Motion in 1D

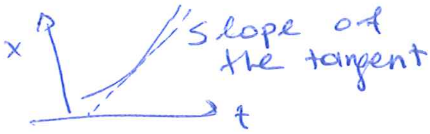
Definitions:

position x in meters (m)

displacement $\Delta x = x_f - x_i$ x_i: initial position, x_f: final "

average velocity $v_{x,avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$ in $\frac{m}{s}$

velocity $v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$



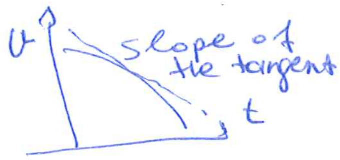
average speed $= \frac{\text{distance}}{\Delta t}$ in $\frac{m}{s}$, positive

speed $|v_x|$ always positive

average acceleration $= \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{\Delta t}$ v_{xi}: initial velocity, v_{xf}: final velocity

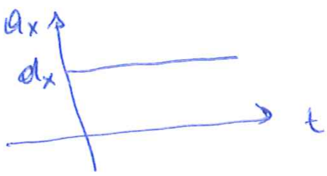
unit: $\frac{m}{s^2}$

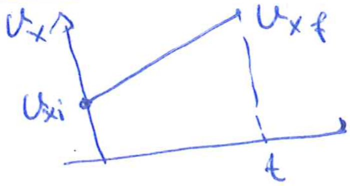
acceleration $a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$

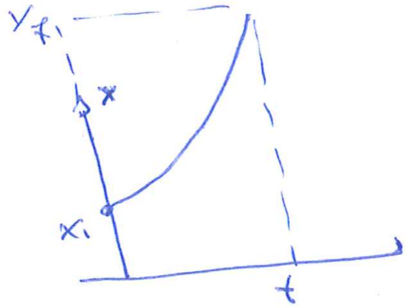


$a_x = \frac{d^2x}{dt^2}$

Constant acceleration motion:







$a_x =$ constant

$v_{xf} = v_{xi} + a_x t$

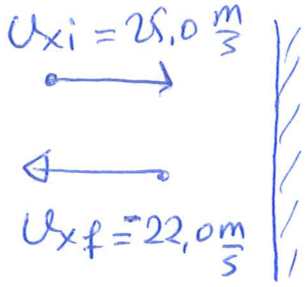
$v_{x,avg} = \frac{v_{xi} + v_{xf}}{2}$

$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$

$v_{xf}^2 = v_{xi}^2 + 2a_x (x_f - x_i)$

$v_{xf}^2 = v_{xi}^2 + 2a_x \Delta x$

14. **W** **Review.** A 50.0-g Super Ball traveling at 25.0 m/s bounces off a brick wall and rebounds at 22.0 m/s. A high-speed camera records this event. If the ball is in contact with the wall for 3.50 ms, what is the magnitude of the average acceleration of the ball during this time interval?



$$\Delta t = 3,50 \text{ ms} = 3,50 \times 10^{-3} \text{ s}$$

$$\Delta v_x = v_{xf} - v_{xi} = -22 - 25 \frac{m}{s} = -47,0 \frac{m}{s}$$

$$a_{x,avg} = \frac{\Delta v_x}{\Delta t} = \frac{-47,0 \frac{m}{s}}{3,50 \times 10^{-3} \frac{m}{s}} = -1,34 \times 10^{4} \frac{m}{s^2}$$

21. M A particle moves along the x axis according to the equation $x = 2.00 + 3.00t - 1.00t^2$, where x is in meters and t is in seconds. At $t = 3.00$ s, find (a) the position of the particle, (b) its velocity, and (c) its acceleration.

(a) $x(t=3{,}0\,s) = 2{,}00 + 3{,}00 \cdot (3.00) - 1.00 \cdot (3.00)^2$

$= 2 + 9 - 9 = 2{,}00\ m$

(b) velocity at any moment

$\frac{dt^n}{dt} = n t^{n-1}$

$v_x = \frac{dx}{dt} = \frac{d}{dt}(2 + 3t - t^2)$

$v_x = 3 - 2t$

at $t = 3{,}0\,s$

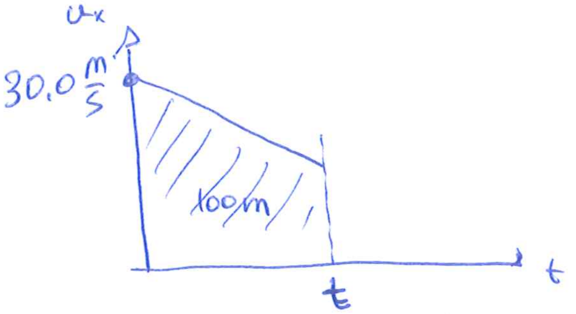
$v_x = 3 - 2 \cdot (3) = -3{,}00\ \frac{m}{s}$

(c) acceleration at any time

constant acceleration

$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(3 - 2t) = -2{,}00\ m/s^2$

26. A speedboat moving at 30.0 m/s approaches a no-wake buoy marker 100 m ahead. The pilot slows the boat with a constant acceleration of $-3.50\ \text{m/s}^2$ by reducing the throttle. (a) How long does it take the boat to reach the buoy? (b) What is the velocity of the boat when it reaches the buoy?



$a_x = -3.50\ \frac{m}{s^2}$

(a) $v_{xf} = 30 - 3.50\, t$

$x_f = x_i + 30t + \frac{1}{2}(-3.50)t^2$

$x_f - x_i = 30t - \frac{3.50}{2}t^2$

$100 = 30t - 1.75t^2$

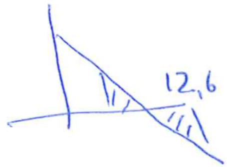
$1.75t^2 - 30t + 100 = 0$

$\sqrt{\Delta} = \sqrt{b^2 - 4ac}$

$\sqrt{\Delta} = \sqrt{30^2 - 4 \cdot (1.75) \cdot 100}$

$\sqrt{\Delta} = \sqrt{200}$

$t_{1,2} = \frac{-b \mp \sqrt{\Delta}}{2a} = \frac{30 \mp \sqrt{200}}{2 \cdot 1.75}$



$t_{1,2} = \frac{30 \mp \sqrt{200}}{3.5} = 12.6\ s$ or $\boxed{4.53\ s}$

physical answer

b) $v_{xf} = 30 - 3.50\, t$

$= 30 - 3.50 \cdot (4.53) = 14.1\ \frac{m}{s}$

29. **M** An object moving with uniform acceleration has a velocity of 12.0 cm/s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00 s later is −5.00 cm, what is its acceleration?

$t = 2{,}00\ s$ $\qquad\qquad$ $t = 0$

$x_f = -5{,}00\ cm$ $\qquad$ x_0 $\qquad$ $x_i = 3{,}00\ cm$

$v_{xi} = 12.0\ \frac{cm}{s}$

$$x_f = x_i + v_{xi} \cdot t + \frac{1}{2} a_x t^2$$

$$-5{,}00\ cm = 3{,}00\ cm + 12{,}00\ \frac{cm}{s} \cdot 2.00\ s + \frac{1}{2} a_x (2)^2$$

$$2 a_x s^2 = -5{,}00\ cm - 3{,}00\ cm - 24.0\ cm = -32{,}0\ cm$$

$$a_x = -16.0\ \frac{cm}{s^2}$$

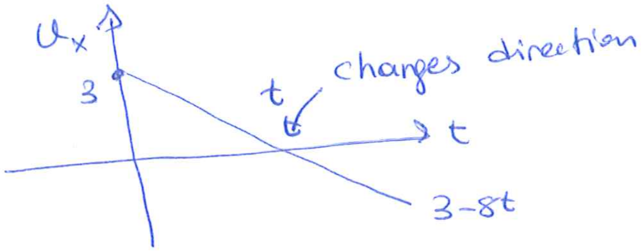
38. A particle moves along the x axis. Its position is given **W** by the equation $x = 2 + 3t - 4t^2$, with x in meters and t in seconds. Determine (a) its position when it changes direction and (b) its velocity when it returns to the position it had at $t = 0$.

$$x = 2 + 3t - 4t^2$$

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(2+3t-4t^2) = 3 - 4 \cdot 2 \cdot t = 3 - 8t$$

$$a_x = \frac{dv_x}{dt} = -8 \text{ m/s}^2$$

(a)



$$3 - 8t = 0$$

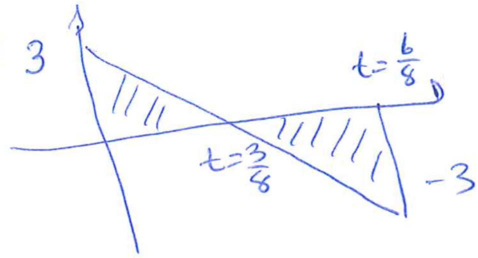
$$\boxed{t = \frac{3}{8} \text{ s}}$$

$$x\left(t=\frac{3}{8}\right) = 2 + 3 \cdot \frac{3}{8} - 4 \cdot \frac{9}{8 \cdot 8}$$

$$x\left(t=\frac{3}{8}\right) = 2 + \frac{9}{8} - \frac{9}{16} = 2 + \frac{9}{16} = \frac{41}{16} \text{ m}$$

$$x\left(t=\frac{3}{8}\text{ s}\right) = 2.56 \text{ m}$$

(b)



$\Delta x = 0$, the two areas are the sam

from the symmetry
-3 m/s

or $x(t=0) = x(t)$

$$2 = 2 + 3t - 4t^2$$

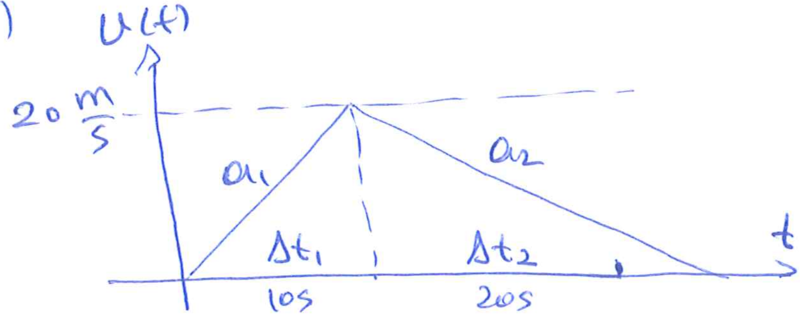
$$4t^2 = 3t$$

$t = 0$, or $t = \frac{3}{4}$ s

$$v_x = 3 - 8t = 3 - 8 \cdot \frac{3}{4} = -3 \text{ m/s}$$

•25 An electric vehicle starts from rest and accelerates at a rate of 2.0 m/s² in a straight line until it reaches a speed of 20 m/s. The vehicle then slows at a constant rate of 1.0 m/s² until it stops. (a) How much time elapses from start to stop? (b) How far does the vehicle travel from start to stop? (c) Graph $x(t)$, $v(t)$, and $a(t)$.

(a)



$a_1 = \frac{\Delta v_1}{\Delta t_1}$

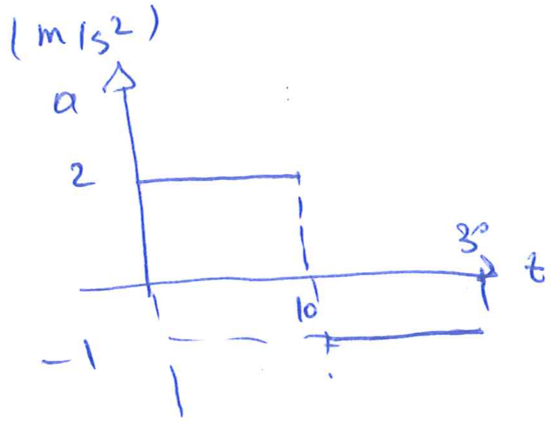
$\Delta t_1 = \Delta v_1 / a_1 = \frac{20}{2} = 10s$

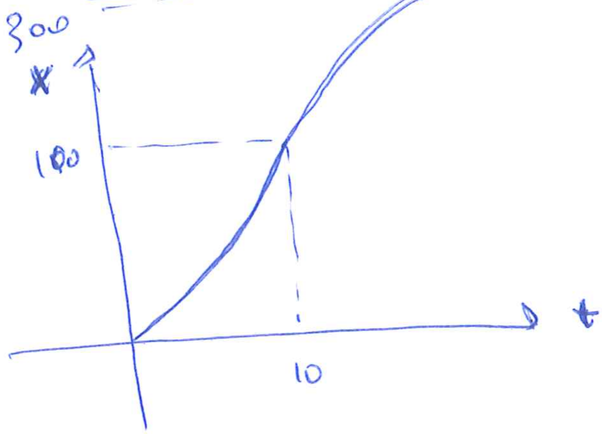
$\Delta t_2 = \Delta v_2 / a_2 = \frac{-20}{-1.0} = 20s$

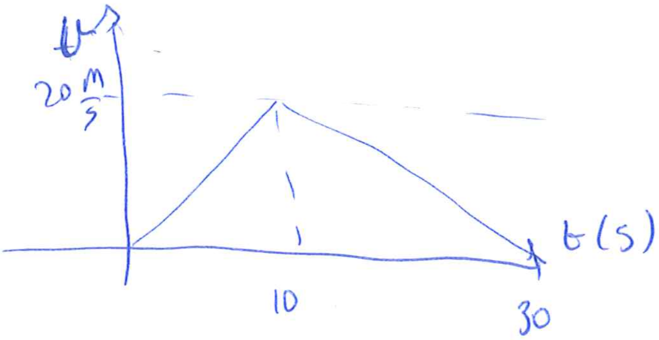
$\Delta t_1 + \Delta t_2 = 30s$

(b) $\Delta x =$ area under the graph

$$\frac{20\cdot(\Delta t_1 + \Delta t_2)}{2} = \frac{20\cdot 30}{2} = 300\ m$$







•33 SSM ILW A car traveling 56.0 km/h is 24.0 m from a barrier when the driver slams on the brakes. The car hits the barrier 2.00 s later. (a) What is the magnitude of the car's constant acceleration before impact? (b) How fast is the car traveling at impact?

$$\overset{\cancel{28}\ 14}{\cancel{56}.0}\ \frac{km}{h} \cdot \frac{100\cancel{0}\ m}{1\ km} \cdot \frac{1\ h}{\underset{18\ 9}{36\cancel{00}\ s}} = \frac{140}{9}\ \frac{m}{s}$$

(a) $x_f = x_i + v_{xi} \cdot t + \frac{1}{2} a_x t^2$

$x_f - x_i = v_{xi}\, t + \frac{1}{2} a_x t^2$

$t = 2.0\,s$

$$24\,m = \frac{140}{9} \cdot (2) + \frac{1}{2} a_x (2)^2$$

$$\overset{12}{\cancel{24}} - \frac{\overset{140}{\cancel{280}}}{9} = \cancel{2} a_x$$

$$a_x = 12 - \frac{140}{9} = -\frac{32}{9}\ \frac{m}{s^2} = -3.56\ \frac{m}{s^2}$$

(b) $v_{xf} = v_{xi} + a_x t$

$$= \frac{140}{9} - \frac{32}{9} \cdot 2 = \frac{140 - 64}{9} = \frac{76}{9} = 8.44\ \frac{m}{s}$$

••41 GO As two trains move along a track, their conductors suddenly notice that they are headed toward each other. Figure 2-31 gives their velocities v as functions of time t as the conductors slow the trains. The figure's vertical scaling is set by v_s = 40.0 m/s. The slowing processes begin when the trains are 200 m apart. What is their separation when both trains have stopped?

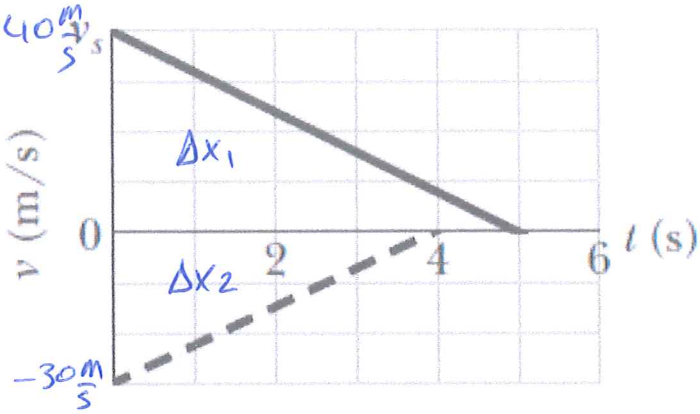


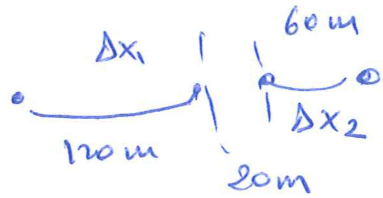
Figure 2-31 Problem 41.

$$\Delta x_1 = \frac{40 \cdot 6}{2} = 120\text{ m}$$

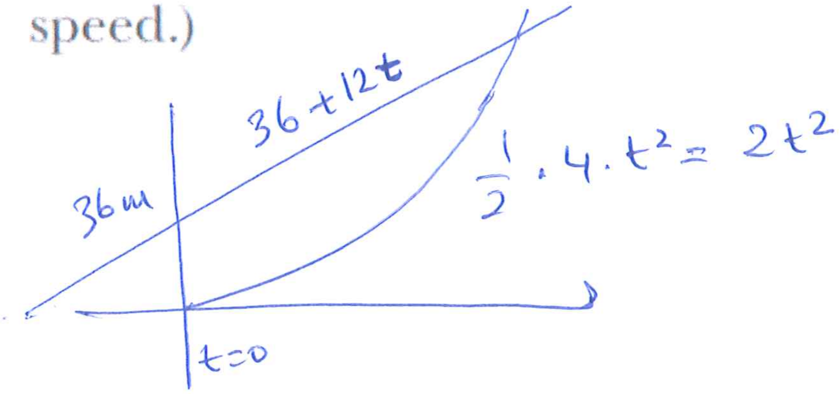
$$|\Delta x_2| = \left|\frac{30 \cdot 4}{2}\right| = 60\text{ m}$$

$$|\Delta x| = |\Delta x_1| + |\Delta x_2| = 180\text{ m}$$

$$200 - 180\text{ m} = 20\text{ m}$$



44. A hockey player is standing on his skates on a frozen pond when an opposing player, moving with a uniform speed of 12.0 m/s, skates by with the puck. After 3.00 s, the first player makes up his mind to chase his opponent. If he accelerates uniformly at 4.00 m/s^2, (a) how long does it take him to catch his opponent and (b) how far has he traveled in that time? (Assume the player with the puck remains in motion at constant speed.)



(a)

$$12(t+3) = 2t^2$$

$$12t+36 = 2t^2$$

$$\Delta = b^2 - 4ac = 36 + 72 = 108$$

$$t^2 - 6t - 18 = 0$$

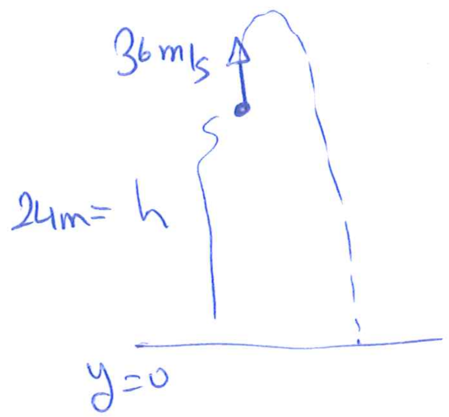
$$t = \frac{6 \mp \sqrt{108}}{2} = \frac{6 \mp 6\sqrt{3}}{2} = 3(1 \mp \sqrt{3})$$

$t_1 = -2{,}20\,s$, $\boxed{t_2 = 8{,}20\,s}$ physical Solution

(b)

$$2t^2 = 36 + 12t = 2.(8{,}20)^2 = 134\ m$$

50. The height of a helicopter above the ground is given by $h = 3.00t^3$, where h is in meters and t is in seconds. At $t = 2.00$ s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?



$t = 2.00\,s$

$h = \cancel{3t^3} = 3\cdot(2)^3 = 24\,m$

$v_y = \frac{dh}{dt} = 9t^2$

$= 9\cdot(2)^2 = 36\,m/s$

$y_f = y_i + v_{yi}\,t - \frac{1}{2}g\,t^2$

$0 = 24 + 36t - 4{,}90\,t^2$

$4{,}90t^2 - 36t - 24 = 0$

$\Delta = b^2 - 4ac$

$\Delta = (36)^2 + 4\cdot 24\cdot 4{,}90$

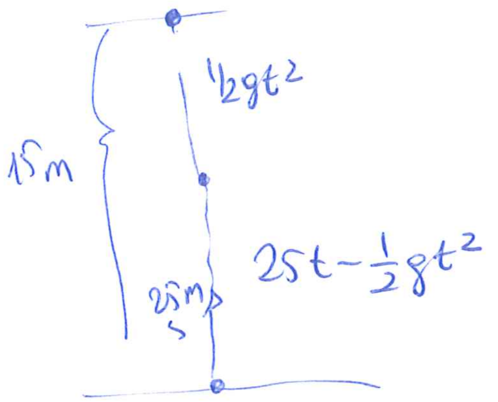
$\Delta = 1766.4$

$t_{1,2} = \frac{36 \mp \sqrt{1766{,}4}}{9{,}80}$

$\boxed{t_{1,2} = 7{,}96\ s}$, $-0{,}615$

$\boxed{7.96\,s}$

52. **M** A ball is thrown upward from the ground with an initial speed of 25 m/s; at the same instant, another ball is dropped from a building 15 m high. After how long will the balls be at the same height above the ground?

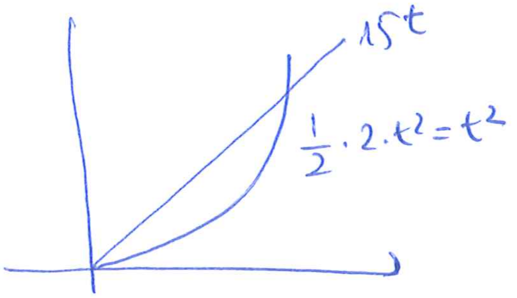


$$25t - \frac{1}{2}gt^2 + \frac{1}{2}gt^2 = 15\text{ m}$$

$$25t = 15\text{ m}$$

$$t = \frac{15}{25} = 0.60\text{s}$$

77. A motorist drives along a straight road at a constant speed of 15.0 m/s. Just as she passes a parked motorcycle police officer, the officer starts to accelerate at 2.00 m/s^2 to overtake her. Assuming that the officer maintains this acceleration, (a) determine the time interval required for the police officer to reach the motorist. Find (b) the speed and (c) the total displacement of the officer as he overtakes the motorist.



(a) $t^2 = 15t$

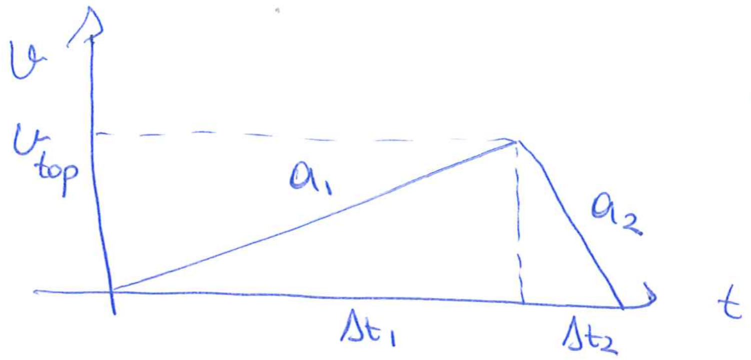
$t = 0, \quad t = 15\,s$

(b) $v_{xf} = v_{xi} + a_x t$

$= 0 + 2 \cdot 15\,s = 30{,}0\,m/s$

(c) $15t = t^2 = (15)^2 = 225\,m$

78. A commuter train travels between two downtown stations. Because the stations are only 1.00 km apart, the train never reaches its maximum possible cruising speed. During rush hour the engineer minimizes the time interval Δt between two stations by accelerating at a rate $a_1 = 0.100\ \text{m/s}^2$ for a time interval Δt_1 and then immediately braking with acceleration $a_2 = -0.500\ \text{m/s}^2$ for a time interval Δt_2. Find the minimum time interval of travel Δt and the time interval Δt_1.



$$v_{top} = a_1 \Delta t_1 = |a_2| \Delta t_2$$

$$0{,}1\, \Delta t_1 = 0{,}5\, \Delta t_2$$

$$\Delta t_1 = 5 \Delta t_2$$

$$\Delta t = \Delta t_1 + \Delta t_2 = 6 \cdot \Delta t_2$$

$$v_{top} = 0{,}5\, \Delta t_2$$

$$\text{area} = \Delta x = \frac{(\Delta t_1 + \Delta t_2) \cdot v_{top}}{2}$$

$$1000\text{ m} = \frac{\cancel{6}^{3}\, \Delta t_2 \cdot 0{,}5\, \Delta t_2}{\cancel{2}}$$

$$\Delta t_2 = \sqrt{\frac{1000}{1{,}5}} = 25{,}82\text{ s}$$

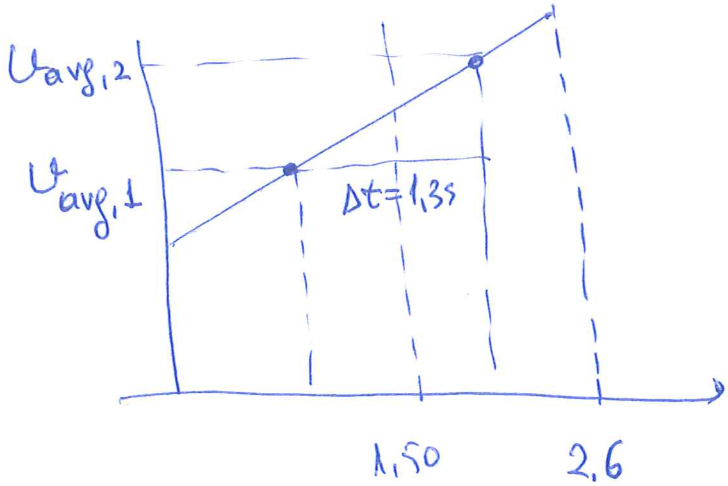
$$\Delta t_1 = 5 \cdot \Delta t_2 = 5 \cdot \sqrt{\frac{2000}{3}}$$

$$= 129\text{ s}$$

$$\Delta t_2 = \sqrt{\frac{2000}{3}} = 25{,}82$$

$$\Delta t = 155\text{ s}$$

79. Liz rushes down onto a subway platform to find her train already departing. She stops and watches the cars go by. Each car is 8.60 m long. The first moves past her in 1.50 s and the second in 1.10 s. Find the constant acceleration of the train.



$$v_{avg,1} = \frac{8,60}{1,50}$$

$$v_{avg,2} = \frac{8,60}{1,10}$$

$$a = \frac{\Delta v_{avg}}{1,35} = \frac{\frac{8,60}{1,10} - \frac{8,60}{1,50} \text{ m/s}}{1,3 \text{ s}}$$

$$= 1,60 \frac{m}{s^2}$$

••69 ILW How far does the runner whose velocity–time graph is shown in Fig. 2-40 travel in 16 s? The figure's vertical scaling is set by $v_s = 8.0$ m/s.

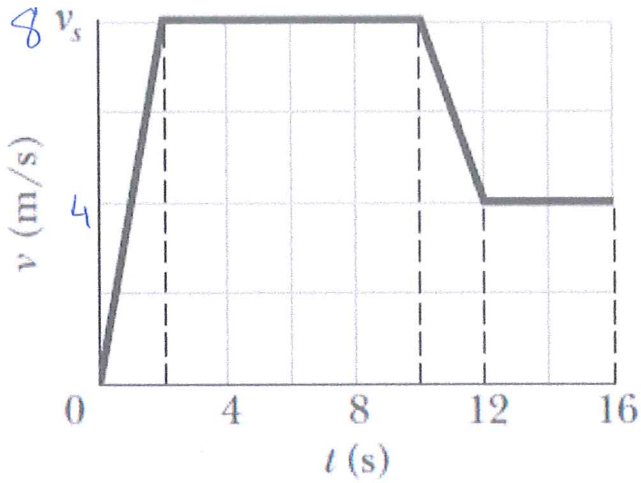


Figure 2-40 Problem 69.

Area under the curve

$$\frac{1}{2}\cdot 2\cdot 8 + 8\cdot 8 + \frac{(8+4)\cdot 2}{2} + 4\cdot 4$$

$$= 8 + 64 + 12 + 16$$

$$= 100 \text{ m}$$