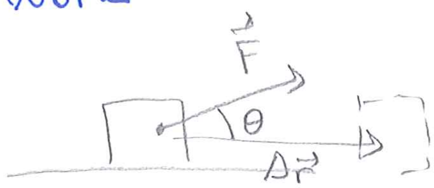
# Work and Energy - Part 1

## Work done by a constant force




$$W = F\Delta r\cos\theta = \vec{F}\cdot\Delta\vec{r}$$

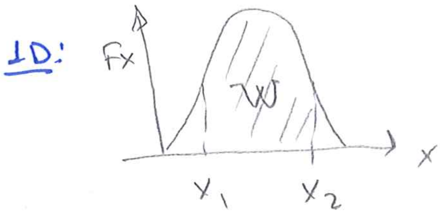
$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

$$W = \vec{F}\cdot\Delta\vec{r} = F_x\Delta x + F_y\Delta y + F_z\Delta z$$

## Work done by a varyin force

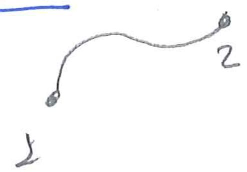
1D:




$$W_{1\to 2} = \int_{x_1}^{x_2} F_x\,dx$$

area under $f_x - x$ curve

2 and 3D

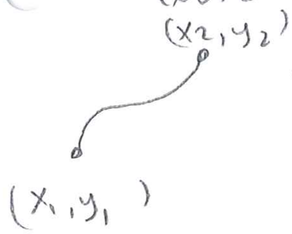



$$W_{1\to 2} = \int_1^2 (F_x dx + F_y dy + F_z dz)$$

examples: work done by a spring

$$W_s(1\to 2) = \int_{x_1}^{x_2} -kx\,dx = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

work done by gravity



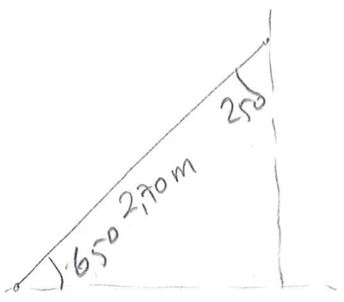

$$W_g = \int_1^2 (\underbrace{F_x}_{0} dx + F_y dy) = \int_{y_1}^{y_2} -mg\,dy = mgy_1 - mgy_2$$

## Work-Energy Theorem:

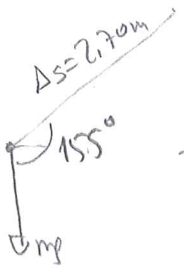
$$W_{total} = \int \sum_i \vec{F}_i \cdot d\vec{r} = \sum_i W_i$$

$$W_{total} = \Delta K \quad , \quad K = \frac{1}{2}mu^2 \text{, kinetic energy.}$$

6.5 •• A 73.0 kg painter climbs a ladder that is 2.70 m long and leans against a vertical wall. The ladder makes a 25.0° angle with the wall. (a) How much work does gravity do on the painter? (b) Does the answer to part (a) depend on whether the painter climbs at constant speed or accelerates up the ladder?




(a)




$$W_g = mg \cdot \Delta s \cdot \cos(155°)$$

$$= 73 \times 9{,}80 \times 2{,}7 \times \cos(155)$$

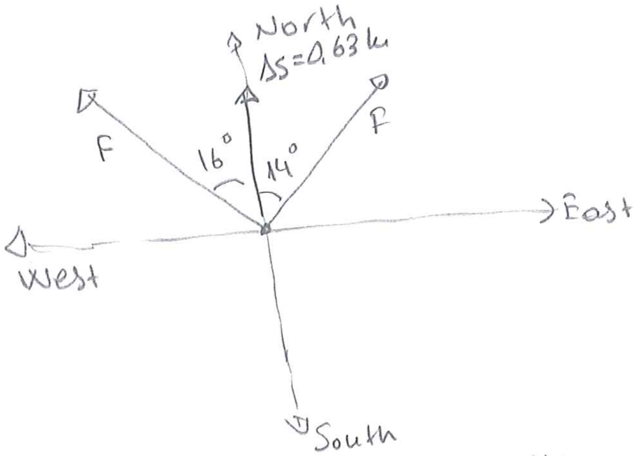
$$W_g = -1750{,}60 \text{ Nm}$$

$$W_g = -1{,}75 \times 10^3 \text{ J}$$

(b) The answer to b does not depend on the acceleration.

Ther force does not depend on the acceleration.

6.6 •• Two tugboats pull a disabled supertanker. Each tug exerts a constant force of $1.80 \times 10^6$ N, one 16° west of north and the other 14° east of north, as they pull the tanker 0.63 km toward the north. What is the total work they do on the supertanker?




$F = 1{,}80 \times 10^6 N$

$\Delta s = 0.63 km = 630 m$

$W_{total} = F \Delta s \cos(16°) + F \Delta s \cos(14°)$

$W_{total} = F \Delta s \left(\cos(16°) + \cos(14°)\right)$

$W_{total} = 1{,}80 \times 10^6 \times 630 \left(\cos(16°) + \cos(14°)\right)$

$= 2{,}1904 \times 10^9 \ Nm$

$W_{total} = 2{,}2 \times 10^9 J$ (The last number of figures is 2)

6.8 •• A loaded shopping trolley is rolling across a parking lot in a strong wind. You apply a constant force $\vec{F} = (30\ \text{N})\hat{\imath} - (37\ \text{N})\hat{\jmath}$ to the trolley as it undergoes a displacement $\vec{s} = (-8.6\ \text{m})\hat{\imath} - (3.8\ \text{m})\hat{\jmath}$. How much work does the force you apply do on the shopping trolley?

$$W = \vec{F} \cdot \Delta\vec{r}$$

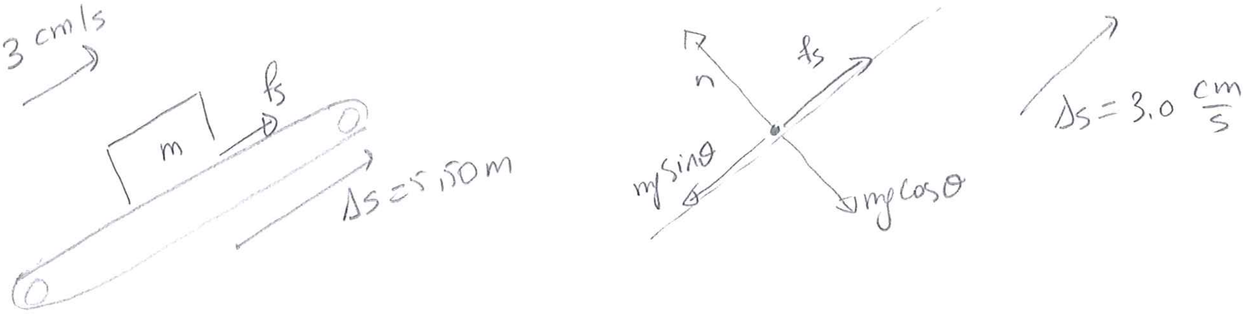
$$= (30\hat{\imath} - 37\hat{\jmath}) \cdot (-8{,}6\,\hat{\imath} - 3{,}8\hat{\jmath})$$

$$= (-30 \times 8{,}6 + 37 \times 3{,}8)\ \text{J}$$

$$W = -117{,}4\ N$$

$$W = -1{,}2 \times 10^2\ N$$ (2 sig figures)

6.12 •• A boxed 11.0-kg computer monitor is dragged by friction 5.50 m upward along a conveyor belt inclined at an angle of 35.6° above the horizontal. If the monitor's speed is a constant 3.00 cm/s, how much work is done on the monitor by (a) friction, (b) gravity, and (c) the normal force of the conveyor belt?




(a) $v$ = Constant

$f_s = mg \sin\theta$

$W_{f_s} = mg \sin\theta \, \Delta s = 11 \times 9.80 \times 5.50 \times \sin(35.6)$

$= 345.141$ J

$W_{f_s} = 345$ J

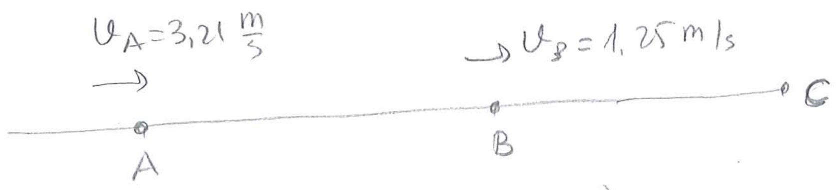
(b) $W_g = mg \, \Delta s \cos(90+35.6)$

$= -345.41$ N m

$W_g = -345$ J

(c) $W_n = n \, \Delta s \cos(90°) = 0$

6.16 •• A 1.50 kg book is sliding along a rough horizontal surface. At point $A$ it is moving at 3.21 m/s, and at point $B$ it has slowed to 1.25 m/s. (a) How much total work was done on the book between $A$ and $B$? (b) If $-0.750$ J of total work is done on the book from $B$ to $C$, how fast is it moving at point $C$? (c) How fast would it be moving at $C$ if $+0.750$ J of total work was done on it from $B$ to $C$?




(a) $W_{ext} = \Delta K = \frac{1}{2} m (v_B^2 - v_A^2)$

$= \frac{1}{2} \cdot 1{,}50 \cdot (1{,}25^2 - 3{,}21^2)$

$W_{ext} = -6{,}5562$ J

$= -6{,}56$ J

(b) $W = -0{,}750 = \frac{1}{2} m (v_c^2 - v_B^2)$

$-0{,}750 = \frac{1{,}50}{2} (v_c^2 - 1{,}25^2)$

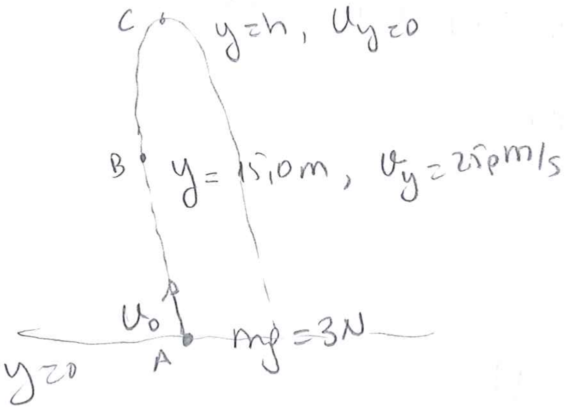
$v_c = \sqrt{(1{,}25)^2 - 1} = 0{,}75$ m/s

(c) $W = 0{,}750 = 0{,}75 (v_c^2 - 1{,}25^2)$

$\sqrt{v_c^2} = \sqrt{1{,}25^2 + 1}$

$v_c = \sqrt{1{,}25^2 + 1} = 1.6$ m/s

**6.24** •• You throw a 3.00 N rock vertically into the air from ground level. You observe that when it is 15.0 m above the ground, it is traveling at 25.0 m/s upward. Use the work–energy theorem to find (a) the rock's speed just as it left the ground and (b) its maximum height.




(a)

$E_A = E_B$

$\frac{1}{2} m v_0^2 = \frac{1}{2} m \cdot (25)^2 + mg \cdot y$

$\frac{1}{2} m v_0^2 = \frac{1}{2} m (25)^2 + mg\,15$

$v_0 = \sqrt{(25)^2 + 2g\,15}$

$v_0 = \sqrt{25^2 + 2 \times 9.80 \times 15} = 30,3\ m/s$

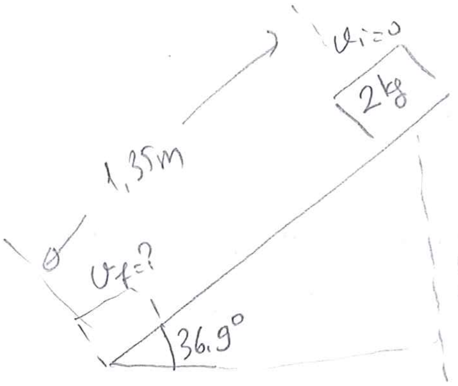
(b)

$E_B = E_C$

$\frac{1}{2} m v_y^2 + mg y = mgh$

$h = \frac{v_y^2}{2g} + y = \frac{25 \cdot 25}{2 \cdot 9.80} + 15 = 46.9\ m$

**6.28** •• A block of ice with mass 2.00 kg slides 1.35 m down an inclined plane that slopes downward at an angle of 36.9° below the horizontal. If the block of ice starts from rest, what is its final speed? Ignore friction.




$y = 1{,}35 \sin(36{,}9°)$

$E_i = E_f$

$\frac{1}{2} m v_i^2 + mgy = \frac{1}{2} m v_f^2 + mg \cdot 0$

$v_i = 0$

$\sqrt{v_f^2} = \sqrt{2gy}$

$v_2 = \sqrt{2 \cdot g \cdot 1{,}35 \sin(36{,}9°)}$

$v = \sqrt{2 \times 9{,}80 \times 1{,}35 \times \sin(36{,}9°)} = 3{,}99 \text{ m/s}$

6.30 •• A 36.0 kg crate is initially moving with a velocity that has magnitude 3.73 m/s in a direction 37.0° west of north. How much work must be done on the crate to change its velocity to 5.40 m/s in a direction 63.0° south of east?

Direction is not important

$$W_{ex} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$= \frac{1}{2} m (v_f^2 - v_i^2)$$

$$= \frac{1}{2} \cdot 36 (5{,}40^2 - 3{,}73^2)$$

$$= 274 J$$

6.32 •• To stretch a spring 9.00 cm from its unstretched length, 19.0 J of work must be done. (a) What is the force constant of this spring? (b) What magnitude force is needed to stretch the spring 9.00 cm from its unstretched length? (c) How much work must be done to compress this spring 4.00 cm from its unstretched length, and what force is needed to compress it this distance?

(a) $W_{ext} = \frac{1}{2}kx^2 = 19\ J$

$$k = \frac{2W}{x^2} = \frac{2 \times 19}{(9 \times 10^{-2})^2} = 4691\ \frac{N}{m}$$

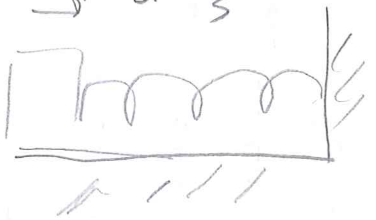
(b) $|F| = k|x| = 4691 \cdot 9 \times 10^{-2}\ m = \frac{2 \times 19}{9 \times 10^{-2}}\ N = 422\ N$

(c) $W_{ext} = \frac{1}{2}kx^2 = \frac{1}{2}\frac{2 \times 19}{(0{,}09)^2} \cdot (0{,}04)^2 = 19\left(\frac{0{,}04}{0{,}09}\right)^2 = 3{,}75\ J$

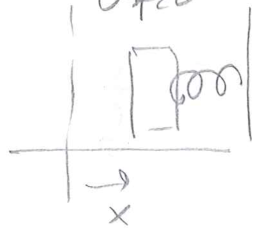
$|F| = k|x| = 4691 \times 0{,}04 = 188\ N$

6.37 •• A 6.0 kg box moving at 3.0 m/s on a horizontal, frictionless surface runs into one end of a light horizontal spring of force constant 75 N/cm that is fixed at the other end. Use the work–energy theorem to find the maximum compression of the spring.

$v_i = 3.0\ \frac{m}{s}$



$v_f = 0$




$k = 75\ \frac{N}{cm} \cdot \frac{100\ cm}{1\ m}$

$= 7500\ \frac{N}{m}$

$\frac{1}{2} m v_i^2 = \frac{1}{2} k x^2$

$x = \sqrt{\frac{m}{k}}\, v_i = \sqrt{\frac{\cancel{6}\ 2}{\cancel{7500}\ 25}}\ 3.0$

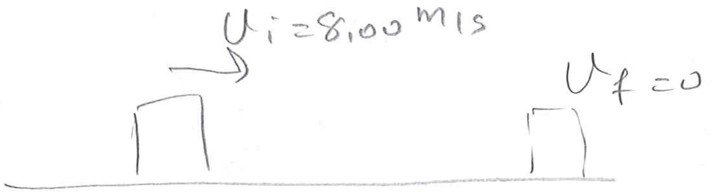
$x = \frac{\sqrt{2}}{50}\ 3 = 0.0848\ m$

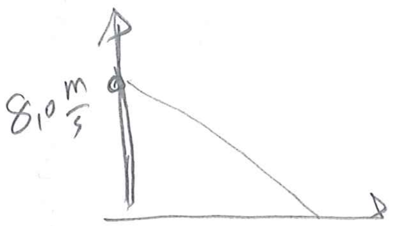
$x = 8.48\ cm$

$x = 8.5\ cm$

(2 sig. figures)

**6.50** •• A 20.0 kg rock is sliding on a rough, horizontal surface at 8.00 m/s and eventually stops due to friction. The coefficient of kinetic friction between the rock and the surface is 0.200. What average power is produced by friction as the rock stops?




$$f_k = mg\mu_k$$

$$a = \frac{f_k}{m} = g\mu_k$$

$$\Delta t = \frac{v_i}{a} = \frac{v_i}{g\mu_k}$$

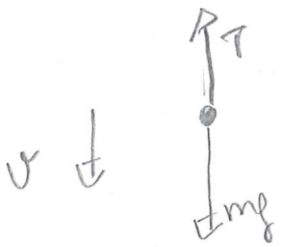
$$W = \frac{1}{2} m v_i^2$$

$$P_{avg} = \frac{W}{\Delta t} = \frac{\frac{1}{2} m v_i^2}{\frac{v_i}{g\mu_k}} = \frac{1}{2} m g \mu_k v_i$$

$$P_{avg} = \frac{1}{2} \cdot 20 \cdot 9{,}80 \cdot 0{,}2 \cdot 8$$

$$= 157 \text{ W}$$

6.54 •• A lift has mass 600 kg, not including passengers. The lift is designed to ascend, at constant speed, a vertical distance of 20.0 m (five floors) in 16.0 s, and it is driven by a motor that can provide up to 30 kW to the lift. What is the maximum number of passengers that can ride in the lift? Assume that an average passenger has mass 65.0 kg.




$$v = \frac{20{,}0\ m}{16{,}0\ s} = 1{,}25\ \frac{m}{s}$$

$$P = Fv = Tv$$

$$T = \frac{P}{v} = \frac{30\times10^3\ \frac{J}{s}}{1{,}25\ m/s}$$

$$m_{total} = \frac{T}{g}$$

$$m_{pasengers} = \frac{T}{g} - 600$$

$$n_{pasenger} = \frac{T/g - 600}{65}$$

$$n_{pas} = \frac{\frac{30\times10^3}{1{,}25\times9{,}80} - 600}{65}$$

$$= 28{,}45$$

28 pasenger can ride