PART 1: The following problem will focus on simple probability rules

Suppose there is a skytrain that travels at a variety of speeds depending on factors such as time of day, weather, etc. The speed at which the skytrain travels at can be modelled by the following random variable:

Speed (km/hr)	Probability
30	0.4
40	0.3
60	0.2
80	0.1

Suppose there are three such skytrains independently travelling at once. After one run of each skytrain, what is the probability that:

A All three skytrains travelled at 40 km/hr?

The probability for 1 skytrain is 0.3, and the 3 of them run independently. To combine independent probabilities, simply multiply them together.

In [2]:
```
a = 0.3*0.3*0.3
a
```

0.027

B All three skytrains travelled at 60 km/hr or less?

So it's going to be 0.2, 0.3 and 0.4 that we have to combine some how (leaving out 80 since it's above 60). So we can use a CDF (cumulative distribution for that). Let's find the probability of 1 train:

$$P(X \leq x) = \sum_{k \leq x} f(x)$$

$f(60) + f(50) + f(40) = 0.4 + 0.3 + 0.2 = 0.9, \therefore$ The chances are 0.9 or 90% for 1 train

for all 3 trains, it would be:

In [2]:
```
b = 0.9*0.9*0.9
b
```

0.729

C. At least one skytrain travelled at 80 km/hr?

To do this, we would have to manually add up:

- p of only first train going 80.
- p of only second train going 80.
- p of only third train going 80.

Wow, we're not even done.

- p of only first and second train going 80.
- p of only first and third train going 80.
- p of only second and third train going 80.
- p of all 3 trains going 80.

add all of these 7 events up, you got a probability! Let's do that!

In [7]:
```
P1 = 0.1 * 0.9 * 0.9
P2 = 0.9 * 0.1 * 0.9
P3 = 0.9 * 0.9 * 0.1

P12 = 0.1 * 0.1 * 0.9
P13 = 0.1 * 0.9 * 0.1
P23 = 0.9 * 0.1 * 0.1

P123 = 0.1 * 0.1 * 0.1

all = c(P1,P2,P3,P12,P13,P23,P123)
sum(all)
```

0.271

tedious as hell! But here's a little secret. If you just do the probability of no train going 80, then subtract by 1, you'll get the probability that 1 train goes 80, watch:

In [6]:
```
no80 = (1 - 0.1) * (1-0.1) * (1-0.1)

# Then 1 - none of them going 80 = at least 1 is going 80.

1 - no80
```

0.271

D. At least two skytrains travelled at 80 km/hr?

Easy

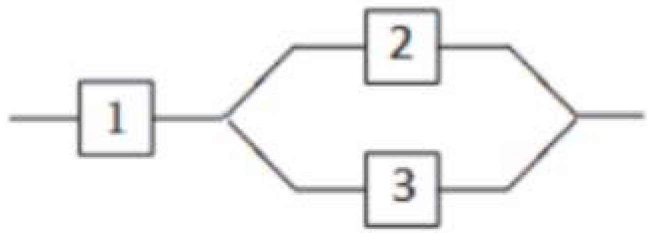
In [1]:
```
P12 = 0.1 * 0.1 * 0.9
P13 = 0.1 * 0.9 * 0.1
P23 = 0.9 * 0.1 * 0.1
P123 = 0.1 * 0.1 * 0.1

at_least_2 = c(P12, P13, P23, P123)
sum(at_least_2)
```

0.028

Part 2: Conditional Probabilites

Suppose we take 3 skytrains and connect them in a chain as shown in the picture below:



The probability that each skytrain breaks down and stops is 0.1

A. Find the reliability of the whole skytrain chain.

to do that, we just have to find the probability that the system will fail (we'll call that F) then to get relaibilty (which we'll call R) we can do $R = 1 - F$.

So how can it fail? Well if 1 breaks down, the thing will die, so if 1 fails,

The other way is if BOTH 2 and 3 fail. If only 1 fails (say 2) then 3 and 1 can still go through.

And remember, they are independent, so we can multiply them together to combine probabilities.

In [3]:
```
#First the failure probability
f1 = 0.1
f23 = 0.1 * 0.1

#Now to get reliability:
r1 = 1 - f1
r23 = 1 - f23

#To get the overall reliability
rt = r1 * r23
rt
```

0.891

This type of stuff is done all the time in electrical engineering, but chemical engineering should see it's fair share fo this, say if it were like a pump or a distiller or something.

B. Given that the chain is broken, find the probability that skytrain 1 broke down.

First, what's the probability of this system breaking down?

$$P(D) = 1 - 0.891 = 0.109$$

Now we can do the $P(T1|D)$ thing.

$$P(T1|D) = \frac{P(T1 \cap D)}{P(D)} = \frac{0.1}{0.109} = 91.74\%$$

C. Suppose you know with 100% certainty that skytrain 3 will not break down. What is the updated reliability of the skytrain chain

Now it doesn't matter if train 2 breaks, because 3 is 100%! So the only way this thing breaks is if 1 breaks. $\therefore$ the new reliability is

$$R = 1 - F1 = 1 - 0.1 = 0.9$$

In [ ]: