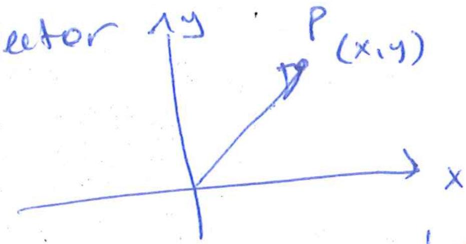
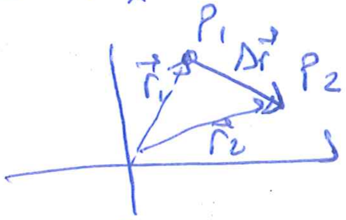
Motion in 2D

Position vector



$$\vec{r} = x\hat{i} + y\hat{j}$$

Displacement Vector



$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$
$$= \Delta x\,\hat{i} + \Delta y\,\hat{j}$$

Average velocity

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t} = \underbrace{\frac{\Delta x}{\Delta t}}_{v_{x,avg}}\hat{i} + \underbrace{\frac{\Delta y}{\Delta t}}_{v_{y,avg}}\hat{j}$$

$$\vec{v}_{avg} = v_{x,avg}\,\hat{i} + v_{y,avg}\,\hat{j}$$

Velocity

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \underbrace{\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}}_{v_x}\hat{i} + \underbrace{\lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}}_{v_y}\hat{j}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j}, \quad v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}$$

average acceleration

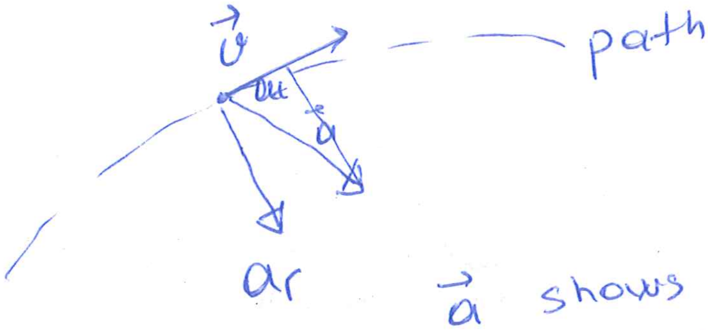
$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \underbrace{\lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}}_{a_x}\hat{i} + \underbrace{\lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t}}_{a_y}\hat{j}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}, \quad a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}$$

Motion in 2D can be modelled as 2 one-dimensional motions, combined.



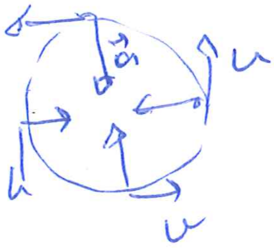
$\vec{v}$ tangential to path

$\vec{a}$ shows the direction of the path curling.

$$a_r = \frac{v^2}{R}, \quad a_t = \frac{dv}{dt}$$ (responsible for speed change)

Uniform Circular Motion:

$$a = a_c = \frac{v^2}{R}$$, T: time for a single complete turn.

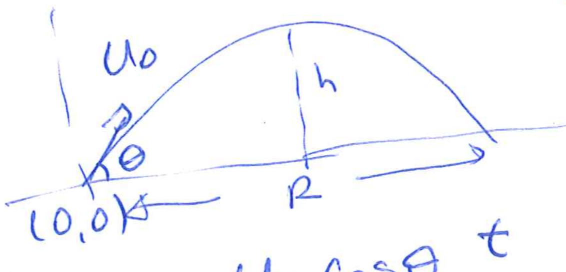


$$v = \frac{2\pi R}{T}, \quad a_c = \frac{4\pi^2 R}{T^2}$$

Projectile Motion

$a_x = 0, \quad a_y = -g$ ↑ +y

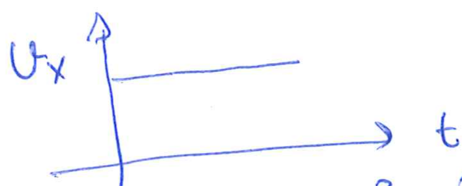
$v_{xi} = v_0 \cos\theta, \quad v_{yi} = v_0 \sin\theta$

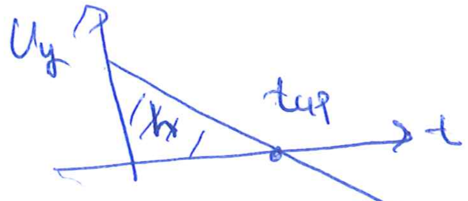


taking $x_i = 0, \; y_i = 0$

$x = v_0 \cos\theta \, t, \quad y = v_0 \sin\theta \, t - \frac{1}{2} g t^2$

$v_x = v_0 \cos\theta$ constant, $\quad v_y = v_0 \sin\theta - g t$





$$t_{up} = \frac{v_0 \sin\theta}{g}$$ when velocity is zero

(max heights) $h = \frac{v_0^2 \sin^2\theta}{2g}, \quad R = \frac{v_0^2 \sin(2\theta)}{g}$ (range)

••15 SSM ILW A particle leaves the origin with an initial velocity $\vec{v} = (3.00\hat{i})$ m/s and a constant acceleration $\vec{a} = (-1.00\hat{i} - 0.500\hat{j})$ m/s^2. When it reaches its maximum x coordinate, what are its (a) velocity and (b) position vector?

$x_i = 0, \quad y_i = 0$

$\vec{v}_i = 3\hat{i}$ m/s $\Rightarrow v_{xi} = 3 \frac{m}{s}, \quad v_{yi} = 0$ m/s

$\vec{a} = (-1\hat{i} - 0{,}5\hat{j}) \frac{m}{s^2} \Rightarrow a_x = -1 \frac{m}{s^2}, \quad a_y = -0{,}5 \frac{m}{s^2}$

$x_f = \underset{0}{\underset{\shortparallel}{x_i}} + v_{xi} t + \frac{1}{2} a_x t^2$

$x_f = 3t - 0{,}5t^2 = t(3 - 0{,}5t)$

$v_{xf} = v_{xi} + a_x t = 3 - t = 0$ is when it changes directi[on]

$t = 3s$ is when x_f should be maximu[m]

$x_f = 3(3 - 0{,}5 \cdot 3) = \underline{4{,}5\ m}$

$y_f = \underset{0}{\underset{\shortparallel}{y_i}} + \underset{0}{\underset{\shortparallel}{v_{yi}}}\, t + \frac{1}{2} a_y t^2 = -0{,}25\, t^2$

$y_f = -0{,}25 \cdot (3)^2 = -2{,}25$ m

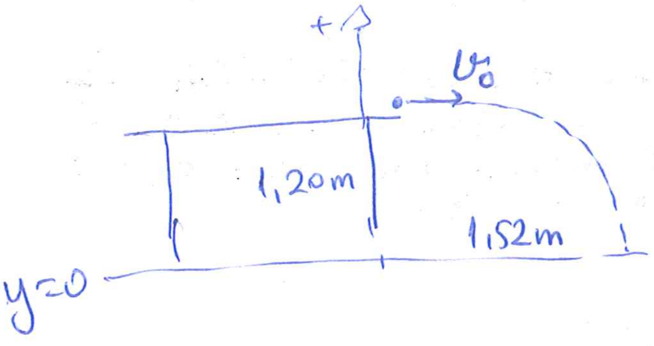
$v_{yf} = v_{yi} + a_y t = -0{,}5t$

$v_{yf} = -0{,}5 \cdot (3) = -1{,}5$ m/s

(a) $\vec{v}_f = v_{xf}\hat{i} + v_{yf}\hat{j} = 0\hat{i} - 1{,}5\hat{j} = (-1{,}5\hat{j})$ m/s

(b) $\vec{r}_f = x_f\hat{i} + y_f\hat{j} = (4{,}5\hat{i} - 2{,}25\hat{j})$ m

•22 A small ball rolls horizontally off the edge of a tabletop that is 1.20 m high. It strikes the floor at a point 1.52 m horizontally from the table edge. (a) How long is the ball in the air? (b) What is its speed at the instant it leaves the table?



$v_{xi} = v_0$, $v_{yi} = 0$

$x_i = 0$, $y_i = 1{,}20\ m$

$a_x = 0$, $a_y = -g = -9{,}80\ \frac{m}{s^2}$

(a) $y_f = y_i + \underbrace{v_{iy}}_{0} t - \frac{1}{2} g t^2$

$0 = 1{,}2 - 4{,}90 t^2$

$t = \sqrt{\frac{1{,}2}{4{,}9}} = 0{,}495\ s$

(b) $x_f = x_i + v_{xi} t$, $a_x = 0$

$1{,}52 = 0 + v_0 \cdot 0{,}495$

$v_0 = \frac{1{,}52}{0{,}495} = 3{,}07\ m/s$

••27 ILW A certain airplane has a speed of 290.0 km/h and is diving at an angle of $\theta = 30.0°$ below the horizontal when the pilot releases a radar decoy (Fig. 4-33). The horizontal distance between the release point and the point where the decoy strikes the ground is $d = 700$ m. (a) How long is the decoy in the air? (b) How high was the release point?

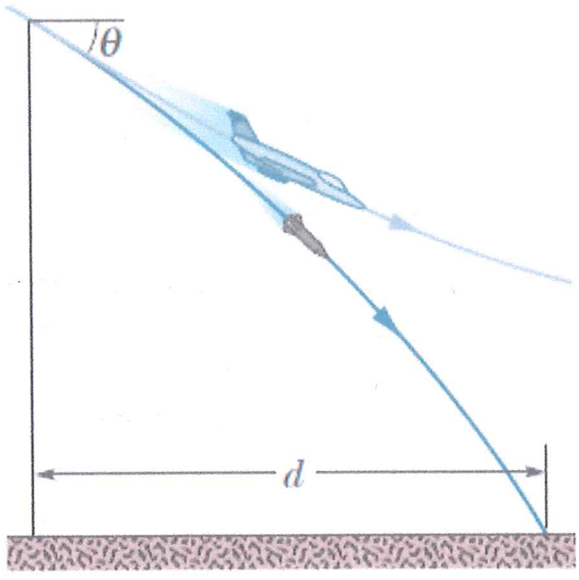
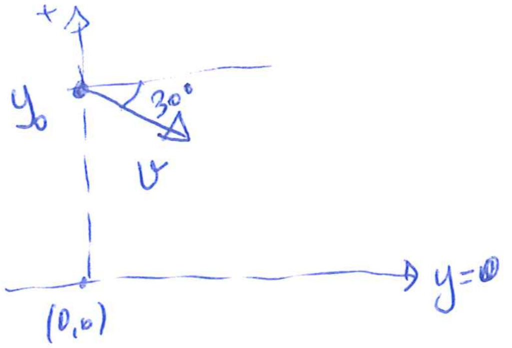


Figure 4-33 Problem 27.

$$290 \frac{km}{h} \cdot \frac{1000\,m}{1\,km} \cdot \frac{1h}{3600\,s} = 80.6\ m/s$$



$v_{xi} = v \cdot \cos(30°) = 69{,}8\ m/s$

$v_{yi} = -v \sin(30°) = -40.3\ m/s$

(a) $x_f = d = x_i + v_{xi} t$, $x_i = 0$

$$t = \frac{d}{v_{xi}} = \frac{700\ m}{69{,}8} = 10.03\ s = 10.0\ s$$

(b) $y_f = 0 = y_i + v_{yi} t - 4{,}90 t^2$

$y_i = 4{,}90 t^2 - v_{yi} t$

$= 4{,}90 \cdot (10.03)^2 + 40.3 \cdot 10{,}03$

$$\boxed{y_i = 897\ m}$$

••28 GO In Fig. 4-34, a stone is projected at a cliff of height h with an initial speed of 42.0 m/s directed at angle $\theta_0 = 60.0°$ above the horizontal. The stone strikes at A 5.50 s after launching. Find (a) the height h of the cliff, (b) the speed of the stone just before impact at A, and (c) the maximum height H reached above the ground.

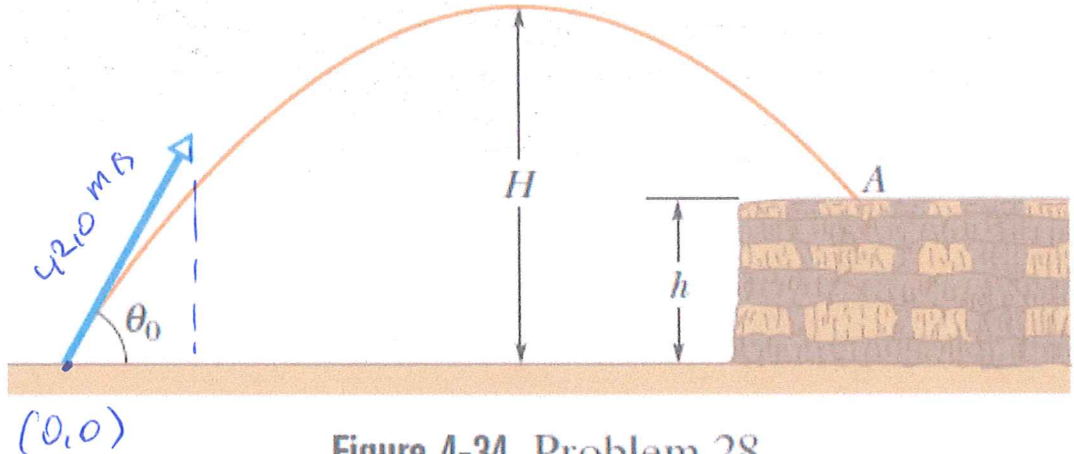


Figure 4-34 Problem 28.

$x_i = 0, \quad y_i = 0$

$v_{xi} = 42 \cdot \cos(60°) = 21$ m/s

$v_{yi} = 42 \cdot \sin(60°) = 36{,}373$ m/s

(a) $h = y_f = \underset{0}{\overset{\shortparallel}{y_i}} + v_{yi}t - \frac{1}{2}gt^2$

$h = (36{,}373) \cdot 5{,}5 - 4{,}90 \cdot (5{,}5)^2$

$h = 51{,}8269$ m $= 51.8$ m (3 significant figures)

(b) $v_{xf} = v_{xi} = 21$ m/s

$v_{yf} = v_{yi} - gt = 36{,}373 - 9{,}80 \cdot 5{,}50$

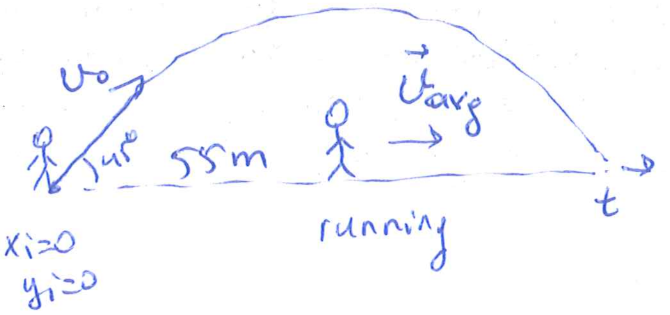
$= -17{,}5269$ m/s

$v = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{21^2 + (-17{,}5269)^2} = 27{,}3531$

$= 27{,}4$ m/s

(c) $H = \frac{v_{yi}^2}{2g} = \frac{(36{,}373)^2}{2 \cdot 9{,}80} = 67{,}5$ m

••30 GO A soccer ball is kicked from the ground with an initial speed of 19.5 m/s at an upward angle of 45°. A player 55 m away in the direction of the kick starts running to meet the ball at that instant. What must be his average speed if he is to meet the ball just before it hits the ground?



they should be here at the same moment

$$y_f = v_{yi}\, t - \frac{1}{2} g t^2 = 0$$

$$t = \frac{2 v_{yi}}{g} = \frac{2 v_0 \sin(45°)}{g} =$$

$$t = \frac{2 \cdot 19{,}5 \cdot \sqrt{2}/2}{9{,}80} = 2{,}814\ s = 2{,}81\ s$$

$$x_f = v_{xi}\, t = 55 + v_{avg}\, t$$

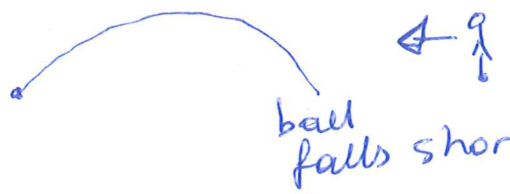
$$v_{avg} = v_{xi} - \frac{55}{t} = v_0 \cos(45°) - \frac{55}{2{,}814}$$

$$= 19{,}5 \cdot \frac{\sqrt{2}}{2} - \frac{55}{2{,}814}$$

$$v_{avg} = -5{,}756\ m/s$$

opposite to what we assumed

$$|v_{avg}| = 5{,}8\ m/s$$



ball falls short

••32 GO You throw a ball toward a wall at speed 25.0 m/s and at angle $\theta_0 = 40.0°$ above the horizontal (Fig. 4-35). The wall is distance $d = 22.0$ m from the release point of the ball. (a) How far above the release point does the ball hit the wall? What are the (b) horizontal and (c) vertical components of its velocity as it hits the wall? (d) When it hits, has it passed the highest point on its trajectory?

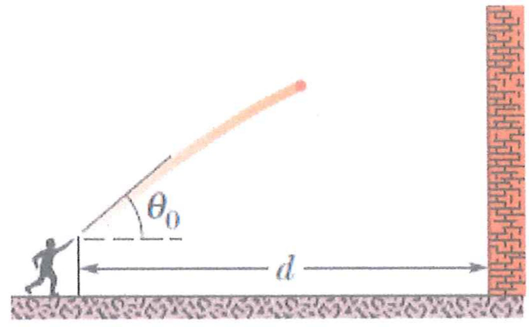
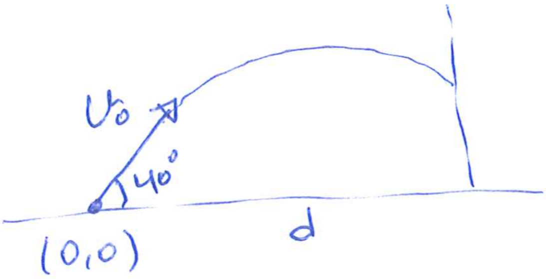


Figure 4-35 Problem 32.



$x_i = 0, \ y_i = 0, \ a_y = -g$

$x_f = d = 22.0$ m

$y_f = ?, \ v_{xf} = ?, \ v_{yf} = ?$

$v_{xi} = v_0 \cos(40) = 25 \cos 40$

$v_{yi} = v_0 \sin(40) = 25 \sin 40$

(a) $\underset{d}{x_f} = \underset{0}{x_i} + v_{xi} t$

$t = \frac{d}{v_{xi}} = \frac{22}{25\cos 40} = 1.14876$ s

$y_f = \underset{0}{y_i} + v_{yi} t - \frac{1}{2} g t^2$

$= \underset{25}{v_0} \sin(40) \cdot \frac{22}{25\cos(40)} - 4.90 \left(\frac{22}{25\cos 40}\right)^2$

$y_f = 22 \tan(40) - 4.90 \left(\frac{22}{25\cos 40}\right)^2 = 11.9939$ m

$y_f = 12.0$ m

(b) $v_{xf} = v_{xi} = 25\cos(40) = 19.2$ m/s

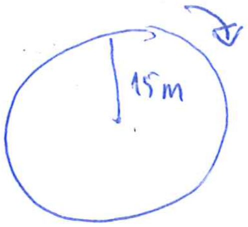
(c) $v_{yf} = v_{yi} - gt = 25\sin(40) - \frac{22 \cdot 9.80}{25\cos 40} = 4.81186 \frac{m}{s}$

$v_{yf} = 4.81$ m/s

(d) $t_{up} = \frac{v_{yi}}{g} = \frac{25\sin 40}{9.80} = 1.6397$ s , not yet!

Also $v_{yf} > 0$ still going up

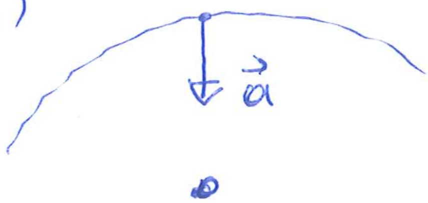
•59 ILW A woman rides a carnival Ferris wheel at radius 15 m, completing five turns about its horizontal axis every minute. What are (a) the period of the motion, the (b) magnitude and (c) direction of her centripetal acceleration at the highest point, and the (d) magnitude and (e) direction of her centripetal acceleration at the lowest point?



(a) $5T = 1\ min = 60\ s$

$T = 12\ s$

(b)



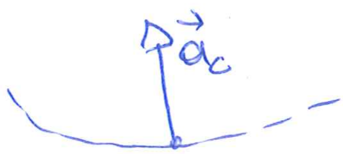
$a = a_c = \frac{v^2}{R}$, $v = \frac{2\pi R}{T}$

$a_c = \frac{(2\pi R/T)^2}{R} = \frac{4\pi^2 R}{T^2}$

$a_c = \frac{4 \cdot \pi^2 \cdot 15}{(12)^2} = 4.11\ m/s^2 = 4.1\ \frac{m}{s^2}$ (2 significant figures)

(c) Pointing dow to the center of the wheel!

(d) $a_c = 4.1\ \frac{m}{s^2}$ the same



(e) Pointing up

40. **W** Figure P4.40 represents the total acceleration of a particle moving clockwise in a circle of radius 2.50 m at a certain instant of time. For that instant, find (a) the radial acceleration of the particle, (b) the speed of the particle, and (c) its tangential acceleration.

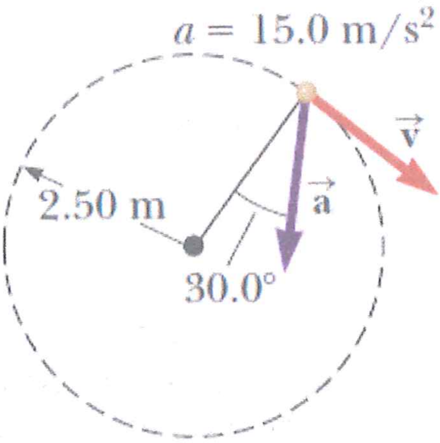
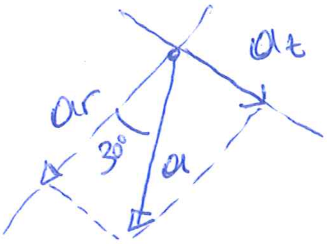


Figure P4.40



(a) $a_r = a\cos(30°)$

$= 15 \cdot \frac{\sqrt{3}}{2} = 12{,}99 \ \frac{m}{s^2}$

$a_r = 13{,}0 \ m/s^2$

(b) $a_r = a_c = \frac{v^2}{R} = 15\frac{\sqrt{3}}{2}$

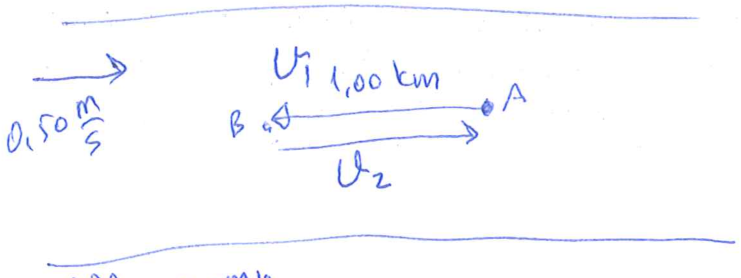
$v = \sqrt{R \cdot 15\frac{\sqrt{3}}{2}} = \sqrt{2{,}50 \cdot 15\frac{\sqrt{3}}{2}}$

$v = 5{,}6987 \ m/s$

$v = 5{,}70 \ m/s$ (3 significant figures)

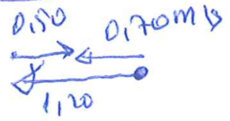
(c) $a_t = a\sin\theta = 15 \cdot \frac{1}{2} = 7{,}50 \ m/s^2$ tangential component

50. M A river has a steady speed of 0.500 m/s. A student swims upstream a distance of 1.00 km and swims back to the starting point. (a) If the student can swim at a speed of 1.20 m/s in still water, how long does the trip take? (b) How much time is required in still water for the same length swim? (c) Intuitively, why does the swim take longer when there is a current?



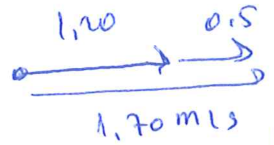
$v_{SR} = 1{,}20\ \frac{m}{s}$ with respect to river

(a)



$v_1 = 1{,}20 - 0{,}50 = 0{,}70\ m/s$

$$t_1 = \frac{1000}{v_1} = \frac{1000}{0{,}70}\ s$$



$v_2 = 1{,}20 + 0{,}50 = 1{,}70\ m/s$

$$t_2 = \frac{1000}{1{,}70}\ s$$

$$t_{total} = \frac{1000}{0{,}70} + \frac{1000}{1{,}70} = 2016{,}8067\ s$$

$$t_{total} = 2{,}02 \times 10^3\ s$$

(b) In still water $t_{total} = \dfrac{2000}{1{,}20} = 1666{,}667\ s$

$= 1{,}67 \times 10^3\ s$

(c) $$t_{total} = \frac{d(v_1+v_2)}{v_1 v_2} = \frac{d(v_1+v_2)^2}{(v_1+v_2)\,v_1 v_2} = \frac{d\,(v_1+v_2)^2}{\left(\frac{v_1+v_2}{2}\right) 2 v_1 v_2}$$

$$t_{total} = \frac{2d}{(v_1+v_2)/2} \cdot \frac{(v_1+v_2)^2}{4v_1v_2} = t_{still} \cdot \frac{(v_1+v_2)^2}{4v_1v_2}$$ and $(v_1+v_2)^2 > 4v_1v_2$ since $(v_1-v_2)^2 > 0$