Laboratory 10

$$\min_{u_A(t)} \frac{m_B(3) - m_B(0)}{m_B(0)}$$

s.t.

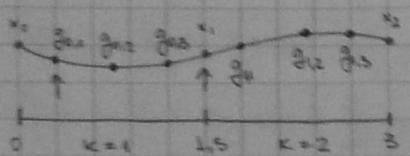
$$\frac{dm_A}{dt} = u_A - k\frac{m_A(t)\, m_B(t)}{V(t)^2}, \quad m_A(0) = 0$$

$$\frac{dm_B}{dt} = -k\frac{m_A(t)\, m_B(t)}{V(t)^2}, \quad m_B(0) = m_{B,0}$$

$$\frac{dV(t)}{dt} = Q_A\, u_A(t), \quad V(0) = V_0$$

$$x_{k+1} = x_k + \tau \sum_{i=1}^{3} b_i\, f(g_{ki}, u_k)$$

$$g_{ki} = x_k + \tau \sum_{j=1}^{3} a_{ij}\, f(g_{kj}, u_k)$$



→ at each finite element is divided through the collocation method

→ order 5 is enough to describe each finite element

$$x_k = \begin{bmatrix} m_{A,k} \\ m_{B,k} \\ V_{B,k} \end{bmatrix}$$

$$x_{A,1} = x_{A,0} + \tau \sum b_i \cdot \left(u_{A,0} - \bar{k} \cdot \frac{g_{A,0,i}\, g_{B,0,i}}{g_{V,0,i}} \right)$$

$\bar{k}$: kinetics constant

$$g_{A,0,i} = x_{A,0} + \tau \sum a_{ij} \cdot \left(u_{A,0} - \bar{k} \cdot \frac{g_{A,0,j} + g_{B,0,j}}{g^2_{V,0,j}} \right)$$

Bands: $0 \le u_k(t) \le \bar{u}$ $\forall k$

→ at each finite element: u_k → const

$$\min \frac{x_{B,2} - x_{B,0}}{x_{B,0}}$$

→ Problem is then reformulated as a nonlinear program

- Variables of the NLP

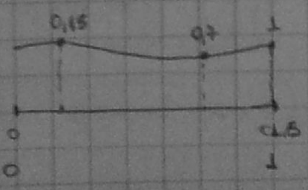
$x_{A,1}$	$x_{A,2}$	u_0	$g_{A,0,1}$	$g_{B,0,1}$
$x_{B,1}$	$x_{B,2}$	u_1	$g_{A,0,2}$	$g_{B,0,2}$
$x_{V,1}$	$x_{V,2}$		$g_{A,0,3}$	

→ 18 g_k variables

→ Provide derivatives of g_k with respect to each $g_{A,0,1}$, ...

$$a = \begin{bmatrix} 88 & 296 & -2 \\ 296 & 88 & -2 \\ 16 & 16 & 1/9 \end{bmatrix} \begin{matrix} \to \Sigma = 0{,}15 \\ \to \Sigma = 0{,}7 \\ \to \Sigma = \underbrace{1}_{\text{sum of the entries}} \end{matrix}$$

Parametric position on the curve



→ the third g is used in the calculation of the final point

As the last collocation point is equal the x_x, b is equal the last line of a.

If we fix the position of the collocation points, a and b can be calculated out of c