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# Adaptive Sliding Mode Observers for Lithium-ion Battery State Estimation Based

# on Parameters Identified Online

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### Highlights

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- 8 An adaptive battery model with parameters estimated online is proposed.
- 9 Parameter adaptive sliding mode observers are proposed for battery states estimation.
- Lyapunov stability criterion has been employed to verify the robustness of the proposed method.
- The proposed approach is verified by lithium-ion battery experiment.
- The proposed method could improve the battery estimation results of state of charge and state of health.

#### 13 Abstract

Simplicity and accuracy are both important factors in real-time battery states estimation applications. 14 However, a battery model initialized with static parameters which are identified in ideal laboratory conditions 15 will not be able to get an accurate estimation in various actual applications. Besides, it is time-consuming 16 and complex in implement. To solve the above problem, a new battery states estimation method is proposed. 17 Firstly, an adaptive battery model is proposed according to a new online parameter estimation algorithm. 18 Based on it, the parameter adaptive sliding mode observer for state of charge is proposed. Thus, the state of 19 charge systematic error led from various work environments could be effectively reduced. The parameter 20 adaptive sliding mode observer for state of health is proposed by tracing the derivative of open circuit voltage 21 estimated online. As the reference open circuit voltage is estimated based on measurable inputs and outputs, 22 rather than conventional observer with an assumed constant capacity. The estimated battery capacity could 23 24 converge to the actual value while the error of battery open circuit voltage converges to zero. The proposed

- 25 method is verified through the urban dynamometer driving schedule driving cycle. The results indicate
- 26 that:1) parameters estimated online are accurate, 2) the absolute error of state of charge is less than 2%, 3) the
- estimated lithium-ion battery capacity could converge to the actual value with small capacity error.

## 28 Keywords

- 29 Lithium-ion battery; Parameters estimation online; Adaptive sliding mode observer; State of charge
- 30 estimation; State of health estimation

error states

### Nomenclature

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| 32 | $E_0$                    | open circuit voltage   |
|----|--------------------------|--|
| 33 | $\hat{E}_{0}$            | estimation of open circuit voltage                                   |
| 34 | $E_{Cn}$                 | filtered output of $\hat{E}_0$                                       |
| 35 | $\hat{E}_{Cn}$           | estimation of filtered output of $\hat{E}_0$                         |
| 36 | $R_1$                    | ohm resistance   |
| 37 | $\hat{R}_{_1}$           | estimation of ohm resistance   |
| 38 | $R_2$                    | polarization resistance  |
| 39 | $\hat{R}_2$              | estimation of polarization resistance                                |
| 40 | $C_2$                    | polarization capacitor   |
| 41 | $\hat{C}_2$              | estimation of polarization capacitor                                 |
| 42 | $V_0$                    | terminal voltage   |
| 43 | $\hat{V_0}$              | estimation of terminal voltage                                       |
| 44 | Ī                        | current  |
| 45 | $V_2$                    | polarization voltage   |
| 46 | $\hat{V_2}$              | estimation of polarization voltage                                   |
| 47 | Z                        | SOC  |
| 48 | $\hat{z}$                | estimation of SOC  |
| 49 | $C_{\rm n}$              | nominal capacity   |
| 50 | $\hat{C}_{_{ m n}}$      | estimation of capacity   |
| 51 | $lpha_{ m n}$            | system parameter   |
| 52 | $oldsymbol{eta}_{ m n}$  | system parameter   |
| 53 | heta                     | a matrix consist of the slow time-varying parameters                 |
| 54 | $\mu$                    | a matrix consist of the parameters could be measured through sensors |
| 55 | Λ                        | positive definite matrix   |
| 56 | $\omega_i (i=1,2,3)$     | noises   |
| 57 | $\delta_i(i=1,2,3), h_1$ | positive feedback coefficients                                       |
| 58 | $\Delta f$               | modeling errors and uncertainties                                    |

Due to the merits in high energy and power density, long cycle life, broad operating temperature range and

extremely low self-discharge rate [1], Lithium-ion battery has been regarded as the most promising

|    |              | $\mathcal{E}$                            |
|----|--------------|--|
| 61 | EKF          | extended Kalman filter                   |
| 62 | EV           | electric vehicle                         |
| 63 | UDDS         | urban dynamometer driving schedule       |
| 64 | HPPC         | hybrid pulse power characterization      |
| 65 | SOC          | state of charge                          |
| 66 | SOH          | state of health                          |
| 67 | <b>PASMO</b> | parameter adaptive sliding mode observer |
| 68 | MAE          | mean absolute error                      |
| 69 | RMSE         | root mean square error                   |
| 70 |              |  |
|    |              |  |

battery management system

#### 71 **1. Introduction**

**BMS** 

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energy storage solution to electrify the transportation sector and reduce the air pollution in urban 74 75 areas [2]. To improve both the operating performance and cycle life of batteries in EVs (electric vehicles), an accurate estimation of the battery states are of great importance. The states emphasized here refer to the 76 SOC (state of charge) and the SOH (state of health). 77 The SOC of a battery is the ratio of remaining capacity to nominal capacity. As an key output of BMS 78 (battery management system), the SOC has an extremely considerable effect on safety and reliability [3]. If a 79 more accurate SOC can be obtained, the SOC range we can use will extends compared with before [4]. It 80 81 means the actual available energy of the same battery pack could be improved without promoting the risk of battery damage caused by over-discharge. Thus, a smaller battery pack will be able to satisfy the power 82 demand of EVs. Accordingly, the cost of the battery pack could be significantly reduced by the saving parts. 83 It will be an useful effort to handle the contradiction between a better mileage and a cheaper price of EVs [5]. 84 A variety of SOC estimation techniques have been proposed, which could be divided into the non-model 85 based methods and the model-based methods. The model-based SOC estimation methods have become the 86 87 research hotspot gradually for the ability of self-correct under unexpected disturbances [3]. In existing studies, three factors are regarded as the key to obtain an accurate SOC, namely an accurate battery model, 88

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accurate battery internal parameters and a robust SOC observer. Thus, a variety of battery models are proposed and used in implementing, for instance, the Rint model, the RC model, the Thevenin model, the PNGV model, the nonlinear equivalent circuit model model and the electrochemical model [6]. HPPC (hybrid pulse power characteristic) and EIS (Electrochemical impedance spectroscopy) methods are used to identify the parameters. Bizhong Xia proposed an improved parameter identification method to analyze the polarization characteristics of lithium-ion batteries [7]. Based on these methods, many SOC observers have been built to optimize the robustness and reduce negative impacts of the estimation. Fuli Zhong proposed a SOC estimation approach based on an adaptive sliding mode observer and a fractional order equivalent circuit model for lithium-ion batteries [8]. Xue Li analyzed the effects of different initial SOC errors and parameters variation on SOC estimation accuracy and robustness with H-infinity observer [9]. Jun Xu proposed a proportional-integral observer to estimate the SOC of lithium-ion batteries in EDVs under unknown initial SOC cases [10]. Di Li proposed a SOC estimation method based on the Strong Tracking Sigma Point Kalman Filter algorithm, which has the advantages of tracking the variables in real-time and adjusting the error covariance by taking the Strong Tracking Factor into account [11]. However, the static battery models initialized with parameters identified in ideal laboratory conditions are unable to satisfy various actual applications. Thus, keeping the coherence of a battery model and its actual characteristics becomes the key to ensure the accuracy of SOC estimation. The SOH is defined as the ability of a cell to store energy, source and sink high currents, and retain charge

The SOH is defined as the ability of a cell to store energy, source and sink high currents, and retain charge over extended periods, relative to its nominal capabilities. An accurate SOH estimation could help to optimize the operation, extend the cycle life and avoid a sudden failure of batteries. The actual capacity variation is the most important and obvious indicator of battery SOH.

The existing researches on SOH estimation are based on chemical analysis or electrical circuits [12]. However, there are still some drawbacks of the chemical analysis methods: 1) Implement to EVs is hard, 2) A lot of history data is needed. Thus, the electrical circuit methods become the research hotspot. Since Plett proposed a dual extended Kalman filter method to estimate the SOC together with the SOH of a battery [13],

| a series of researches based on the cyclic-state-estimation method have been proposed. Haifeng Dai et al.     |
|---|
| improved the cyclic-state-estimation method with dual time-scale theory to lithium-ion battery packs. [14]    |
| Song et al. proposed a dual sliding mode observer [15]. Lei Pei et al. proposed a real-time peak power/SOP    |
| prediction method for lithium-ion batteries based on DEKF [16]. However, some serious issues are ignored      |
| in the cyclic-state-estimation based methods. 1) The error propagation between parameter estimation and       |
| battery state estimation is ignored. As we know, The inputs and the outputs are interdependent in cyclic-     |
| state-estimation methods. It means accurate initial battery parameters are needed to prevent an inaccurate    |
| battery state estimation. Similarly, an accurate battery state estimation is necessary for battery parameters |
| estimation. However, it is hard to guarantee. 2) The estimated battery capacity will fluctuate around the     |
| assumed constant capacity, rather than converge to its actual value. Because the actual capacity is estimated |
| according to a closed-loop based on a quantitative relationship of SOC error. However, the actual SOC value   |
| needed in the SOH observer is estimated based on an assumed constant capacity.                                |
| To solve the problem stated above, a new electrical circuit based battery state estimation method is          |
| proposed in this paper. In the first place, a new online parameter estimation algorithm is proposed.          |
| Additionally, parameters estimated online are filtered by moving average filter to update the battery model.  |
| Based on it, two independent PASMOs (parameter adaptive sliding mode observers) for SOC and SOH               |
| estimation have been proposed. As parameters estimation, model updating and SOC estimation are almost         |
| synchronous, the system error of adaptive battery model can be eliminated effectively. Combined with          |
| sliding mode observers for battery states estimation, many significant advantages are performed: 1) online    |
| shaing mode observers for battery states estimation, many significant advantages are performed. If online     |
| estimation of parameters, 2) no need of accurate initial parameters, 3) simple mathematical operation, 4)     |

A battery experimental platform is constructed to acquire the experimental data. The UDDS (urban dynamometer driving schedule) current profile is used to verify the algorithm for online parameters identification, SOC and SOH estimation.

accurate SOC and SOH estimation, 5) cheap hardware cost.

As shown in Fig. 1, the proposed method includes three synchronous parts, namely the parameter adaptive model specification of a battery, the estimation of SOC and the estimation of SOH. The paper is organized as follows to explain it. In section 2, the new online parameter estimation algorithm is proposed based on a simple first order RC battery model, then the adaptive battery model is built according to the parameters estimated online. In section 3 and 4, PASMOs for SOC and SOH estimation are built based on the online parameter estimation algorithm proposed in section 2, respectively. In section 5, under the UDDS current profile, experiments are performed on the lithium-ion battery to verify the reliability and robustness of the proposed method. The results indicated that accurate SOC and SOH estimation results can be obtained by the

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#### 2. Battery Modeling

- In the model specification part, a first order RC equivalent circuit model is adopted as the base. Ohm
- resistance, polarization resistance, polarization capacity and open circuit voltage are parameters of the model.
- They are assumed to be dynamic to reflect actual characteristics of a battery.

# 2.1 Equivalent Circuit of A Battery

- As shown in Fig. 2, the first order RC battery model consists of a voltage source ( $^{E_0}$ ), a resistor ( $^{R_1}$ ), and a
- parallel capacitor  $\binom{C_2}{2}$  and resistor  $\binom{R_2}{2}$ .
- $R_1$  represents the battery ohm resistance.  $C_2$  and  $R_2$  are utilized to model the polarization effect with the
- batteries.  $E_0$  which represents the open circuit voltage of the battery is a nonlinear function of SOC. In
- general, the  $E_0$ -SOC curve of a battery changes slowly over time when battery electrochemistry properties
- vary due to aging or other factors [17]. The relationship between  $E_0$  and SOC could be decomposed as
- 159  $E_0 = \alpha_n \times z + \beta_n$  by interpolation, where z represents SOC. The related parameters  $\alpha_n$  and  $\beta_n$  are listed in
- Table 1 through linear interpolation.

- According to Kirchhoff voltage law in circuit theory, terminal voltage  $V_0$  shown in Fig. 2 could be written 161
- 162 as Eq. (1).
- $V_0 = E_0 + IR_1 + V_2 \ (1)$ 163
- where  $V_2$  denotes the electrochemical polarization voltage. The derivative of it could be expressed as Eq. (2) 164
- according to Kirchhoff current law. 165
- $\dot{V}_2 = -V_2/(R_2C_2) + I/C_2$  (2) 166
- The SOC of a battery is the ratio of the remaining capacity to the nominal capacity. Assume the initial 167
- SOC and the current SOC to be z(0) and z(t) respectively. The SOC could be written as Eq. (3) through the 168
- Ah counting approach. 169

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$$z(t) = z(0) + \Delta z = z(0) + \int_0^t (I(\tau)/C_n) d\tau$$
 (3)

- where z denotes SOC,  $\Delta z$  is the variation of battery SOC during time period 0 to t,  $I(\tau)$  is the instantaneous 171
- battery current,  $C_n$  is the nominal battery capacity. Thus, the derivative of SOC could be written as Eq. (4). 172

$$\dot{z} = I/C_{\rm n} \quad (4)$$

#### **Online Parameter Estimation**

- Parameters of a battery model, such as ohm resistance, polarization resistance, polarization capacity and 175
- open circuit voltage, vary with operating environment and battery state. However, the variation is slight and 176
- slow in a normal operating condition [15]. Thus,  $E_0$  together with  $R_1$ ,  $R_2$ ,  $C_2$  are assumed to be slow time-177
- varying parameters.  $I \setminus I \setminus V_0$  are set to be measurable inputs and response of a battery system. The 178
- derivative of  $V_0$  in Eq. (1) can be rewritten by substituting Eq. (2) and Eq. (4) into itself. 179

180 
$$\dot{V_0} = R_1 \dot{I} + [(R_1 + R_2)I]/(R_2C_2) - V_0/(R_2C_2) + E_0/(R_2C_2)$$

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$$\dot{V}_{0} = R_{1}\dot{I} + [(R_{1} + R_{2})I]/(R_{2}C_{2}) - V_{0}/(R_{2}C_{2}) + E_{0}/(R_{2}C_{2})$$

$$= [R_{1} (R_{1} + R_{2})/(R_{2}C_{2}) 1/(R_{2}C_{2}) E_{0}/(R_{2}C_{2})][\dot{I} I - V_{0} 1]^{T} = [\theta_{1} \theta_{2} \theta_{3} \theta_{4}][\mu_{1} \mu_{2} \mu_{3} \mu_{4}]^{T} = \theta \mu^{T} (5)$$

- where  $\theta(E_0, R_1, R_2, C_2)$  is a matrix of the slow time-varying parameters needed to be identified.  $\mu$  is the 182
- symbol of the matrix consist of the inputs could be measured through sensors. 183

184 According to Eq. (5),  $\dot{V}_0$  observer is constructed as follow.

$$\dot{\hat{V}}_0 = \hat{\boldsymbol{\theta}} \boldsymbol{\mu}^{\mathrm{T}} + \lambda e_{V_0}$$

$$= [\hat{\theta}_{1} \ \hat{\theta}_{2} \ \hat{\theta}_{3} \ \hat{\theta}_{4}][\dot{I} \ I \ -V_{0} \ 1]^{T} + \lambda e_{V_{0}} \ (6)$$

- where  $e_{V_0} = V_0 \hat{V_0}$  is battery terminal voltage error.
- Define  $e_{\theta} = \theta \hat{\theta}$ . Eq. (7) and Eq. (8) are needed for the convergence of the  $E_0$ ,  $R_1$ ,  $R_2$ ,  $C_2$  observed online.

$$\lim_{t \to \infty} \mathbf{e}_{\theta} = \mathbf{0} \quad (7)$$

$$\lim_{t \to \infty} e_{V_0} = 0 \ \ (8)$$

Construct Lyapunov function as Eq. (9).  $\Lambda$  is a positive definite matrix, therefore V is positive definite.

$$V = \frac{1}{2}e_{V_0}^2 + \frac{1}{2}e_{\theta}\Lambda e_{\theta}^{\mathrm{T}}$$
 (9)

In order to satisfy the Lyapunov stability criterion,  $\dot{V}$  expressed as Eq. (10) should be negative definite.

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$$\dot{V} = e_{V_0} \dot{e}_{V_0} + e_{\theta} \mathbf{\Lambda} \dot{e}_{\theta}^{\mathrm{T}}$$

$$= e_{V_0} [\theta \mu^{\mathrm{T}} - (\hat{\theta} \mu^{\mathrm{T}} + \lambda e_{V_0})] + e_{\theta} \mathbf{\Lambda} (\dot{\theta}^{\mathrm{T}} - \dot{\hat{\theta}}^{\mathrm{T}}) \quad (10)$$

$$\approx e_{\theta} (\mu^{\mathrm{T}} e_{V_0} - \mathbf{\Lambda} \dot{\hat{\theta}}^{\mathrm{T}}) - \lambda e_{V_0}^{2}$$

195 Thus,

196
$$\begin{bmatrix}
\dot{\hat{\theta}}_{1} \\
\dot{\hat{\theta}}_{2} \\
\dot{\hat{\theta}}_{3} \\
\dot{\hat{\theta}}_{4}
\end{bmatrix} = \begin{bmatrix}
\rho_{1}\dot{I}(V_{0} - \hat{V}_{0}) \\
\rho_{2}I(V_{0} - \hat{V}_{0}) \\
-\rho_{3}V_{0}(V_{0} - \hat{V}_{0}) \\
\rho_{4}(V_{0} - \hat{V}_{0})
\end{bmatrix} (11)$$

- where  $\rho_i$  (i = 1,2,3,4) are the feedback parameters which constitute the positive matrix  $\Lambda$ .
- It means the accurate quantitative relationship between the unknown  $\theta(E_0, R_1, R_2, C_2)$  and the measurable
- inputs, response of a battery system is constructed.
- Thus, slow time-varying parameters of a battery could be estimated as follows.

$$\hat{R}_1 = \hat{\theta}_1 \ (12)$$

$$\hat{R}_{2} = \hat{\theta}_{2} / \hat{\theta}_{3} - \hat{\theta}_{1}$$
 (13)

$$\hat{E}_0 = \hat{\theta}_4 / \hat{\theta}_3 \quad (14)$$

$$\hat{C}_2 = [(\hat{R}_1 + \hat{R}_2)I - V_0 + \hat{E}_0]/[(\dot{V}_0 - i\hat{R}_1)\hat{R}_2]$$
 (15)

- The parameters estimated online need to be filtered by a moving average filter to reduce the impacts of 205 noise before the next step of operation. It won't affect the estimation accuracy as the parameters vary slowly 206 in a short period of time.
- **Parameter Adaptive Battery Model** 208

- The continuous state space equation of a static first-order RC battery model can be constructed as shown in 209
- Eq. (16) based on Eq. (1), Eq. (2) and Eq. (4). 210

$$\dot{V}_{0} = -V_{0}/(R_{2}C_{2}) + E_{0}/(R_{2}C_{2}) + [I(R_{1} + R_{2})]/(R_{2}C_{2}) + \omega_{1}$$

$$\dot{z} = (V_{0} - E_{0} - V_{2})/(R_{1}C_{n}) + \omega_{2}$$

$$\dot{V}_{2} = -V_{2}/(R_{2}C_{2}) + I/C_{2} + \omega_{3}$$
(16)

- where I is the input,  $V_0$  is the response, z and  $V_2$  are the state variables of the model,  $\omega_i(i=1,2,3)$  are noises. 212
- The battery model could be updated by using the parameters estimated online. It means that a dynamical 213
- battery model is built as Eq. (17). 214

$$\dot{V}_{0} = -V_{0}/(\hat{R}_{2}\hat{C}_{2}) + \hat{E}_{0}/(\hat{R}_{2}\hat{C}_{2}) + [I(\hat{R}_{1} + \hat{R}_{2})]/(\hat{R}_{2}\hat{C}_{2}) + \omega_{1}$$

$$\dot{z} = (V_{0} - \hat{E}_{0} - V_{2})/(\hat{R}_{1}C_{n}) + \omega_{2}$$

$$\dot{V}_{2} = -V_{2}/(\hat{R}_{2}\hat{C}_{2}) + I/\hat{C}_{2} + \omega_{3}$$
(17)

- 3. Adaptive Sliding Mode SOC Observer 216
- In the SOC estimation part, the PASMO is proposed based on parameters (ohm resistance, polarization 217
- resistance, polarization capacity) identified online. The systematic error led from variation of work 218
- environment could be effectively reduced. 219
- As the observation matrix is positive definite, state variables of the battery model Eq. (17) are observable. 220
- Based on Eq. (17), state observer of the battery could be constructed as the style shown in Eq. (18). 221

$$\dot{\hat{V}}_{0} = -V_{0}/(\hat{R}_{2}\hat{C}_{2}) + \hat{E}_{0}/(\hat{R}_{2}\hat{C}_{2}) + [I(\hat{R}_{1} + \hat{R}_{2})]/(\hat{R}_{2}\hat{C}_{2}) + \delta_{1} \operatorname{sgn}(e_{V_{0}})$$

$$\dot{\hat{z}} = (V_{0} - \hat{E}_{0} - \hat{V}_{2})/(\hat{R}_{1}C_{n}) + \delta_{2} \operatorname{sgn}(e_{Z})$$

$$\dot{\hat{V}}_{2} = -\hat{V}_{2}/(\hat{R}_{2}\hat{C}_{2}) + I/\hat{C}_{2} + \delta_{3} \operatorname{sgn}(e_{V_{2}})$$
(18)

- where  $\hat{V_0}$ ,  $\hat{E_0}$ ,  $\hat{z}$ ,  $\hat{V_2}$  are the estimated values of  $V_0$ ,  $E_0$ , z,  $V_2$  respectively. Feedbacks are constructed
- to guarantee the robustness and eliminate the noise.  $\delta_i(i=1,2,3)$  are positive feedback coefficients.  $e_{V_0}$ ,  $e_Z$ ,  $e_{V_0}$
- are defined as follows respectively.

$$\begin{aligned}
e_{V_0} &= V_0 - \hat{V_0} \\
e_Z &= z - \hat{z} \\
e_{V_2} &= V_2 - \hat{V_2}
\end{aligned}$$

- Eq. (19), Eq. (20) and Eq. (21) are needed for the convergence of the SOC estimation.
- $\lim_{t \to \infty} e_{V_0} = \mathbf{0} \ (19)$
- $\lim_{t \to \infty} e_z = 0 \quad (20)$
- $\lim_{t \to \infty} e_{v_2} = 0 \quad (21)$
- Among them,  $V_0$  could be accurately measured, however z and  $V_2$  could not be measured through sensors
- directly. Therefore, the quantitative relationship between  $e_Z$ ,  $e_{V_2}$  and  $e_{V_0}$  are important for the effective
- feedback of the battery model.
- Thus, construct the following error equations based on Eq. (17) and Eq. (18) in the first place.

$$\dot{e}_{V_0} = (E_0 - \hat{E}_0) / (\hat{R}_2 \hat{C}_2) + [\omega_1 - \delta_1 \operatorname{sgn}(e_{V_0})] 
\dot{e}_z = -(E_0 - \hat{E}_0) / (\hat{R}_1 C_n) - (V_2 - \hat{V}_2) / (\hat{R}_1 C_n) + [\omega_2 - \delta_2 \operatorname{sgn}(e_z)]$$

$$\dot{e}_{V_2} = -(V_2 - \hat{V}_2) / (\hat{R}_2 \hat{C}_2) + [\omega_3 - \delta_3 \operatorname{sgn}(e_{V_2})]$$
(22)

- Definite Lyapunov function  $V_{V_0} = \frac{1}{2}e_{v_0}^2$ , to guarantee the convergence of  $e_{V_0}$ ,
- 237  $\dot{V}_{V_0} = e_{V_0} \dot{e}_{V_0} = e_{V_0} \{(E_0 \hat{E}_0)/(\hat{R}_2 \hat{C}_2) + [\omega_1 \delta_1 \operatorname{sgn}(e_{V_0})]\}$  should be negative definite. As
- 238  $(E_0 \hat{E}_0)/(\hat{R}_2\hat{C}_2) < (4.2 2.8)/(0.015*730) = 0.128$ ,  $\delta_1$  is assumed to be larger than 0.128. After a period of time of
- observation, there will be

$$e_{V_0} = 0$$

$$\dot{e}_{V_0} = 0 \quad (23)$$

$$\omega_1 \to 0$$

Thus, the relationship between  $e_Z$  and  $e_{V_0}$  could be obtained

$$(z - \hat{z}) = \frac{R_2 C_2 \delta_1 \operatorname{sgn}(e_{V_0})}{\alpha_n} \tag{24}$$

- Similarly, definite Lyapunov function  $V_z = \frac{1}{2}e_z^2$ , to guarantee the convergence of  $e_z$ ,  $\dot{V}_z$  should be negative
- definite. It means that  $\delta_2$  should be larger than  $w_2$ . After a period of time of observation, there will be

$$e_{z} = 0$$

$$\dot{e}_{z} = 0 \quad (25)$$

$$\omega_{2} \to 0$$

Thus, the relationship between  $e_{V_2}$  and  $e_Z$  could be obtained,

$$(V_2 - \hat{V_2}) = -R_1 C_n \delta_2 \operatorname{sgn}(z - \hat{z}) = -R_1 C_n \delta_2 \operatorname{sgn}(\frac{R_2 C_2 \delta_1 \operatorname{sgn}(e_{V_0})}{\alpha_n})$$
 (26)

- According to the Lyapunov stability criterion, while  $\delta_3$  is assumed to be larger than  $w_3$ , the convergence
- of  $e_{y_2}$  is guaranteed in observation as the same way.
- Based on the quantitative relationship between  $e_z$ ,  $e_{v_2}$  and  $e_{v_0}$  constructed before, the adaptive sliding
- 251 mode SOC observer could be rewritten as follow.

$$\dot{\hat{V}}_{0}^{'} = -V_{0}/(\hat{R}_{2}\hat{C}_{2}) + \hat{E}_{0}/(\hat{R}_{2}\hat{C}_{2}) + [I(\hat{R}_{1} + \hat{R}_{2})]/(\hat{R}_{2}\hat{C}_{2}) + \delta_{1} \operatorname{sgn}(e_{V_{0}})$$

$$\dot{\hat{z}} = (V_{0} - \hat{E}_{0} - \hat{V}_{2})/(\hat{R}_{1}C_{n}) + \delta_{2} \operatorname{sgn}[\hat{R}_{2}\hat{C}_{2} \operatorname{sgn}(e_{V_{0}})/\alpha_{n}]$$

$$\dot{\hat{V}}_{2} = -\hat{V}_{2}/(\hat{R}_{2}\hat{C}_{2}) + I/\hat{C}_{2} + \delta_{3} \operatorname{sgn}\{-\hat{R}_{1}C_{n} \operatorname{sgn}[\hat{R}_{2}\hat{C}_{2} \operatorname{sgn}(e_{V_{0}})/\alpha_{n}]\}$$

$$\begin{cases}
\delta_{1} >> 0 \\
\delta_{2} >> 0 \\
\delta_{3} >> 0
\end{cases}$$

$$\operatorname{sgn}(e_{V_{0}}) = \begin{cases} +1, e_{V_{0}} > 0 \\
-1, e_{V_{0}} < 0
\end{cases}$$

#### 253 4. Sliding Mode SOH Observer

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As capability fading is the indicator of battery SOH, estimating an accurate actual capacity in variable operating conditions is very important. By tracing the continuous derivate of battery open circuit voltage

which is identified online in section 2.2, an ASMO for battery capacity estimation is constructed in this section. The estimated battery capacity will converge to its actual value when the error of battery open circuit voltage converges to zero. The reference open circuit voltage is estimated only based on measurable inputs and outputs, rather than an assumed constant capacity value. Thus, the actual capacity could be obtained by feedback based on a quantitative relationship of the open circuit voltage error.

Based on the open circuit voltage  $\hat{E}_0$  identified online before and the definition of SOC, a new state variable  $E_{Cn}$  which is assigned as the filtered output of  $\hat{E}_0$  could be defined as follow.

$$\dot{E}_{C_n} = \alpha_n I/C_n + \Delta f_{E_{C_n}}$$
 (28)

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where  $\Delta f_{E_{Cn}}$  is noise. The observer equation could be expressed as follow.

$$\dot{\hat{E}}_{C_n} = \alpha_n I / \hat{C}_n + h_1 (E_{C_n} - \hat{E}_{C_n})$$
 (29)

- where  $\hat{E}_{C_n}$ ,  $\hat{C}_n$  are the estimated values of  $E_{C_n}$  and  $C_n$ .  $h_1$  is a constant positive-feedback gain. When the  $E_{C_n}$  error is defined as  $e_{E_{C_n}} = (E_{C_n} \hat{E}_{C_n})$ , the following error equation is obtained.
- $\dot{e}_{E_{C_n}} = (\alpha_n I/C_n \alpha_n I/\hat{C}_n) + \Delta f_{E_{C_n}} h_1(E_{C_n} \hat{E}_{C_n})$  (30)
- Define the Lyapunov candidate function  $V_{E_{Cn}} = \frac{1}{2}e_{E_{Cn}}^2$ . To guarantee the convergence of  $e_{E_{Cn}}$ ,  $\dot{V}_{E_{Cn}} = e_{E_{Cn}}\dot{e}_{E_{Cn}} = e_{E_{Cn}}[(\alpha_n I/C_n \alpha_n I/\hat{C}_n) + \Delta f_{E_{Cn}} h_I(E_{Cn} \hat{E}_{Cn})]$  should be negative definite. The nominal  $C_n$  of the battery tested in the paper is 3Ah (10800 As). As actual  $C_n$  slowly monotonically decreases with battery aging over time, the estimation error  $(I/C_n I/\hat{C}_n)$  must be a very small value. When  $h_I >> \Delta f_{E_{Cn}}$ , the signs of  $\dot{e}_{E_{Cn}}$  and  $e_{E_{Cn}}$  are opposing. The condition for convergence of the error equation is satisfied. As in the previous case, after a period of time, there will be the following results.

$$\begin{aligned}
\dot{e}_{E_{Cn}} &= 0 \\
e_{E_{Cn}} &= 0 \\
\Delta f_{E_{Cn}} &\to 0
\end{aligned} (31)$$

According to equation (31), after a period of time there will be the following equation

$$(\alpha_{n}I/C_{n} - \alpha_{n}I/\hat{C}_{n}) - h_{1}(E_{C_{n}} - \hat{E}_{C_{n}}) = 0 \quad (32)$$

Define  $e_{Cn} = C_n - \hat{C}_n$ , the relationship between  $e_{Cn}$  and  $e_{E_{Cn}}$  could be obtained based on equation (32). 278

$$e_{C_n} = (C_n - \hat{C}_n) = -C_n \hat{C}_n (1/C_n - 1/\hat{C}_n)$$

$$= -C_n \hat{C}_n [h_1(E_{C_n} - \hat{E}_{C_n})] / \alpha_n I = -C_n \hat{C}_n (h_1 e_{E_{C_n}}) / \alpha_n I$$
(33)

- As mentioned before,  $C_n$  which represents the actual capacity of a battery decreases slowly over time. 280
- Therefore, it could be rewritten in derivation as follow. 281

$$\dot{C}_{\rm n} = \Delta f_{\rm c.} \ (34)$$

- According to the relationship between  $e_{Cn}$  and  $e_{E_{Cn}}$  constructed previously, define the observer equation for 283
- 284  $C_{\rm n}$  as.

$$\dot{\hat{C}}_{n} = -h_{2}C_{n}\hat{C}_{n}[h_{1}(E_{Cn} - \hat{E}_{Cn})]/\alpha_{n}sgn(I)$$
 (35)

Define the Lyapunov function as follow. 286

$$V_{C_n} = \frac{1}{2} e_{C_n}^2$$
 (36)

- As the signs of  $\dot{e}_{C_n}$  and  $e_{C_n}$  are opposing,  $\dot{V}_{C_n} = \dot{e}_{C_n} e_{C_n} < 0$ . Thus, after a period of time of observation, there 288
- will be. 289

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289 will be. 
$$\hat{C}_{n} \rightarrow C_{n}$$
 (37)

Thus, the observer equations for battery actual capacity are obtained. 291

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$$\dot{\hat{E}}_{Cn} = I/\alpha_n \hat{C}_n + h_1(E_{Cn} - \hat{E}_{Cn}) \\
\dot{\hat{C}}_n = -h_2 C_n \hat{C}_n [h_1(E_{Cn} - \hat{E}_{Cn})] / \alpha_n sgn(I)$$

#### 5. Validation of The Proposed Algorithm: Set-up Parameters

A battery experimental platform is established to obtain the experimental data (battery current, terminal voltage) which are needed for the verification of the proposed algorithm. As shown in Fig. 3(a), the experimental platform consists of a monitoring computer, a battery test instrument and a thermostat. The monitoring computer is used to control the experiment and record experimental data. The battery test

| instrument is used to load a battery according to the EV operating modes, including starting, running and         |
|---|
| braking. The thermostat is used to set the battery in the certain experimental temperature. To effectively        |
| compare the results of the algorithm proposed in this paper with the traditional one, the test temperature is set |
| to be 20°C by the thermostat. A simple flow chart of the platform is shown in Fig. 3(b). NCR 18650 lithium-       |
| ion batteries are adopted in the experiment. Each cell has a nominal voltage of 3.62 V and a nominal capacity     |
| of 3 Ah. It has an upper voltage limit of 4.2V and a lower cut-off voltage of 2.8V.                               |

The whole experiment includes two main parts, namely the HPPC test and the UDDS cycles test. The HPPC test is used to evaluate the accuracy of the parameters identified online. The RLS (recursive least squares) method is used to identify parameters offline. Then, the parameters identified offline will be used as reference to compare with our online estimated values.

The UDDS cycles test is used to simulate the actual operating and accelerate the decline of the battery. The UDDS current profile is shown in Fig. 4. Based on it, we verify our proposed algorithm of online parameters identification, SOC and SOH estimation. In the test, the battery's terminal voltage and current are measured per 50 millisecond. Negative current represents battery discharge while positive current means battery charge.

To apply the algorithm in implementation, a hardware platform including current censors, voltage sensors and a digital signal processor is needed. As the variation of the actual capacity is small in a short period, it could be updated periodically using a multi-time dimension method to reduce the computation of DSP. At the beginning of the estimation period, the estimated capacity is not stable, so the historical parameter could be used to replace it.

#### 6. Validation of The Proposed Algorithm: Results and Discussion

The proposed method is verified through the UDDS driving cycle. The validation procedure consists of three main parts as below.

#### **6.1 Parameters Estimation Online**

Computation cost is critical to online estimation test. To guarantee the effectiveness of online estimation, a good sampling precision and a sufficient sampling frequency are needed. In our test, the sampling frequency is set to be 50 ms. It means we have to solve a five-element differential equation every 50 ms. Then the parameters estimated online will be filtered by a moving average filter to reduce the impacts of noise before the next step of operation. As the parameters vary slowly in a short period of time, the sampling frequency maybe modified in actual operation to reduce the computing cost.

Initial parameters and feedback coefficients needed for parameters estimation online are listed in Table 2.

Comparison between curves of online estimated parameters and the reference ones are shown in Fig. 5. The red points plotted in Fig. 5 are identified offline by applying the RLS method with the actual data of HPPC test. According to piecewise linear interpolation, the reference curves battery parameters are obtained. As shown in Fig. 5, the estimated curves of parameters quickly converge to the reference ones with small fluctuations. It indicates the online parameters estimation algorithm proposed in this paper is useful and accurate. The MAE (mean absolute error) and RMSE (root mean square error) of parameters estimated

After being filtered, they are used to dynamically update the first order RC model. The terminal voltage based on model and the actual measured one are compared in Fig. 6. MAE and RMSE of parameters estimated online. The curves are almost the same. It indicates that the proposed adaptive battery model could accurately reflect the characteristics of the battery.

#### **6.2** State of Charge Estimation Online

online are shown in Table 3.

Initial parameters and feedback coefficients which are needed for SOC estimation based on the parameter adaptive battery model proposed previously are listed in Table 4. In order to verify the estimation accuracy of

- 344 the SOC under unknown initial situation, the initial error of estimated SOC are set to be 20% and 40%, 345 respectively. The estimated SOC curve based on the method proposed previously, the estimated SOC curve based on the 346 typical static one-order RC equivalent circuit battery model and the actual one are compared in Fig. 7 (a), (c). 347 The results indicate that the estimated SOC curves estimated by the proposed method rise rapidly and 348 converges to the actual quickly. Then the estimated SOC curves tend to be stable with few minor 349 fluctuations. The error of SOC estimation is less than 2% as shown in Fig. 7 (b), (d). 350 To verify the accuracy of the proposed method, we add a traditional static RC model as comparison. In 351 Fig.7, we can see that the SOC estimated by the proposed method is much closer to the reference SOC. 352 Almost in any points, the estimation error of the proposed method is smaller. It means the proposed method 353 is more suitable and effective than the traditional method by taking advantage of the updated battery model. 354 As a feedback structure is built for keeping the robustness in SOC estimation, the estimated SOC could 355 converge to its reference value in a period of time no matter what the initial SOC value is. It is obvious that 356 the more initial estimation error there is, the more time it needs to converge. Under normal conditions, the 357
- Additionally, the comparisons of MAE and RMSE based on the proposed method and the typical static one-order RC equivalent circuit battery model under dynamic current condition are shown in Table 5.

initial SOC error of a battery is from self-discharge. In general, it is much smaller than 20%, So the

#### 6.3 State of Health Estimation Online

convergence time should be less than 400 seconds generally.

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To obtain the battery actual capacity in working condition, the current profile used for SOH estimation is designed based on UDDS current profile. It includes two main parts. The former part is UDDS cycles, the latter one is a constant-current discharge process. Between the upper voltage limit of 4.2V and the lower cut-off voltage of 2.8V, the UDDS cycles test is operated on the lithium-ion battery. When the battery terminal voltage reaches the cut-off voltage, the UDDS cycles test ends to protect it from over-discharging. After 10

- minutes rest, the constant-current discharge process is taken on the lithium-ion battery. 400 cycles of UDDS are taken on the battery to obtain the actual capacity data of each time for the verification of the proposed SOH estimation method.
- Initial parameters and feedback coefficients which are needed for SOH estimation are listed in Table 6.

- To verify the accuracy of the actual capacity estimation method proposed previously under unknown initial condition, the initial capacity  $\hat{C}_n$  is set to be 4 Ah while the actual capacity is much less than it. The comparison of the estimated capacity curve and the actual one is shown in Fig. 8.
- The open circuit voltage identification online needs a period of time to converge to its actual value. As the capacity is estimated by feedback based on a quantitative relationship of the open circuit voltage error, there will be influence on our estimation value in the beginning. However, the convergence time is always less than 500s. The estimated capacity starts to calculate after 500 seconds. In the beginning, the estimated capacity has a larger error for its initial setting. In about 2500 seconds, the estimated capacity converges to the actual one and tends to be stable with few minor fluctuations. The rapid convergence and continuous stability reflect the SOH estimation algorithm proposed in this paper is feasible and effective under variable conditions, though initial capacity is unknown.
- To apply the algorithm in implementation, a hardware platform including current censors, voltage sensors and a digital signal processor is needed. As the variation of the actual capacity is small in a short period, it could be updated periodically using a multi-time dimension method to reduce the computation of DSP. At the beginning of the estimation period, the estimated capacity is not stable, so the historical parameter could be used to replace it.
- As we all know, temperature is one of the factors which could influent the parameters values of a battery. However, the temperature chosen in our experiment (20°C) cloud be thought as a random value for validation. It means the conclusion, our online estimated values will converge to the actual reference values has an extensive adaptability.

#### 7. Conclusion

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- A new battery states estimation method has been proposed in this paper. It consists of a new online parameter estimation algorithm, a parameter filtering element, an adaptive battery model, and two PASMOs for SOC and SOH estimation, respectively. The ohm resistance, polarization resistance, polarization capacity, open circuit voltage could be estimated online synchronously. The battery capacity is adopted as the direct indicator of battery SOH. Based on these, the SOC and the SOH (battery capacity) could be obtained through the proposed PASMOs, respectively.

  Compared with the previous researches, the SOC systematic error led from various work environments
- could be effectively reduced. The estimated battery capacity could converge to the actual value, rather than fluctuate around the assumed constant capacity. The validity of the proposed method was verified through the UDDS driving cycle. The results indicated that: 1) parameters estimated online are accurate, 2) the absolute error of SOC is less than 2%, 3) the estimated lithium-ion battery capacity could converge to the actual value in 2500s and the estimated battery capacity error rate was less than 3%.

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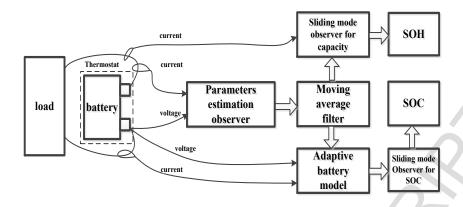


Figure 1 Schematic of the parameter adaptive sliding mode SOC and SOH estimation method

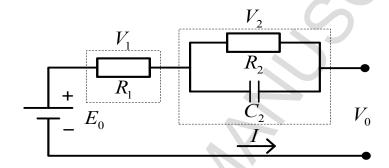


Figure 2 First order RC equivalent circuit battery model

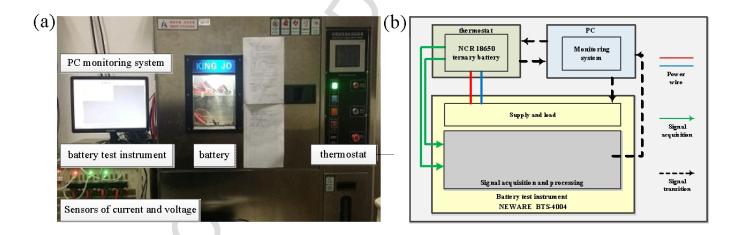


Figure 3 Experimental platform: (a) Experimental instruments; (b) Experimental flow chart.

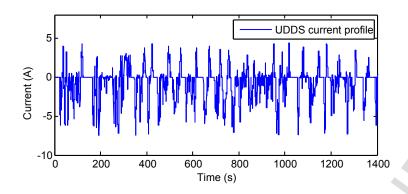
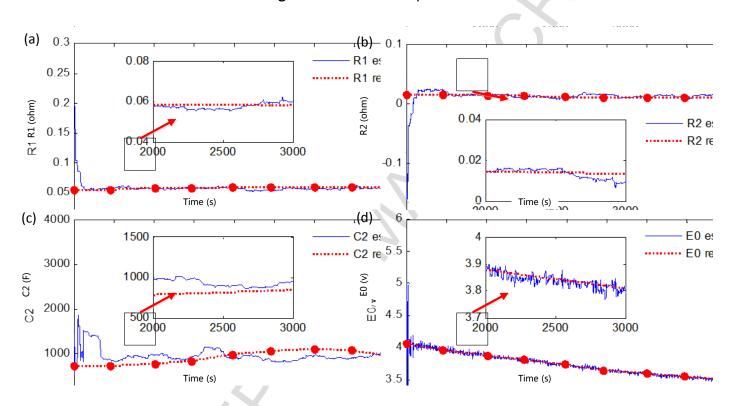


Figure 4 UDDS current profile



**Figure 5** Online parameter estimation: (a) Ohm resistance estimation curve; (b) Polarization resistance estimation curve; (c) Polarization capacitance estimation curve; (d) Open circuit voltage estimation curve

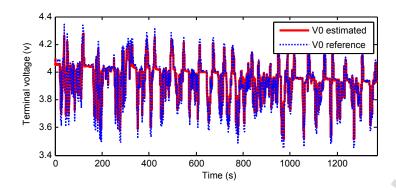
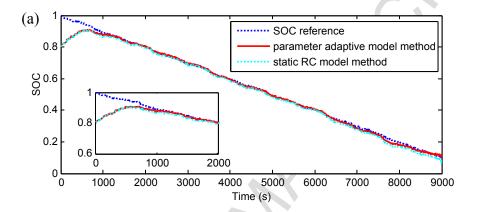
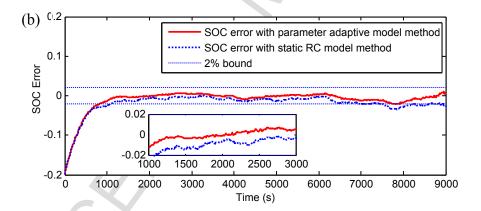
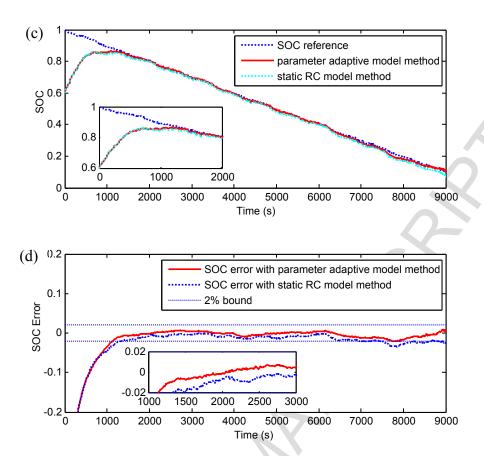


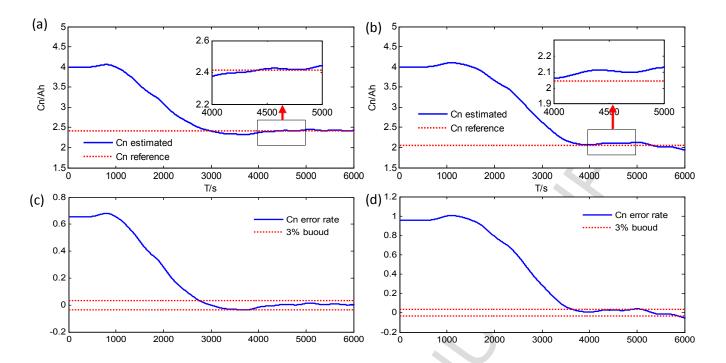
Figure 6 Terminal voltage under UDDS current







**Figure 7** SOC estimation: (a) SOC estimation curve (initial SOC error: 0.2); (b) SOC estimation error curve (initial SOC error: 0.2); (c) SOC estimation curve (initial SOC error: 0.4); (d) SOC estimation error curve (initial SOC error: 0.4).



**Figure 8** Battery actual capacity estimation: (a) 1th cycle actual capacity(2415mAh); (b) 400th cycle actual capacity (2046mAh); (c) 1th cycle actual capacity error(2415mAh); (d) 400th cycle actual capacity error(2046mAh).

**TABLE 1** Relationship between  $E_{\scriptscriptstyle 0}$  and SOC

| Z                                    | 0-0.1   | 0.1-0.2 | 0.2-0.3 | 0.3-0.4 | 0.4-0.5 |
|--------------------------------------|---------|---------|---------|---------|---------|
| $\alpha_{_{ m n}}$                   | 1.011   | 0.663   | 0.406   | 0.434   | 0.608   |
| $oldsymbol{eta}_{	ext{n}}^{	ext{"}}$ | 3.326   | 3.351   | 3.417   | 3.419   | 3.354   |
| Z                                    | 0.5-0.6 | 0.6-0.7 | 0.7-0.8 | 0.8-0.9 | 0.9-1   |
| $\alpha_{\scriptscriptstyle  m n}$   | 0.902   | 0.759   | 0.642   | 0.843   | 0.952   |
| $\beta_{r}^{"}$                      | 3.197   | 3.283   | 3.365   | 3.204   | 3.106   |

 TABLE 2 Initial parameters and coefficients

| parameter  | value |
|--|-------|
| $\hat{	heta}_{1(0)}$   | 0.2   |
| $\hat{	heta}_{2(0)}$   | 0.01  |
| $\hat{	heta}_{3(0)}$   | 0.2   |
| $egin{aligned} \hat{	heta}_{	ext{I}(0)} \ \hat{	heta}_{2(0)} \ \hat{	heta}_{3(0)} \ \hat{	heta}_{4(0)} \ \hat{	heta}_{t(0)} \ \hat{	heta}_{t} \end{aligned}$ | 1     |
| $\hat{V}_{t(0)}$   | 4     |
| λ  | 0.2   |
| $ ho_{	ext{l}}$  | 0.01  |
| $ ho_2$  | 0.002 |
| $ ho_3$  | 0.02  |
| $ ho_4$  | 0.005 |
|  |       |

Table 3 MAE and RMSE of parameters estimated online

|    | MAE      | RMSE    |
|----|----------|---------|
| R1 | -0.00024 | 0.0024  |
| R2 | 0.0045   | 0.0065  |
| C2 | 5.7978   | 188.765 |
| E0 | 0.0014   | 0.0209  |

**TABLE 4** Initial parameters and coefficients

| parameter                       | value |
|---------------------------------|-------|
| $\overline{z_0}$                | 0.8   |
| $V^{}_2$                        | 0.0   |
| $V_{0}$                         | 4.0   |
| $\delta_{_{1}}$                 | 0.2   |
| $\delta_{\scriptscriptstyle 2}$ | 0.001 |
| $\delta_3$                      | 0.005 |

Table 5 Comparisons of MAE and RMSE under dynamic current condition

|      | Parameter adaptive model (%) | Traditional static model (%) |
|------|------------------------------|------------------------------|
| MAE  | -0.0031                      | -0.0133                      |
| RMSE | 0.0071                       | 0.0155                       |

**TABLE 6** Initial parameters and coefficients

| value |
|-------|
| 3.9   |
| 0.005 |
| 500   |
|       |