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ASSIGNMENT-II

M. Polireddy
192323056

6) Big omega notation: prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

Sol $g(n) \geq c \cdot n^3$

$g(n) = n^3 + 2n^2 + 4n$

To find constants and n_0

$n^3 + 2n^2 + 4n \geq cn^3$

Divide both sides with n^3

$$1 + \frac{2n^2}{n^3} + \frac{4n}{n^3} \geq c$$

$$1 + \frac{2}{n} + \frac{4}{n^2} \geq c$$

Here $\frac{2}{n}$ and $\frac{4}{n^2}$ approaches 0

$1 + 2/n + 4/n^2$.

Example $c = 1/2$

$$1 + \frac{2}{n} + \frac{4}{n^2} \geq \frac{1}{2}$$

$1 + 2/n + 4/n^2 \geq 1$ $\quad (1 \geq 1/2, n \geq 1)$

$1 + 2/n + 4/n^2 \geq 1/2$ $\quad (n \geq 1, n_0 = 1)$

Thus, $g(n) = n^3 + 2n^2 + 4n$ is indeeded $\Omega(n^3)$

7) Big theta notation: Determine wheather $h(n) = 4n^2 + 3n$ is $\Theta(n^2)$ or not.

$c_1 n^2 \leq h(n) \leq c_2 n^2$

In upper bound $h(n)$ is $O(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$

Upper bound $f(n) = O(g(n))$:

$h(n) = 4n^2 + 3n$

$h(n) \leq C_2 n^2$

$4n^2 + 3n \leq C_2 n^2$

$4n^2 + 3n \leq 5n^2$

let's $C_2 = 5$

Divide both sides by n^2

$4 + 3/n \leq 5$

$h(n) = 4n^2 + 3n$ is $O(n^2)$ $(C_2 = 5, n_0 = 1)$

lower bound:-

$h(n) = 4n^2 + 3n$

$h(n) \geq C_1 n^2$

$4n^2 + 3n \geq C_1 n^2$

let's $C_1 = 4 \Rightarrow 4n^2 + 3n \geq 4n^2$

pivide both sides by n^2

$4 + 3/n \geq 4$

$h(n) = 4n^2 + 3n$ $(C_1 = 4, n_0 = 1)$

$h(n) = 4n^2 + 3n$ is $\Theta(n^2)$

(3) let's $f(n) = n^3 - 2n^2 + n$ and $g(n) = n^2$ show wheather $f(n) = \Omega(g(n))$ is true or false and justify your answer.

$f(n) \geq C \cdot g(n)$

substituting $f(n)$ and $g(n)$ into this inequality we get

$n^3 - 2n^2 + n \geq C(n^2)$

Find cond no hols $n \geq n_0$

$n^3 - 2n^2 + n \geq -cn^2$

$n^3 - 2n^2 + n + cn^2 \geq 0$

$n^3 + (c-2)n^2 + n \geq 0$

$n^3 + (c-2)n^2 + n \geq 0 \quad (n^3 \geq 0)$

$n^3 + (1-2)n^2 + n = n^3 - n^2 + n \geq 0 \quad (c=2)$

$f(n) = n^3 - 2n^2 + n$ is $\Omega(g(n)) = \Omega(-n^2)$

Therefore the statement $f(n) = \Omega(g(n))$ is true.

9) Determine wheather $h(n) = n\log n$ is in $O(n\log n)$ Prove a Rigorous proof your conclusion.

Sol $c_1 n\log n < h(n) \leq c_2 n\log n$

Upper bound:

$h(n) \leq c_2 n \log n$

$h(n) = n\log n + n$

$n\log n + n \leq c_2 n\log n$

Divide both sides by $n\log n$

$1 + \frac{n}{n\log n} \leq 2$

$1 + \frac{1}{\log n} \leq 2$

$1 + \frac{1}{\log n} \leq 2$ (simplify)

Then $h(n)$ is $\Theta(n \log n)$ $(c_1 = 2, n_0 = 2)$

lower bound:

$h(n) \geq c_1 n \log n$

$h(n) = n \log n + n$

$n \log n + n \geq c_1 n \log n$

divide both sides by $n \log n$

$1 + \frac{n}{n \log n} \geq c_1$

$1 + \frac{1}{\log n} \geq c_1$ (simplify)

$1 + \frac{1}{\log n} \geq 1 \quad (c_1 = 1)$

$\frac{1}{\log n} \geq 0$ (true) for all $n \geq 1$

$h(n)$ is $\Omega(n \log n)$ $(c_1 = 1, n_0 = 1)$

$h(n) = n \log n + n$ is $\Theta(n \log n)$

10. Solve the following recurrence relations and find the order of growth for solutions. $T(n) = 4T(n/2) + n^2, T(1) = 1$

$T(n) = 4T(n/2) + n^2, T(1) = 1$

$T(n) = aT(n/b) + f(n)$

$a = 4, b = 2, f(n) = n^2$

Applying master theorem

$T(n) = aT\left(\frac{n}{b}\right) + f(n)$ $\left(\begin{matrix} \epsilon > 0 \\ T(n) = \Theta(n^{\log_b a}) \end{matrix}\right)$

$f(n) = O(n^{\log_b a - \epsilon})$

$f(n) = \Theta(n^{\log_b a})$ then $T(n) = \Theta(n^{\log_b a} \log n)$

$f(n) = \Omega(n^{\log_b a + \epsilon})$, then $T(n) = f(n)$

calculating $\log_b a$:

$\log_b a = \log_2 4 = 2$

$f(n) = n^2 = \Theta(n^2)$

$f(n) = \Theta(n^2) = \Theta(n^{\log_b a})$

$T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$T(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^2 \log n)$

order of growth

$T(n) = 4T\left(\frac{n}{2}\right) + n^2$ with $T(1) = 1$ is $\Theta(n^2 \log n)$