Statistical Parameter Estimation of Power Consumption Data: A Comparative Study of Maximum Likelihood and Bayesian Approaches

P Harish Ragavender, S Murugan, S Yashwanth
IIIT Sri City
S20220020301, S20220020306, S20220020323

May 2025

Abstract

This report presents a project focused on statistical estimation of real-world power consumption data using Maximum Likelihood Estimation (MLE) and Bayesian Estimation. The project aims to estimate the mean and variance of the dataset under the assumption of a Gaussian distribution and evaluates the estimator performance using the Cramér-Rao Bound (CRB).

1 Objective

The objective of this project is to estimate statistical parameters (mean and variance) of a power consumption dataset using MLE and Bayesian approaches. The performance of these estimators is evaluated and compared using the CRB.

2 Dataset Description

The dataset contains power consumption values sampled over time. It is assumed to follow a Gaussian distribution, verified through histogram plotting and Gaussian curve fitting. **Range:**

- Raw Power Values: 12,464 W to 15,480 W
- Normalized Range: 0 to 1 (after min-max scaling)

3 Visualization of Power Consumption Data

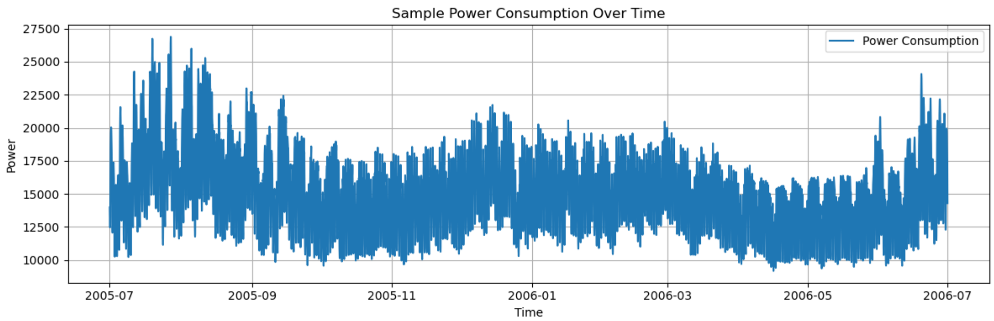

Figure 1: Sample Power Consumption Over Time

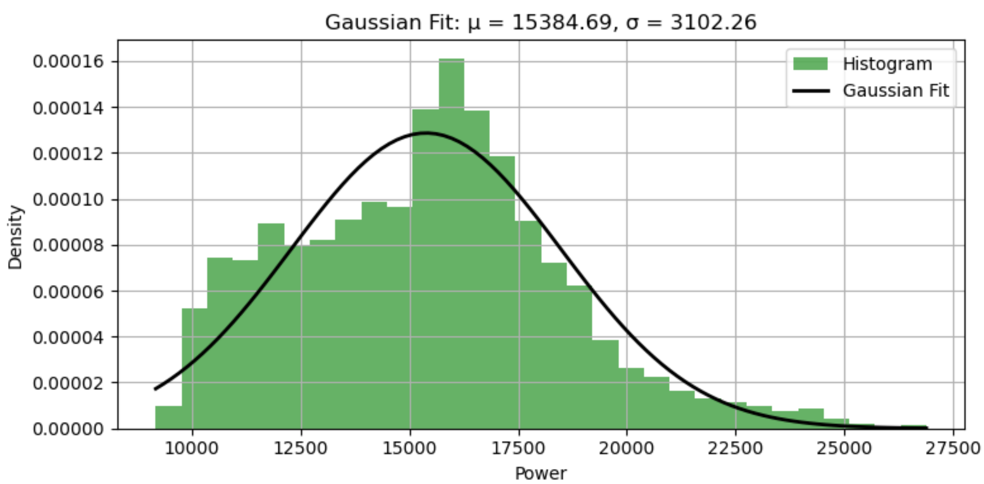

Figure 2: Histogram of raw power consumption data with Gaussian fit.

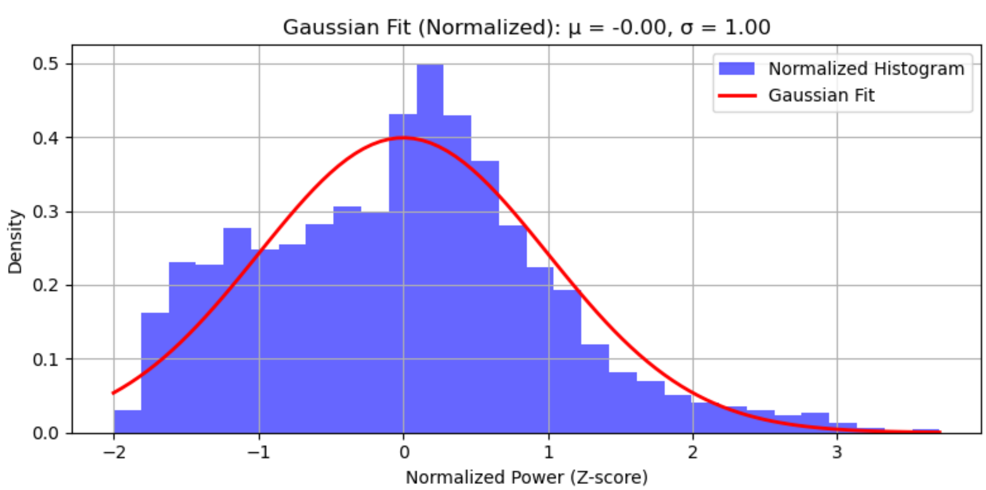

Figure 3: Histogram of normalized data with Gaussian fit

4 Methodology

4.1 Maximum Likelihood Estimation (MLE)

Under the Gaussian model:

$$\hat{\mu}_{MLE} = \frac{1}{n}\sum x_i, \quad \hat{\sigma}^2_{MLE} = \frac{1}{n}\sum (x_i - \hat{\mu})^2$$

- Estimated Mean (): 0.350778
- Estimated Variance (^{2}): 0.030671
- CRB: 0.00000350

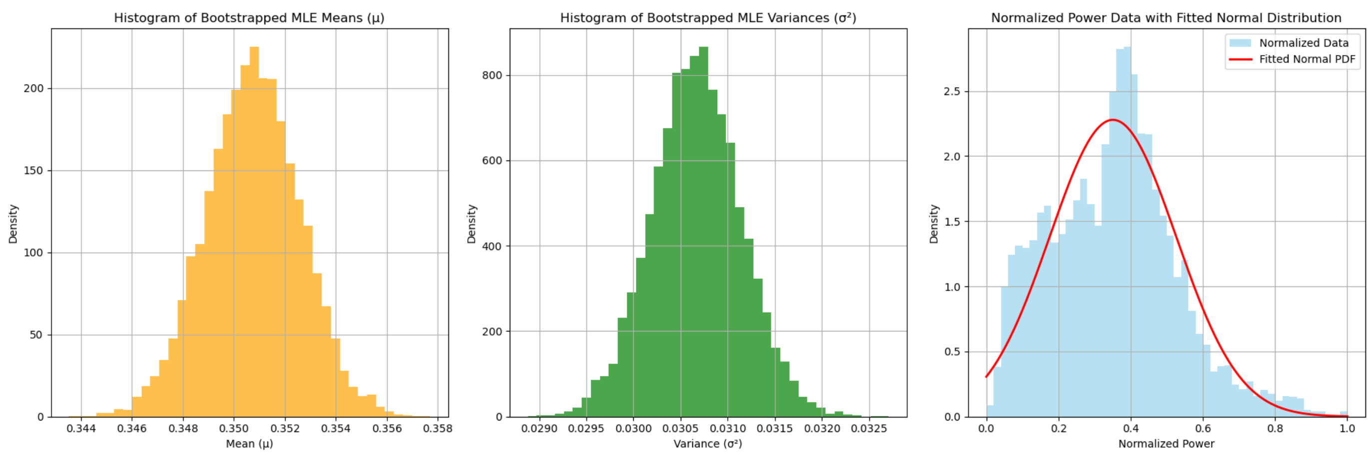

Figure 4: MLE fit on normalized data.

4.2 Bayesian Estimation

Assuming a Gaussian prior:

$$\mu \sim \mathcal{N}(\mu_0, \sigma_0^2)$$

$$\mu_{post} = \frac{\sigma^2\mu_0 + n\sigma_0^2\bar{x}}{n\sigma_0^2 + \sigma^2}, \quad \sigma^2_{post} = \left(\frac{1}{\sigma_0^2} + \frac{n}{\sigma^2}\right)^{-1}$$

- Posterior Mean (): 0.350738
- Posterior Variance (^{2}): 0.030906
- CRB: 0.00000353

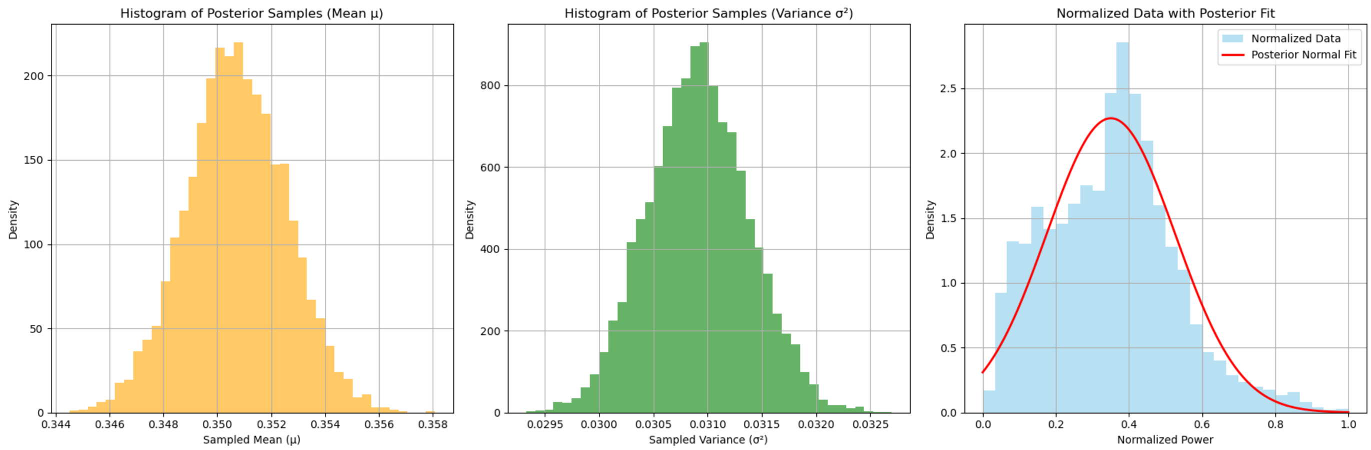

Figure 5: Bayesian posterior distribution.

5 Comparison of Estimators

Metric	MLE Estimate	Bayesian Estimate
Mean (μ)	0.350778	0.350738
Variance (σ^2)	0.030671	0.030906
CRB (μ)	0.00000350	0.00000353

Table 1: Comparison of MLE and Bayesian estimates

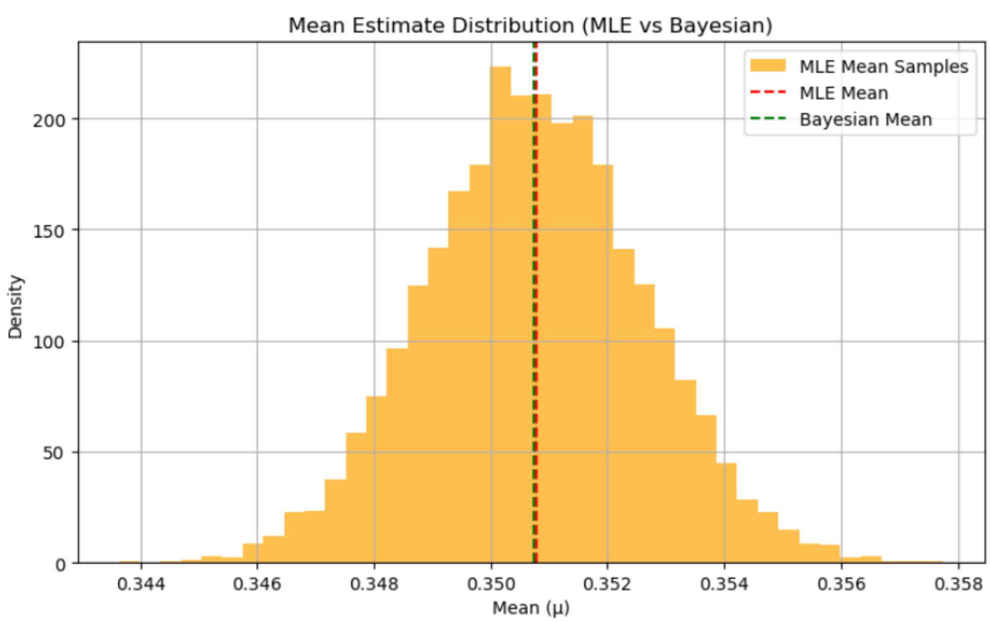

Figure 6: MLE vs Bayesian on Mean Estimate Distribution.

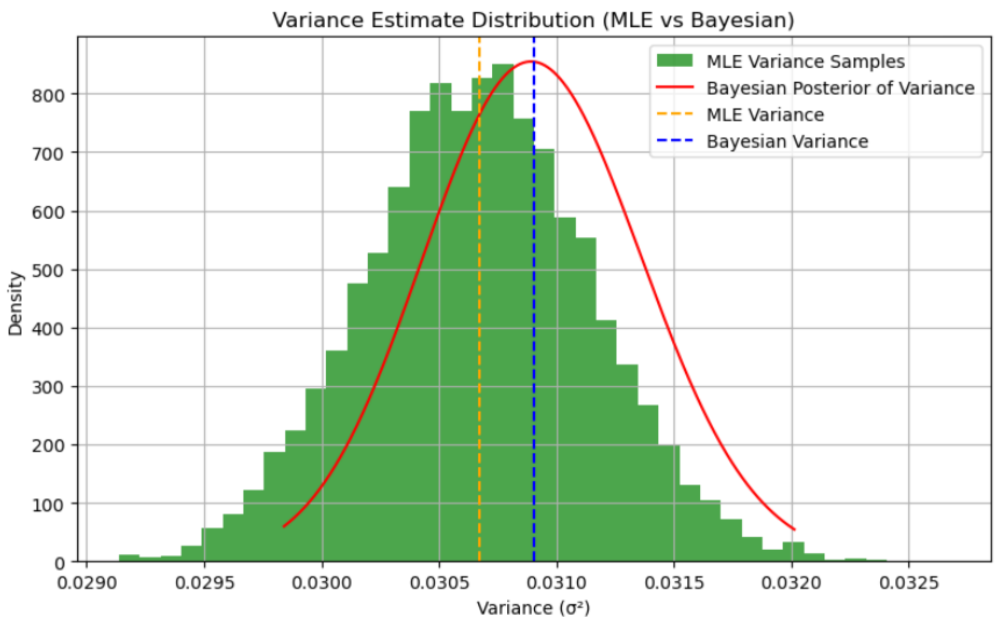

Figure 7: MLE vs Bayesian on Variance Estimate Distribution.

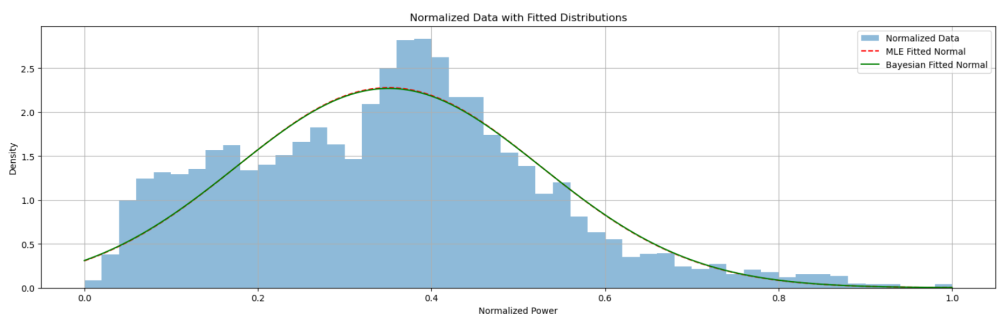

Figure 8: Histogram of Normalized Data and Fitted Normal Distribution

6 Conclusion

MLE and Bayesian estimation were successfully applied to power consumption data. The dataset was confirmed to be approximately Gaussian. MLE showed slightly lower CRB, indicating higher efficiency, whereas Bayesian estimation offered robustness when priors are valid. Both methods are valuable in signal processing and estimation problems.

References

1. Kay, S. M. *Fundamentals of Statistical Signal Processing: Estimation Theory*,
2. Bishop, C. M. *Pattern Recognition and Machine Learning*, Springer, 2006.
3. Gelman, A. et al., *Bayesian Data Analysis*, CRC Press, 2013.