Question 2

The number of patients get tested at a COVID-19 drive through test site follows a Poisson process with parameter $\lambda = 4$ per minute. Suppose the probability that a patient is tested positive for COVID is $p = 0.9$

A) Find the probability that no patients get tested in 30 seconds

$X_1 \sim Poisson(4)$ where X_1 means 1 minute. $\therefore X_t \sim Poisson(4t)$. So for a 30 second interval (or half a minute) then $X_{0.5} \sim Poisson(4 * 0.5)$.

So what is the probability that we get 0 tests in 30 seconds?

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

In this case, x=0

In [1]:

```
rate = 4 * 0.5
x = 0

P0 = rate^(x)*exp(-rate) / (factorial(x))

paste("The probability of getting 0 patients in 30 seconds is ",round(P0,4))
```

'The probability of getting 0 patients in 30 seconds is 0.1353'

B) Find the distribution of the number of patients that are tested positive in l minutes. (5 marks)

We have to make a new distribution that involves the number of people being tested and the probability of getting a positive test. Since it wants the number of positive patients in l minutes (a specific period of time) it wants another poisson distribution. We could find the PDF of such a distribution like so:

Consider the poisson distribution:

$$X_t \text{~} Poisson(4t)$$

A patient is found to be positive 90 percent of the time, so:

$$X_l \text{~} Poisson((4 * 0.9)l)$$

so the actual distribution is:

$$X_l \text{~} Poisson(3.6l)$$

We just need to sum all the integers up to get teh cumulative denisity function:

$$P(X = i) = \frac{(3.6l)^i e^{-3.6l}}{i!}$$

C) Find the probability that the first patient is tested positive shows up at least 1 minute after the test center opens. (3 marks)

$$P(X \geq 1) = 1 - P(X > 1) = 1 - P(X = 0)$$

Given that it is 1 minute, l must be 1.

In [2]:

```
rate = 3.6
i = 0

p = ((rate^i)*exp(-rate)) / factorial(i)
print(paste("the probability is ",round(1-p,4)*100,'%', sep = ''))
```

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[1] "the probability is 97.27%"
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