ASSIGNMENT - I

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1.
a) Solve ~~the~~ the following recurrence relations

(a) $x(n) = x(n-1)+5$ for $n>1$ with $x(1)=0$

1) write decon the first two items to identify the pattern

$x(1)=0$

$x(2)=x(1)+5=5$

$x(3)=x(2)+8=10$

$x(4)=x(3)+5=15$

2) identify the pattern (or) the general term

-> The first term $x(1)=0$

The common difference $d=5$

The genaral formola for the n^{th} term of an AP is

$x(n)=x(1)+(n-1)d$

Substitoting the given values

$x(n)=0+(n-1).5=5(n-1)$

The solution is

$x(n)=5(n-1)$

(b) $x(n)=3x(n-1)$ for $n>1$ with $x(1)=4$

1) write down the first two terms to identify the pattern

$x(1)=4$

$x(2)=3x(1)=3.4=12$

$x(3)=3x(2)=36$

$x(4)=3x(3)=108$

2) Identify the general term

-> The first term $x(1)=4$

-> The common ration $r=3$

The G.F for the n^{th} term of a g p is $x(n)=x(1).r^{n-1}$

substituting the given values

$x(n) = 4 \cdot 3^{n-1}$

The solution is

$x(n) = 4 \cdot 3^{n-1}$

c) $x(n) = x(n/2) + n$ for $n > 1$ with $x(1) = 1$ (solve for $n = 2^k$)

for $n = 2^k$, we can write recurrence in term of k.

1) substitute $n = 2^k$ in the recurrence

$x(2^k) = x(2^{k-1}) + 2^k$

2) write down the first few terms to identify the pattern

$x(1) = 1$

$x(2) = x(2^1) = x(1) + 2 = 1 + 2 = 3$

$x(4) = x(2^2) = x(2) + 4 = 3 + 4 = 7$

$x(8) = x(2^3) = x(4) + 8 = 7 + 8 = 15$

3) Identify the general term by finding the pattern we observe that :-

$x(2^k) = x(2^{k-1}) + 2^k$

we sum the series:

$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + \dots$

Since $x(1) = 1$:

$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + \dots$ (additional +1 term)

The sum of a geometric series with ratio $r = 2$ is given by

$$S = a\frac{r^n - 1}{r - 1}$$

Here $a = 2$, $r = 2$ and $n = k$

$$S = 2\frac{2^k - 1}{2 - 1} = 2(2^k - 1) = 2^{k+1}$$

Adding the +1 term

$x(2^k) = 2^{k+1} - 2 + 1 = 2^{k+1} - 1$

Solution is

$x(2^k) = 2^{k+1} - 1$

(d) $x(n) = x(n/3) + 1$ for $n > 1$ with $x(1) = 1$ (solve for $n = 3^k$)

for $n = 3^k$, we can write recurrence in terms of k

1) substitute $n = 3^k$ in the recurrence

$$x(3^k) = x(3^{k-1}) + 1$$

2) write down the first few terms to identify the pattern

$x(1) = 1$

$x(3) = x(3^1) = x(1) + 1 = 1 + 1 = 2$

$x(9) = x(3^2) = x(3) + 1 = 2 + 1 = 3$

$x(27) = x(3^3) = x(9) + 1 = 3 + 1 = 4$

3) identify the general term:

we observe that:

$$x(3^k) = x(3^{k-1}) + 1$$

Summing up the series

$x(3^k) = 1 + 1 + 1 + \dots + 1$

$x(3^k) = k + 1$

The solution is

$x(3^k) = k + 1$

② Evaluate the following recurrences complexity.

(i) $T(n) = T(n/2) + 1$, wherere $n = 2k$ for all $k \geq 0$

The recorrence relation can be solved using iteration method

① substitute $n = 2^k$ in the recoirence

2) Iterate the recorrence

for $k=0$ $T(2^0) = T(1) = T(1)$

$k=1 : T(2^1) = T(1)+1 =$

$k=2 : T(2^2) = T(8) = T(n)+1 = (T(1)+2)+1 = T(1)+2$

$k=3 : T(2^3) = T(8) = T(n)+1 = (T(1)+2)+1 = T(1)+3$

3) generalize the pattern

$T(2^k) = T(1)+k$

since $n = 2^k$, $k = \log_2 n$

$T(n) = T(2^k) = T(1) + \log_2 n$

4) Assume $T(1)$ is a constant c

$T(n) = c + \log_2 n$

The solution is

$T(n) = O(\log n)$

(ii) $T(n) = T(n/3) + T(2n/3) + cn$. where c is constant and n is input size

$T(n) = aT(n/2) + f(n)$

where $a=2$, $b=3$ and $f(n) = cn$

lets determine the value of $\log_b^a$

$\log_b^a = \log^2$

(b) set up a recorrence relation for the algorithm basic operation count and solve it

The solution is

$T(n) = n$

This means the algorithm performs 'n' basic operations for an input array of size n

(c) f

(4) Analize the order of Path.

(i) $f(n) = 2n^2+5$ and $g(n) = 7n$ use the $\Omega(g(n))$ notation

To analize the order of growth and use the Ω notation we need to compare the given function f(n) and g(n) order given functions:

$$f(n) = 2n^2+5$$

$$g(n) = 7n$$

Order of growth using $\Omega(g(n))$ Notation;

The notation $\Omega(g(n))$ describes a lower bound on the growth rate that for sufficiently large n, f(n), grows atleast as fast as g(n)

lets analyse $f(n) = 2n^2+5$ with respect to $g(n) = 7n$

1) Identify Dominant terms:

-> The dominant terms in f(n) is $2n^2$ since it grow's faster then the constant terms as n increases.

-> The dominant term in g(n) is 7n.

using the propertis of logarithms

$$\log_3 2 = \frac{\log 2}{\log 3}$$

Now we compare $f(n) = cn$ with $n^{\log_3 2}$

$f(n) = O(n)$

$n = n^1$

since $\log_3 2$ are in the third case of the master's theorem

$f(n) = O(n^c)$ with $c > \log_b a$

The solution is

$T(n) = O(f(n)) = O(cn = O(n)$

(8) consider the following recorrence algorithm:

min [A(0.....n-2)]

if n=1 return A[0]

else temp = min(A[0...n-2])

if temp <= A(n-1) return temP else

Return A[n-1]

(a) What does the algorithm compute?

The given algorithm, min [A(0....n+3) computes the min values in the array A, from index 0' for (n-1' it does this by recorrentively finding the min value in the sub array A[0...n-2] and the comparing it with the last element A[n-1] to determine the overal max value....

(b) set up a recurrence relation for the algorithm basic operation count and solve it

The solution is

$T(n) = n$

This means the algorithm performs 'n' basic operations for an input array of size n

(c) f

(4) Analyze the Order of Path.

(i) $f(n) = 2n^2+5$ and $g(n) = 7n$ use the $\Omega(g(n))$ notation

To analyze the order of growth and use the Ω notation we need to compare the given function $f(n)$ and $g(n)$

order given functions:

$f(n) = 2n^2+5$

$g(n) = 7n$

Order of growth using $\Omega(g(n))$ Notation;

The notation $\Omega(g(n))$ describes a lower bound on the growth rate that for sufficiently large n, $f(n)$, grows atleast as fas as $g(n)$

lets analyse $f(n) = 2n^2+5$ with respect to $g(n) = 7n$

i) Identify Dominant terms:

-> The dominant terms in $f(n)$ is $2n^2$ since it grow's faster then the constant terms as n increases.

-> The dominant term in $g(n)$ is $7n$.

2) establish the inequality

-> we want to find contants c and n_0 such that:

$$2n^2+5 \geq c \cdot 7n \cdot \text{ for all } n \geq n_0$$

3) simplify the inequality,

-> ignore the lower order term 5 for larger

$$2n^2 \geq 7cn$$

-> Divide both sides n:

$$2n \geq 7c$$

-> solve for n:

$$n > 7/2$$

4) Choose constants

let $c = 1$

$$n \geq 7 \cdot \frac{1}{2} = 3.5$$

∴ for $n \geq n_r$ the inequality hold:

$$2n^2+5 \geq 7n \cdot \text{ for all } n \geq n$$

we have shown that there exists contants $c = 1$ and $n_0 = n$

such that for all $n \geq n_0 = 2n^2+5 \geq 7n$

Thus, we can conclude that:

$$f(n) = 2n^2+5 = \Omega(7n)$$

in Ω notation, the domains term $2n^2$ in $f(n)$ clearly

grows faster that $7n$ Hence

$f(n) = \Omega(n)$

However for the specific comparision asked $f(n) = \Omega(7n)$ is also correct

showing that $f(n)$ grows at least as fast as $7n$.