Révision

* Crédibilité de stabilité : tien compte de l'expérience individuelle si et seulement si elle est stable dans le temps

↳ 2 type : Partiel et Complète

L'idée : est ce que l'exprience est utile?

↳ On cherche le nombre minimum pour dire que c'est stable

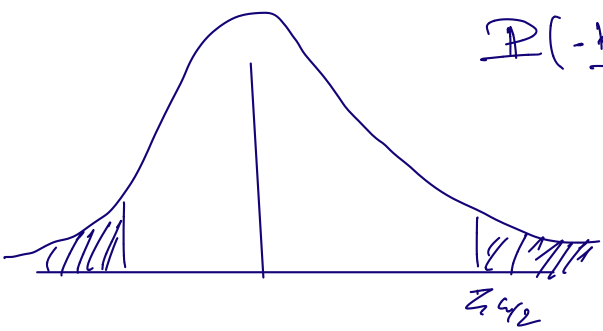
Complète : l'exprience du contrat, en fonction de la taille (+ c'est mieux)

Partiel : fait attention à l'exprience individuelle

Crédibilité :

$$\mathbb{P}\left((1-k)\mathbb{E}[S] \leq S \leq (1+k)\mathbb{E}[S] \right) \geq p$$

$$\mathbb{P}\left(-\frac{k\mathbb{E}[S]}{\sigma_S} \leq Z(0,1) \leq \frac{k\mathbb{E}[S]}{\sigma_S} \right) \geq p$$



$$\mathbb{E}[S] \geq \left(\frac{z_{\alpha/2}}{k}\right) \sqrt{Var(S)}$$

. la taille est liée à la fréquence est non la sévérité

$$\mathbb{E}[S] \geq \left(\frac{z_{\alpha/2}}{k}\right) \sqrt{Var(S)}$$

Si loi Composée :

$$S = \sum_{i=1}^{N} X_i \qquad N \sim Poi(\lambda) \quad X_i \sim f_X(x)$$

$$\mathbb{E}[S] = \lambda \mathbb{E}[X] \qquad \mathbb{E}[Var(X|N)] + Var(\mathbb{E}[X|N])$$

$$Var(S) = \lambda \mathbb{E}[X^2] \qquad Var(X)\mathbb{E}[N] + \mathbb{E}^2[X]Var(N)$$

$$\mathbb{E}[S] \geq \left(\frac{z_{\varepsilon/2}}{k}\right)\sqrt{Var(S)} \Rightarrow \lambda \mathbb{E}[X] \geq \left(\frac{z_{\varepsilon/2}}{k}\right)\sqrt{\lambda \mathbb{E}[X^2]}$$

$$\lambda \mathbb{E}[X]^2 \geq \left(\frac{z_{\varepsilon/2}}{k}\right)^2 \mathbb{E}[X^2] \rightarrow \lambda \geq \left(\frac{z_{\varepsilon/2}}{k}\right)^2 \frac{\mathbb{E}[X^2]}{\mathbb{E}^2(X)}$$

$$\lambda \geq \left(\frac{z_{\varepsilon/2}}{k}\right)^2 \left(\frac{Var(S) + \mathbb{E}^2(X)}{\mathbb{E}^2(X)}\right) \rightarrow \lambda \geq \left(\frac{z_{\varepsilon/2}}{k}\right)^2 \left(1 + CV(X)^2\right)$$

$$CV(X) = \frac{\sigma_X}{\mu_X}$$

Si $\mathbb{P}(X = M) = 1 \rightarrow Var(X) = 0 \quad \rightarrow \lambda \geq \left(\frac{z_{\varepsilon/2}}{k}\right)^2 (1+0)$

$Var(X) = M^2 \cdot 1 - (M \cdot 1)^2 = 0$

le tout en haut pour une période

* En fonction de nombre d'année d'experience :

$$W = \frac{1}{n}\sum_{i=1}^{n} S_i \qquad \mathbb{E}[W] = \mathbb{E}[S]$$

$$Var(W) = \frac{Var(S)}{n}$$

$$\mathbb{E}[S] \geq \left(\frac{z_{\varepsilon/2}}{k}\right)\sqrt{\frac{Var(S)}{n}} \qquad n \geq \left(\frac{z_{\varepsilon/2}}{k}\right)^2 \frac{Var(S)}{\mathbb{E}^2[S]}$$

on est interesser par # années d'experience

* Crédibilité partielle — m'importe l'expérience individuelle.

$$\pi = Z S + (1-Z) m$$

$$Z = \min\left\{ \sqrt{\frac{n}{n_0}}, 1 \right\}$$ n_0 : crédibilité complète

$$Z = \min\left\{ \left(\frac{n}{n_0}\right)^{2/3}, 1 \right\}$$

2 : #5

$N \sim Poi(256)$ $X \sim Pareto(3, 0.05)$ $S = \sum_{i=1}^{N} X_i$

a) on cherche le k.

$$\mathbb{E}[S] \geq \left(\frac{Z_{\alpha/2}}{k}\right) \sqrt{Var(S)}$$

$Var(X) = 0.001875$

$\mathbb{E}[X] = 0.025$

$$\lambda^2 \mathbb{E}[X]^2 \geq \left(\frac{Z_{\alpha/2}}{k}\right)^2 \left(Var(X).\lambda + \mathbb{E}^2[X]\lambda \right)$$

$$\lambda \cancel{\mathbb{E}[X]^2} \geq \left(\frac{Z_{\alpha/2}}{k}\right)^2 \left(1 + \frac{Var(X)}{\mathbb{E}(X)^2} \right)$$

$$k \geq \sqrt{\frac{(Z_{\alpha/2})^2}{\lambda}(1+3)} = 0.205625$$

b) $n = 10$

$$W = \sum_{i=1}^{10} \frac{S_i}{10}$$

$$\mathbb{E}[W]^2 \geq \left(\frac{Z_{\alpha/2}}{k}\right)^2 \frac{\lambda \mathbb{E}[X^2]}{10}$$

$$\lambda \geq \frac{(1.645)^2}{k^2} \frac{\mathbb{E}[X^2]}{10\,\mathbb{E}(X)^2}$$

$$K \geq \sqrt{\frac{(1.645)^2}{10.256} . (1+3)} = 0.0650$$

2: #8

$N \sim BinNeg(r, \theta = 0.01)$ $\qquad$ $X_i \sim Gamma(0.02, \lambda = 1)$

$0.95\mathbb{E}[S] \quad \mathbb{E}[S] \rightarrow 1.05\mathbb{E}[S]$ $\qquad$ $K = 0.05$

$$\mathbb{P}(0.95\mathbb{E}[S] \le S \le 1.05\mathbb{E}[S]) \ge 19/20 = 0.95$$

$$S = \sum_{i=1}^{N} X_i \qquad \mathbb{E}[S] = \mathbb{E}[N]\mathbb{E}[X] = r \cdot \frac{1-0.01}{0.01} \times 0.02 = 1.98r$$

$$Var(S) = \mathbb{E}[Var(\Sigma X \mid N)] + Var(\mathbb{E}[\Sigma X \mid N])$$

$$= Var(X)\mathbb{E}[N] + \mathbb{E}^2[X]Var(N)$$

$$= 0.02 \cdot r \frac{1-0.01}{0.01} + (0.02)^2 \cdot r \frac{1-0.01}{0.01^2}$$

$$= 1.98r + 3.96r = 5.94r$$

$$1.98r \ge \left(\frac{1.96}{0.05}\right)\sqrt{5.94r} \rightarrow (1.98)^2 r \ge \left(\frac{1.96}{0.05}\right)^2 5.94$$

$$r \ge 2328.24$$

2: #11

A: 50 \$ $\qquad$ $\mathbb{P}(X = 50) = 1$

B: $X \sim Gamma$ $\quad \frac{\alpha}{\lambda} = 50$

A: $\lambda \ge 1000$

B: $\lambda \ge 3000$

$$\lambda \ge \left(\frac{z_{\alpha/2}}{k}\right)^2 \left(1 + \overset{=0}{CV(X)^2}\right) = 1000$$

$$\left(\frac{z_{\alpha/2}}{k}\right)^2 = 1000 \quad ✓$$

$$\lambda \ge \left(\frac{z_{\alpha/2}}{k}\right)^2 \left(1 + CV(X)^2\right) = 3000$$

$$\left(1 + \frac{\alpha/\beta^2}{(50)^2}\right) = 3 \rightarrow \left(1 + \frac{\cancel{50}/\beta}{\cancel{50^2}} = 3\right)$$

$$\alpha = 50\beta$$

$$\frac{1}{50\beta} = 2 \rightarrow \beta = 0.01 \rightarrow X \sim Gamma(1/2, 0.01)$$

2 : #13

fréquence = 6494 $\qquad q = 0.04 \qquad CV(X) = 2$

$$n q \geq \left(\frac{Z_{\alpha/2}}{k}\right)^2 nq(1-q) \qquad n \geq \left(\frac{Z_{\alpha/2}}{k}\right)^2 \frac{1-q}{q} = 6494$$

$$(nq\,\mathbb{E}(X))^2 \geq \left(\frac{6494}{24}\right)\left(\mathbb{E}(N)\,\mathrm{Var}(X) + \mathbb{E}^2(X)\,\mathrm{Var}(N)\right) \qquad \left(\frac{Z_{\alpha/2}}{k}\right)^2 = \frac{6494}{24}$$

$$(nq\,\mathbb{E}(X))^2 \geq \left(\frac{6494}{24}\right)\left(nq\,\mathrm{Var}(X) + \mathbb{E}^2(X)\,n(1-q)q\right)$$

$$CV(X)^2 = \frac{\sigma^2}{\mathbb{E}(X)^2}$$

$$nq \geq \left(\frac{6494}{24}\right)\left(\frac{\mathrm{Var}(X)}{\mathbb{E}(X)^2} + \frac{\mathbb{E}^2(X)\,(1-q)}{\mathbb{E}^2(X)}\right)$$

$$2\,\mathbb{E}(X)^2 = \sigma^2$$

$$n \geq \left(\frac{6494}{24}\right)\left(4 + (1-q)\right)/q \qquad n \geq 33482.$$

2 : #16

a) $$\lambda \geq \left(\frac{1.645}{0.05}\right)^2 = 1082.41$$

b) $$\lambda \geq \left(\frac{1.96}{0.04}\right)^2\left(1 + \frac{1/4}{1/4}\right) = 4802$$

c) $$\min\left(\sqrt{\frac{\lambda}{4802}},\, 1\right) = Z$$

#17

$$\lambda \geq \left(\frac{Z_{\alpha/2}}{k}\right)^2 = 3000$$

$Z = 0.5$ $$\lambda \geq 3000\,(1 + 3) = 12000$$

$$0.5 = \sqrt{\frac{\lambda}{12000}} \rightarrow (\lambda = 3000)$$

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- $N \sim Poi$
- $X \sim LN \quad CV(X) = 3$
- $n = 1 \quad \lambda \geq 1000$
- $S_1 = 6.75$ M\$
- $m = 8$ M\$
- 5% de moyen $\quad k = 5\% \quad p = 0.95$

$$\lambda \geq 15366.4 \qquad Z = \min\left(\sqrt{\frac{n}{n_0}}, 1\right) = \min\left(\sqrt{\frac{1000}{15366.4}}, 1\right)$$

$$\pi_1 = Z\bar{S}_1 + m(1-Z) = 5.45 \text{ M\$} ✓$$

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$$\lambda = 0.035 \qquad n \geq 103500$$

$$Z = 0.67 = \sqrt{\frac{n}{103500}} \longrightarrow n = 46461 ✓$$

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n pour crédi- complète

$k = 0.05$
$p = 0.9$

$$n_2 \geq \left(\frac{1.645}{0.05}\right)^2 0.125$$

$$n_2 \geq \left(\frac{1.645}{0.1}\right)^2 0.125 = \longrightarrow n = 9.86 ✓$$

Chapitre 3

Porte feuille de I contrat :

Θ \ S_{ij}			
1	S_{11}	....	S_{1n}
2	S_{21}	....	S_{2n}
⋮	⋮	⋱	⋮
I	S_{I1}	....	S_{In}

Θ_i (inconnu) : niveau de risque

1. $S \mid \Theta_i \sim f(x \mid \Theta_i)$
2. $\Theta_1, \dots \Theta_I$: identiquement distribuer
3. Contrat $\perp\!\!\!\perp$

* iid seulement lorsque on connais Θ_i sinon Dépendants

* voir graph (...)

* meilleur prévision : $\mu(\Theta_i) = \mathbb{E}[S_{it} \mid \Theta_i = \theta_i] = \int_0^{\infty} x \, f(x \mid \theta_i) \, dx$

* prime Collective : $m = \mathbb{E}[\mu(\Theta_i)] = \int_{-\infty}^{\infty} \mu(\theta) \, u(\theta) \, d\theta$

$$m = \mathbb{E}[\mu(\Theta_i)] = \mathbb{E}[\mathbb{E}[S_{it} \mid \Theta_i]] = \mathbb{E}[S_{it}]$$

meilleur prevision : $\mathbb{E}[(\mu(\Theta_i) - g(X_{i1}, \dots X_{in}))^2]$

$$g(X_{i1}, \dots X_{in}) = \mathbb{E}[\mu(\Theta_i) \mid X_{i1}, \dots, X_{in}]$$

Prime Bayésienne : $B_{i,n+1} = \mathbb{E}[\mu(\Theta_i) \mid S_{i1} = x_{i1}, \dots, S_{in} = x_{in}]$

$$B_{i,n+1} = \mathbb{E}[\mu(\Theta_i) \mid S_{i1} = x_{i1}, \dots, x_{in}] = \int_{-\infty}^{\infty} \mu(\theta) \, u(\theta \mid x_1 \dots x_n) \, d\theta$$

$$\mu(\theta) = \mathbb{E}[S_{it} \mid \theta] \qquad = \int_{-\infty}^{\infty} \mathbb{E}[S_{it} \mid \theta] \, u(\theta \mid x_1 \dots x_n) \, d\theta$$

- distribution à postériori : $\pi(\theta \mid x_1, \dots, x_n)$

$$\pi(\theta \mid x_1, \dots, x_n) = \frac{\pi(x_1, \dots, x_n \mid \theta)\, \pi(\theta)}{\int_\theta \pi(x_1 \dots x_n \mid \theta)\, \pi(\theta)\, d\theta}$$

$$m = \int_{-\infty}^{\infty} \mu(\theta)\, \pi(\theta)\, d\theta$$

$$B_{i,n+1} = \int_{-\infty}^{\infty} \mu(\theta)\, \pi(\theta \mid x_{i1}, \dots, x_{in})\, d\theta$$

* distribution prédictive

$$B_{i,n+1} = \mathbb{E}[\mu(\Theta) \mid S_{i1}, \dots, S_{in}] = \mathbb{E}[S_{i,n+1} \mid S_{i1}, \dots, S_{in}]$$

$$f(x \mid x_1, \dots, x_n) = \int_{-\infty}^{\infty} f(x \mid \theta)\, \pi(\theta \mid x_1, \dots, x_n)\, d\theta$$

$$f(x \mid x_1, \dots, x_n) = \frac{f(x, x_1, \dots, x_n)}{f(x_1, \dots, x_n)} = \frac{\int_{-\infty}^{\infty} f(x \mid \theta)\, f(x_1, \dots, x_n \mid \theta)\, \pi(\theta)\, d\theta}{\underbrace{\int_{-\infty}^{\infty} f(x_1, \dots, x_n \mid \theta)\, \pi(\theta)\, d\theta}_{\text{(constante)}}}$$

$$f(x \mid x_1, \dots, x_n) = \int_{-\infty}^{\infty} f(x \mid \theta)\, \pi(\theta \mid x_1, \dots, x_n)\, d\theta$$

$$f(x) = \int_{-\infty}^{\infty} f(x \mid \theta)\, \pi(\theta)\, d\theta$$

$$B_{i,n+1} = \int_{-\infty}^{\infty} x\, f(x \mid x_1, \dots, x_n)\, dx$$

** **

Si distribution à postériori Θ est de m̂ type que à priorie

↳ Alors la distribution prédictive est m̂ que marginale

* Distribution qui donnent une prime linéaire:

- $S|\theta \sim Poi(\theta)$ $\quad \theta \sim Gamma(\alpha, \lambda)$; $\rightarrow$ $S|\theta \sim \sum Poi(\theta)$
- $S|\theta \sim Exp(\theta)$ $\quad \theta \sim Gamma(\alpha, \lambda)$; $\rightarrow$ $S|\theta \sim Gamma(n, \theta)$, $\theta \sim Gamma(\alpha, \lambda)$
- $S|\theta \sim N(\theta, \sigma_2^2)$ $\quad \theta \sim N(\mu_1, \sigma_1^2)$
- $S|\theta \sim Ber(\theta)$, $\theta \sim Beta(a,b)$ $\rightarrow$ $S|\theta \sim Bin(n,\theta)$ $\quad \theta \sim Beta(a,b)$
- $S|\theta \sim Geo(\theta)$, $\theta \sim Beta(a,b)$ $\rightarrow$ $S|\theta \sim BinNeg(r,\theta)$ $\quad \theta \sim Beta(a,b)$

* Démo Exp - Gamma

$$f(x|\theta) = \theta e^{-\theta x} \qquad f(\theta) = \frac{\theta^{\alpha-1} e^{-\theta\beta} \beta^{\alpha}}{\Gamma(\alpha)}$$

$$u(\theta | x_1, \dots, x_n) = \frac{u(x_1, \dots, x_n | \theta)\, u(\theta)}{\int u(x_1 \dots | \theta)\, u(\theta)\, d\theta} \qquad \mu(\theta) = \left(\frac{1}{\theta}\right)$$

$$\propto \frac{\theta^{n} e^{-\sum x_i \theta}\, \theta^{\alpha-1} e^{-\theta\beta} \cancel{\beta^{\alpha}}}{\cancel{\Gamma(\alpha)}}$$

$$\propto \theta^{n+\alpha-1} e^{-\theta(\sum x_i + \beta)}$$

$$u(\theta | x_1, \dots, x_n) \sim Gamma(n + \alpha, \sum x_i + \beta)$$

$$f(x) = \int \theta e^{-\theta x} \frac{\theta^{\alpha-1} e^{-\theta\beta} \beta^{\alpha}}{\Gamma(\alpha)} d\theta = \int \frac{\theta^{\alpha+1-1} e^{-\theta(x+\beta)} \beta^{\alpha}}{\Gamma(\alpha)} d\theta$$

$$f(x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \frac{\Gamma(\alpha+1)}{(x+\beta)^{\alpha+1}} \times 1 = \frac{\alpha \beta^{\alpha}}{(x+\beta)^{\alpha+1}}$$

$$f(x \mid x_1 \ldots x_n) \sim \text{Pareto}\left(n+\alpha, \sum x_i + \beta\right)$$

$$P_{i,n+1} = \mathbb{E}(S_{i,n+1} \mid X_1, \ldots, X_n) = \frac{\sum x_i + \beta}{n+\alpha-1}$$

$$B_{i,n+1} = \mathbb{E}[\mu(\theta) \mid x_1 \ldots x_n] = \int \mu(\theta)\, u(\theta \mid x_1 \ldots x_n)\, d\theta$$

$$= \int \frac{1}{\theta} \frac{\theta^{n+\alpha-1} e^{\theta(\sum x_i + \beta)} \left(\sum x_i + \beta\right)^{n+\alpha}}{\Gamma(\alpha+n)}$$

$$= \frac{\sum x_i + \beta}{(\alpha+n-1)}$$

m̂ chose

$*$ Si $u(\theta \mid x_1, \ldots, x_n)$ appartient à la m̂ famille que $u(\theta)$, on dit que $u(\theta)$ et $f(x \mid \theta)$ sont conjuguées naturelles

Chapitre 4

$$m = \mathbb{E}[\mu(\Theta_i)]$$

$$s^2 = \mathbb{E}[\mathrm{Var}(S|\Theta_i)]$$

$$a = \mathrm{Var}(\mathbb{E}[S|\Theta_i])$$

$$\mathrm{Cov}(X,Y) = \mathbb{E}[(X-\mathbb{E}[X])(Y-\mathbb{E}[Y])]$$

$$= \mathbb{E}[(X-\mathbb{E}[X|\Theta]+\mathbb{E}[X|\theta]-\mathbb{E}[X])(Y-\mathbb{E}[Y|\theta]+\mathbb{E}[Y|\theta]-\mathbb{E}[Y])]$$

$$= \mathbb{E}[(X-\mathbb{E}[X|\theta])(Y-\mathbb{E}[Y|\theta])] + \mathbb{E}[(X-\mathbb{E}[X|\theta])(\mathbb{E}[Y|\theta]-\mathbb{E}[Y])]$$

$$+ \mathbb{E}[(\mathbb{E}[X|\theta]-\mathbb{E}[X])(Y-\mathbb{E}[Y|\theta])] + \mathbb{E}[(\mathbb{E}[X|\theta]-\mathbb{E}[X])(\mathbb{E}[Y|\theta]-\mathbb{E}[Y])]$$

$$= \mathbb{E}[\mathrm{Cov}(X,Y|\Theta)] + 0 + 0 + \mathrm{Cov}(\mathbb{E}[X|\theta], \mathbb{E}[Y|\theta])$$

$$\mathrm{Cov}(S_k,S_u) = \mathbb{E}[\mathrm{Cov}(S_k,S_u|\Theta_i)] + \mathrm{Cov}(\mathbb{E}[S_k|\Theta_i], \mathbb{E}[S_u|\Theta_i])$$

$$\mathrm{Cov}(S_k,S_u) = \overbrace{\mathbb{E}[\sigma^2(\Theta_i)]}^{= s^2}\mathbb{1}_{\{k=u\}} + \overbrace{\mathrm{Var}(\mu(\Theta_i))}^{= a}$$

$$\cdot\ \mathrm{Cov}(S_k,S_u) = a + s^2\,\mathbb{1}_{\{k=u\}}$$

$$\begin{cases} a = \mathrm{Var}(\mathbb{E}[S_t|\Theta_i]) \\ s^2 = \mathbb{E}[\mathrm{Var}(S_t|\Theta_i)] \end{cases}$$

$$\mathrm{Cov}(\mu(\Theta_i), S_u) = \mathrm{Cov}(\mathbb{E}[\mu(\Theta_i)|\Theta_i], \mathbb{E}[S_u|\Theta_i]) + \mathbb{E}[\mathrm{Cov}(\mathbb{E}[S|\Theta], S_u|\Theta_i)]$$

$$\mathrm{Cov}(\mu(\Theta_i), S_u) = \mathrm{Cov}(\mu(\Theta_i), \mu(\Theta_i)) + \mathbb{E}[0]$$

$$\mathrm{Cov}(\mu(\Theta_i), S_u) = a = \mathrm{Var}(\mu(\Theta_i))$$

. les contrats (Θ_i, S_i) sont $\perp\!\!\!\perp$, Θ_i sont identiquement distribuer

. $$\pi^B_{i,n+1} = z\bar{S}_i + (1-z)m$$

$$z = \frac{n}{n+K}, \quad K = \frac{s^2}{a}$$

$$\mathbb{E}\left[\left(\mu(\Theta_i) - c_0 - \sum_{t=1}^{n} c_t S_{it}\right)^2\right]$$

c_0:

$$\frac{\partial}{\partial c_0}(\dots) = 0 \longrightarrow c_0 = \mathbb{E}[\mu(\Theta_i)] - \sum_{t=1}^{n} c_t \mathbb{E}[S_{it}]$$

c_u:

$$\frac{\partial}{\partial c_u} f(\dots) = 0$$

$$\mathbb{E}\left[\left(\mu(\Theta_i) - \mathbb{E}[\mu(\Theta_i)] + \sum_{t=1}^{n} c_t \mathbb{E}[S_t] - \sum_{t=1}^{n} c_t S_t\right)(\mathbb{E}[S_u] - S_u)\right]$$

$$\mathbb{E}[(\mu(\Theta_i) - \mathbb{E}[\mu(\Theta_i)])(\mathbb{E}[S_u] - S_u)] = -\mathbb{E}\left[\left(\sum c_t \mathbb{E}[S_t] - \sum c_t S_t\right)(\mathbb{E}[S_u] - S_u)\right]$$

$$+\mathbb{E}[(\mu(\Theta_i) - \mathbb{E}[\mu(\Theta_i)])(S_u - \mathbb{E}[S_u])] = +\mathbb{E}\left[\left(\sum c_t S_t - \sum c_t \mathbb{E}[S_t]\right)(S_u - \mathbb{E}[S_u])\right]$$

$$\mathrm{Cov}(\mu(\Theta_i), S_u) = \mathrm{Cov}\left(\sum_{t=1}^{n} c_t S_t, S_u\right)$$

$$\mathbb{E}[\mathrm{Cov}(\mu(\Theta_i), S_u \mid \Theta_i)] + \mathrm{Cov}(\mathbb{E}[\mu(\Theta) \mid \Theta], \mathbb{E}[S_u \mid \Theta]) = \mathrm{Var}(\mu(\Theta_i)) = a$$

$$a = \sum_{t=1}^{n} c_t \, Cov(S_t, S_u)$$

$$a = \sum_{t=1}^{n} c_t \left(a + s^2 \mathbb{1}_{\{u=t\}}\right)$$

$$a = a \sum_{t=1}^{n} c_t + c_u s^2 \qquad c_1 = c_2 = c_3 = \dots = c_n$$

$$a = a n c_u + c_u s^2 \longrightarrow c_u = \frac{a}{an + s^2}$$

$$c_0 = \mathbb{E}[\mu(\Theta_i)] - \sum_{t=1}^{n} c_t \mathbb{E}[S_i]$$

$$c_0 = m - \sum \frac{a}{an+s^2} m = m\left(1 - \frac{na}{an+s^2}\right)$$

$$\pi^B_{i,n+1} = \sum_{t=1}^{n} \frac{a}{an+s^2} S_t + m\left(1 - \frac{n}{n+s^2/a}\right)$$

$$\pi^B_{i,n+1} = \frac{n}{n+s^2/a} \bar{S}_i + m\left(1 - n/_{n+s^2/a}\right)$$

$$\pi^B_{i,n+1} = \frac{n}{n+s^2/a} \bar{S}_i + m\left(1 - \frac{n}{n+\frac{s^2}{a}}\right)$$

$$z = \frac{n}{n+s^2/a} \qquad \mathbb{E}[\pi^B_{i,n+1}] = m$$

Qd $n \to \infty$ $\pi^B_{i,n+1} = \mu(\Theta_i)$ on accorde plus à la prime individuelle

* meilleur approximation de la prime de Risque $\mu(\Theta_i)$, est aussi meilleur approximation Bayes $B_{i,n+1}$

$$\mathbb{E}\left[\left(\mu(\Theta_i) - c_0 - \sum c_t S_t\right)^2\right] = \mathbb{E}\left[\left(\mu(\Theta_i) - B_{i,n+1}\right)^2\right] + \mathbb{E}\left[\left(B_{i,n+1} - c_0 - \sum c_t S_t\right)^2\right]$$

$$\mathbb{E}\left[\left(\mu(\Theta_i) - \pi^B_{i,n+1}\right)^2\right] \geq \mathbb{E}\left[\left(\mu(\Theta_i) - B_{i,n+1}\right)^2\right]$$

↳ Qd $\pi^B_{i,n+1} = B_{i,n+1} \longrightarrow$ égalité meilleur scénario

↳ la prime de crédibilité ne sera jamais meilleure que la prime Bayes

* Approche paramétrique :

↳ on trouve juste les diff moment puis :

$m = \mathbb{E}[\mu(\Theta)]$

$s^2 = \mathbb{E}[\mathbb{V}ar(S_i|\Theta)]$ $\quad K = \dfrac{s^2}{a}$ $\quad z = \dfrac{n}{n+K}$

$a = \mathbb{V}ar(\mathbb{E}[S_i|\Theta_i])$

* Approche non-Paramétrique

l'idée est de trouver m, a, s^2

Θ_i	S_i			
	S_{11}	S_{12}	...	S_{1n}
	S_{21}	....		S_{2n}
	⋮	⋱		
	S_{I1}	...		S_{In}

$$\hat{m} = \frac{1}{In}\sum_{j=1}^{I}\sum_{t=1}^{n} S_{jt}$$

$\mathbb{E}[\hat{m}] = m \longrightarrow$ sans Biais

$$s^2 = \mathbb{E}\left[\mathbb{V}ar(S|\Theta)\right] \longrightarrow \hat{s}^2 = \frac{1}{I}\frac{1}{n-1}\sum_{j=1}^{I}\sum_{t=1}^{n}(S_{jt} - \bar{S}_j)^2$$

$$\mathbb{E}\left[(S_{jt} - \bar{S}_j)^2\right] = \mathbb{E}\left[\mathbb{E}\left[(S_{jt} - \bar{S}_j)^2 | \Theta_i\right]\right]$$

$$\hookrightarrow \mathbb{E}\left[(S_{jt} - \bar{S}_j)^2 | \Theta_i\right] = \mathbb{V}ar\left(S_{jt} - \bar{S}_j | \Theta\right)$$

$$= \mathbb{V}ar(S_{jt}|\Theta_i) + \mathbb{V}ar(\bar{S}_j|\Theta_i) - 2\,\mathbb{C}ov(S_{jt}, \bar{S}_j | \Theta_i)$$

$$= \mathbb{V}ar(S_{jt}|\Theta_i) + \frac{\mathbb{V}ar(S_{jt}|\Theta)}{n} - \frac{2}{n}\mathbb{V}ar(S_{jt})$$

$$= \mathbb{V}ar(S_{ij}|\Theta_i)\left(\frac{n-1}{n}\right)$$

$$\hookrightarrow \mathbb{E}\left[\mathbb{V}ar(S_{ij}|\Theta_i)\right]\frac{n-1}{n} = s^2\frac{n-1}{n}$$

$$\hookrightarrow \mathbb{E}[\hat{s}^2] = \frac{1}{I(\cancel{n-1})}\sum_{j=1}^{I}\sum_{t=1}^{n} s^2 \frac{\cancel{n-1}}{n} = \frac{\cancel{n}\cancel{I}s^2}{\cancel{n}\cancel{I}} = s^2 \checkmark \text{ sans biais}$$

$$a = \mathbb{V}ar\left(\mathbb{E}[S|\Theta]\right) \longrightarrow \hat{a} = \frac{1}{I-1}\sum_{j=1}^{I}(\bar{S}_j - \bar{S})^2$$

$$\bar{S} = \sum\sum S_{ij} \cdot \frac{1}{In}$$

<u>$\mathbb{E}[\hat{a}]$:</u>

$$\mathbb{E}\left[(\bar{S}_j - \bar{S})^2\right] = \mathbb{V}ar(\bar{S}_j) + \mathbb{V}ar(\bar{S}) - 2\,\mathbb{C}ov(\bar{S}_j, \bar{S})$$

$$\frac{1}{I}\mathbb{C}ov\left(\bar{S}_j, \sum_{I=1}^{I}\bar{S}_j\right) = \frac{1}{I}\mathbb{V}ar(\bar{S}_j)$$

$$\hookrightarrow \mathbb{V}ar(\bar{S}_j) + \frac{1}{I}\mathbb{V}ar(\bar{S}_j) - \frac{2}{I}\mathbb{V}ar(\bar{S}_j) = \mathbb{V}ar(\bar{S}_j)\left(\frac{I-1}{I}\right)$$

$$\mathbb{E}[\hat{a}] = \mathbb{V}ar(\bar{S}_j) = \mathbb{E}\left[\mathbb{V}ar(\bar{S}|\Theta_i)\right] + \mathbb{V}ar\left(\mathbb{E}[\bar{S}|\Theta_i]\right) =$$

$$\mathbb{E}[\hat{a}] = \frac{s^2}{n} + a$$

donc : $\hat{a} = \frac{1}{I-1} \sum_{j=1}^{I} (\bar{S}_j - \bar{S}) - \frac{s^2}{n}$ ↙ pour enlever le Biais

$$\pi^B_{i,n+1} = \hat{z}\,\bar{S}_i + \hat{m}\,(1-\hat{z}) \quad , \quad \hat{z} = \frac{n}{n + \frac{\hat{s}^2}{\hat{a}}}$$

- Qd $n \to \infty \to$ on accorde $\mu(\Theta_i)$ . . . . .
- s^2 petit $\to$ experience stable dans le portfeuile . . . . . .
  ↳ $\bar{S}_i$ sera utile , plus Besoin de m

- $a \to \infty$: heterogène , meilleur approximation prime individuel
- $a = 0 \to$ $\bar{S}_1 = \bar{S}_2 = \dots = \bar{S}_I$: la prim $\bar{S}_i = m$, plus Besoin de l'expérience individuel $\to$ (IARD I)