Abstract

This project aims to implement True Modal Control, whereby every controllable mode of a dynamic system will be modelled and controlled independently. Dynamic systems can be represented by spring-mass-viscous-damper system via the Lumped Parameter strategy, but we would have to decouple the equations so obtained to obtain the individual modes. Decoupling classically damped systems is easily done, however not much has been done in the case of non-classical damping. We aim to make our method as general as possible, so we shall focus on general damping in this project. There have been recent advances in this field by Garvey et al. (2002a,b) and Chu and Buono (2008b,a). We shall review these advances, and build upon them, principally on the work of Chu and Buono (2008b).

Nearly everything is lifted from my BTP report.

Why, Oh! Why?

Murukesh Mohanan
133059001

August 18, 2013

Contents

List of Figures

1 Eternal damnation

Insert random metaphysical stuff here.

2 Non-eternal damnation

Insert non-random metaphysical stuuf here.

2.1 Non-eternal non-damnation

Insert non-random non-metaphysical stuff here.

Contact Bertrand Russell. If not, then please submit your geek card and proceed to Processing. Something's wrong with atl. Abandon latex plugins, ye who enter vim! [1]

1. For $j = 1, \ldots, \rho$,

$$\{\mathrm{r}_j, \mathrm{i}_j\} \in \mathbf{C}_a \Leftrightarrow \lambda_{\mathrm{r}_j} \lambda_{\mathrm{i}_j} > 0 \tag{1}$$

$$\{\mathrm{r}_j, \mathrm{i}_j\} \in \mathbf{C}_f \Leftrightarrow \lambda_{\mathrm{r}_j} \lambda_{\mathrm{i}_j} < 0 \tag{2}$$

2. For $j = 1, \ldots, (n(\mathfrak{R}) - \rho)/2$,

$$\{\mathrm{r}_{\rho+2j-1}, \mathrm{r}_{\rho+2j}\} \in \mathbf{C}_b \Leftrightarrow \lambda_{\mathrm{r}_{\rho+2j-1}} \lambda_{\mathrm{i}_{\rho+2j-1}} < 0 \tag{3}$$

$$\{\mathrm{r}_{\rho+2j-1}, \mathrm{r}_{\rho+2j}\} \in \mathbf{C}_d \Leftrightarrow \lambda_{\mathrm{r}_{\rho+2j-1}} \lambda_{\mathrm{i}_{\rho+2j-1}} > 0 \tag{4}$$

3. For $j = 1, \ldots, (n(\mathfrak{I}) - \rho)/2$,

$$\{\mathrm{i}_{\rho+2j-1}, \mathrm{i}_{\rho+2j}\} \in \mathbf{C}_c \Leftrightarrow \lambda_{\mathrm{i}_{\rho+2j-1}} \lambda_{\mathrm{i}_{\rho+2j-1}} < 0 \tag{5}$$

$$\{\mathrm{i}_{\rho+2j-1}, \mathrm{r}_{\rho+2j}\} \in \mathbf{C}_e \Leftrightarrow \lambda_{\mathrm{i}_{\rho+2j-1}} \lambda_{\mathrm{i}_{\rho+2j-1}} > 0 \tag{6}$$

[1] Why is :MakeLatex using Python? And more python errors when downloaded from the site.

3 LitRev

Theories on Modal Control have been around for around half a century, initially suggested by Rosenbrock (1962). Simon and Mitter (1968) expanded on this work. Modal control theories emerged independently in two fields - chemical engineering, and structural dynamics. The concepts which evolved in structural engineering were formulated into Independent Modal Space Control (IMSC) by Prof Meirovitch and his team in the 1980s, and this work has been summarized in Meirovitch (1990).

4 Independent Modal Space Control

In IMSC, one decouples the equations (**??**) by unitary similarity transforms. In the event that $\mathbf{C}$ is not diagonalized, we ignore the off-diagonal terms. Further, the control forces are only applied to the lower modes, since the higher modes are difficult to physically monitor or control. Thus, IMSC is most effective in situations where only a few critical modes need be studied and controlled. However, this strategy exposes itself by two grievous flaws in the general case. Firstly, when the off-diagonal terms are of comparable magnitude to the diagonal terms, ignoring them is a very poor approximation. In rotating systems, for example, the skew-symmetry of the damping matrix is due to the gyroscopic nature of the system, and ignoring off-diagonal terms here is ignoring the gyroscopic effects themselves – and the very essence of the problem. It may happen that the decoupled equations thus obtained have quite different eigenvalues from the original. There is also the problem of deciding how many modes one must model, how many to leave unmodelled, and how many of the modelled modes be controlled. This question is further complicated by the different needs of open and closed loop control systems. What may be satisfactory for one may not be so for the other.

Secondly, trying to control only the lower modes raises the problem of spill-over. Spill-over refers to applying a control law designed around a limited model of a system, to the entire system and inadvertently exciting that part of the model discarded (called residual modes in a modal model of a structure). Since the higher modes are not in the model, the effect will be different from that predicted by the control design. In some cases, the modal control forces may significantly increase the contributions of uncontrolled modes to the vibration of the system, especially in cases like flexible structures, in which the contributions of its higher modes cannot be ignored. Thus while using a modal model one must be careful not to excite the unused modes with the control system designed to improve the structure's response. Otherwise, control energy will 'spill over' into the residual modes and excite these modes, spoiling the response and desired performance is not achieved.

As mentioned earlier, IMSC converts the n-DOF second-order system to a $2n$-DOF first-order system. That is, the system is linearised. Useful characteristics of the system matrices such as symmetry and definiteness are lost. A Modified Independent Modal Space Control (MIMSC) theory as presented by Fang et al. (2003) seeks to address shortcomings of IMSC, such as spill over.

5 Structure Preserving Equivalences (SPEs)

The quadratic pencil (??) can be linearized to different forms. Consider the following matrices, called Lancaster Augmented Matrices:

Undefined references are to sections that I couldn't be bothered to copy.

$$\mathbf{A} = \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & -\mathbf{M} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} \mathbf{C} & \mathbf{M} \\ \mathbf{M} & \mathbf{0} \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} \mathbf{0} & \mathbf{K} \\ \mathbf{K} & \mathbf{C} \end{bmatrix} \tag{7}$$

and the *companion matrices*, often seen in the state-space form of system equations:

$$\mathbf{C}_R = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \qquad \mathbf{C}_L = \begin{bmatrix} \mathbf{0} & \mathbf{K}\mathbf{M}^{-1} \\ \mathbf{I} & \mathbf{C}\mathbf{M}^{-1} \end{bmatrix} \tag{8}$$

If we let $\mathbf{p} = \dot{\mathbf{r}}$ and $\mathbf{P} = \dot{\mathbf{f}}$, then we can write equation (??) in any of the following state-space forms:

$$\mathbf{D}\begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} - \mathbf{A}\begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}\mathbf{f} \tag{9}$$

$$\mathbf{A}\begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} + \mathbf{B}\begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}\mathbf{f} \tag{10}$$

$$\mathbf{D}\begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} + \mathbf{B}\begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{P} \\ \mathbf{f} \end{bmatrix} \tag{11}$$

Now, two non-singular matrices T_L and $T_R \in \mathbb{C}^{2n\times 2n}$, can be considered to be Structure Preserving Equivalences (SPEs)[2], if the isospectral transforms $\mathbf{A}_0 = \mathbf{T}_L\mathbf{A}_\mathbf{0}\mathbf{T}_R, \mathbf{B}_0 = \mathbf{T}_L\mathbf{B}_\mathbf{0}\mathbf{T}_R$ and $\mathbf{D}_0 = \mathbf{T}_L\mathbf{D}_\mathbf{0}\mathbf{T}_R$, retain their block-structure, that is, if

$$\mathbf{A}_\mathbf{0} = \begin{bmatrix} \mathbf{K}_0 & \mathbf{0} \\ \mathbf{0} & -\mathbf{M}_0 \end{bmatrix} \quad \mathbf{B}_\mathbf{0} = \begin{bmatrix} \mathbf{C}_0 & \mathbf{M}_0 \\ \mathbf{M}_0 & \mathbf{0} \end{bmatrix} \quad \mathbf{D}_0 = \begin{bmatrix} \mathbf{0} & \mathbf{K}_0 \\ \mathbf{K}_0 & \mathbf{C}_0 \end{bmatrix} \tag{12}$$

Utilisation of the SPEs permits the diagonalization of the system mass, damping and stiffness matrices for non-classically damped systems, as shown by Garvey et al. (2002a,b). A modal control method is presented by Houlston (2007), which exploits this diagonalization. The method introduces independent modal control in which a separate modal controller is designed in modal space for each individual mode or pair of modes. The theory of decoupling used is presented in a concise form in Garvey et al. (2001). We shall present the basic equations here.

Except for defective systems, it is possible to find matrices $\mathbf{W}_L$, $\mathbf{X}_L$, $\mathbf{Y}_L$,

[2] As used by Garvey et al. (2002a,b), there is a restriction on the above definition: that $\mathbf{M}_0$ not be singular. We shall modify this restriction: $\mathbf{M}_0 \neq \mathbf{0}$. The reason shall be apparent later.

$\mathbf{Z}_L$ and $\mathbf{W}_R$, $\mathbf{X}_R$, $\mathbf{Y}_R$, $\mathbf{Z}_R$ in $\mathbb{R}^{n\times n}$ such that

$$\begin{bmatrix}\mathbf{W}_L & \mathbf{X}_L\\ \mathbf{Y}_L & \mathbf{Z}_L\end{bmatrix}^T \begin{bmatrix}\mathbf{0} & \mathbf{K}\\ \mathbf{K} & \mathbf{C}\end{bmatrix} \begin{bmatrix}\mathbf{W}_R & \mathbf{X}_R\\ \mathbf{Y}_R & \mathbf{Z}_R\end{bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{\Omega}^2\\ \mathbf{\Omega}^2 & (2\zeta\mathbf{\Omega})\end{bmatrix} \tag{13}$$

$$\begin{bmatrix}\mathbf{W}_L & \mathbf{X}_L\\ \mathbf{Y}_L & \mathbf{Z}_L\end{bmatrix}^T \begin{bmatrix}\mathbf{K} & \mathbf{0}\\ \mathbf{0} & -\mathbf{M}\end{bmatrix} \begin{bmatrix}\mathbf{W}_R & \mathbf{X}_R\\ \mathbf{Y}_R & \mathbf{Z}_R\end{bmatrix} = \begin{bmatrix}\mathbf{\Omega}^2 & \mathbf{0}\\ \mathbf{0} & -\mathbf{I}\end{bmatrix} \tag{14}$$

$$\begin{bmatrix}\mathbf{W}_L & \mathbf{X}_L\\ \mathbf{Y}_L & \mathbf{Z}_L\end{bmatrix}^T \begin{bmatrix}\mathbf{C} & \mathbf{M}\\ \mathbf{M} & \mathbf{0}\end{bmatrix} \begin{bmatrix}\mathbf{W}_R & \mathbf{X}_R\\ \mathbf{Y}_R & \mathbf{Z}_R\end{bmatrix} = \begin{bmatrix}(2\zeta\mathbf{\Omega}) & \mathbf{I}\\ \mathbf{I} & \mathbf{0}\end{bmatrix} \tag{15}$$

where, both $\mathbf{\Omega}^2$ and $2\zeta\mathbf{\Omega}$ are real-valued and diagonal. Now,

$$\mathbf{Y}_L = -\mathbf{X}_L\mathbf{\Omega}^2, \quad \mathbf{W}_L = \mathbf{Z}_L + \mathbf{X}_L(2\zeta\mathbf{\Omega}) \tag{16}$$

A similar relation holds for the right matrices. Consider the eigenvalue problem obtained from the first of the state-space forms in (9). Let $\mathbf{X}$ be the matrix composed of right eigenvectors and $\mathbf{Y}$ be the matrix composed of left eigenvectors. Considering a block structure using matrices similar to $\mathbf{W}_L$, $\mathbf{X}_L$, $\mathbf{Y}_L$,, etc., let $\mathbf{\Phi}_{L1}$, $\mathbf{\Phi}_{L2}$, $\mathbf{\Theta}_{L2}$, $\mathbf{\Theta}_{L2}$ form the block structure of $\mathbf{X}$, with a similar structure for $\mathbf{Y}$. Then, rewriting (**??**) using the block structure:

$$\begin{bmatrix}\mathbf{\Phi}_{L1} & \mathbf{\Phi}_{L2}\\ \mathbf{\Theta}_{L1} & \mathbf{\Theta}_{L2}\end{bmatrix}^T \begin{bmatrix}\mathbf{0} & \mathbf{K}\\ \mathbf{K} & \mathbf{C}\end{bmatrix} \begin{bmatrix}\mathbf{\Phi}_{R1} & \mathbf{\Phi}_{R2}\\ \mathbf{\Theta}_{R1} & \mathbf{\Theta}_{R2}\end{bmatrix} = \begin{bmatrix}\mathbf{\Lambda}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{\Lambda}_2\end{bmatrix} \tag{17}$$

$$\begin{bmatrix}\mathbf{\Phi}_{L1} & \mathbf{\Phi}_{L2}\\ \mathbf{\Theta}_{L1} & \mathbf{\Theta}_{L2}\end{bmatrix}^T \begin{bmatrix}\mathbf{K} & \mathbf{0}\\ \mathbf{0} & -\mathbf{M}\end{bmatrix} \begin{bmatrix}\mathbf{\Phi}_{R1} & \mathbf{\Phi}_{R2}\\ \mathbf{\Theta}_{R1} & \mathbf{\Theta}_{R2}\end{bmatrix} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix} \tag{18}$$

Of the $2n$ eigenvalues obtained, counting multiplicity, let $2p \leq 2n$ be the number of complex eigenvalues with $2q$ real eigenvalues. The matrix which we now define will also be used later, since a modified version plays the same role in the theory presented by Chu and Buono (2008b).

$$\mathbf{J} = \begin{bmatrix}\frac{1}{\sqrt{2}}\mathbf{I}_p & \mathbf{0} & \frac{-\iota}{\sqrt{2}}\mathbf{I}_p & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_q & \mathbf{0} & \mathbf{0}\\ \frac{1}{\sqrt{2}}\mathbf{I}_p & \mathbf{0} & \frac{\iota}{\sqrt{2}}\mathbf{I}_p & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \iota\mathbf{I}_q\end{bmatrix} \tag{19}$$

$$\mathbf{J}^T\mathbf{\Lambda}\mathbf{J} = \begin{bmatrix}\mathbf{\Lambda}_x & \mathbf{\Lambda}_y\\ \mathbf{\Lambda}_y & \mathbf{\Lambda}_z\end{bmatrix} \tag{20}$$

Now select a diagonal matrix $\gamma \in \mathbb{C}^{n\times n}$, such that

$$\begin{bmatrix}\cosh\gamma & \sinh\gamma\\ \sinh\gamma & \cosh\gamma\end{bmatrix} \begin{bmatrix}\mathbf{\Lambda}_x & \mathbf{\Lambda}_y\\ \mathbf{\Lambda}_y & \mathbf{\Lambda}_z\end{bmatrix} \begin{bmatrix}\cosh\gamma & \sinh\gamma\\ \sinh\gamma & \cosh\gamma\end{bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{\Omega}^2\\ \mathbf{\Omega}^2 & (2\zeta\mathbf{\Omega})\end{bmatrix} \tag{21}$$

Then the following relation (with its equivalent for the right eigenvectors) give us the required diagonalizing SPEs:

$$\begin{bmatrix}\mathbf{W}_L & \mathbf{X}_L\\ \mathbf{Y}_L & \mathbf{Z}_L\end{bmatrix} = \begin{bmatrix}\mathbf{\Phi}_{L1} & \mathbf{\Phi}_{L2}\\ \mathbf{\Theta}_{L1} & \mathbf{\Theta}_{L2}\end{bmatrix} \mathbf{J} \begin{bmatrix}\cosh\gamma & \sinh\gamma\\ \sinh\gamma & \cosh\gamma\end{bmatrix} \begin{bmatrix}\mathbf{\Omega} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix} \tag{22}$$

There are two points to consider in this rather straightforward relation. First is obtaining an appropriate γ, which is where the bulk of the computation lies. Secondly, there is no simple way whereby this method could be adapted for systems with eigenvalue zero. Chu and Buono (2008b) derived a modified version of this relation, which, as we shall show in the next chapter, is relatively easily adapted for this case. The transformation derived above is not unique, and can be arrived via a different route, albeit sharing a few steps. Another route to decoupling transformations is also discussed in Garvey et al. (2001), which uses Clifford algebra, specifically, $\mathcal{C}\ell_2$. We shall not discuss this method, since Clifford algebras are beyond the scope of this work.

6 Dunno.

- Movement of armies:
  - Over land: Slower, delicate units such as siege weapons and monks need to be protected. Exploit the speed of mounted units to ward off attacks on the main army.
  - Over sea: Transport ships have limited capacity and units in ships cannot defend themselves. If the enemy is across a body of water, one must set up a base on enemy territory.
- Replenishment of armies after battles and skirmishes:
  - Maintaining a level of resources, to allow for quick replenishment.
  - Ensuring proper timing of arrival of reinforcements.
- Retreating under fire while minimizing losses.

7 The table that you requested, milord.

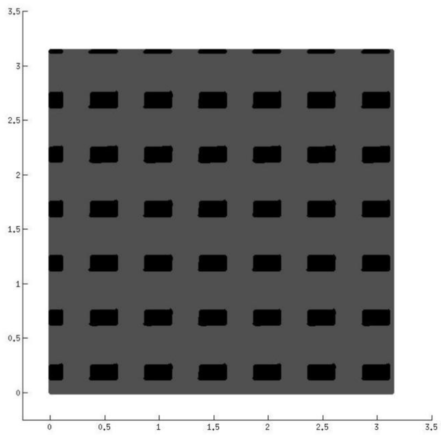
I hope milord is pleased with my offering.[3]

Day	Min Temp	Max Temp	Summary
Monday	11C	22C	A clear day with lots of sunshine. However, the strong breeze will bring down the temperatures.
Tuesday	9C	19C	Cloudy with rain, across many northern regions. Clear spells across most of Scotland and Northern Ireland, but rain reaching the far northwest.
Wednesday	10C	21C	Rain will still linger for the morning. Conditions will improve by early afternoon and continue throughout the evening.

[3]Lifted from Wikibooks.

8 Unreachable Regions for Automorphic Transforms

Before we focus on the area of good filters, let us study the area of available filters.



Now, for a quadratic polynomial, the coefficients corresponding to complex roots lie within a parabola. This is easily established, since, for complex roots of a monic polynomial $P(\lambda) = \lambda^2 + x\lambda + y$, we must have $x^2 - 4y < 0$. Whether an eigenvalue is complex or not is not of much importance. But for this system of matrices, and indeed it seems for most such systems, there appear to be regions inaccessible via automorphic SPEs. And, in the 2-dimensional case, such regions seem to share two common properties:

1. They are conic sections;

2. They lie within this parabola.

For certain systems, for example the system in the previous example, the entire parabola is inaccessible. A plot of the corresponding eigenvalues reveals that these inaccessible regions have their analogues in the eigenspace. For the present system, since the entire parabola was inaccessible by automorphic transforms, eigenvalues of all stable filters are necessarily real.

These special areas also exist for the 3-dimensional case, and presumably for all dimensions. In the 3-dimensional case, contrary to what one might expect, these regions are not cones or revolutions of conic sections, but have quite arbitrary shapes, with symmetry a common feature in the cases examined so far. The high number of variables involved have made analytical derivation of the equations of these regions very difficult. One way to do it numerically would be to evaluate the Jacobian at each point. Anywhere outside the unreachable region, a point can move in any direction. However, at the surface of the region, the point is constrained from moving further within the region. Thus the derivative of the coordinate vector along these directions would be zero. Hence the Jacobian would be rank-deficient, and its determinant would be zero. This is true independent of the basis chosen for obtaining the Jacobian. Thus, but determining where the Jacobian is zero, one can approximate the surface of the unreachable region.

This method was applied to the present example. For evaluating the partial derivatives that make up the Jacobian, a first-order finite difference method was used. Then a root-finding algorithm was applied on the Jacobian determinant for each grid line. The resultant boundary points are shown in Figure 3. The fit (quadratic) is reasonably accurate, as can be inferred from

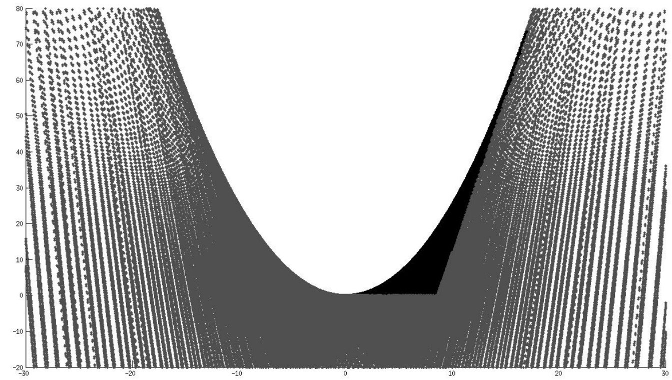


Figure 1: Good CPCs (shown in black) and reachable CPCs of a 2D system

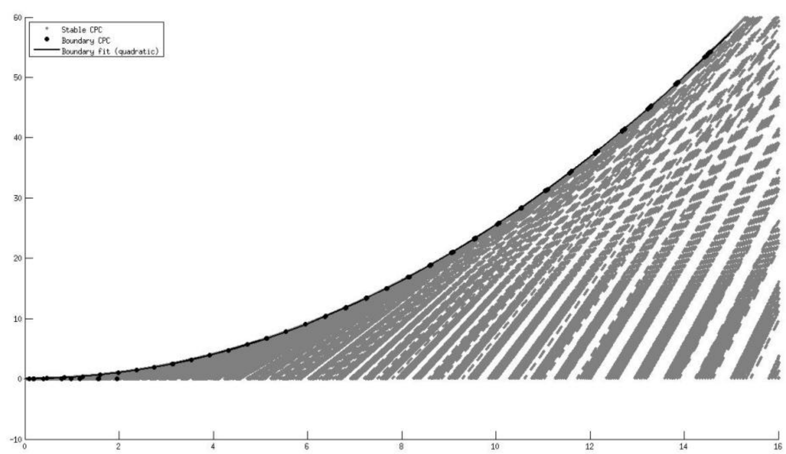


Figure 2: Scatter of (θ_1, θ_2) for good CPCs (in black) over reachable CPCs

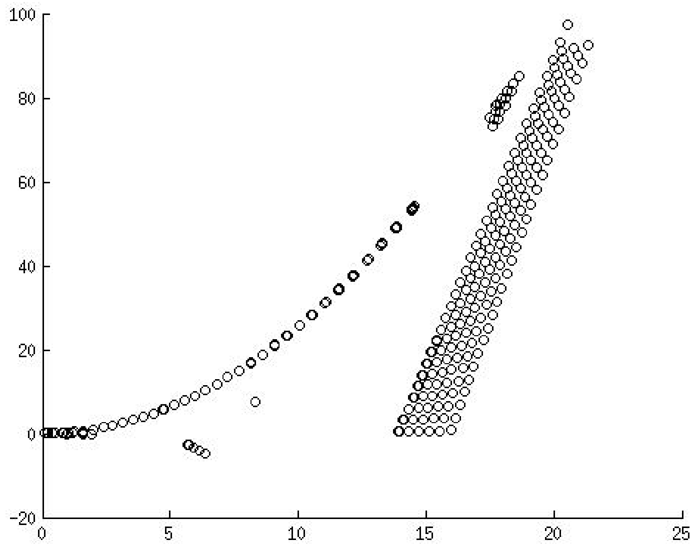
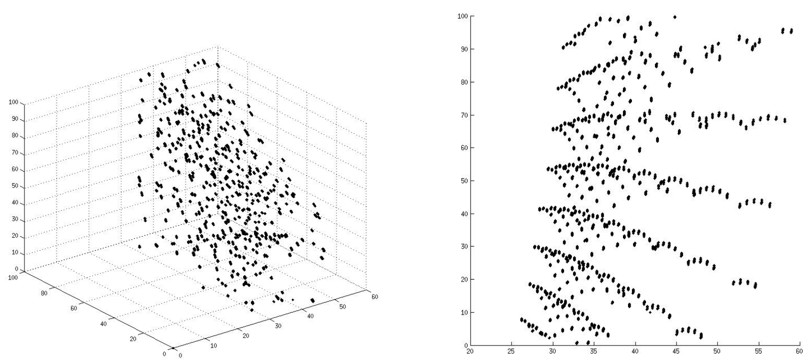


Figure 3: Calculated boundary points of unreachable CPC region

These figures are inside a multicolumn area (using the multicol package. So technically, TECHNICALLY, I have satisfied the requirement of having a figure span both columns.



(a) A 3D scatter, seemingly random

(b) Projection on the x-y plane, revealing a pattern.

Figure 4: Good CPCs for a 3D system

the figure. The extra points are due to the gaps in calculated points, which arise due to the uniform intervals in which the range of values of $\mathbf{\Theta}$ was divided. The bottom edge, and some of the gaps in the space of stable points, which may or may not be unreachable, were also detected by this method.

References

M. T. Chu and N. D. Buono. Total decoupling of general quadratic pencils, part II: Structure preserving isospectral flows. *Journal of Sound and Vibration*, 309(1-2):112 – 128, 2008a. ISSN 0022-460X. doi: 10.1016/j.jsv.2007.05.052. URL http://www.sciencedirect.com/science/article/pii/S0022460X07004245. 1

M. T. Chu and N. D. Buono. Total decoupling of general quadratic pencils, part I: Theory. *Journal of Sound and Vibration*, 309(1-2):96 – 111, 2008b. ISSN 0022-460X. doi: 10.1016/j.jsv.2007.05.058. URL http://www.sciencedirect.com/science/article/pii/S0022460X07004233. 1, 6, 7

J. Fang, Q. Li, and A. Jeary. Modified independent modal space control of m.d.o.f. systems. *Journal of Sound and Vibration*, 261(3):421 – 441, 2003. ISSN 0022-460X. doi: 10.1016/S0022-460X(02)01085-4. URL http://www.sciencedirect.com/science/article/pii/S0022460X02010854. 4

S. Garvey, U. Prells, and M. Friswell. Diagonalising coordinate transformations for systems with general viscous damping. *Proceedings of the International Modal Analysis Conference - IMAC*, 1:622–627, 2001. URL http://www.scopus.com/inward/record.url?eid=2-s2.0-0035051399&partnerID=40&md5=3c7a57da010dcd776db7932565d83b37. cited By (since 1996) 0. 5, 7

S. Garvey, M. Friswell, and U. Prells. Co-ordinate Transformations for Second Order Systems. part I: General Transformations. *Journal of Sound and Vibration*, 258(5):885 – 909, 2002a. ISSN 0022-460X. doi: 10.1006/jsvi.2002.5165. URL http://www.sciencedirect.com/science/article/pii/S0022460X02951655. 1, 5

S. Garvey, M. Friswell, and U. Prells. Co-ordinate Transformations for Second Order Systems. part II: Elementary Structure-Preserving Transformations. *Journal of Sound and Vibration*, 258(5):911 – 930, 2002b. ISSN 0022-460X. doi: 10.1006/jsvi.2002.5166. URL http://www.sciencedirect.com/science/article/pii/S0022460X02951667. 1, 5

P. R. Houlston. *Active Vibration Control of Rotating Machines*. PhD thesis, University of Nottingham, 2007. URL http://etheses.nottingham.ac.uk/275/. 5

L. Meirovitch. *Dynamics and control of structures*. Wiley-Interscience publication. Wiley, 1990. ISBN 9780471628583. URL http://books.google.com/books?id=Yz1HjDS9q2sC. 4

H. Rosenbrock. Distinctive problems of process control. *Chemical Engineering Progress*, 58(9), 1962. 4

J. Simon and S. Mitter. A theory of modal control. *Information and Control*, 13(4):316 – 353, 1968. ISSN 0019-9958. doi: 10.1016/S0019-9958(68)90834-6. URL http://www.sciencedirect.com/science/article/pii/S0019995868908346. 4

List of Symbols

Matrices

0 Null matrix. 5, 6

A LAM comprising **K** and **M**. 5

C Damping matrix. 4–6

I Identity matrix of size n, $\mathbf{I}_i$ is the identity matrix of size i. 5, 6

K Stiffness matrix. 5, 6

M Mass matrix. 5, 6

X Matrix composed of right eigenvectors. 6

Y Matrix composed of left eigenvectors. 6

$\boldsymbol{\Lambda}$ Diagonal matrix comprising the eigenvalues. 6

γ Diagonal matrix used in decoupling. 6, 7

Other symbols

$\mathbb{C}^{n\times n}$ The set of complex-valued n-by-n matrices. 6

$\mathbb{R}^{n\times n}$ The set of real-valued n-by-n matrices. 6

$\mathcal{C}\ell_2$ Clifford Algebra over $\mathbb{C}^{n\times n}$. 7

Scalars

λ Eigenvalue. 3

Vectors

$\mathfrak{I}$ Index set of purely imaginary eigenvectors, When used as a subscript to a matrix, refers to the submatrix formed from the corresponding index set. 3

$\mathfrak{R}$ Index set of real eigenvectors, When used as a subscript to a matrix, refers to the submatrix formed from the corresponding index set. 3

Acronyms

IMSC Independent Modal Space Control. 4, 5

LAM Lancaster Augmented Matrix. 5

MIMSC Modified Independent Modal Space Control. 5

SPE Structure Preserving Equivalence. 5, 6