Question 3

The expected grades of two independent STAT courses (A and B, say) are 86 and 82. The respective standard deviations are 5 and 7. Let $\bar{X}$ and $\bar{Y}$ be the sample average grades of 38 students of course A and 32 students of course B, respectively.

A) What is the approximate distribution of $\bar{X}$? Of $\bar{Y}$? (3 mark)

Each one of the 38 and 32 students are going to act as a random variable. Since all students from group A all come from the same normal distribution, and because the sample size is sufficiently large $(n \geq 20)$ then CLT can be applied to this scenario and the approximate distribution for group A would be:

$A \sim N(86, \frac{5^2}{38})$

The same logic is applied to group B:

$B \sim N(82, \frac{7^2}{32})$

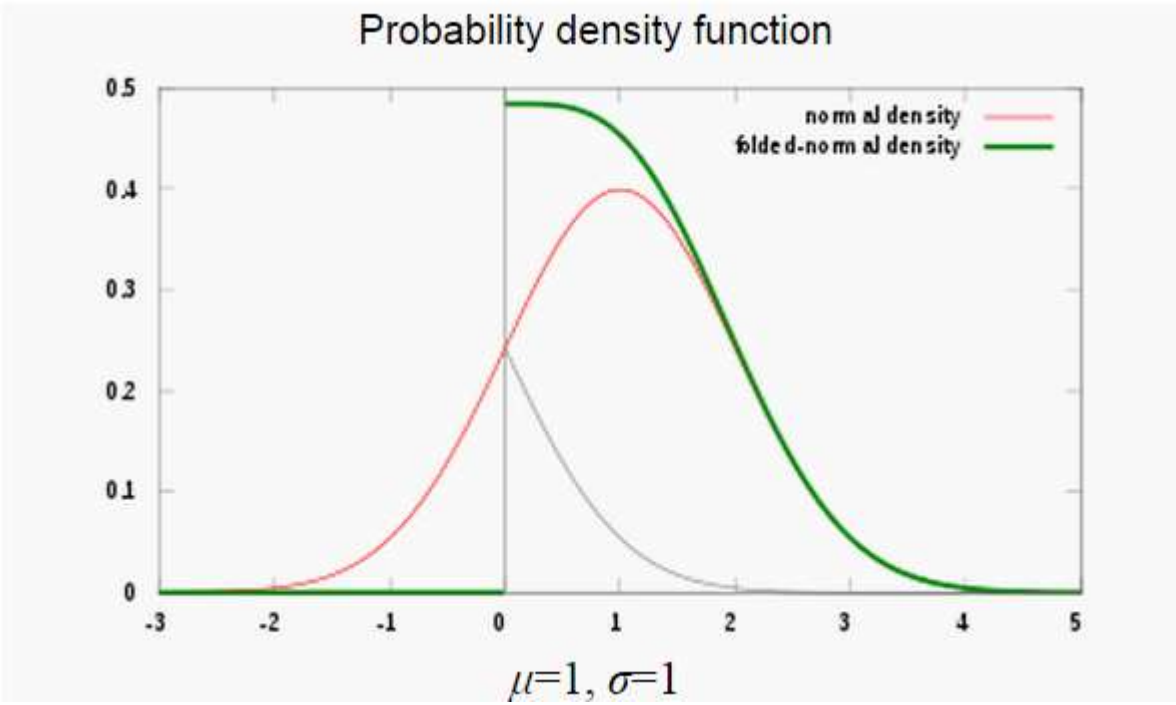
B) What is the approximate distribution of $\bar{X} - \bar{Y}$? Why? (3 mark)

We can simply take the difference of these sums. They act approximatley like random variables who vary normally, so this is valid:

$$\mu = \mu_A - \mu_B = 86 - 82 = 4$$
$$\sigma^2 = \sigma_A^2 + \sigma_B^2 = \frac{5^2}{38} + \frac{7^2}{32} \approx 2.1891$$

C) Calculate (approximately) P(|X¯ − Y¯ | < 1). (4 mark)

Since the absolute value is applied to the normal distribution, it has been folded in half, making any values less than 1 invalid:



So I'll find

$P(|\bar{X} - \bar{Y}| < 1) - P(|\bar{X} - \bar{Y}| < 0)$

In [1]:

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sigma = (5^2)/38 + (7^2)/32

less1 = pnorm(1, mean = 4, sd = sigma)
less0 = pnorm(0, mean = 4, sd = sigma)
less1 - less0
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0.0514462847137594

The probability is approimatley 5%