$$A_1^0 = \begin{bmatrix} -s\theta_1 & 0 & c\theta_1 & 0 \\ c\theta_1 & 0 & s\theta_1 & 0 \\ 0 & 1 & 0 & \ell_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2^1 = \begin{bmatrix} -s\theta_2 & -c\theta_2 & 0 & -\ell_2 s\theta_2 \\ c\theta_2 & -s\theta_2 & 0 & \ell_2 c\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3^2 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & \ell_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & \ell_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

last column of A_3^0

$$= \begin{bmatrix} \ell_3 s\theta_1 s\theta_2 c\theta_3 + \ell_3 s\theta_1 c\theta_2 s\theta_3 + \ell_2 s\theta_1 s\theta_2 \\ -\ell_3 c\theta_1 s\theta_2 c\theta_3 - \ell_3 c\theta_1 c\theta_2 s\theta_3 - \ell_2 c\theta_1 s\theta_2 \\ \ell_3 c\theta_2 c\theta_3 - \ell_3 s\theta_2 s\theta_3 + \ell_1 + \ell_2 c\theta_2 \\ 1 \end{bmatrix}$$

$$J_{v_1} = \frac{\partial O_3^0}{\partial \theta_1} = \begin{bmatrix} \ell_3 c\theta_1 (s(\theta_2+\theta_3)) + \ell_2 c\theta_1 s\theta_2 \\ \ell_3 s\theta_1 (s(\theta_2+\theta_3)) + \ell_2 s\theta_1 s\theta_2 \\ 0 \end{bmatrix}$$

$$J_{v_2} = \frac{\partial O_3^0}{\partial \theta_2} = \begin{bmatrix} \ell_3 s\theta_1 \, c(\theta_2+\theta_3) + \ell_2 s\theta_1 c\theta_2 \\ -\ell_3 c\theta_1 \, c(\theta_2+\theta_3) - \ell_2 c\theta_1 c\theta_2 \\ -\ell_3 \, s(\theta_2-\theta_3) - \ell_2 s\theta_2 \end{bmatrix}$$

$$J_{v_3} = \frac{\partial O_3^0}{\partial \theta_3} = \begin{bmatrix} \ell_3 s\theta_1 \, c(\theta_2+\theta_3) \\ -\ell_3 c\theta_1 \, c(\theta_2+\theta_3) \\ -\ell_3 \, s(\theta_2+\theta_3) \end{bmatrix}$$

$$J_{\omega_1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad J_{\omega_2} = R_1^0 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -s\theta_1 & 0 & c\theta_1 \\ c\theta_1 & 0 & s\theta_1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} c\theta_1 \\ s\theta_1 \\ 0 \end{bmatrix}$$

$$J_{\omega_3} = R_2^0 \, z_2^2 = R_1^0 R_2^1 \cdot z_2^2 = \begin{bmatrix} -s\theta_1 & 0 & c\theta_1 \\ c\theta_1 & 0 & s\theta_1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -s\theta_2 & -c\theta_2 & 0 \\ c\theta_2 & -s\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_1 \\ s\theta_1 \\ 0 \end{bmatrix}$$

$$\xi = J(q) \cdot \dot{q} \rightarrow$$ Calculate velocity kinematic map:

$$J(q) = \left[\begin{array}{ccc} c\theta_1[\ell_3 s(\theta_2+\theta_3) + \ell_2 s\theta_2] & s\theta_1[\ell_3 c(\theta_2+\theta_3) + \ell_2 c\theta_2] & \ell_3 s\theta_1 \, c(\theta_2+\theta_3) \\ s\theta_1[\ell_3 s(\theta_2+\theta_3) + \ell_2 s\theta_2] & -c\theta_1[\ell_3 c(\theta_2+\theta_3) + \ell_2 c\theta_2] & -\ell_3 c\theta_1 \, c(\theta_2+\theta_3) \\ 0 & -\ell_3 s(\theta_2-\theta_3) - \ell_2 s\theta_2 & -\ell_3 s(\theta_2+\theta_3) \\ \hline 0 & c\theta_1 & c\theta_1 \\ 0 & s\theta_1 & s\theta_1 \\ 1 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} V \\ \omega \end{bmatrix} = \xi = J(q) \cdot \dot{q} \qquad \dot{q} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

$$\omega = \begin{bmatrix} c\theta_1(\dot{\theta}_2 + \dot{\theta}_3) \\ s\theta_1(\dot{\theta}_2 + \dot{\theta}_3) \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad V = \begin{bmatrix} J_{00} \cdot \dot{\theta}_1 + J_{01} \dot{\theta}_2 + J_{02} \dot{\theta}_3 \\ J_{10} \cdot \dot{\theta}_1 + J_{11} \dot{\theta}_2 + J_{12} \dot{\theta}_3 \\ J_{20} \dot{\theta}_1 + J_{21} \dot{\theta}_2 + J_{22} \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$$

Calculate Position and orientation change

$\delta t = 0{,}01$ is given.

$$d_x = V_x \cdot \delta t \qquad \delta \phi_x = \omega_x \, \delta t$$
$$d_y = V_y \cdot \delta t \qquad \delta \phi_y = \omega_y \cdot \delta t$$
$$d_z = V_z \cdot \delta t \qquad \delta \phi_z = \omega_z \cdot \delta t$$

Update Kinematic Map

$$A(t+\delta t) = T_{dx,dy,dz} \cdot R_{\delta\phi_x, \delta\phi_y, \delta\phi_z} \cdot A(t)$$

$$A(0) = \theta_1 = \theta_2 = \theta_3 = 0$$

$$A(0) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{dx,dy,dz} = \begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & -\delta\phi_z & \delta\phi_y & 0 \\ \delta\phi_z & 1 & -\delta\phi_x & 0 \\ -\delta\phi_y & \delta\phi_x & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse Velocity Kinematics:

Aim: Find $\dot{\theta}$ values from w, v and J.

$\xi = J_{(q)} \cdot \dot{q}$ → $J_{(q)}$ may be non-invertible matrix, to find $\dot{q}$

so;

In fact;

$$\xi = J_{(q)} \cdot \dot{q} \qquad \underset{3\times1}{\dot{q}} = \left(\underset{3\times6}{J_{(q)}^{T}}\ \underset{6\times3}{J_{(q)}}\right)^{-1} \underset{3\times6}{J_{(q)}^{T}}\ \underset{6\times1}{\xi}$$

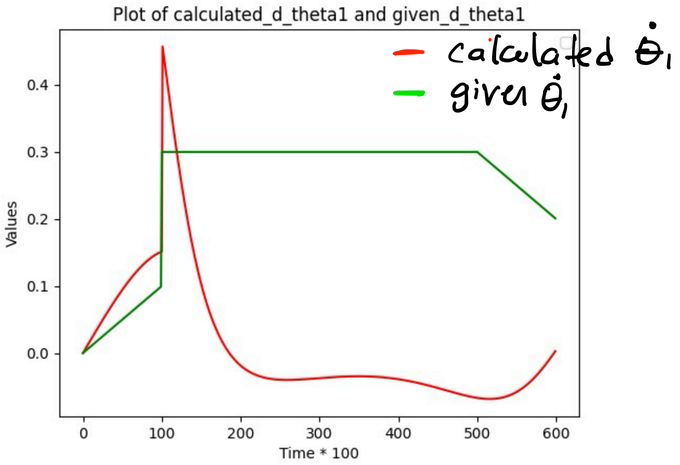
$$\dot{q} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

I used Euler method to calculate θ values.

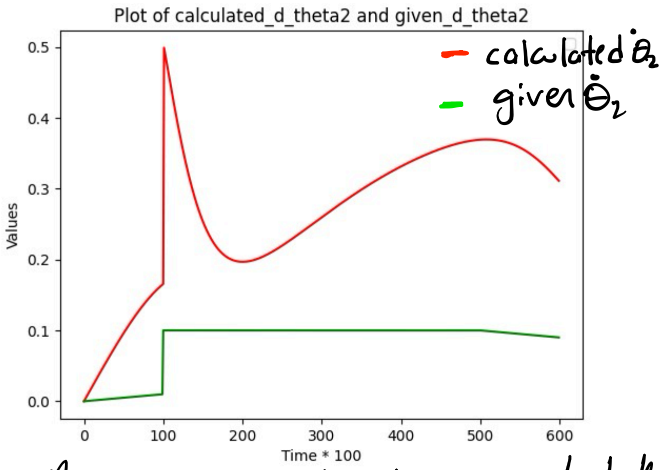
$$\theta_{new} = \theta_{old} + \left(\frac{d\theta}{dt}\right) \cdot \Delta t \qquad \Delta t = 0{,}01 \text{ is given.}$$

```
Printing linear velocity:
0.00472567 0.158093 -0.145229
Printing angular velocity:
0.00815092 0.309893 0.3
Orientation change:
8.15092e-05 0.00309893 0.003
Position change:
4.72567e-05 0.00158093 -0.00145229
Printing final(updated) kinematic map
0.928318 -0.0794238 0.17029 1.89399
-0.0244199 0.00208632 0.998128 0.471623
-0.160193 -1.07144 0.41007 -0.998296
0 0 0 1
Printing Jacobian:
0.012281 0.48329 -0.0158199
0.527068 -0.0112609 0.000368612
0 0.471009 -0.49975
0 0.0232942 0.998491
0 0.0549224 0.999729
1 0 0
```

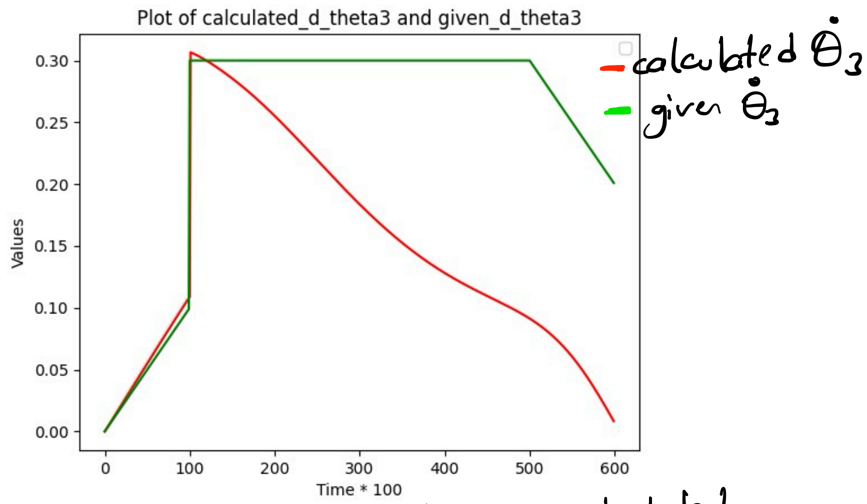
This image shows the terminal, as shown, the linear and angular velocity, Jacobian matrix and changes in positions are printed.



As shown in the figure, calculated and given $\dot{\theta}_1$ plotted.



As shown in the figure, calculated and given $\dot{\theta}_2$ are plotted.



As shown in the figure, calculated and given $\dot{\theta}_3$ are plotted.

Given $\dot{\theta}$'s

$$\dot{\theta}_1 = \dot{\theta}_3 = \begin{cases} 0{,}1t & , 0 \leq t \leq 1 \\ 0{,}3 & , 1 < t < 5 \\ 0{,}3 - 0{,}1(t-5) & , 5 < t < 6 \end{cases}$$

$$\dot{\theta}_2 = \begin{cases} 0{,}01t & , 0 \leq t \leq 1 \\ 0{,}1 & , 1 < t < 5 \\ 0{,}1 - 0{,}01(t-5) & , 5 < t < 6 \end{cases}$$