EE 576 Project: # 2

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I. Problem

The project aims to explore transformations between images using homography mapping. Given a sequence of images, we are tasked with understanding and implementing a homography transformation between a pair of consecutive images. The homography transformation is represented by the following equation:

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

where (x, y) refers to the homogeneous representation of a pixel from the first image and (x', y') refers to the homogeneous coordinates of the corresponding pixel in the second image.

To determine the homography matrix H, we define the equation $Ah = a$ to be solved, where A is the matrix of coefficients formed from the pairs of corresponding matched points, h is the vector to be solved representing the elements of the homography matrix, and a is the vector of known values.

1) Pseudo-inverse calculation to solve $Ah = a$:

$$h = (A^T A)^{-1} A^T a$$

2) Reshaping h into a 3×3 homography matrix H:

$$H = \begin{pmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & 1 \end{pmatrix}$$

3) Matrix form of A:

$$A = \begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1 x'_1 & -y_1 x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1 y'_1 & -y_1 y'_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n & y_n & 1 & 0 & 0 & 0 & -x_n x'_n & -y_n x'_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -x_n y'_n & -y_n y'_n \end{pmatrix}$$

4) Matrix form of a:

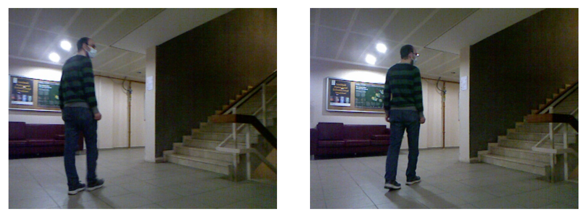
$$a = \begin{pmatrix} x'_1 \\ y'_1 \\ \vdots \\ x'_n \\ y'_n \end{pmatrix}$$

5) Matrix form of h:

$$h = \begin{pmatrix} h_1 \\ h_2 \\ \vdots \\ h_7 \\ h_8 \end{pmatrix}$$

II. Results

Fig. 1: Test Images



(a) First image (b) Second image

Figure 1 shows the consecutive images that are used for the test.

III. Conclusion

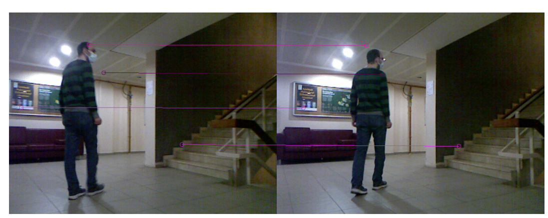


Fig. 3: Matched Images

Figure 3 shows the matched points between the images. The Homography matrix is calculated using pseudo-inverse method of OpenCV. The calculated Homagraphy matrix is:

$$H = \begin{bmatrix} 1.0437713 & 0.075590581 & 16.422955 \\ 0.054039776 & 1.0577009 & -10.013304 \\ 0.00016356152 & 4.9446287 \times 10^{-5} & 1 \end{bmatrix}$$

The average error is $e = 0.578058$.

Figure 4 shows the matched points between the images. The Homography matrix is calculated using findHomography

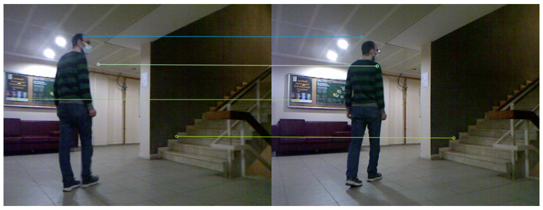


Fig. 4: Matched Images

method of OpenCV which is a built-in function. The Homagraphy matrix is:

$$H = \begin{bmatrix} 1.042761799100274 & 0.07619661581954505 & 16.53525685510474 \\ 0.05407120731817169 & 1.057441134298677 & -9.961777008805736 \\ 0.0001638749929677742 & 4.891028178658599 \times 10^{-5} & 1 \end{bmatrix}$$

The average error is $e = 4.84706e - 13$.