STAT 251 Written Assignment-2

(40 TOTAL MARKS)

Note 1: Please show steps with proper justifications in your solutions. Partial credits are given to intermediate steps and reasoning. Also define any event and notation that you use in your solutions. Use 3 decimal points in calculation. Don't excessively round off in intermediate steps.

Note 2: Discussion of ideas learned in class is encourage (with other students, TAs or the instructor). This helps the leaning process. But individual work turned in by each student should be your own work. Do not copy or paraphrase solutions from other students or from other sources. DO NOT provide your solutions to another student. Failure to comply with these rules will result in an automatic 0 for your work, and additional academic penalties.

Question 1

Two gamblers decided to play a game involving rolling a *fair* die. Player A agreed to pay Player B \$10 as an "entry fee" to play. The game consists of Player A rolling a die until he rolls an 1 for the first time. An example of a possible sequence of rolls in this game is $\{(2, 6, 3, 5, 1)\}$, with the number of rolls $N = 5$. Assume the rolls are independent.

a) Player A wins the game if it takes at least 3 rolls for the first 1 to show up. Find out the probability that player A wins one round of the game.*(3 marks)*

b) For each round of the game, if Player A wins according to the condition in part (a), then he gets \$30 from Player B. Find the expected value of Player A's profit. Is this game a good deal for Player A? Briefly explain your answer. *(3 marks)*

c) Player A decides to play the game for 100 rounds. Find the expectation and variance of the number of games that player A wins according to the condition in part (a). *(4 marks)*

Question 2

The number of patients get tested at a COVID-19 drive through test site follows a Poisson process with parameter $\lambda = 4$ per minute. Suppose the probability that a patient is tested positive for COVID is $p = 0.9$.

a) Find the probability that no patients get tested in 30 seconds. *(2 marks)*

b) Find the distribution of the number of patients that are tested positive in ℓ minutes. *(5 marks)*

c) Find the probability that the first patient is tested positive shows up at least 1 minute after the test center opens. *(3 marks)*

Question 3

The expected grades of two independent STAT courses (A and B, say) are 86 and 82. The respective standard deviations are 5 and 7. Let $\bar{X}$ and $\bar{Y}$ be the sample average grades of 38 students of course A and 32 students of course B, respectively.

a) What is the approximate distribution of $\bar{X}$? Of $\bar{Y}$? *(3 mark)*

b) What is the approximate distribution of $\bar{X} - \bar{Y}$? Why? *(3 mark)*

c) Calculate (approximately) $P(|\bar{X} - \bar{Y}| < 1)$. *(4 mark)*

Question 4

Mechanics agree that the chain on a bicycle should be replaced after covering an average of 3000 miles. StakRide is a new bicycle manufacturing company and it wants to check if the chains it produces for bicycles have a longer life span than the existing average. The lead mechanic randomly selects 25 of the new bicycles and test runs them. The resulting sample mean and standard deviation are 3050 miles and 85 miles, respectively.

a) What hypothesis should be tested to determine whether the life span of StakRide bicycle chains is longer than the known average? *(3 marks)*

b) Assuming that the life span of the bicycle chains is approximately normal, what test statistic would you use to test the hypotheses in part(a)? What is the value of the test statistic for this data? *(3 marks)*

c) What conclusion would you reach for a significance level of 0.05. *(4 marks)*