Journal of Applied Developmental Psychology 41 (2015) 46–59

Contents lists available at ScienceDirect

Journal of Applied Developmental Psychology

Developmental patterns in the associations between instructional practices and children's math trajectories in elementary school

Melissa Castle Heatly *, Heather J. Bachman, Elizabeth Votruba-Drzal

University of Pittsburgh, United States

ARTICLE INFO

Article history:
Received 22 April 2014
Received in revised form 12 May 2015
Accepted 5 June 2015
Available online xxxx

Keywords:
Math achievement
Conceptual & procedural instruction
ECLS-K
Longitudinal within-child analysis

ABSTRACT

Although procedural and conceptual math instruction have been linked to children's math achievement in elementary school, the extant research provides an inconsistent developmental picture of how children respond to a variety of instructional inputs from kindergarten through 5th grade. Using data from a large, longitudinal sample, the ECLS-K (n = ~7600), this study considered how within-child changes in exposure to procedural and conceptual approaches are additively and interactively linked to corresponding changes in children's math achievement across elementary school. Specifically, this investigation examined whether these instructional associations with math achievement change as children progress from kindergarten to 5th grade. Significant two- and three-way interactions between instructional approach and longitudinal time were detected. Although an emphasis on procedural instruction was most positively linked to achievement for kindergarteners, a combination of both conceptual and procedural instruction was more beneficial for 5th graders' achievement. Implications for practice are discussed.

Students experience significant growth in their mathematical skills during elementary school (Fuson, Kalchman, & Bransford, 2005). Yet considerable heterogeneity exists in the way mathematics is taught during this period, and debate over the optimal instructional content and approach has been intense (Gamoran, 2001; Shrouse, 2001). Procedural instruction emphasizes basic skill acquisition and calculation activities to convey math facts (Gamoran, 2001; Shrouse, 2001), while conceptual instruction emphasizes the development of analytic and reasoning skills (Fuson et al., 2005; Griffin, 2005). Though both have been associated with math learning (Fuson et al., 2005), less is known about how associations between math instruction and achievement change as children develop across elementary school. Instead, the previous literature typically examines these associations within grades (e.g., 1st grade) rather than across grades (e.g., K–5th grade). As a result, the literature paints an inconsistent picture of how children respond to instructional input as they progress through elementary school. There is also uneven consideration both for how teachers combine procedural and conceptual instruction, and no studies have formally assessed how this combination shifts as children develop more skills and the work becomes more advanced and problem-based. Accounting for how procedural and conceptual instruction are interactively associated with math development across elementary school therefore represents an important next step.

The purpose of the present study is to consider how additive and interactive associations between math instruction and math achievement change as children develop across elementary school. This investigation follows children from kindergarten through 5th grade, and specifically focuses on within-child associations to track how change in exposure to conceptual and procedural instruction is associated with change in individual children's math achievement over time. The study uses a large, longitudinal, and nationally representative dataset, the Early Childhood Longitudinal Study — Kindergarten cohort (ECLS-K) to investigate associations (U.S. Department of Education, National Center for Education Statistics, 2006). As students' math skills grow from kindergarten through 5th grade, understanding how instruction can be leveraged to promote growth is central to understanding how children acquire mathematical skills and competency throughout the elementary school years (NMAP, 2008).

A developmental perspective on elementary math instruction

Contemporary thinking about developmentally appropriate instructional input has been heavily influenced by Piagetian and Vygotskian theories of learning and development. Piagetian theory argues that children must obtain and then master a basic body of facts, procedures, and skills before extending or applying such skills to more advanced mathematics. This approach is reflected by instruction in procedural math, which focuses upon calculation and basic number concepts as important tools for math achievement (Gamoran, 2001; Rosenshine & Stevens, 1986). In contrast, Vygotskian theory argues for the utilization of higher-order problem-solving and reasoning skills inherent in conceptual instruction (Goodman, 1989; Vygotsky, 1978; Xue & Meisels, 2004). Although it is less clear how Piagetian or Vygotskian

* Corresponding author. Tel.: +1 585 746 2893; fax: +1 412 624 4428.
E-mail address: mhc14@pitt.edu (M.C. Heatly).

http://dx.doi.org/10.1016/j.appdev.2015.06.002
0193-3973/

perspectives may apply as children move through middle childhood, it may be that instruction in math facts and computation skills are more beneficial for younger children who have not yet acquired the basic math skills needed for more advanced application and problem-solving (Piaget, 1977). In contrast, conceptual instructional approaches that emphasize applied and problem-solving skills may be more effective for older children who are increasingly adept at thinking symbolically and representationally (Burns & Sibley, 2001). As reviewed below, the empirical evidence supporting either approach is limited and mixed.

Empirical associations between pedagogical approach and math achievement

Several studies suggest that procedural instruction is beneficial for children's math development during the early years of elementary school (Bodovski & Farkas, 2007a; Byrnes & Wasik, 2009; Georges, 2009). For example, kindergarteners whose teachers spent more time on procedural skills such as advanced counting, addition and subtraction facts, single-digit operations, worksheet, and chalkboard activities demonstrated greater achievement gains across the school year than those whose teachers spent less time on these activities (Bodovski & Farkas, 2007a; Georges, 2009). Byrnes and Wasik (2009) demonstrated that these early positive associations continued through 3rd grade, with the strongest associations detected in kindergarten. However, examinations of procedural instruction and achievement in later elementary-school are more mixed. For example, Crosnoe et al. (2010) found that more procedural instruction in 3rd and 5th grade was not significantly associated with children's math achievement trajectories. Moreover, several correlational studies suggest that procedural instruction is negatively associated with student achievement once children reach 5th grade (Crosnoe et al., 2010; Hamilton, McCaffrey, Klein, Stecher, Robyn & Bugliari, 2003; Klein, Hamilton, McCaffrey, Stecher, Robyn & Burroughs, 2000; Le et al., 2006).

Exposure to conceptual math instruction is also inconsistently associated with children's math achievement throughout elementary school. For instance, studies conducted in the early years of elementary school (e.g., Pre-K–3) find that greater (rather than lesser) amounts of conceptual instruction are associated with math achievement, but with very small (Bodovski & Farkas, 2007a; Georges, 2009; Mashburn et al., 2008) or non-significant (Byrnes & Wasik, 2009; Howes et al., 2008; Le et al., 2006) effect sizes. Others have found negative associations for conceptual strategies that utilize manipulatives in early elementary school (Bodovski & Farkas, 2007a; Georges, 2009). In contrast, greater amounts of conceptual instruction in 4th and 5th grade are more consistently positively associated with children's math skills than are lower amounts (e.g., Hecht & Vagi, 2010, 2012). For example, Crosnoe et al. (2010) found that 3rd and 5th graders who experienced high amounts of conceptual instruction showed greater growth than their peers with low levels of conceptual instruction, though only when instruction was accompanied by non-conflictual teacher–child relationships. Experimental evidence also revealed that 4th graders who received conceptual instruction were just as likely to develop basic skills and were more adept at applying and extending their knowledge to novel situations than those who received procedural instruction (Rittle-Johnson & Alibali, 1999). These mixed findings leave an unclear picture of how procedural and conceptual instruction are associated with math achievement through elementary school.

Balanced, dynamic, and generalizable associations across elementary school

In addition to the mixed empirical support for conceptual and procedural math instruction from kindergarten through 5th grade, three issues remain understudied. First, the extent to which the combination of conceptual and procedural instruction promotes math achievement remains unclear. Evidence from a large, nationally-representative sample finds that a combination of instructional styles, defined as high amounts of both conceptual and procedural instruction, are more strongly linked to children's math achievement in kindergarten, 1st, and 3rd grade than disproportionate reliance upon either conceptual or procedural instruction alone (Byrnes & Wasik, 2009). However this investigation only considered additive effects of teachers who used primarily conceptual, primarily procedural, and both forms of instruction, and no formal tests of moderation have been conducted. Since synergistic instructional effects may be operating to enhance both domains of math competence, accounting for naturalistic variation in instruction is an important step.

Second, there are no studies to date that follow children throughout elementary school to examine how associations between instructional exposure and math achievement trajectories may change in meaningful ways over time. Although math proficiency involves both procedural and conceptual abilities (Fuson et al, 2005), children's math reasoning becomes more sophisticated and problems become more challenging and complex as they approach 5th grade. In contrast, young children who are learning basic skills and routines may benefit from an emphasis on procedural rather than conceptual instruction. While several studies are suggestive of developmental patterns for conceptual, procedural, and balanced instruction (e.g., Bodovski & Farkas, 2007b; Byrnes & Wasik, 2009; Georges, 2009), these investigations only considered additive associations over 2 to 4 years and did not procedurally test two- or three-way interactions between exposure to pedagogical approaches as children move through elementary school. Thus, considering how different combinations of instruction differentially promote achievement in early versus late elementary school is also a major interest of this investigation.

Third, much of the work on conceptual and procedural instruction is subject to external validity and endogeneity concerns. For instance, the majority of the extant research has been conducted on small or regional samples, which limits the generalizability of past findings. Previous research has also relied on between-child comparisons that may be affected by complex selection processes that could bias associations (Byrnes & Wasik, 2009; Desimone & Long, 2010; Georges, 2009). In contrast, within-child analyses make it possible to examine whether children's math achievement is higher at times when their teachers use relatively higher procedural or constructivist math instruction. These processes effectively control for all time-invariant child, family, teacher, and school characteristics and results in a fairly conservative model. Investigating within-child associations would therefore provide a more robust assessment of causal inference when examining additive, balanced, and dynamic associations between instruction and achievement across elementary school.

The present study

To address these important considerations, the present study uses data from a large, longitudinal, and nationally representative dataset, the Early Childhood Longitudinal Study, Kindergarten Cohort (ECLS-K) to examine how procedural and conceptual math instruction are associated with children's math achievement in elementary school. This investigation makes use of the ECLS-K's rich longitudinal design and hierarchical linear modeling (HLM) to examine within-child differences in exposure to instructional approach and corresponding math achievement from kindergarten through 5th grade. This approach holds constant time-invariant characteristics of children and their environments that may bias estimates (Singer & Willett, 2003) while controlling for important time-varying characteristics of schools, families, and children.

This investigation has two primary goals. The first is to identify how differences in exposure to procedural and conceptual instruction across elementary school are additively and interactively linked to change in individual children's math achievement during this time. Given inconsistent patterns between each respective instructional approach and lack of prior research considering interactive associations, the first investigative goal is exploratory in nature. The second goal is to consider whether passing time moderates associations between instructional approach and math achievement from kindergarten through 5th grade. In line with previous literature (Bodovski & Farkas, 2007a; Byrnes & Wasik, 2009; Georges, 2009), it is expected that procedural instruction will be most beneficial when children are younger and developing more basic math skills, but less so as they progress through elementary school. In contrast, conceptual instruction is expected to show stronger associations as children move into later elementary school and increasingly rely on conceptually-driven math abilities.

Methods

Participants

This study uses data from the Early Childhood Longitudinal Study — Kindergarten Cohort (ECLS-K), a longitudinal study of 20,000 children entering kindergarten in the fall of 1998 (U.S. Department of Education, National Center for Education Statistics, 2006). The ECLS-K was designed to document the educational status and progress of children from kindergarten through eighth grade. A multistage probability sampling design was utilized to select a nationally representative sample of children across the United States in its base year. The ECLS-K selected 1280 public and private schools in approximately 3500 classrooms offering kindergarten programs across the United States using primary sampling units (PSU) of counties or groups of counties. A target sample of approximately 24 children from each public school and 12 children from each private school was drawn. At the time of recruitment, 51.7% of children were identified as non-Hispanic Caucasian, 14.7% were African American, 7% were Hispanic, 6% were Asian, and 11% as "other".

This study is based on a nationally-representative subsample of approximately 10,700 children from the ECLS-K for whom longitudinal item response theory (IRT) scores through fifth grade, and a valid sampling weight (C2_6WC0) was available. The sample also consisted of approximately 10,900[1] teachers across the 4 sampling periods, 39% of whom held a Masters degree and had an average of 14.2 years of experience. Missing data due to attrition and nonresponse within the child subsample (n ~ = 10,700) was addressed through the use of the longitudinal direct child assessment weight C2_6WC0. The C2_6WC0 weight indicates the children with complete assessment data (spring- kindergarten, spring-first grade, spring-third grade, and spring-fifth grade), or if the child was excluded from direct assessment in all four of these rounds of data collection due to a disability. This weight is provided by the ECLS-K to compensate for differential probabilities of selection at each sampling stage, and to adjust for differential nonresponse so that the sample is nationally-representative of the cohort of children who were in kindergarten in 1998–99 despite missingness.

Approximately 7600[1] children were included in our analysis. Within the omitted cases, 72% of excluded children were missing information from the school and teacher questionnaires. An additional 4% were missing data on their achievement scores, 10% were missing data on the cognitive controls taken at the fall of Kindergarten, and 15% of children were missing data from the parent questionnaires. Comparisons of the children in our sample with valid weights and full data (n = ~7600) to those who were excluded from analysis (n = ~3100) revealed that children with complete data were more likely to be non-Hispanic White, to be first-time kindergartners from smaller schools, to have slightly higher math skills upon entry to kindergarten, and less likely to speak English in the home than those who were excluded from analysis. Each of these characteristics were included as covariates in our multivariate models to reduce the threat of biased estimates. Though these differences suggest that results may be slightly less generalizable than the fully weighted sample, it is important to note that those with missing data did not differ from the other children on math test scores or classroom instruction.

Measures

Measures used in the current study are described below. Unless otherwise specified, each measure was completed in spring of kindergarten, first, third, and fifth grades (U.S. Department of Education, National Center for Education Statistics, 2006). Descriptive statistics regarding the sample and outcome measures are listed in Table 1.

Math achievement

Children's math achievement in this study was obtained through standardized batteries administered by ECLS-K research associates in the fall and spring of kindergarten, and in the spring of 1st, 3rd, and 5th grades (U.S. Department of Education, National Center for Education Statistics, 2006). The assessments included items created for the ECLS-K by a panel of experts, as well as items adapted from well-validated and reliable measures of children's cognitive and academic development, including the Woodcock–Johnson Tests of Achievement—Revised (WJ-R; Woodcock & Johnson, 1989, 1990), the Peabody Picture Vocabulary Test—Revised (PPVT-R; Dunn & Dunn, 1981), and the Peabody Individual Achievement Test—Revised (PIAT-R; Dunn & Markwardt, 1970; Markwardt, 1989). New sets of assessment instruments were developed for the third- and fifth- grade assessments to capture advances in children's developing math skills. At each timepoint, the assessment evaluated children's skills in problem solving, number concepts, procedural knowledge, and measurement skills. Some of the assessment items were retained across rounds to support the development of the longitudinal IRT scale scores utilized in this study. IRT uses the pattern of right, wrong, and omitted responses to items administered in an assessment, along with indicators of difficulty, discriminability, and "guess-ability" of each item to place each child on a continuous ability scale. The resulting longitudinal IRT scale scores represent estimates of the number of items each child would have answered correctly at each point in time if they had taken all of the 174 questions in all of the mathematics forms, and was highly reliable ($\alpha_{fallK} = .92$ $\alpha_{springK} = .94$ $\alpha_1 = .94$, $\alpha_3 = .95$, $\alpha_5 = .92$) (U.S. Department of Education, National Center for Education Statistics, 2006).

Pedagogical approach

Instructional measures were obtained from teachers' report in spring of kindergarten, 1st, 3rd, and 5th grades (U.S. Department of Education, National Center for Education Statistics, 2006), and were created from teachers' reports regarding how often (*never*, <1×/*week*, 1–2×/*week*, 3–4×/*week*, *daily*), how much time (1–30 *min*/*day*, 31–60 *min*/*day*, 61–90 *min*/*day*, >90 *min*/*day*) and frequency (*not available*, *never*, >1×/*month*, 2–3×/*month*, 1–2×/*week*, 3–4×/*week*, *daily*) of their instructional and curricular practices. These items were developed for the ECLS-K through a consultative process involving a panel of curriculum experts comprised of university and school personnel. These experts intentionally included activities that were known to occur in classrooms and which largely reflected either a more interactive, conceptual style or a more didactic, procedural style of instruction (Rock & Pollack, 2002). The items were then pilot tested with the teachers of 1000 children.

A confirmatory factor analysis of these items was conducted in order to define dimensions of math instruction across elementary school. Items placed in initial CFAs were informed by previous work utilizing the ECLS-K to measure instructional practices (Bodovski & Farkas, 2007a; Byrnes & Wasik, 2009; Xue & Meisels, 2004), the literature on conceptual and procedural instructional approaches (Gamoran, 2001; Shrouse, 2001; Vygotsky, 1978), and consultation with instruction specialists in math (M. Smith, personal communication, June 15, 2010).

[1] Due to NCES's confidentiality policy, all unweighted sample sizes are rounded to the nearest 50.

Table 1
Descriptive statistics presented for time-varying measures of math achievement & level-1 covariates, as well as averaged covariates at level-2.

	Level 2			Level 1, wave-by-wave								
Direct assessment	~L2 Obs.	Avg. (L2)	*SD*	~ L1 Obs.	Kindergarten	*SD*	1st grade	*SD*	3rd grade	*SD*	5th grade	*SD*
Direct Assessment												
Time of spring assessment	–	–	–	17800	−26.64	*1.39*	−14.57	*1.38*	9.07	*0.91*	32.32	*0.87*
Quadratic time of spring assessment	–	–	–	17800	711.63	*52.26*	214.30	*21.33*	83.10	*17.74*	1045.64	*50.63*
Math IRT	–	–	–	17800	32.68	*11.59*	57.10	*16.70*	90.88	*21.96*	111.73	*22.21*
Reading IRT	7600	91.50	*21.57*	17800	40.51	*13.58*	71.19	*22.78*	116.88	*25.81*	137.42	*24.09*
Day of kindergarten fall assessment	7600	59.22	*16.9*	–	–	–	–	–	–	–	–	–
Math IRT — fall of kindergarten	7600	20.97	*7.39*	–	–	–	–	–	–	–	–	–
Child characteristics												
Boy	7600	50%	–	–	–	–	–	–	–	–	–	–
Low birthweight	7600	7%	–	–	–	–	–	–	–	–	–	–
Not a 1st-time kindergartener	7600	4%	–	–	–	–	–	–	–	–	–	–
Child health	7600	1.6		–	–	–	–	–	–	–	–	–
Ethnicity												
Black	7600	10%	–	–	–	–	–	–	–	–	–	–
White	7600	67%	–	–	–	–	–	–	–	–	–	–
Native	7600	3%	–	–	–	–	–	–	–	–	–	–
Asian	7600	5%	–	–	–	–	–	–	–	–	–	–
Hispanic	7600	13%	–	–	–	–	–	–	–	–	–	–
Multi-racial	7600	3%		–	–	–	–	–	–	–	–	–
Childcare characteristics												
Childcare hours before K	7600	25.37	*21.03*	–	–	–	–	–	–	–	–	–
Center-based childcare	7600	46%	–	–	–	–	–	–	–	–	–	–
Headstart childcare	7600	7%	–	–	–	–	–	–	–	–	–	–
Relative childcare	7600	14%	–	–	–	–	–	–	–	–	–	–
Non-relative childcare	7600	11%	–	–	–	–	–	–	–	–	–	–
Multiple childcare centers	7600	5%	–	–	–	–	–	–	–	–	–	–
Parental childcare	7600	16%	–	–	–	–	–	–	–	–	–	–
Parent & home characteristics												
English not spoken in home	7600	7%	–	–	–	–	–	–	–	–	–	–
Mother's age	7600	34.05	*6.03*	–	–	–	–	–	–	–	–	–
Number of kids in home	7600	2.46	*1.00*	–	–	–	–	–	–	–	–	–
Socioeconomic status												
Linear income	7600	10.54	*0.93*	17800	10.40	*0.95*	10.51	*0.92*	10.61	*0.91*	10.64	*0.94*
Occupational prestige	7600	45.62	*11.66*	17800	45.06	*11.64*	45.67	*11.50*	45.83	*11.69*	45.92	*11.80*
Maternal highest education												
Advanced degree	7600	11.03%	–	17800	10.26%	–	10.46%	–	11.33%	–	12.08%	–
Bachelors degree	7600	19.94%	–	17800	19.19%	–	19.13%	–	20.60%	–	20.83%	–
Vocational degree	7600	33.91%	–	17800	32.85%	–	33.75%	–	33.98%	–	35.08%	–
High school degree	7600	24.30%	–	17800	27.00%	–	25.03%	–	22.82%	–	22.36%	–
Less than high school degree	7600	10.81%	–	17800	10.70%	–	11.63%	–	11.27%	–	9.65%	–
Maternal employment												
Part time	7600	22.02%	–	17800	20.54%	–	22.43%	–	23.34%	–	21.79%	–
Unemployed — not looking	7600	24.96%	–	17800	29.18%	–	26.08%	–	22.51%	–	22.07%	–
Unemployed — looking	7600	3.17%	–	17800	3.64%	–	2.62%	–	3.16%	–	3.27%	–
Full time	7600	49.84%	–	17800	46.64%	–	48.87%	–	51.00%	–	52.87%	–
Maternal marital status												
Married	7600	69.16%	–	17800	68.85%	–	69.51%	–	70.12%	–	68.17%	–
Separated/divorced	7600	16.17%	–	17800	14.84%	–	14.57%	–	16.37%	–	18.89%	–
Never married	7600	13.43%	–	17800	15.36%	–	15.15%	–	12.15%	–	11.05%	–
Widowed	7600	1.24%	–	17800	0.95%	–	0.78%	–	1.36%	–	1.89%	–
Learning-related parenting practices												
Cognitive stimulation	7600	−0.02	*0.55*	17800	0.00	*0.56*	−0.01	*0.56*	−0.03	*0.53*	−0.04	*0.53*
Parental involvement	7600	−0.02	*0.54*	17800	−0.01	*0.52*	0.00	*0.54*	−0.03	*0.54*	−0.03	*0.54*
Family routines	7600	−0.03	*0.54*	17800	−0.01	*0.54*	−0.02	*0.54*	−0.04	*0.55*	−0.04	*0.54*
Classroom characteristics												
Teacher has an MA	7600	40.20%	–	17800	–	*0.48*	38%	–	43.52%	–	43.24%	–
Teacher has a BA	7600	59.80%	–	17800	–	*0.48*	62%	–	56.48%	–	56.76%	–
Years teaching experience	7600	13.81	*10.01*	17800	12.80	*9.54*	13.19	*10.00*	14.98	*10.22*	14.28	*10.27*
Class size	7600	21.25	*5.41*	17800	20.78	*4.32*	20.78	*4.32*	21.08	*4.55*	22.35	*8.46*
Hours of math groups/week	7600	2.14	*1.01*	17800	1.82	*1.00*	2.53	*1.02*	2.26	*1.06*	1.95	*0.97*
Hours of math instruction/week	7600	8.49	*2.96*	17800	6.90	*3.13*	9.18	*2.57*	9.31	*2.62*	8.58	*3.51*
School characteristics												
Percent minority	7600	19.01%	–	–	–	–	–	–	–	–	–	–
Free/reduced lunch	7600	33.64%	–	–	–	–	–	–	–	–	–	–
Private school	7600	21%	–	–	–	–	–	–	–	–	–	–
School size	7600	511.2	*259.07*	–	–	–	–	–	–	–	–	–
Instruction												
Math instruction												
Procedural instruction	7600	0.03	*0.51*	17800	0.02	*0.48*	0.01	*0.52*	0.04	*0.45*	0.06	*0.58*
Conceptual instruction	7600	0.01	*0.56*	17800	0.00	*0.55*	0.02	*0.53*	0.02	*0.53*	−0.01	*0.62*

Although the questionnaire changed between grades to reflect developmentally appropriate changes in instruction (e.g,. counting in early grades vs. multiplication in later grades), factors were designed to maintain conceptual consistency across waves. As Hair et al. (1998) suggest that factor loadings above .300 are sufficient for sample sizes above 350, items that did not load above .300 on either scale were dropped from analysis and are not shown. Guided by resulting factor loadings, separate procedural and conceptual math instruction scales were identified. Items included in each scale and their respective factor loadings can be found in Appendices A and B. Each item was standardized, and composite variables were calculated by averaging the standardized items. Higher values on each of these composite measures of instruction represent more frequent use of the particular approach.

The procedural math instruction composites (e.g., adding single digit numbers, writing numbers between 1–100, learning math facts; see Appendix A) included 16 items in kindergarten and first grade, 8 in third grade, and 8 items in fifth grade. Alphas were generally high for this measure, with $\alpha_K = 80$, $\alpha_1 = 82$, $\alpha_3 = .60$, and $\alpha_5 = .70$. Lower reliabilities in 3rd and 5th grade were likely due to fewer items in each scale (Cortina, 1993). The conceptual math scales (e.g., math games, solving real life problems, using creative movement for math; see Appendix B) include 17 items in kindergarten and first grade, 12 items in third grade, and 7 items in fifth grade. Alphas were again generally high: $\alpha_K = 87$, $\alpha_1 = 85$, $\alpha_3 = .80$, and $\alpha_5 = .82$. Conceptual and procedural composites were moderately correlated at each wave: $r_K = .48$, $r_1 = .57$, $r_3 = .32$, and $r_5 = .38$.

Child covariates

Several child characteristics that are consistently linked to achievement trajectories were included in these analyses as control variables, and were obtained during parent interviews and the Kindergarten child assessments. A measure of time was obtained from a variable representing the amount of time, in months, that has passed since the child's first math assessment in the fall of kindergarten and the date of subsequent math assessments in the spring of kindergarten, 1st, 3rd, and 5th grades. Child gender was represented by an indicator variable (1 = *male*; 0 = *female*). Children's race/ethnicity was classified into six categories: non-Hispanic White (*reference*), non-Hispanic African American, non-Hispanic Asian or Pacific Islander, Hispanic or Latino, Native American, or of multi-racial background. Indicators of overall child health and low birth weight status represented whether a child weighed less than 5 lb, 8 oz at birth. An indicator of whether the child repeated kindergarten was also included in the analyses. Early childcare experiences can be quite heterogeneous, and can set the stage for later achievement trajectories (Duncan & Gibson-Davis, 2006). As such, a control variable was included that indicated the average number of hours of childcare each child experienced before entry into kindergarten, along with an indicator of whether the child experienced general center-based (*reference*), Head Start, relative care that occurred within the child's or relative's home, non-relative care in the child or caregiver's home, or multiple childcare settings prior to kindergarten. Children who only experienced parental care were identified by a 'parental childcare' indicator. Notably, these children experienced 0 hours of childcare.

Two key achievement-related skills were also included in this analysis. First, because reading skills can be associated with children's ability to comprehend and respond to mathematical word-problems and text, we included a time-varying measure of reading achievement. This measure included items created for the ECLS-K by a panel of experts, as well as items adapted from well-validated and reliable measures of children's cognitive and academic development including the Woodcock–Johnson Tests of Achievement—Revised (WJ-R; Woodcock & Johnson, 1989, 1990), the Peabody Picture Vocabulary Test—Revised (PPVT-R; Dunn & Dunn, 1981), and the Peabody Individual Achievement Test—Revised (PIAT-R; Dunn & Markwardt, 1970; Markwardt, 1989). The reading assessment included 72 items (alpha range .93–.96) and was designed to measure children's language and literacy skills, including basic skills, vocabulary, and reading proficiency and comprehension. Second, because the heterogeneity in children's math skills when beginning kindergarten may relate to the type of instruction they receive, an assessment of children's math IRT scores at school entry (fall of kindergarten, $\alpha = 96$) was included as a baseline control. This measure assessed children's initial problem solving, number concepts, procedural knowledge, and measurement skills at entry to kindergarten (U.S. Department of Education, National Center for Education Statistics, 2006). This was assessed using the same procedures as measures of time-varying math achievement, described above.

Family covariates

Several control variables about the family environment were obtained during parent interviews in kindergarten, including an indicator of whether English was the primary language spoken in the home when the child was in kindergarten, and maternal age at the birth of her child. Separate indicators of parental socioeconomic status were obtained during parent interviews in K, first, third, and fifth grade, including the number of children in the household, family income, maternal education (less than a high school diploma (*reference*), high school, some college, college degree, advanced degree), and maternal occupational prestige. Finally, parental marital status was represented with separate indicators of whether parents were married (*reference*), separated or divorced, widowed, or never married. All measures were obtained through direct interviews with independent research assistants employed by the ECLS-K.

Classroom covariates

Several characteristics of the children's classrooms were also included in the analyses. Continuous variables representing class size and teachers' total years of experience were included, as was a dichotomous variable representing whether a teacher had a bachelor's degree or an advanced degree (*reference*). Variables representing weekly instructional time and teachers' use of small groups are also included as controls in this analysis. Both the weekly instructional time and small groups measures are derived from teachers' reports of the number of minutes per day and times per week they "typically" spend teaching math, as well as teachers' reports of how often they divide their class into small groups for math activities or lessons. Teachers reported times per week on a five-point scale (*never*, *less than once/week*, 1–2 *times/week*, 3–4 *times/week*, and *daily*), and minutes per day on a four-point scale (1–30 *min/day*, 31–60 *min/day*, 61–90 *min/day*, and *more than* 90 *min/day*). A midpoint was assigned for each of these variables to meaningfully represent the approximate number of days per week and minutes per day math was taught, and that small groups were utilized. The days per week and minutes per day measures were then multiplied together to form a variable that represents the number of hours per week teachers report spending on math instruction, and a second variable was calculated to represent the number of hours per week teachers reported using small groups for math instruction.

School characteristics

Several school-level characteristics were obtained from the Principal Questionnaire in kindergarten, 1st, 3rd, and 5th grades and included in this analysis to account for aggregate differences in school disadvantage. Covariates included continuous measures of the size of the school, the percent of children registered for free or reduced-priced lunch, an indicator of whether the school qualifies for Title-I assistance, and a dichotomous variable representing whether the school is public (*reference*) or private. School characteristics displayed little variation across grades ($rs = .66$–.96), and were averaged across the four waves of data.

Analytic plan

Three-level Hierarchical Models (HLM, Raudenbush & Bryk, 2002) were used to examine the extent to which exposure to procedural and conceptual math instruction explain changes in children's math achievement, and how these associations may change over the course of elementary school. All models were estimated in HLM 6.06 using full information maximum likelihood estimation (Raudenbush, Bryk, & Congdon, 2008). Our analysis began by estimating unconditional growth models of children's math achievement (Raudenbush & Bryk, 2002). Next, a series of conditional growth models were estimated to address the study's aims. Level 1 contained repeated measures of math achievement scores taken in the spring of kindergarten, 1st, third, and fifth grades, which were nested within individual children at level 2 and within kindergarten school ID at level 3. A random effect (ε_{ijt}) was included for the intercept and linear slope terms at levels 1, 2, and 3. The quadratic slope term was fixed at levels 1, 2, and 3, as there was insufficient variability in the quadratic term for meaningful interpretation. All other parameters were estimated as random.

Repeated measures of conceptual and procedural instruction were included as predictors of children's math achievement at level 1 using Eq. (1):

$$\begin{aligned} \mathit{Math\ Achievement}_{ijt} = {} & \pi_{0ij} + \pi_{1ij}\left(\mathit{Time}_{ijt} - \overline{\mathit{Time}}_{\cdot jk}\right) + \pi_{2ij}\left(\mathit{Time}_{ijt} - \overline{\mathit{Time}}_{\cdot jk}\right)^2 \\ & + \pi_{3ij}\left(\mathit{Child}_{ijt} - \overline{\mathit{Child}}_{\cdot jk}\right) + \pi_{4ij}\left(\mathit{Family}_{ijt} - \overline{\mathit{Family}}_{\cdot jk}\right) \\ & + \pi_{5ij}\left(\mathit{Classroom}_{ijt} - \overline{\mathit{Classroom}}_{\cdot jk}\right) \\ & + \pi_{6ij}\left(\mathit{Instruction}_{ijt} - \overline{\mathit{Instruction}}_{\cdot jk}\right) + \varepsilon_{ijk}. \end{aligned} \tag{1}$$

Math achievement for child i at time t was modeled as a function of passing time (Time_{ijt}) and instruction ($\mathit{Instruction}_{ijt}$), as well as child ($\mathit{Child}_{ijt}$), family ($\mathit{Family}_{ijt}$), and classroom ($\mathit{Classroom}_{ijt}$) characteristics. All predictors at level 1 were group-mean centered, also known as within-person centering (Raudenbush & Bryk, 2002; Singer & Willett, 2003). Thus, the academic achievement of child i in school j at time t was modeled as a function of his or her average academic achievement (π_{0ij}), linear growth in achievement per month increase in time from kindergarten through 5th grade (π_{1ij}), and quadratic growth trajectory from the spring of kindergarten to the spring of fifth grade (π_{2ij}). Coefficients on level-1 predictors therefore represent within-child associations over time. Correspondingly, instructional coefficients assess whether increases or decreases in conceptual or procedural instruction are associated corresponding changes in math achievement, relative to what the child experiences on average over the course of elementary school.

Group-mean centering is useful for addressing bias due to unobserved heterogeneity or unmeasured time-invariant child and contextual characteristics. This is an important advantage over between-child analyses, which are more susceptible to time-invariant characteristics (e.g., child intelligence, temperament). However, level-1 predictors are still susceptible to time-varying omitted variable bias. To determine the extent to which within-child associations mirror between-child differences, between-child associations between average classroom conceptual and procedural instruction across elementary school and children's average level and growth of academic achievement from kindergarten through 5th grade will be considered at level-2, with Eqs. (2)–(4) estimating variability in the level-1 parameters:

$$\begin{aligned} \pi_{0ij} = {} & \beta_{ooj} + \beta_{01j}\mathit{Ach}_{fk} + \beta_{02j}\mathit{Family} + \beta_{03j}\mathit{Child} + \beta_{04j}\left(\mathit{AvChild}_i - \overline{\mathit{AvChild}}_{\cdot\cdot}\right) \\ & + \beta_{05j}\left(\mathit{AvFamily}_i - \overline{\mathit{AvFamily}}_{\cdot\cdot}\right) + \beta_{06j}\left(\mathit{AvTeacher}_i - \overline{\mathit{AvTeacher}}_{\cdot\cdot}\right) \\ & + \beta_{07j}\left(\mathit{AvClass}_i - \overline{\mathit{AvClass}}_{\cdot\cdot}\right) + \beta_{08j}\left(\mathit{AvSchool}_i - \overline{\mathit{AvSchool}}_{\cdot\cdot}\right) \\ & + \beta_{09j}\left(\mathit{AvInstruction}_i - \overline{\mathit{AvInstruction}}_{\cdot\cdot}\right) + r_{0ij} \end{aligned} \tag{2}$$

$$\begin{aligned} \pi_{1ij} = {} & \beta_{10j} + \beta_{11j}\mathit{Ach}_{fk} + \beta_{12j}\mathit{Fam} + \beta_{13j}\mathit{Child} + \beta_{14j}\left(\mathit{AvChild}_i - \overline{\mathit{AvChild}}_{\cdot\cdot}\right) \\ & + \beta_{15j}\left(\mathit{AvFamily}_i - \overline{\mathit{AvFamily}}_{\cdot\cdot}\right) + \beta_{16j}\left(\mathit{AvTeacher}_i - \overline{\mathit{AvTeacher}}_{\cdot\cdot}\right) \\ & + \beta_{17j}\left(\mathit{AvClass}_i - \overline{\mathit{AvClass}}_{\cdot\cdot}\right) + \beta_{18j}\left(\mathit{AvSchool}_i - \overline{\mathit{AvSchool}}_{\cdot\cdot}\right) \\ & + \beta_{19j}\left(\mathit{AvInstruction}_i - \overline{\mathit{AvInstruction}}_{\cdot\cdot}\right) + r_{1ij} \end{aligned} \tag{3}$$

$$\pi_{2ij} = \beta_{30j}. \tag{4}$$

Variation in mean levels of achievement (π_{0ij}) and in the growth of achievement over time (π_{1ij}) was explained with individual-level averages across elementary school for instruction ($\mathit{AvInstruction}_i$), as well as child, family, teacher, classroom characteristics. This model also includes a lagged measure of children's initial math achievement at school entry (Ach_{FK}), as well as all time-invariant child characteristics. Exploratory analyses determined that there were insufficient students within each classroom to estimate classroom effects, and that school characteristics displayed little variation across grades (rs = .66–.96). Thus, averaged classroom and school characteristics were included as covariates in the level-2 models (Snijders & Bosker, 1993).

Level-2 equations were grand-mean centered. The level 2 coefficients on instruction therefore consider whether children who are exposed to higher average levels of either instructional approach throughout elementary school display better average levels of mathematic skills as well as greater improvement in math over time. Data dependence due to children nested within schools is further accounted for by including school specific random effects at level-3 based on spring of kindergarten school ID. These models therefore capture both between-child variations in children's experiences, while the level-1 coefficient on instruction examines whether within-child changes in exposure to instructional approaches are associated with improvements in academic development.

To consider interactive associations between instructional approaches, a 2-way interaction between conceptual and procedural instruction was added into level-1 and level-2 equations. To consider dynamic associations across elementary school, 2-way interactions between conceptual instruction and time, and procedural instruction and time were then stepped into the level-1 equation. Finally, a 3-way interaction representing conceptual instruction by procedural instruction by time was added to the level 1 model.

Results

Time-varying descriptive statistics are presented in Table 1. Children's math IRT scores were lowest at entry to kindergarten (average = 20.97), but grew to an average of 111.73 in 5th grade. Teachers reported that children received an average of 8.49 hours of math instruction and 2.14 hours of small math-groups per week. Bivariate correlations between time-varying achievement and instructional approach measures are presented in Table 2. As can be seen, conceptual instruction is negatively associated with math achievement in kindergarten and 1st grade (rs = −.004 & −.024, *respectively*) — but positively associated with achievement as children move into 3rd and 5th grades (rs = .040 & .110, *respectively*). Alternatively, procedural math instruction is positively associated with children's math achievement at each time-point, but associations are strongest in kindergarten (r = .121) and at 5th grade (r = .256). Time-varying reading achievement was also strongly correlated with children's math achievement at each time-point, with rs ranging from .691 to .759.

Table 2
Time-varying bivariate correlations between achievement and instructional approach in math.

	Kindergarten	1	2	3	4
1	Math IRT — kindergarten	1			
2	Reading IRT — kindergarten	.706**	1		
3	Conceptual math — kindergarten	−.004**	.014**	1	
4	Procedural math — kindergarten	.121**	.160**	.557**	1
	1st grade				
1	Math IRT — 1st grade	1			
2	Reading IRT — 1st grade	.691**	1		
3	Conceptual math — 1st grade	−.024**	−.029**	1	
4	Procedural math — 1st grade	.045**	.041**	.642**	1
	3rd grade				
1	Math IRT — 3rd grade	1			
2	Reading IRT — 3rd grade	.743**	1		
3	Conceptual math — 3rd grade	.040**	.015**	1	
4	Procedural math — 3rd grade	.057**	.046**	.438**	1
	5th grade				
1	Math IRT — 5th grade	1			
2	Reading IRT — 5th grade	.759**	1		
3	Conceptual math — 5th grade	.110**	.099**	1	
4	Procedural math — 5th grade	.256**	.253**	.370**	1

Note. ***$p = .001$. **$p = .01$. *$p = .05$. + $p < .10$

Children's math achievement trajectories from kindergarten to fifth grade

This analysis began by estimating unconditional growth models to determine whether children had significant variability in their average levels and growth of math achievement. All models report robust standard errors, and Table 3 contains the results of these models. For all models, effect sizes were calculated using the standard deviations taken from children's baseline math achievement in the spring of kindergarten ($SD = 11.59$). The intercept term placed the average child's math achievement at 83.48, which is approximately half-way through elementary school (between 2nd and 3rd grade). The positive and significant coefficient on the linear slope terms indicates that children's achievement skills were improving over time. Specifically, a one-month change in time was associated with 12% of a *SD*, or 1.43-point improvement in children's average math achievement. The negative and significant coefficient on the quadratic slope terms suggests that the rate of growth decreased over time, where every passing month was associated with a very small ($\beta = > .01$; .014 point) but significant decrease in growth.

Table 3
Unconditional growth curves of child achievement from kindergarten through fifth grade.

Fixed effect	Coefficient	SE	t-Score	*p*-Value
Intercept, π_0				
Intercept, β_{00}				
Intercept, γ_{000}	83.488***	0.37	255.488	0
Linear time, π_1				
Intercept, β_{10}				
Intercept, γ_{100}	1.435***	0.005	242.174	0
Quadratic time, π_1				
Intercept, β_{10}				
Intercept, γ_{100}	−0.014***	0	−60.376	0
Levels 1 & 2 random effects	**Variance component**	**~*df***	**x^2**	***p***
Intercept, r_0	190.078	5650	46,034.263	0
Linear time, r_1	0.033	5650	8560.048	0
Level-1 effect, e_{ij}	60.125			
Level 3 random effects	**Variance component**	**~*df***	**x^2**	***p***
Intercept, u_{00}	54.236	900	2747.83	0
Linear time, u_{10}	0.014	900	1821.5	0

Note. ***$p < .001$. **$p < .01$. *$p < .05$. + $p < .10$; degrees of freedom rounded to nearest 10 to conform to NCES guidelines.

Main effects of procedural and conceptual math instruction

The first goal of this investigation was to identify whether conceptual and procedural instructional approaches are additively associated with children's math achievement from Kindergarten through 5th grade. Table 4 presents the results of these conditional growth models.

In this table, Model 1 presents results with child, family, class, and school controls, a measure of time-varying reading achievement, and conceptual and procedural instruction. As before, effect sizes were calculated using standard deviations taken from the spring of kindergarten ($SD_{Procedural} = .47$; $SD_{Conceptual} = .54$). Effect sizes are also placed in the context of monthly math learning, which is represented by the coefficient representing linear time. Here, each month is associated with 11% of a *SD*, or 1.29-point improvement in average math achievement.

An examination of the level-1 coefficients suggests that both conceptual and procedural instruction were associated with math achievement — though in different ways. Here, children's math achievement was relatively higher in years they experienced more procedural instruction than they experienced on average throughout elementary school. Specifically, each SD increase in procedural instruction was associated with 6% of a SD greater math learning. When compared to children's average gain in math achievement (1.29 points/month), 6% of a SD is associated with approximately 54% of a month's math learning. In contrast, children demonstrated relatively lower levels of math achievement in years they were exposed to more conceptual instruction than they typically experienced throughout elementary school. Specifically, each SD increase in conceptual instruction was related to 4% of a SD, or 36% of a month's decrease in achievement scores. Notably, these associations were significant even after accounting for children's time-invariant differences (e.g., math skills prior to school entry), as well as time-varying measures of reading achievement and other child, parent, teacher, classroom, and school characteristics.

To determine the extent to which within-child associations mirror between-child differences, Table 4 also presents level-2 instructional coefficients. An examination of these between-child comparisons revealed that level-2 intercept coefficients paralleled level-1 findings. Here, students who experienced higher average levels of procedural math instruction across elementary school, compared to children who experienced lower levels, had higher average math achievement scores average across children — even after controlling for children's initial math achievement at entry to kindergarten and average reading scores. Specifically, a standard deviation increase in average exposure to procedural math was associated with 8% of a SD, or 72% of a month's greater average math achievement across elementary school. However, the level-2 slope term for procedural instruction was not significantly associated with growth in math from kindergarten through 5th grade. In contrast, neither intercept nor slope terms for conceptual instruction paralleled negative level-1 within-child associations. Here, average levels of conceptual instruction were not significantly associated with children's average math achievement. In addition, students who experienced higher average levels of conceptual math instruction showed slightly greater growth in math achievement from kindergarten through 5th grade ($\beta = .03$) than children who experienced lower average levels of conceptual instruction.

Table 4
Within- and between-child associations between instructional approach and math achievement from kindergarten through fifth grade.

Item	Model 1: achievement by instruction						Model 2: two-way interactions						Model 3: conceptual × procedural × time					
	Level 1[1]		Level 2[2]				Level 1[1]		Level 2[2]				Level 1[1]		Level 2[2]			
	Slope	*SE*	Intercept	*SE*	Slope	*SE*	Slope	*SE*	Intercept	*SE*	Slope	*SE*	Slope	*SE*	Intercept	*SE*	Slope	*SE*
Intercept term	81.48***	*.42*					81.48***	*.41*					81.49***	*.41*				
Passing time																		
Linear time	1.29***	*.02*					1.29***	*.02*					1.29***	*.02*				
Quadratic time	−.01***	*.00*					−.01***	*.000*					−.01***	*.00*				
Achievement controls																		
Lagged math skills — fall of K			1.10***	*.04*	.00	*.00*			1.10***	*.04*	.00	*.00*			1.10***	*.04*	.00	*.00*
Reading achievement	.06***	*.01*	.36***	*.02*	.01***	*.00*	.06***	*.01*	.35***	*.02*	.01***	*.00*	.06***	*.001*	.36	*.02*	.01***	*.00*
Instruction																		
Conceptual	−.77*	*.33*	.52	*.88*	.07*	*.03*	−.41	*.38*	.501	*.88*	.06*	*.02*	−.44	*.38*	.492	*.05*	.06*	*.03*
Procedural	1.32**	*.38*	1.98*	*1.06*	.02	*.03*	.97*	*.44*	1.97*	*.106*	.02	*.034*	1.12**	*.43*	4.188***	*1.019*	.02	*.03*
Interactions																		
Conceptual × time							.06*	*.02*					.031	*.016*				
Procedural × time							−.06*	*.02*					−.053*	*.02*				
Conceptual × procedural													.057*	*.02*	−2.44	*2.24*	−.09^{+}	*.07*
Conceptual × procedural × time													.134**	*.04*				

Note. $^{***}p < .001$. $^{**}p < .01$. $^{*}p < .05$. $+\ p < .10$; All models include covariates found in Table 1. Degrees of freedom rounded to nearest 50 to conform to NCES guidelines.
[1]Level 1 covariates include: time-varying measures of child reading achievement; maternal income, prestige, education, employment, cognitive stimulation, involvement, and family routines; teachers' years of experience, education, instructional time, and math groups.
[2]Level 2 covariates include: averaged measures of all level 1 covariates, as well as time-invariant measures of: lagged math achievement at the fall of kindergarten; child gender, ethnicity, childcare, birthweight, 1st time kindergartener status, and health; parental age at kindergarten, number of children, and an indicator if English is spoken in the home; school size, percent minority, percent free/reduced lunch, and an indicator if the school is private.

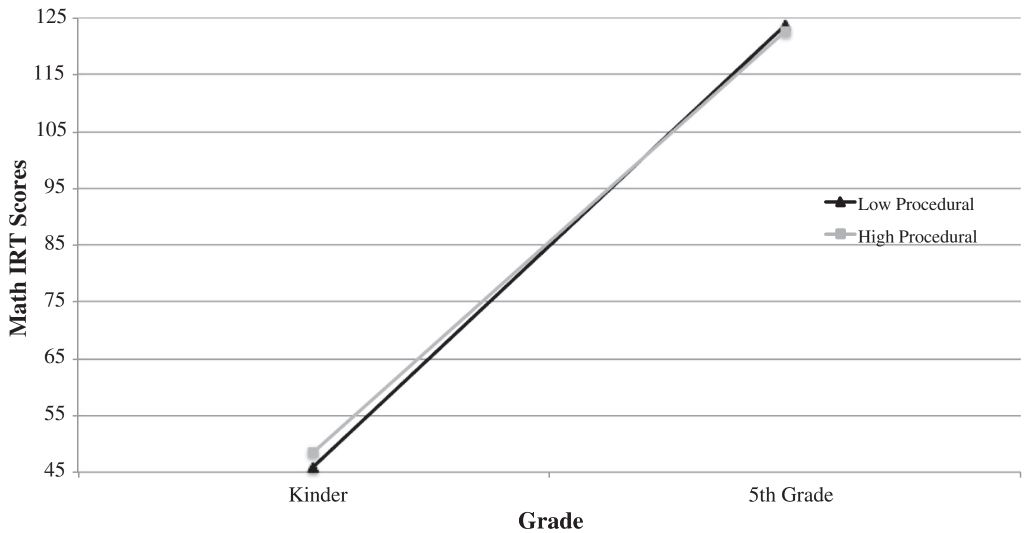

Fig. 1. Two-way interaction between procedural instruction and passing time, graphed at kindergarten and 5th grade.

Dynamic associations of instruction with achievement across elementary school

To consider whether the additive effects of conceptual or procedural instruction became more or less influential as children moved from kindergarten through 5th grade, two-way interactions between Conceptual x Time and Procedural x Time were added to level-1 models. These are presented in Model 2 of Table 4. An examination of simple slopes reveals a significant and negative interaction between procedural math instruction and time. This interaction was subsequently probed (Preacher, Curran, & Bauer, 2006), and graphs of simple slopes can be found in Fig. 1. As can be seen, the benefits of procedural instruction were attenuated as children progressed from kindergarten through 5th grade. In kindergarten, a SD increase procedural instruction was associated with 12% of a SD greater growth in math achievement. This accounted for more than a month's math learning across elementary school (1.07 of a month's learning). By the time children reached 5th grade however, a SD increase in procedural instruction was only associated with 6% of a SD, or 70% of a month's increase in math achievement. This suggests that high amounts of procedural instruction have a positive association with children's achievement in early elementary school, but a diminished yet still positive and significant association in later elementary school. In contrast, the significant positive interaction between conceptual instruction and time suggests that the negative association between conceptual instruction and math trajectories diminished as they moved through elementary school (Fig. 2). In kindergarten, a SD increase conceptual instruction was associated with 9% (1.04 of a month's math learning) of a SD decrease in achievement, but these differences narrowed to 4% of a SD (70% of a month) difference in math achievement by 5th grade that was not significantly different than zero. This suggests that

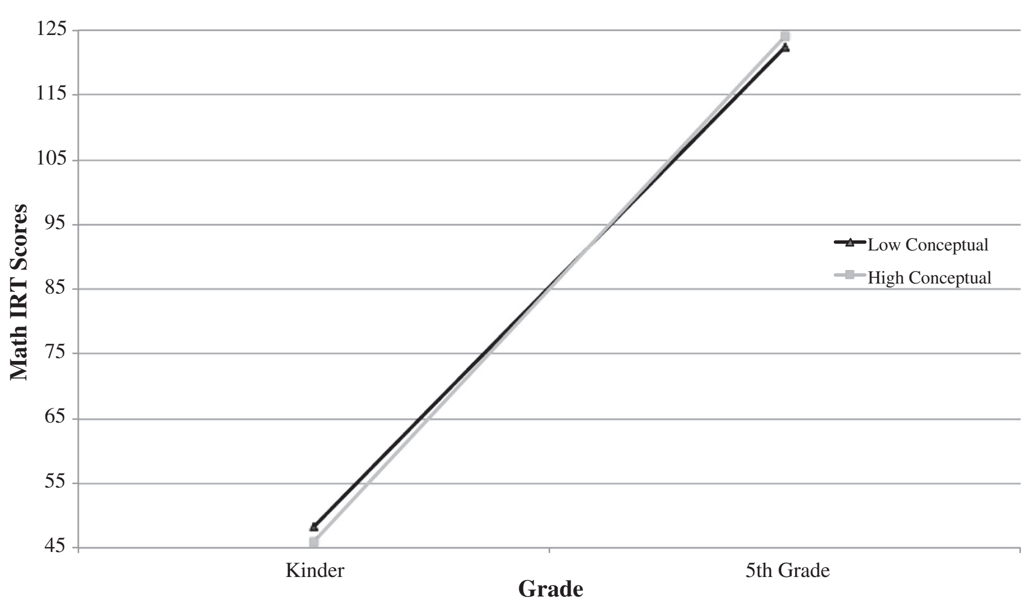

Fig. 2. Two-way interaction between conceptual instruction and passing time, graphed at kindergarten and 5th grade.

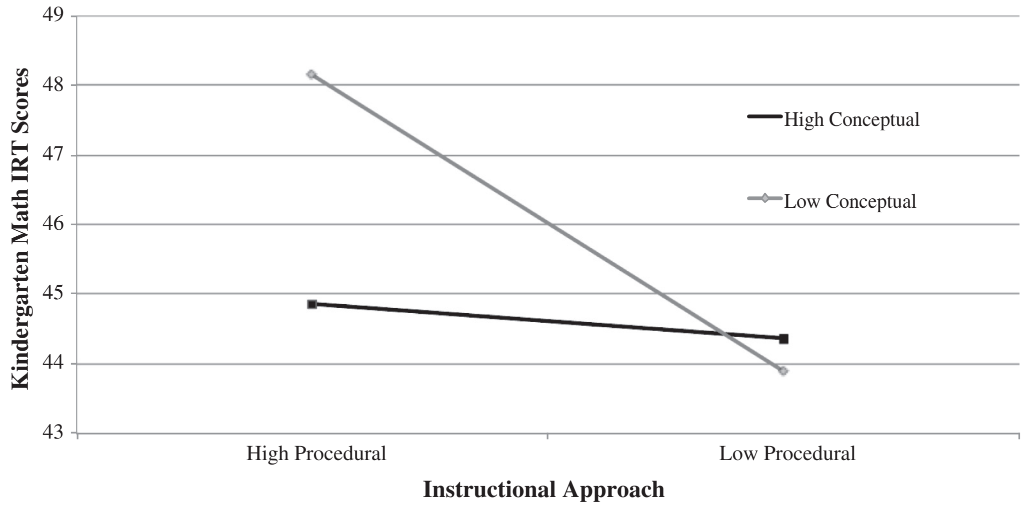

Fig. 3. Three-way interaction between conceptual instruction, procedural instruction, and time — graphed at kindergarten.

conceptual instruction has a small but negative association with children's achievement in early elementary school that diminishes to a non-significant association by fifth grade.

Balanced math instructional approaches on achievement across elementary school

In order to examine the combination of conceptual and procedural approaches, a two-way interaction between conceptual and procedural instruction was added to both level-1 and level-2 of Model 2. However, this interaction term was not significant at either level. For parsimony, this step was not displayed in Table 4.

To investigate developmental differences in associations between balanced instruction and math achievement, a three-way interaction of conceptual x procedural x time was added to level-1, and is presented in Model 3 of Table 4. A significant interaction was detected, and graphs representing this three-way interaction term can be found in Figs. 3 (kindergarten) and 4 (5th grade). Low and high amounts of each instructional approach are graphed as a SD below and above the mean, respectively. As shown in the figures, the combination of low amounts of conceptual but high amounts of procedural instruction was most beneficial for children's math achievement in Kindergarten, and resulted in 8% greater growth in math achievement than children who received low amounts of procedural and high amounts of conceptual instruction. In 5th grade, children that received high amounts of both conceptual and procedural instruction displayed 1.5–3% greater growth in math achievement than those who received instruction that emphasized one approach over another (e.g., high conceptual, but low procedural; low conceptual but high procedural). Unexpectedly, students with teachers reporting low use of both conceptual and procedural approaches only scored 1.5–3% better than those who emphasized either conceptual or procedural instruction.

Discussion

The present study considered how procedural and conceptual instructional approaches are additively and interactively linked to math achievement in elementary school, and whether these associations change from kindergarten to 5th grade. In examining these associations, this study addresses the current literature's reliance on cross-sectional

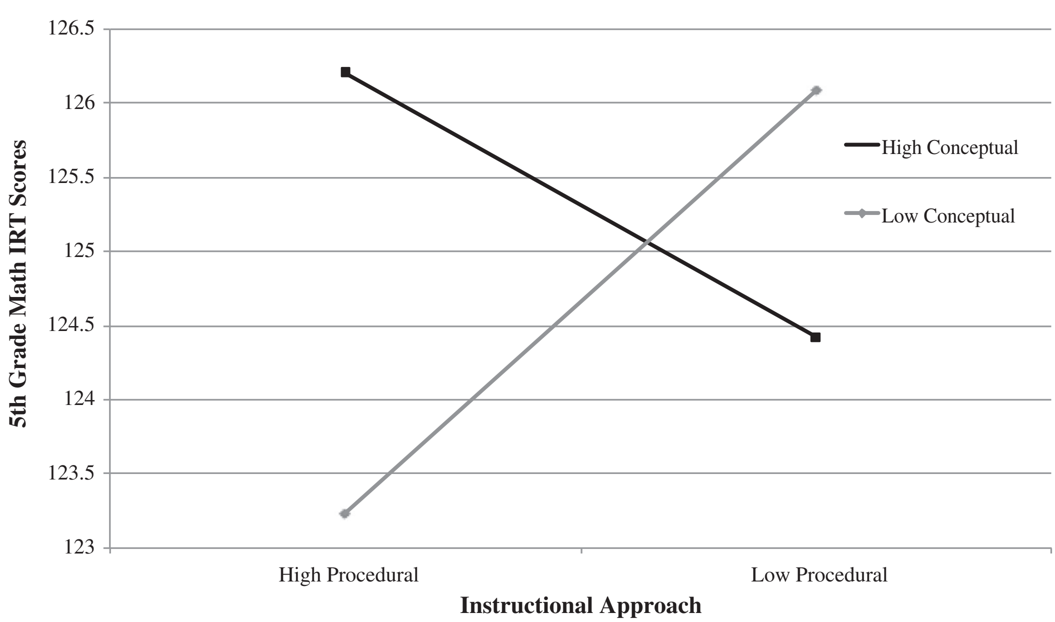

Fig. 4. Three-way interaction between conceptual instruction, procedural instruction, and time — graphed at 5th grade.

and between-child designs by employing longitudinal data to examine within-child associations, and draws attention to developmental differences in response to instruction as children progress through elementary school. Using data from a large, nationally representative, longitudinal sample, this investigation demonstrated that procedural and conceptual instruction are both independently and jointly associated with changes in children's math achievement during elementary school. In addition, dynamic and complex associations between pedagogical approach and time emerged as children moved from kindergarten through 5th grade. Associations are discussed in further detail below.

Pedagogical approach & math skills

This investigation began by examining direct associations between instruction and math achievement over the course of elementary school. A focus on within-child associations allows for an examination of whether children are performing better or worse at times they experience more or less conceptual or procedural instruction, relative to what that individual child has already experienced or will go on to experience throughout elementary school. Correspondingly, this allows for an examination of how change in instructional input is associated with corresponding change in math achievement over time. This within-child approach also allows for greater causal inference by comparing each child to themselves, rather than between-child comparisons that evaluate associations relative to other children (Duncan, Magnuson, & Ludwig, 2004; Duncan & Raudenbush, 1999).

When considering procedural instruction, within-child coefficients suggest that children who experienced greater amounts of procedural instruction also demonstrated small but positive changes in their own math achievement, as compared to times when they received lesser amounts of procedural math instruction. In addition, the directionality of within- and between-child associations for procedural math is consistent with each other. For example, children who received greater amounts of procedural math across elementary school performed better on average. Allison (2005) suggests that if within- and between-child findings are similar, then the within estimator is preferred. As such, this lends credence to the beneficial associations of procedural math instruction during elementary school.

In contrast, within-child findings for conceptual instruction suggest that children demonstrated less growth when they experienced greater amounts of conceptual instruction than times that they experienced relatively less. Yet, between-child findings for conceptual instruction did *not* align with within-child associations. This inconsistency between within- and between-child findings suggests that important time-invariant selection effects such as the quality of instruction may be operating to bias between-child findings. Indeed, some evidence suggests children with more skilled teachers receive higher-quality conceptual instruction than those with lower-quality instructors (Smith, Desimone, & Ueno, 2005). In this case, Allison (2005) suggests the within estimator is preferred and the between estimator should not be interpreted. Thus, the results of this study contribute to a small but converging body of evidence that demonstrates beneficial associations of procedural instruction and math achievement, but few positive associations for conceptual instruction and math achievement during elementary school (Bodovski & Farkas, 2007a; Byrnes & Wasik, 2009; Crosnoe et al., 2010; Georges, 2009).

Dynamic associations of instruction with achievement across elementary school

A major aim of the current paper was to examine whether associations between instructional inputs and children's math achievement significantly change during the early school years. Indeed, the main effect instructional patterns were qualified by significant interactions with time. Significant two-way interactions at level-1 in the HLM analyses revealed that procedural instruction was positively associated with math growth in kindergarten, but associations became significantly less positive as children approached 5th grade. Conversely, conceptual instruction appeared to be more negatively associated with growth in math achievement for young children, but less so as they progressed through school. Such findings correspond with Piagetian theories of cognitive development, which posit that young children must acquire and master a basic set of procedures, facts, and algorithms before applying or extending them (Piaget, 1977; Rittle-Johnson & Alibali, 1999). Moreover, previous experimental work suggests that children utilize relatively simple strategies most frequently when they are learning new mathematical skills, and early in development (Carpenter & Moser, 1984). As procedural instruction promotes basic skill acquisition and computational skill development for young children (Bodovski & Farkas, 2007a; Georges, 2009), it is possible that an emphasis on this approach best promotes mastery of these simple yet necessary mathematical strategies (e.g., counting, carrying, and borrowing). Indeed, one limitation of discovery-based learning is that children need a strong knowledge base in which to perform effective problem solving to answer questions (Casey & Howson, 1993; Casey & Tucker, 1994). If conceptual instruction asks young children to problem-solve without sufficient content or procedural knowledge, such lessons may be less successful in promoting math learning.

In contrast, procedural instruction became less beneficial for achievement as children approached 5th grade, while associations between conceptual instruction and achievement went from negative to null. This suggests that procedural instruction may be less effective for older children, while conceptual instruction becomes less related, as opposed to negatively related, during this time period. Although 5th graders are increasingly asked to apply their knowledge to novel situations and problems (Hecht & Vagi, 2010, 2012), previous work finds that children have difficulty applying simple computational skills when problems change in even minor ways (Larkin, 1989), or when they are presented out-of-context (Fuchs et al., 2003). Procedural instruction may therefore be less strongly associated with achievement in later elementary school because it is less effective in promoting problem-solving skills, and more beneficial for basic calculation skills (Georges, 2009).

Balanced associations across elementary school

Although previous within-grade (Bodovski & Farkas, 2007a) and short-term (Byrnes & Wasik, 2009) investigations have demonstrated that elementary school teachers who spend time on both procedural and conceptual approaches tend to best facilitate algorithmic and applied forms of student math learning, the present study did not detect a significant 2-way interaction between conceptual and procedural instruction at either level-1 or level-2. However, complex associations were detected when examining the three-way interaction with conceptual instruction, procedural instruction, and time. In kindergarten, children exposed to the greatest amounts of procedural and least amounts of conceptual instructional strategies were the highest performing students; students whose teachers emphasized conceptual and de-emphasized procedural approaches achieved nearly 8% less and were the poorest-performing group. Moreover, procedural instruction was less strongly associated with kindergarteners' achievement when it was accompanied by high amounts of conceptual instruction. Yet, these associations substantially changed as children aged across elementary school. By the time children reached 5th grade, those who experienced high amounts of both conceptual and procedural instruction performed better than those exposed to a less balanced instructional format, with one approach favored over another. One unexpected pattern was found: students with teachers reporting balanced but low use of both approaches scored slightly better than those who more heavily favored one or the other. Teachers reporting lower levels of both instructional approaches may be under-reporters in general, or may utilize instructional techniques not identified by our factor analysis. However it is also important to note that this interaction term compares children who receive balanced vs. unbalanced instruction, rather than high or low amounts of conceptual or procedural input. This pattern therefore reaffirms the idea that a balanced approach to instruction (rather than an emphasis on either conceptual or procedural) may be most strongly associated with achievement in 5th grade.

Taken together, these findings indicate that high amounts of conceptual instruction may only be positively linked to young children's achievement when also presented in conjunction with high amounts of procedural instruction. This may be because 5th grade students are still developing the proficiency and automaticity in basic math skills and procedures (e.g., long division; pre-algebra) necessary for further applied work. In addition, results also suggest that procedural instruction remains an important component of children's math achievement throughout elementary school. Children may therefore continue to benefit from procedural approaches that are linked to smaller achievement gaps in basic calculation skills throughout the duration of elementary school (Desimone & Long, 2010; Georges, 2009). Alternatively, little evidence supports positive links between conceptual instruction and achievement, especially for younger children. Instead, conceptual instruction may be more beneficial for older children, especially in middle and high school (see Slavin, Lake, & Groff, 2009, for review), because it encourages the use of complex and efficient problem-solving strategies that can build upon, extend and ease transfer of prior knowledge acquired in earlier years (Hecht & Vagi, 2010, 2012; Karmiloff-Smith, 1986, 1992; Piaget, 1977; Rittle-Johnson & Alibali, 1999). Further investigation is needed to determine whether conceptual instruction becomes even more important in middle and high school, once children are called upon to demonstrate higher-order problem solving skills.

Limitations

Although the current investigation has many strengths, including the use of a large, longitudinal dataset with direct assessments of children's math achievement analyzed within a multi-level HLM framework, there are also several limitations. First, instructional approach was measured with annual teacher report instead of through observational assessment. The factor analyses of instructional items showed acceptable reliability, were conceptually rooted in theoretical writings on conceptual and procedural math instruction, and were comparable to past investigations on instructional style and achievement using the ECLS-K (Bodovski & Farkas, 2007a; Byrnes & Wasik, 2009; Xue & Meisels, 2004). Moreover, teachers have been considered to be reliable reporters of their own instructional practices (Porter, Kirst, Osthoff, Smithson, & Schneider, 1993). However, identifying convergent validity with regular logs of instruction or observational measures of teacher practice would be useful in future work (Rowan, Harrison & Hayes, 2004). Second, this study's measures of math instruction and achievement were assessed between 1998 and 2004, and thus, during the dawn of No Child Left Behind and far before implementation of common core standards. The common core represents considerable change in state-wide standards and assessment, and places further emphasis upon higher-order thinking skills (Porter, McMaken, Hwang, & Yang, 2011). As these changes likely filter into teachers' instructional style and delivery, it is important that future research takes advantage of newly emerging longitudinal and nationally representative data (e.g., ECLS-K:2011; U.S. Department of Education, National Center for Education Statistics, 2013). Third, this study was unable to paint a complete picture of children's learning experiences throughout the duration of elementary school because instructional data was unavailable in 2nd and 4th grades. Although this is the current state of large multi-state datasets that contain instructional information (e.g., ECLS-K; NICHD SECCYD), teachers do change from year-to-year and may utilize heterogeneous instructional styles that are lowly correlated with teacher practices from previous years. In the newly released ECLS-K:2011 Kindergarten Cohort 2nd grade data are forthcoming which will allow for a more thorough examination of longitudinal changes during the early grades. In the meantime, we believe this limitation is outweighed by the benefits of this nationally-representative longitudinal analysis of math instruction.

Finally, it is important to note that this study was based on correlational data, and that we cannot draw causal conclusions. At the same time, we have taken several steps that greatly enhance our ability to draw causal *inference* despite the correlational nature of the study. This includes the use of a 3-level HLM design with focus on the within-child effects, lagged covariates, and a host of controls. This methodology is widely utilized in econometric analyses, as such models permit far greater causal inference than traditional between-child models (Duncan, Magnuson, & Ludwig, 2004). At the same time, it is important to note that time-varying omitted variables may still bias estimates. For instance, children's early experiences with math learning and instruction can be quite heterogeneous at school entry (Magnuson, Ruhm, & Waldfogel, 2007), and may relate to how children experience procedural or conceptual math instruction. Though we controlled for children's preschool experiences and math skills at entry to kindergarten, assessments of early math and instructional exposure are unavailable within the ECLS-K and should be considered with further research. It is also possible that this study's pedagogical approach composites do not capture the full extent of instructional activities present within classrooms, especially during reading, writing, social studies, or science activities. In order to more accurately and thoroughly capture teachers' broader instructional approach, future investigations may consider conducting observational assessments of instruction within a wide swath of subjects throughout the school day.

Practical implications

The results of this study contribute to growing evidence that procedural skill development is promotive of children's math learning during this time, regardless of children's age, and that balanced instruction is associated with gains as children approach 5th grade (Byrnes & Wasik, 2009). Teachers may consider providing an emphasis upon foundational skills for young children in particular, while increasingly mixing in conceptual approaches in the later years of elementary school. Programs that have proven success in facilitating a balanced instructional approach, for example, techniques implemented in de-tracked classrooms (Crosnoe et al., 2010; Cohen, 1994), may be particularly useful later grades.

A key factor here, of course, is the effective delivery of well-implemented, engaging, and challenging instruction. Several large-scale observational studies of the classroom environment (e.g., the CLASS; Pianta, Karen, Paro, & Hamre, 2008) suggest that features of high-quality instructional support, characterized by frequent and effective use of instruction, regular provision of evaluative feedback, ongoing instructional conversations, and encouragement of child responsibilities are associated with engagement and achievement from pre-k through elementary school (Hamre & Pianta, 2005; Howes et al., 2008; La Paro et al., 2004; Mashburn et al., 2008). Exposing children to disorganized, poorly planned, low-quality, and overly simplistic instructional techniques may simply result in ineffective or lost instructional time. As children enter school with heterogeneous math skills that may affect how they respond to different forms of instruction, teachers should continue differentiating instruction to best meet student needs. Children may become bored and disengaged without high-quality and well-differentiated instruction, in which case even the most optimally balanced approach would fail to promote math learning. Disengagement may be a particular challenge in the older grades, as procedural complexity increases and teachers struggle to deliver procedural steps in appropriately differentiated, exciting, meaningful, or interesting way (Hill, Rowan, & Ball, 2005). Yet conceptual instruction is also challenging to implement well, and several studies find that teachers are not adequately prepared to implement high-quality conceptual pedagogy in their classrooms (Brown & Borko, 1992; Brown, Cooney, & Jones, 1990; Cooney, Shealy, & Arvold, 1998; Frykholm, 1999; Weiss, 1995). Crosnoe et al. (2010) further suggests that conceptual instruction could lead to increased frustration and conflict in the teacher–child relationships, particularly for those with fewer foundational math skills. Thus, teachers implementing a balanced approach may consider additional training in how to best navigate and integrate procedural and conceptual instruction for high-quality and supportive instructional delivery.

Conclusions

The results of this study represent an important step in understanding the developmental combination of instructional approaches that are dynamically associated with child achievement in elementary school. Though effect sizes typically only accounted for approximately half- to one-month's difference in math learning between children, it is compelling that key associations were largely detected at level-1 where the conservative within-child design accounted for time-invariant omitted variables such as child IQ or school readiness as well as a host of time-varying covariates. Between-child associations also addressed questions of simultaneity by including a lagged assessment of children's math achievement at school entry along with a variety of additional controls. Moreover, post-hoc analyses of this data suggest that main effects of procedural and conceptual instruction were not moderated by initial math skills at school entry, race/ethnicity, or public/private school status. An important next step is to determine whether observational measures of classroom instructional content are associated with child achievement, and to determine the extent to which quality of instruction moderates findings. It may also be beneficial to identify the combination of approaches different children need, as lower-skilled students may benefit from an emphasis on procedural instruction while highly skilled students may experience greater gains with an emphasis in conceptual instruction (Finn, 1993; Stipek, Feiler, Daniels, & Milburn, 1995). To our knowledge, this investigation is the first to consider how conceptual and procedural instruction interact over time to promote student achievement across six years of schooling. These findings provide important new insight into how both conceptual and procedural math instruction promote student achievement throughout elementary school, and raise a number of new questions regarding the most effective combination of instructional approaches that can be used to promote higher achievement for all children.

Appendix A

Procedural math instruction.

Math instruction: procedural items	Grade			
	K	First	Third	Fifth
	$\alpha = .80$	$\alpha = .82$	$\alpha = .60$	$\alpha = .70$
Adding single-digit numbers	0.69	0.65		
Subtracting single-digit numbers	0.67	0.65		
Adding double-digit numbers	0.49	0.62		
Subtracting two-digit numbers w/o regrouping	0.47	0.58		
Reading two-digit numbers	0.53	0.65		
Counting by 2s, 5s, 10s	0.53	0.55		
Counting beyond 100	0.52	0.56		
Reading three-digit numbers	0.53	0.57		
Writing numbers between 1 and 10	0.31	0.24		
Writing numbers between 1 and 100	0.54	0.46		
Mixed operations $(2 - 3 + 5)$	0.41	0.43		
Carrying numbers in addition	0.42	0.35		
Writing math equations to solve problems	0.51	0.49		
Recognizing value of coins & currency	0.51	0.60		
Complete problems on chalkboard	0.40	0.29		
Place value	0.53	0.64	0.61	0.72
Take math test			0.36	
Solve math problems in worksheet			0.37	
Skills & procedures to solve routine problems			0.67	0.77
Learning math facts/concepts			0.68	0.74
Number operations			0.53	0.67
Algebra & functions			0.31	0.36
Solve math problems in textbooks			0.31	0.31

Appendix B

Conceptual math instruction.

Math instruction: conceptual items	Grade			
	K	First	Third	Fifth
	$\alpha = .87$	$\alpha = .85$	$\alpha = .80$	$\alpha = .82$
Identifying relative quantity (estimation)	0.57	0.55		
Sorting objects into subgroups by rules	0.71	0.70		
Ordering objects by size or other properties	0.68	0.69		
Making, copying, or extending patterns	0.56	0.61		
Work in mixed-achievement groups	0.50	0.53		
Peer tutoring	0.51	0.52		
Use music to understand math	0.46	0.41		
Creative movement or drama	0.51	0.49		
Explain how problem is solved	0.58	0.48		
Work with geometric manipulatives	0.60	0.52		
Play math games	0.60	0.55		
Performing simple data collection	0.50	0.56		
Correspondence between number/quantity	0.54	0.42		
Estimating quantities	0.57	0.55	0.48	
Solve problems with partners/groups	0.62	0.62	0.62	
Use measuring instruments (i.e. rulers)	0.55	0.52	0.50	0.50
Work on real-life problems	0.62	0.60	0.63	0.67
Explain answers to math problems			0.61	
Write reports or do math projects			0.49	
Use a computer for math			0.33	
Conduct data analysis			0.52	0.60
Work with math manipulatives			0.62	0.50
Develop mathematical reasoning			0.64	0.77
Communicate math ideas effectively			0.65	0.72
Discuss math solutions with other kids			0.56	0.65
Use visual representations for solutions				0.58

References

Allison, P. D. (2005). *Fixed effects regression methods for longitudinal data using SAS*. SAS Institute.
Bodovski, K., & Farkas, G. (2007-aa). Do instructional practices contribute to inequality in achievement? The case of mathematics instruction in kindergarten. *Journal of Early Childhood Research*, *5*(3), 301–322.
Bodovski, K., & Farkas, G. (2007-bb). Mathematics growth in early elementary school: The roles of beginning knowledge, student engagement, and instruction. *The Elementary School Journal*, *108*, 115–130.
Brown, C. A., & Borko, H. (1992). Becoming a mathematics teacher. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 209–242). New York: Macmillan.
Brown, S. I., Cooney, T. J., & Jones, D. (1990). *Mathematics teacher education*. Handbook of research on teacher education, 639–656.
Burns, M., & Sibley, R. (2001). Math journals boost real learning. *Scholastic Instructor*, *36*(2), 18–20.
Byrnes, J. P., & Wasik, B. A. (2009). Factors predictive of math achievement in the primary grades: An opportunity-propensity analysis. *Contemporary Educational Psychology*, *34*, 167–183.
Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education*, *15*(3), 179–202.

Casey, M. B., & Howson, P. (1993). Educating pre-service students based on a problem-centered approach to teaching. *Journal of Teacher Education*, *44*(5), 361–369.
Casey, M. B., & Tucker, E. C. (1994). Problem-centered classrooms: Creating lifelong learners. *Phi Delta Kappan*, *76*(2), 139–143.
Cohen, E. G. (1994). *Designing groupwork: Strategies for the heterogeneous classroom.* New York, NY: Teachers College.
Cooney, T. J., Shealy, B. E., & Arvold, B. (1998). Reformizing belief structures of preservice secondary mathematics teachers. *Journal for Research in Mathematics Education*, *29*(3), 306–333.
Cortina, J. M. (1993). What is coefficient alpha? An examination of theory and applications. *Journal of Applied Psychology*, *78*(1), 98.
Crosnoe, R., Morrison, F., Burchinal, M., Pianta, R., Keating, D., Friedman, S. L., et al. (2010). Instruction, teacher–student relations, and math achievement trajectories in elementary school. *Journal of Educational Psychology*, *102*(2), 407.
Desimone, L. M., & Long, D. (2010). Teacher effects and the achievement gap: Do teacher and teaching quality influence the achievement gap between Black and White and high-and low-SES students in the early grades? *Teachers College Record*, *112*(12), 3024–3073.
Duncan, G. J., & Gibson-Davis, C. M. (2006). Connecting childcare quality to child outcomes: Drawing policy lessons from nonexperimental data. *Evaluation Review*, *30*(5), 611–630.
Duncan, G. J., Magnuson, K. A., & Ludwig, J. (2004). The endogeneity problem in developmental studies. *Research in Human Development*, *1*(1–2), 59–80.
Duncan, G. J., & Raudenbush, S. W. (1999). Assessing the effects of context in studies of child and youth development. *Educational Psychologist*, *34*, 29–41.
Dunn, L. M., & Dunn, L. M. (1981). *PPVT — revised manual.* Circle Pines, MN: American Guidance Service, Inc.
Dunn, L. M., & Markwardt, F. C. (1970). *Peabody Individual Achievement Test Manual.* Circle Pines, MN: American Guidance Service, Inc.
Finn, J. D. (1993). *School engagement and students at risk.* Washington, DC: National Center for Education Statistics.
Frykholm, J. A. (1999). The impact of reform: Challenges for mathematics teacher preparation. *Journal of Mathematics Teacher Education*, *2*(1), 79–105.
Fuchs, L. S., Fuchs, D., Prentice, K., Burch, M., Hamlett, C. L., Owen, R., et al. (2003). Explicitly teaching for transfer: Effects on third-grade students' mathematical problem solving. *Journal of Educational Psychology*, *95*(2), 293.
Fuson, K. C., Kalchman, M., & Bransford, J. D. (2005). Mathematical understanding: An introduction. In M. S. Donovan, & J. D. Bransford (Eds.), *How students learn: History, mathematics, and science in the classroom* (pp. 217–256). Washington D.C.: The National Academies Press.
Gamoran, A. (2001). Beyond curriculum wars: Content and understanding in mathematics. In T. Loveless (Ed.), *The great curriculum debate: How should we teach reading and math?* (pp. 134–142). Washington, DC: Brookings Institution Press.
Georges, A. (2009). Relation of instruction and poverty to mathematics achievement gains during kindergarten. *Teachers College Record*, *111*(9), 2148–2178.
Goodman, K. S. (1989). Whole-language research: Foundations and development. *The Elementary School Journal*, *90*(2), 207–221.
Griffin, S. (2005). Fostering the development of whole-number sense: Teaching mathematics in the primary grades. In M. S. Donovan, & J. D. Bransford (Eds.), *How students learn: History, mathematics, and science in the classroom* (pp. 257–307). Washington D.C.: The National Academies Press.
Hair, J. F., Tatham, R. L., Anderson, R. E., & Black, W. (1998). *Multivariate data analysis* (5th ed.). London: Prentice-Hall.
Hamilton, L. S., McCaffrey, D., Klein, S. P., Stecher, B. M., Robyn, A., & Bugliari, D. (2003). Studying large-scale reforms of instructional practice: An example from mathematics and science. *Educational Evaluation and Policy Analysis*, *25*(1), 1–29.
Hamre, B. K., & Pianta, R. C. (2005). Can instructional and emotional support in the first-grade classroom make a difference for children at risk of school failure? *Child Development*, *76*(5), 949–967.
Hecht, S. A., & Vagi, K. J. (2010). Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology*, *102*(4), 843.
Hecht, S. A., & Vagi, K. J. (2012). Patterns of strengths and weaknesses in children's knowledge about fractions. *Journal of Experimental Child Psychology*, *111*(2), 212–229.
Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371–406.
Howes, C., Burchinal, M., Pianta, R., Bryant, D., Early, D., Clifford, R., et al. (2008). Ready to learn? Children's pre-academic achievement in pre-kindergarten programs. *Early Childhood Research Quarterly*, *23*(1), 27–50.
Karmiloff-Smith, A. (1986). From meta-processes to conscious access: Evidence from children's metalinguistic and repair data. *Cognition*, *23*(2), 95–147.
Karmiloff-Smith, A. (1992). *Beyond modularity: A developmental perspective on cognitive science.* Cambridge, MA: MIT Press.
Klein, S. P., Hamilton, L. S., McCaffrey, D. F., Stecher, B. M., Robyn, A., & Burroughs, D. (2000). Teaching practices and student achievement. *Report of first-year results from the Mosaic study of systemic initiatives in mathematics and science (MR-1233-EDU)*. Santa Monica, CA: RAND.
La Paro, K. M., Pianta, R. C., & Stuhlman, M. (2004). The classroom assessment scoring system: Findings from the prekindergarten year. *The Elementary School Journal*, 409–426.
Larkin, J. H. (1989). What kind of knowledge transfers? In L. B. Renick (Ed.), *Knowing, learning, and instruction* (pp. 283–305). Hillsdale, NJ: Erlbaum.
Le, V., Stecher, B. M., Lockwood, J. R., Hamilton, L. S., Robyn, A., Williams, V., et al. (2006). Improving mathematics and science education. *A longitudinal investigation of the relationship between reform-oriented instruction and student achievement*. Santa Monica, CA: RAND.
Magnuson, K. A., Ruhm, C., & Waldfogel, J. (2007). The persistence of preschool effects: Do subsequent classroom experiences matter? *Early Childhood Research Quarterly*, *22*(1), 18–38.
Markwardt, F. C. (1989). *Peabody individual achievement test — Revised.* Circle Pines, MN: American Guidance Service, Inc.
Mashburn, A. J., Pianta, R. C., Hamre, B. K., Downer, J. T., Barbarin, O. A., Bryant, D., et al. (2008). Measures of classroom quality in prekindergarten and children's development of academic, language, and social skills. *Child Development*, *79*(3), 732–749.
National Mathematics Advisory Panel (NMAP) (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* Washington, DC: U.S. Department of Education.
Piaget, Jean (1977). *The development of thought: Equilibrium of cognitive structures.* New York: Viking Press.
Pianta, R. C., Karen, M., Paro, L., & Hamre, B. K. (2008). *Classroom Assessment Scoring System (CLASS) Manual: K-3.* Paul H: Brookes Publishing Company.
Porter, A. C., Kirst, M. W., Osthoff, E. J., Smithson, J. S., & Schneider, S. A. (1993). Reform up close: An analysis of high school mathematics and science classrooms. *Final report to the National Science Foundation on grant no. SPA-8953446 to the Consortium for Policy Research in Education*. Madison, WI: University of Wisconsin-Madison, Wisconsin Center for Education Research.
Porter, A., McMaken, J., Hwang, J., & Yang, R. (2011). Common core standards the new US intended curriculum. *Educational Researcher*, *40*(3), 103–116.
Preacher, K. J., Curran, P. J., & Bauer, D. J. (2006). Computational tools for probing interaction effects in multiple linear regression, multilevel modeling, and latent curve analysis. *Journal of Educational and Behavioral Statistics*, *31*, 437–448.
Raudenbush, S., & Bryk, A. (2002). *Hierarchical linear models: Applications and data analysis methods* (2nd ed.). Newbury Park, CA: Sage.
Raudenbush, S. W., Bryk, A. S., & Congdon, R. (2008). *HLM 6.06.* Lincolnwood, IL: Scientific Software International.
Rittle-Johnson, B., & Alibali, M. W. (1999). Reform and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology*, *91*(1), 175.
Rock, D. A., & Pollack, J. M. (2002). *Psychometric report for kindergarten through first grade.* Early childhood longitudinal study — Kindergarten class 1998–1999 (ECLS-K). Washington, DC: US Department of Education, National Center for Educational Statistics.
Rosenshine, B., & Stevens, R. (1986). Teaching functions. In M. Wittrock (Ed.), *Handbook of Research on Teaching* (pp. 376–391). Macmillan Publishing Co.
Rowan, B., Harrison, D. M., & Hayes, A. (2004). Using instructional logs to study mathematics curriculum and teaching in the early grades. *The Elementary School Journal*, *105*(1), 103–127.
Shrouse, R. (2001). The impact of traditional and reform-style practices on student mathematics achievement. In T. Loveless (Ed.), *The great curriculum debate* (pp. 108–133). Washington, DC: Brookings.
Singer, J., & Willett, J. B. (2003). *Applied longitudinal data analysis: Modeling change and event occurrence.* Oxford, England: University Press.
Slavin, R. E., Lake, C., & Groff, C. (2009). Effective programs in middle and high school mathematics: A best-evidence synthesis. *Review of Educational Research*, *79*(2), 839–911.
Snijders, T. A., & Bosker, R. J. (1993). Standard errors and sample sizes for two-level research. *Journal of Educational and Behavioral Statistics*, *18*(3), 237–259.
Smith, T. M., Desimone, L. M., & Ueno, K. (2005). "Highly qualified" to do what? The relationship between NCLB teacher quality mandates and the use of reform-oriented instruction in middle school mathematics. *Educational Evaluation and Policy Analysis*, *27*(1), 75–109.
Stipek, D., Feiler, R., Daniels, D., & Milburn, S. (1995). Effects of different instructional approaches on young children's achievement and motivation. *Child Development*, *66*(1), 209–223.
U.S. Department of Education, National Center for Education Statistics. (2013) (2013)). *Early Childhood Longitudinal Study, Kindergarten Class of 2010–11 (ECLS-K:2011), Restricted-Use Kindergarten Data File and Electronic Codebook (NCES 2013-060). CD-ROM.* Washington, D.C.: U.S. Department of Education, National Center for Education Statistics.
U.S. Department of Education, National Center for Education Statistics (2006). *Early Childhood Longitudinal Study, Kindergarten Class of 1998–99: ECLS-K longitudinal kindergarten–fifth grade public-use data file and electronic code book.* NCES Number: 2006035 Washington, D.C.: U.S. Department of Education, National Center for Education Statistics.
Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Boston: Harvard University Press.
Weiss, I. R. (1995). *A profile of science and mathematics education in the United States.* Chapel Hill, NC: Horizon Research, Inc.
Woodcock, R. W., & Johnson, M. B. (1989, 1990). *Woodcock–Johnson psycho-educational battery — Revised.* Itasca, IL: Riverside Publishing.
Xue, Y., & Meisels, S. J. (2004). Early literacy instruction and learning in kindergarten: Evidence from the Early Childhood Longitudinal Study — Kindergarten Class of 1998–1999. *American Educational Research Journal*, *41*(1), 191–229.