x_i \in \mathbb{R}$) imply a special symmetry

$$X_{f_k} = X^*_{-f_k} \quad \Rightarrow \quad P_{f_k} = P_{-f_k}$$

- restrict to **one-sided spectrum**

```
>> Pk(fk < 0) = []; % remove negative frequency components
>> Xk(fk < 0) = [];
>> fk(fk < 0) = [];
>> Pk(2:end) = 2 * Pk(2:end); % rescale to match total power
>> Xk(2:end) = sqrt( 2 ) * Xk(2:end);
```

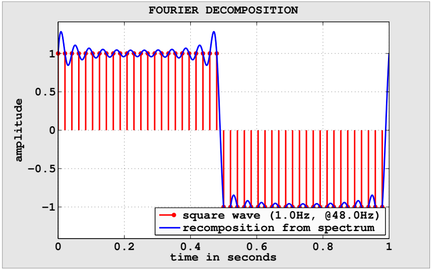
- P_1 is **DC offset**, $P_{k>1}$ are **contributions of sines** with frequencies $f_{k>1}$

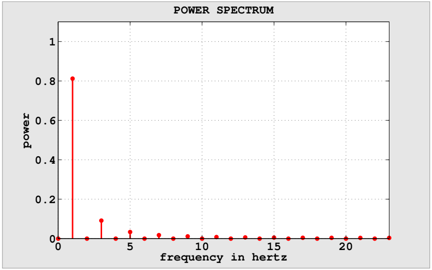
$$x(t) = \sqrt{P_1} + \sqrt{2} \sum_{k>1} \sqrt{P_k} \sin(2\pi f_k t)$$

- complex valued but without loss of phase information

$$x(t) = X_1 + \sqrt{2}\sum_{k>1} X_k e^{2\pi i f_k t}$$

- **example:** `matlab/fdomain.m`

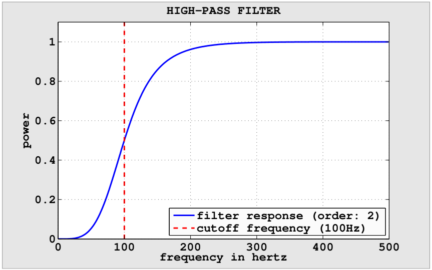


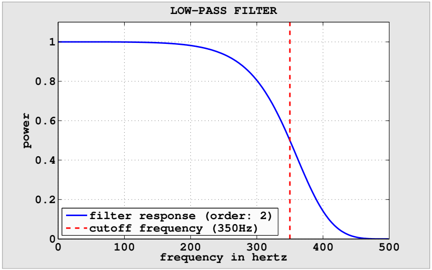


- **exercise:**
  - is there any loss of information by changing to real values?
  - TODO!

Digital signals/Filters

- general **filter types**:
  - **low-pass**: passes low frequencies (cuts high ones)
  - **high-pass**: passes high frequencies (cuts low ones)
  - **band-pass**: passes a range of frequencies (combination of low- and high-pass)
  - **band-stop** (notch): cuts a range of frequencies (opposite of band-pass)
- **cutoff frequency** at which output power is (generally) reduced by −3 dB
- **example**: `matlab/filters.m`

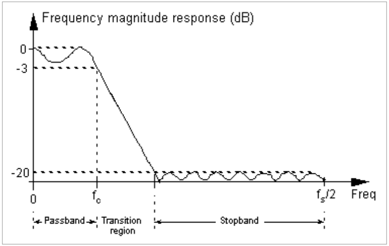


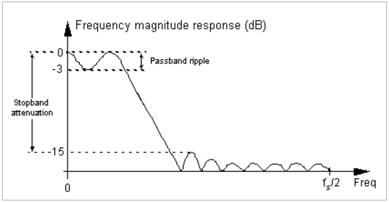


- filters are represented by **filter coefficients** b_j (feedforward) and a_j (feedback)
- high **filter order** m increases computational complexity but thereby quality

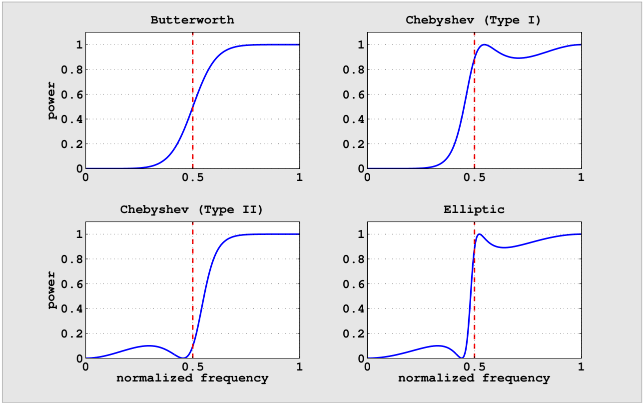
$$y_i = \overbrace{\frac{1}{a_1}\left(\underbrace{\sum_{j=0}^{m} b_{j+1} x_{i-j}}_{\text{FIR}} - \sum_{j=1}^{m} a_{j+1} y_{i-j}\right)}^{\text{IIR}} \quad \text{with} \quad i \in \{1, \ldots, N\}$$

- **FIR filters** (finite impulse response) are slow to compute but stable
- **IIR filters** (infinite impulse response) are fast to compute but might be unstable
- some often used **additional terms** (images from `http://dspguru.com`)





- many **filter families** with different characteristics (`matlab/filters2.m`), e. g.



- **normalized frequency**

$$\tilde{f}_k = \frac{f_k}{f_{\mathrm{Ny}}} = \frac{2f_k}{f_{\mathrm{S}}} \in [0, 1] \quad \text{with} \quad k \in \{1, \ldots, N\}$$

Filters

- **Butterworth filter** (high-pass, second-order, 100 Hz cutoff)

```
>> m = 2; % filter order
>> cutoff = 100; % cutoff frequency
>> [b, a] = butter( m, cutoff / (fS/2), 'high' );
```

- **Chebyshev filter** (high-pass, 1 dB ripple, 40 dB attenuation, 100 Hz cutoff)

```
>> cutoff = 100; % cutoff frequency
>> stopband = 90; % stopband frequency
>> ripple = 1; % passband ripple
>> attenuation = 40; % stopband attenuation
>> m = cheb2ord( cutoff / (fS/2), stopband / (fS/2), ripple, attenuation );
>> [b, a] = cheby2( m, attenuation, stopband / (fS/2) );
```

- **apply any filter**

```
>> y = filter( b, a, x ); % filter signal x using coefficients a, b
```

- or in zero-phase version (without filter delay)

```
>> y = filtfilt( b, a, x ); % zero-phase filtering
```

Filters

- **exercise**:
  - Can you image what **filter delay** means?
  - TODO: home audio/hifi, bass and treble

Digital signals/Noise

- **spectral flatness measure**

$$SFM = \dots$$

- **spectral entropy** using probability densities p_k and skipping constant DC

$$H = -\sum_{k>1} p_k \log(p_k) \quad \text{with} \quad p_k = \frac{P_k}{\sum_{k>1} P_k}$$

```
>> pk = Pk(2:end) / sum( Pk(2:end) ); % probability densities
>> H = -sum( log2( pk .^ pk ) ); % entropy in bits
```

- additive noise
- signal-to-noise ratio
- spectral subtraction

Acoustic signals/Short-time analysis

Acoustic signals/Spectrograms

Acoustic signals/Activity detection

Acoustic signals/Landmarks detection

Acoustic signals/Formants detection