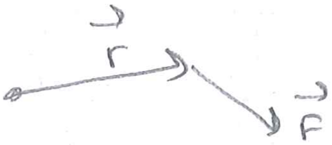
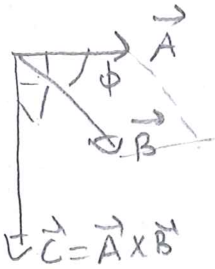
Angular Momentum



Torque $\vec{\tau} = \vec{r} \times \vec{F}$

Vector Product:

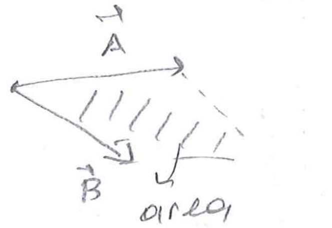
$\vec{C} = \vec{A} \times \vec{B}$



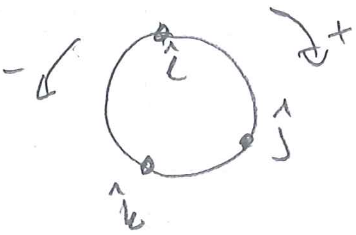
$\vec{C} \perp \vec{A}$, $\vec{C} \perp \vec{B}$

$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$

$|\vec{C}| = C = |\vec{A}|\ |\vec{B}| \sin\phi$



$\vec{A} \times \vec{A} = 0$



$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{i} \times \hat{k} = -\hat{j}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{j} \times \hat{i} = -\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= \hat{i}\,(A_y B_z - B_y A_z) - \hat{j}\,(A_x B_z - B_x A_z) + \hat{k}\,(A_x B_y - B_x A_y)$$

Angular Momentum

$\vec{L} = \vec{r} \times \vec{p}$

$\vec{L} \perp \vec{r}$

$\vec{L} \perp \vec{p}$

$\sum \vec{\tau} = \dfrac{d\vec{L}}{dt}$ similar to $\sum \vec{F} = \dfrac{d\vec{p}}{dt}$

$\sum \vec{\tau}_{ext} = \dfrac{d\vec{L}_{tot}}{dt}$

$\Delta \vec{L}_{tot} = \int \vec{\tau}_{ext}\, dt$

$\sum \vec{\tau}_{int} = 0$ internal forces produce zero net torque

Angular Momentum of Rigid Object

$$L = I\omega$$

$$\sum \tau_{ex} = \frac{dL}{dt} = I\,\frac{d\omega}{dt} = I\alpha$$

Isolated system (or $\sum \vec{\tau}_{ext} = 0$)

$$\frac{d\vec{L}_{tot}}{dt} = 0 \Rightarrow \vec{L}_{total} = \text{constant}$$

conservation of angular momentum

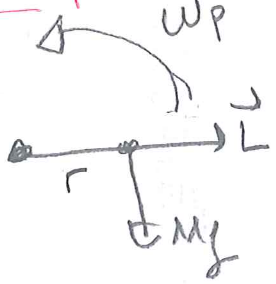
If I changing

$$I_i\omega_i = I_f\omega_f$$

Inelastic Angular Collision

$$(I_1 + I_2)\,\omega_f = I_1\omega_1 + I_2\omega_2$$

Gyroscope

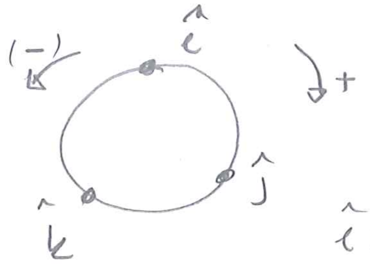


$$\frac{Mgr}{L} = \omega_p \quad \text{precession frequency}$$

$$L = I\omega$$

$$\boxed{\omega_p = \frac{Mgr}{I\omega}}$$

1. Given $\vec{\mathbf{M}} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\vec{\mathbf{N}} = 4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, calculate the vector product $\vec{\mathbf{M}} \times \vec{\mathbf{N}}$.



$\hat{i} \times \hat{j} = \hat{k}$

$\hat{i} \times \hat{k} = -\hat{j}$

$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

$\hat{i} \times \hat{k} = -\hat{j}$

$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{k} \times \hat{j} = -\hat{i}$

1st method:

$$\vec{M} \times \vec{N} = (2\hat{i} - 3\hat{j} + \hat{k}) \times (4\hat{i} + 5\hat{j} - 2\hat{k})$$

$$= 8\underbrace{\hat{i}\times\hat{i}}_{0} + 10\underbrace{\hat{i}\times\hat{j}}_{\hat{k}} - 4\underbrace{\hat{i}\times\hat{k}}_{-\hat{j}}$$

$$-12\underbrace{\hat{j}\times\hat{i}}_{-\hat{k}} - 15\underbrace{\hat{j}\times\hat{j}}_{0} + 6\underbrace{\hat{j}\times\hat{k}}_{\hat{i}}$$

$$+4\underbrace{\hat{k}\times\hat{i}}_{+\hat{j}} + 5\underbrace{\hat{k}\times\hat{j}}_{-\hat{i}} - 2\underbrace{\hat{k}\times\hat{k}}_{0}$$

$$= 10\hat{k} + 4\hat{j} + 12\hat{k} + 6\hat{i} + 4\hat{j} - 5\hat{i}$$

$$= \hat{i} + 8\hat{j} + 22\hat{k}$$

2nd method

$$\vec{M} \times \vec{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 4 & 5 & -2 \end{vmatrix} = \hat{i}\begin{vmatrix} -3 & 1 \\ 5 & -2 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & 1 \\ 4 & -2 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -3 \\ 4 & 5 \end{vmatrix}$$

$$= \hat{i}(6-5) - \hat{j}(-4-4) + \hat{k}(10+12)$$

$$= \hat{i} + 8\hat{j} + 22\hat{k}$$

11. **M** A light, rigid rod of length $\ell = 1.00$ m joins two particles, with masses $m_1 = 4.00$ kg and $m_2 = 3.00$ kg, at its ends. The combination rotates in the xy plane about a pivot through the center of the rod (Fig. P11.11). Determine the angular momentum of the system about the origin when the speed of each particle is 5.00 m/s.

+ direction

$\odot \ \vec{L}$

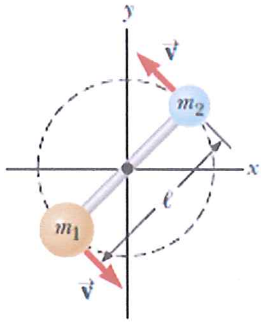
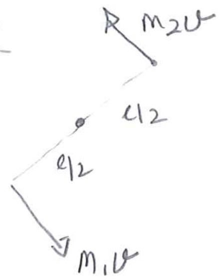


Figure P11.11

1st method



$L_1 = m_1 v \frac{\ell}{2}$ $\boxed{L = rp}$

$L_2 = m_2 v \, \ell/2$

$L = L_1 + L_2 = (m_1 + m_2) v \frac{\ell}{2}$

$L = (4+3) \cdot 5 \cdot \frac{1}{2} = 17{,}5 \ \text{kg} \frac{m^2}{s}$

$(+z)$ yönünde $\vec{L} = (17{,}5 \, \hat{k}) \ \text{kg} \frac{m^2}{s}$

2nd method:

$I = m_1 \left(\frac{\ell}{2}\right)^2 + m_2 \left(\frac{\ell}{2}\right)^2 = \frac{(m_1 + m_2)\ell^2}{4} = \frac{7 \cdot 1}{4} = 1{,}75 \ \text{kg} \cdot m^2$

$\frac{\ell}{2} \omega = v \Rightarrow \omega = \frac{2v}{\ell} = \frac{2 \cdot 5}{1} = 10.0$ rad/s

$L = I \omega = \frac{(m_1 + m_2)\ell^{\cancel{2}}}{\cancel{4}\, 2} \cdot \frac{\cancel{2} v}{\cancel{\ell}} = \frac{(m_1 + m_2) v \ell}{2}$ the same result

$L = 1{,}75 \times 10 = 17{,}5 \ \text{kg} \frac{m^2}{s}$

12. A 1.50-kg particle moves in the xy plane with a velocity of $\vec{v} = (4.20\hat{\imath} - 3.60\hat{\jmath})$ m/s. Determine the angular momentum of the particle about the origin when its position vector is $\vec{r} = (1.50\hat{\imath} + 2.20\hat{\jmath})$ m.

$$\vec{p} = m\vec{v} = 1{,}5\,(4{,}20\hat{\imath} - 3{,}60\hat{\jmath})$$
$$= (6{,}30\hat{\imath} - 5{,}40\hat{\jmath})\ \text{kg m/s}$$

$$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1{,}5 & 2{,}2 & 0 \\ 6{,}3 & -5{,}4 & \end{vmatrix} = \hat{\imath}\underbrace{\begin{vmatrix} 2{,}2 & 0 \\ -5{,}4 & 0 \end{vmatrix}}_{0} - \hat{\jmath}\underbrace{\begin{vmatrix} 1{,}5 & 0 \\ 6{,}3 & 0 \end{vmatrix}}_{0} + \hat{k}\begin{vmatrix} 1{,}5 & 2{,}2 \\ 6{,}3 & -5{,}4 \end{vmatrix}$$

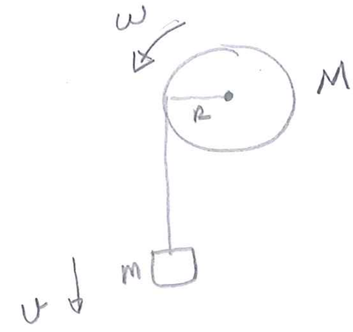
$$\vec{L} = \hat{k}\left(-1{,}5 \times 5{,}4 - 6{,}3 \times 2{,}2\right)\ \text{kg}\,\frac{m^2}{s}$$

$$= \hat{k}\left(-22{,}0\ \text{kg}\,\frac{m^2}{s}\right)$$

$$= \left(-22{,}0\,\hat{k}\right)\ \text{kg}\,\frac{m^2}{s}$$

18. A counterweight of mass $m = 4.00$ kg is attached to a light cord that is wound around a pulley as in Figure P11.18. The pulley is a thin hoop of radius $R = 8.00$ cm and mass $M = 2.00$ kg. The spokes have negligible mass. (a) What is the magnitude of the net torque on the system about the axle of the pulley? (b) When the counterweight has a speed v, the pulley has an angular speed $\omega = v/R$. Determine the magnitude of the total angular momentum of the system about the axle of the pulley. (c) Using your result from part (b) and $\vec{\tau} = d\vec{L}/dt$, calculate the acceleration of the counterweight.

AMT W



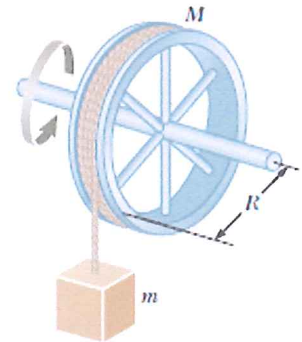
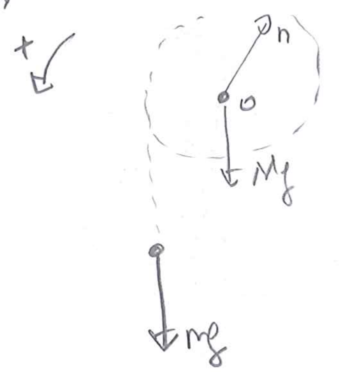


Figure P11.18

$I = MR^2$ loop

(a) For the system, only the external fors



$$\Sigma\tau = mgR = 4 \times 9{,}80 \times 8 \times 10^{-2}$$
$$= 3{,}14 \text{ Nm}$$

(b) $L = I\omega + mvR = MR^2 \cdot \frac{v}{R} + mvR$

$$L = (M+m)\,vR = (2+4) \cdot v \cdot 8 \cdot 10^{-2}$$
$$= 0{,}48v$$

(c) $a = \frac{dv}{dt}$

$$\tau = \frac{dL}{dt}$$

$$3{,}14 = \frac{d}{dt}(0{,}48v)$$

$$3{,}14 = 0{,}48 \underbrace{\frac{dv}{dt}}_{a}$$

$$a = \frac{3{,}14}{0{,}48} = 6{,}53 \text{ m/s}^2$$

19. **M** The position vector of a particle of mass 2.00 kg as a function of time is given by $\vec{r} = (6.00\hat{i} + 5.00t\hat{j})$, where $\vec{r}$ is in meters and t is in seconds. Determine the angular momentum of the particle about the origin as a function of time.

$$\vec{r} = 6\hat{i} + 5t\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 5\hat{j}$$

$$\vec{p} = m\vec{v} = 2 \cdot 5\hat{j} = (10\hat{j})\ kg\ \frac{m}{s}$$

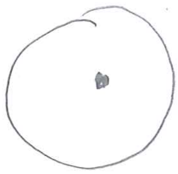
$$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 5t & 0 \\ 0 & 10 & 0 \end{vmatrix} = \hat{i}\underbrace{\begin{vmatrix} 5t & 0 \\ 10 & 0 \end{vmatrix}}_{0} - \hat{j}\underbrace{\begin{vmatrix} 6 & 0 \\ 0 & 0 \end{vmatrix}}_{0} + \hat{k}\begin{vmatrix} 6 & 5t \\ 0 & 10 \end{vmatrix}$$

$$= \hat{k}(60 - 0.5t) = 60\hat{k}$$

constant

25. A uniform solid disk of mass $m = 3.00$ kg and radius **W** $r = 0.200$ m rotates about a fixed axis perpendicular to its face with angular frequency 6.00 rad/s. Calculate the magnitude of the angular momentum of the disk when the axis of rotation (a) passes through its center of mass and (b) passes through a point midway between the center and the rim.

(a)



$\omega = 6,00 \frac{rad}{s}$

$I = I_{CM} = \frac{1}{2} m r^2 = \frac{3 \cdot (0,2)^2}{2} = 0,06 \ kg\, m^2$

$L = I\omega = 0,06 \times 6 = 0,36 \ kg \frac{m^2}{s}$

$D = \frac{r}{2} = 0,1$

(b)

$I = I_{CM} + m D^2$

$I = 0,06 \ kg\, m^2 + 3 \cdot (0,1)^2 = 0,09 \ kg\, m^2$

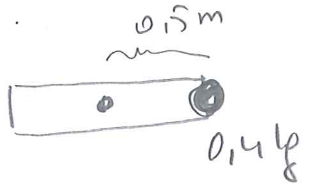
$L = I\omega = 0,09 \cdot 6 \ \frac{kg\, m^2}{s} = 0,54 \ kg \frac{m^2}{s}$

27. A particle of mass 0.400 kg is attached to the 100-cm mark of a meterstick of mass 0.100 kg. The meterstick rotates on the surface of a frictionless, horizontal table with an angular speed of 4.00 rad/s. Calculate the angular momentum of the system when the stick is pivoted about an axis (a) perpendicular to the table through the 50.0-cm mark and (b) perpendicular to the table through the 0-cm mark.

$\omega = 4{,}00 \frac{rad}{s}$

$I_{CM} = \frac{1}{12} M L^2$

(a)



$I = 0{,}4 \cdot (0{,}5)^2 + \frac{1}{12} 0{,}1 \cdot (1)^2$

$I = 0{,}10 + \frac{0{,}1}{12} = \frac{1{,}3}{12} \text{ kg m}^2 = 0{,}108 \text{ kg m}^2$

$L = I\omega = \frac{1{,}3}{12} \cdot 4 = \frac{1{,}3}{3} \text{ kg} \frac{m^2}{s} = 0{,}433 \text{ kg} \frac{m^2}{s}$

(b)

$I_s' = I_{CM} + M \left(\frac{L}{2}\right)^2 = \frac{1}{3} M L^2$

$I = 0{,}4 (L)^2 + \frac{1}{3} 0{,}1 \cdot (1)^2$

$= 0{,}4 + \frac{0{,}1}{3} = \frac{1{,}3}{3} \text{ kg m}^2$

$L = I\omega = \frac{1{,}3}{3} \cdot 4 \frac{\text{kg m}^2}{s} = \frac{5{,}2}{3} \text{ kg} \frac{m^2}{s} = 1{,}73 \text{ kg} \frac{m^2}{s}$

30. A disk with moment of inertia I_1 rotates about a frictionless, vertical axle with angular speed ω_i. A second disk, this one having moment of inertia I_2 and initially not rotating, drops onto the first disk (Fig. P11.30). Because of friction between the surfaces, the two eventually reach the same angular speed ω_f. (a) Calculate ω_f. (b) Calculate the ratio of the final to the initial rotational energy.

Angular momentum conserved. External forces go through the axle

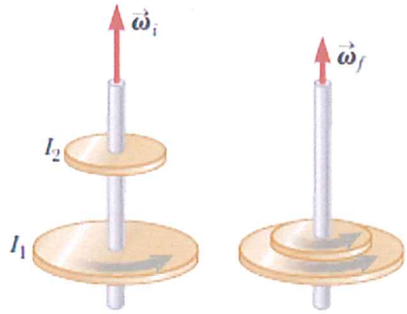


Figure P11.30

(a) $L_i = L_f$

$$I_1 \omega_i + I_2 \cdot 0 = (I_1 + I_2)\,\omega_f$$

$$\omega_f = \frac{I_1}{I_1 + I_2}\,\omega_i$$

(b) $K_i = \frac{1}{2} I_1 \omega_i^2$, $K_f = \frac{1}{2}(I_1 + I_2)\,\omega_f^2$

$$\frac{K_f}{K_i} = \frac{I_1 + I_2}{I_1} \cdot \left(\frac{\omega_f}{\omega_i}\right)^2 = \frac{I_1 + I_2}{I_1}\left(\frac{I_1}{I_1 + I_2}\right)^2 = \frac{I_1}{I_1 + I_2}$$

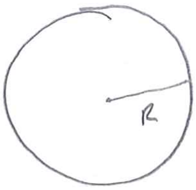
kinetic energy converted into interal energy

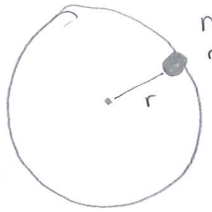
31. A playground merry-go-round of radius $R = 2.00$ m has a moment of inertia $I = 250$ kg · m² and is rotating at 10.0 rev/min about a frictionless, vertical axle. Facing the axle, a 25.0-kg child hops onto the merry-go-round and manages to sit down on the edge. What is the new angular speed of the merry-go-round?

AMT W

$w_i = 10 \dfrac{rev}{min}$

$w_1 = 10 \dfrac{rev}{min}$





$I_i = 250\ kg \cdot m^2$

$I_f = 250 + mR^2$

$= 250 + 25 \cdot (2)^2 = 350\ kg \cdot m^2$

$I_i w_i = I_f w_f$

$w_f = \dfrac{I_i}{I_f} w_i = \dfrac{25\cancel{0}}{\cancel{35}\cancel{0}_{7}} \cdot \cancel{10}^{2} \dfrac{rev}{min} = \dfrac{50}{7} \dfrac{rev}{min}$

$w_f = 7{,}14 \dfrac{rev}{min}$

34. A student sits on a freely rotating stool holding two **W** dumbbells, each of mass 3.00 kg (Fig. P11.34). When his arms are extended horizontally (Fig. P11.34a), the dumbbells are 1.00 m from the axis of rotation and the student rotates with an angular speed of 0.750 rad/s. The moment of inertia of the student plus stool is 3.00 kg · m² and is assumed to be constant. The student pulls the dumbbells inward horizontally to a position 0.300 m from the rotation axis (Fig. P11.34b). (a) Find the new angular speed of the student. (b) Find the kinetic energy of the rotating system before and after he pulls the dumbbells inward.

$m = 3{,}00\,kg.$

$I_{base} = 3{,}00\ kg\cdot m^2$

$I_i = I_{base} + 2\cdot m\cdot r_i^2$

$= 3{,}00\,kg\,m^2 + 2\cdot 3\cdot 1$

$\boxed{I_i = 9.00\ kg\,m^2}$

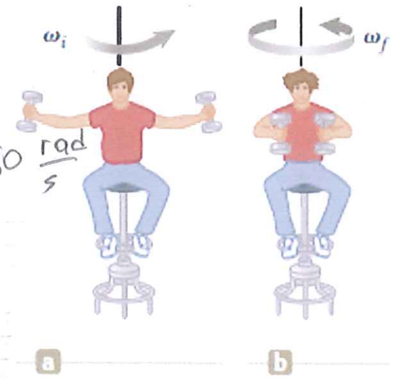


Figure P11.34

$w_i = 0{,}750\ \frac{rad}{s}$

$I_f = I_{base} + 2\,m\,r_f^2$

$= 3{,}00\,kg\,m^2 + 2\cdot 3\cdot (0{,}3)^2$

$I_f = 3{,}54\ kg\,m^2$

(a) $I_i w_i = I_f w_f$

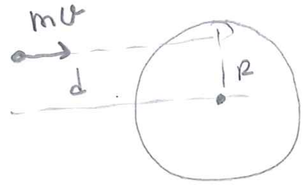
$w_f = \frac{I_i}{I_f}\,w_i = \frac{9}{3{,}54}\cdot 0{,}750\ \frac{rad}{s} = 1{,}91\ \frac{rad}{s}$

(b) $K_i = \frac{1}{2} I_i w_i^2 = \frac{1}{2}\cdot 9\cdot (0{,}75)^2 = 2{,}53\ J$

$K_f = \frac{1}{2} I_f w_f^2 = \frac{1}{2}\ 3{,}54\ (1.91)^2 = 6{,}44\ J$

student does work

39. A wad of sticky clay with mass m and velocity $\vec{v}_i$ is fired at a solid cylinder of mass M and radius R (Fig. P11.39). The cylinder is initially at rest and is mounted on a fixed horizontal axle that runs through its center of mass. The line of motion of the projectile is perpendicular to the axle and at a distance $d < R$ from the center. (a) Find the angular speed of the system just after the clay strikes and sticks to the surface of the cylinder. (b) Is the mechanical energy of the clay–cylinder system constant in this process? Explain your answer. (c) Is the momentum of the clay–cylinder system constant in this process? Explain your answer.



$I = \frac{1}{2} MR^2$

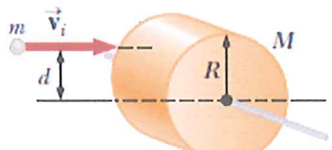


Figure P11.39

Angular momentum conserved

sticks to the surface

(a) $I_f = \frac{1}{2} MR^2 + mR^2$

$L_i = mvd$, $L_f = I_f \omega_f$

$mvd = \left(\frac{1}{2} MR^2 + mR^2\right) \omega_f$

$$\omega_f = \frac{2mvd}{(M+2m)R^2}$$

(b) Mechanical energy not conserved

$K_i = \frac{1}{2} mv^2$, $K_f = \frac{1}{2}\left(\frac{M}{2} + m\right)R^2 \left[\frac{2mvd}{(M+2m)R^2}\right]^2$

$K_f = \frac{1}{2} mv^2 \frac{md^2}{(M+2m)R^2}$

$K_f = K_i \cdot \underbrace{\frac{m}{(M+2m)} \cdot \left(\frac{d}{R}\right)^2}_{<1}$

(c) $P_i = mv$, $P_f = 0$

total momentum not conserved, axle apply an external force

41. A 0.005 00-kg bullet traveling horizontally with speed 1.00×10^3 m/s strikes an 18.0-kg door, embedding itself 10.0 cm from the side opposite the hinges as shown in Figure P11.41. The 1.00-m wide door is free to swing on its frictionless hinges. (a) Before it hits the door, does the bullet have angular momentum relative to the door's axis of rotation? (b) If so, evaluate this angular momentum. If not, explain why there is no angular momentum. (c) Is the mechanical energy of the bullet–door system constant during this collision? Answer without doing a calculation. (d) At what angular speed does the door swing open immediately after the collision? (e) Calculate the total energy of the bullet–door system and determine whether it is less than or equal to the kinetic energy of the bullet before the collision.

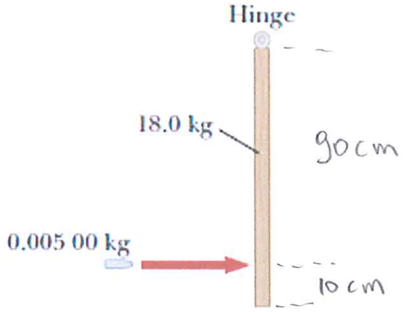


Figure P11.41 An overhead view of a bullet striking a door.

(a)(b) $L_{bullet} = mvr = 5\times10^{-3} \cdot 1\times10^{3} \cdot 0{,}9 = 4{,}5 \ kg\frac{m^2}{s}$

(c) Inelastic, mechanical energy not conserved

(d) Angular momentum conserved

$$I_f = 5\times10^{-3}\,(0{,}9)^2 + \frac{1}{3}\cdot 18 \cdot (1)^2 = (6 + 45\times10^{-5})\ kg\,m^2$$

$$L_f = L_i = 4{,}5\ kg\frac{m^2}{s}$$

$$w_f = \frac{L_f}{I_f} = \frac{4{,}5\ kg\ m^2/s}{(6+45\times10^{-5})\ kg\,m^2} = 0{,}749\ \frac{rad}{s}$$

(e) $K_f = \frac{1}{2} I_f w_f^2 = \frac{1}{2}(6+45\times10^{-5}) \cdot (0{,}749)^2 \doteq 1{,}68\ J$

$K_i = \frac{1}{2} m v^2 = \frac{1}{2} \cdot 5\cdot10^{-3} \cdot (1\times10^3)^2 = 2{,}50\times10^3\ J$

large amount of energy converted into internal energy

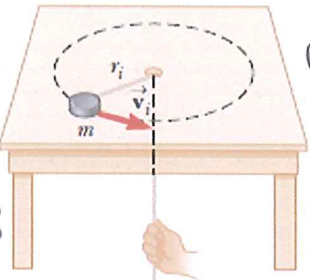
52. AMT M A puck of mass $m = 50.0$ g is attached to a taut cord passing through a small hole in a frictionless, horizontal surface (Fig. P11.52). The puck is initially orbiting with speed $v_i = 1.50$ m/s in a circle of radius $r_i = 0.300$ m. The cord is then slowly pulled from below, decreasing the radius of the circle to $r = 0.100$ m. (a) What is the puck's speed at the smaller radius? (b) Find the tension in the cord at the smaller radius. (c) How much work is done by the hand in pulling the cord so that the radius of the puck's motion changes from 0.300 m to 0.100 m?

external force goes throug the axis. angular momentum conserved.

$r_i = 0{,}3$ m

$v_i = 1{,}50 \frac{m}{s}$

$m = 50 \times 10^{-3}$ kg



$r_f = 0{,}1$ m

Figure P11.52 Problems 52 and 53.

(a) $L_i = L_f$

$\cancel{m} v_i r_i = \cancel{m} v_f r_f$

$v_f = v_i \frac{r_i}{r_f} = 1{,}50 \cdot \frac{0{,}3}{0{,}1}$

$v_f = 4{,}50$ m/s

(b) $T = F_c = m a_c = m \frac{v_f^2}{r_f} = 50 \times 10^{-3} \cdot \frac{(4{,}50)^2}{0{,}1} = 10{,}1$ N

(c) $W = \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$= \frac{1}{2} \cdot 50 \times 10^{-3} (4{,}5^2 - 1{,}50^2)$

$= 0{,}450$ J

60. The puck in Figure P11.60 has a mass of 0.120 kg. The distance of the puck from the center of rotation is originally 40.0 cm, and the puck is sliding with a speed of 80.0 cm/s. The string is pulled downward 15.0 cm through the hole in the frictionless table. Determine the work done on the puck. (*Suggestion:* Consider the change of kinetic energy.)

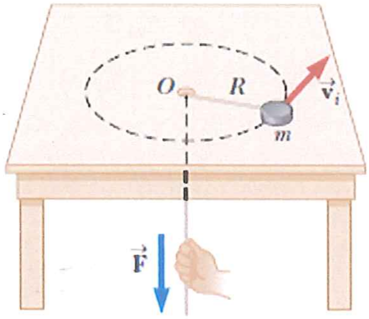


Figure P11.60

$m = 0{,}120\,kg$

$r_i = 0{,}4\,m$

$r_f = 0{,}25\,m$

$v_i = 0{,}8\,m/s$

$L_i = L_f$

$m v_i r_i = m v_f r_f$

$v_f = v_i \dfrac{r_i}{r_f}$

$\Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} m v_i^2 \frac{r_i^2}{r_f^2} - \frac{1}{2} m v_i^2$

$= \frac{1}{2} m v_i^2 \left(\frac{r_i^2}{r_f^2} - 1 \right) = \frac{1}{2} m v_i^2 \left(\frac{r_i^2 - r_f^2}{r_f^2} \right)$

$W = \Delta K = \frac{1}{2} \cdot 0{,}120 \cdot (0{,}8)^2 \left(\underbrace{\left(\frac{0{,}4}{0{,}25}\right)^2 - 1}_{\frac{39}{25}} \right)$

$W = 5{,}99 \times 10^{-2}\,J$