LAB 4

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1. $A \sim Exp(\frac{1}{80})$
$B \sim Exp(\frac{1}{80})$
$C \sim Exp(\frac{1}{80})$

$Y = \max(A, B, C)$

first, the CDF

$$F(y) = P(Y \leq y)$$
$$= P(A \leq y \,\&\, B \leq y \,\&\, C \leq$$
(independent) $$= P(A \leq y) \cdot P(B \leq y) \cdot P($$
$$= (1-e^{-y/80}) \cdot (1-e^{-y/80}) \cdot ($$
$$= (1-e^{-y/80})^3$$

CDF of Y $\longrightarrow$ $F(y) = 3e^{\frac{-y}{40}} - 3e^{\frac{-y}{80}} - e^{\frac{-3y}{80}} +$

$$PDF = \frac{d}{dx} CDF$$
$$= \frac{d}{dx}\left(3e^{-y/40} - 3e^{-\frac{y}{80}} - e^{-3y/80}\right.$$
$$f(y) = \frac{-3}{40}e^{\frac{-y}{40}} + \frac{3}{80}e^{-\frac{y}{80}} + \frac{3}{80}e^{\frac{-3y}{80}}$$ (PDF of Y)

In [1]:
```
options(repr.plot.width=4, repr.plot.height=4) #Something to modify the size of the plot
```

Question 2: What is the probability that the system fails before 70 hours based on your cdf in Question 1?

In [2]:
```
cdf = function(y){(1-exp(-y/80))^3}
```

This function is the probability that a certain value falls between 0 and y, where $y = 70$ according to the question:

In [3]:
```
cat(round(cdf(70),4)*100,"%")
```

```
19.83 %
```

Which makes sense given that the means are 80.

Question 3:Generate a random sample of size 10,000 for the lifetime of System 1

i) Draw a histogram representing the probability density of the sample. On top of the histogram, draw the pdf calculated in Question 1. Does the probability density of the sample follow similar pattern as the pdf?

ii) Estimate the probability that the system fails before 70 hours using the sampled data. Is the result close to the true probability value?

i)

In [4]:
```
n = 10000

A = rexp(n, rate = 1/80)
B = rexp(n, rate = 1/80)
C = rexp(n, rate = 1/80)

#I copy it 3 times with the expectation that the random element will make them different
#Despite the same parameters

Y = cbind(A,B,C)
head(Y)

Y1 = apply(Y,1,max)
head(Y1)
```

A	B	C
177.90208	37.66048	138.942161
14.05924	268.37468	48.547797
96.34598	462.07320	154.529644
80.70542	23.62402	12.047823
73.69495	46.14321	1.204341
25.77810	48.03577	154.740564

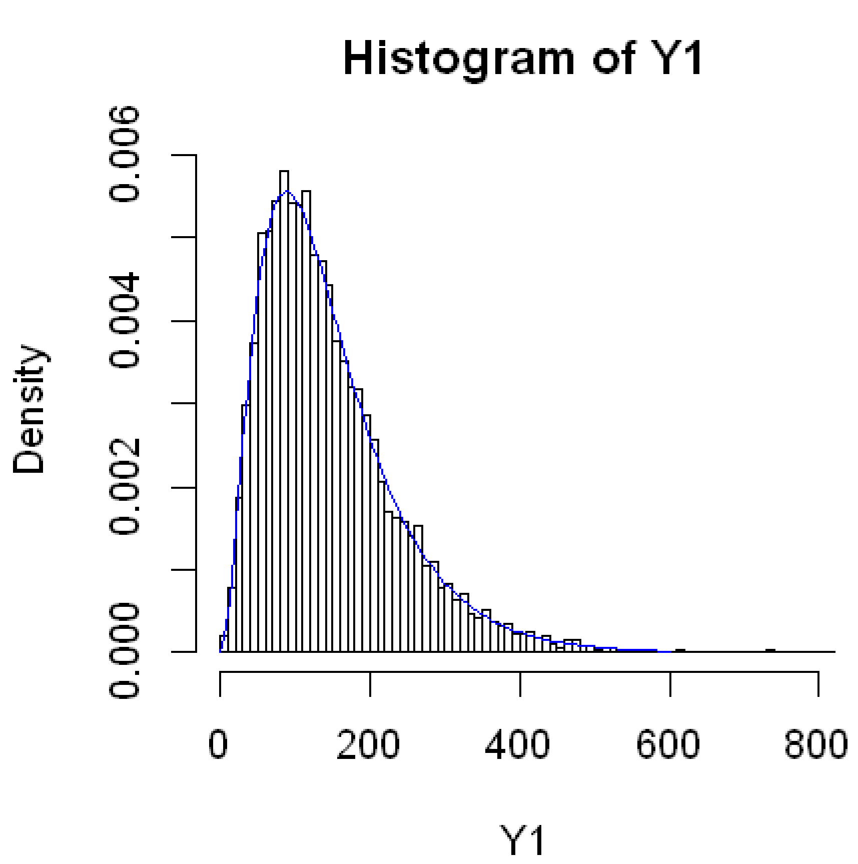
1. 177.902081624757
2. 268.374676690964
3. 462.073199080202
4. 80.7054182108412
5. 73.694950562335
6. 154.740564220383

Now we have the data, we can easily generate histogram

In [5]:
```
#First, let's get the PDF function up and running
pdf = function(y){(-3/40)*exp(-y/40) + (3/80)*exp(-y/80) + (3/80)*exp(-3*y/80)}

#Now the histogram
hist(Y1, prob=TRUE, breaks = 100)

#Now the curve
curve(pdf, from = 0, to = 600, add=TRUE, col="blue")
```



And yes, the randomly generated curve does seem to follow the PDF quite closley!

ii)

In [6]:
```
cat(round(sum(Y1<70)/n,4)*100,"%")
```

```
19.71 %
```

Which is close enough.

Part 2:

$F(Y) = P(Y \leq y)$

but we are looking for the min, which means we need the P that it w[ill] last LONGER than the other va[lues]

$= 1 - P(Y \geq y)$

$= 1 - P(A \geq y \;\&\; B \geq y \;\&\; C \geq y)$

$= 1 - P(A \geq y) \cdot P(B \geq y) \cdot P(C \geq y)$ (independent)

$= 1 - F(A) * F(B) \cdot F(C)$

$= 1 - \left(e^{\frac{-y}{80}}\right)^3$

$F(y) = 1 - e^{\frac{-3y}{80}}$ (CDF of y)

$\text{pdf} = \frac{d}{dx}(\text{CDF})$

$f(Y) = \frac{3}{80} e^{\frac{-3Y}{80}}$ (follows an exponential curve)

In [1]:
```
options(repr.plot.width=4, repr.plot.height=4) #Something to modify the size of the plot
```

Question 2: What is the probability that the system fails before 70 hours based on your cdf in Question 1?

In [2]:
```
cdf = function(y){1-exp(-(3*y)/80)}
cat(round(cdf(70),4)*100,"%")
```

```
92.76 %
```

Question 3:Generate a random sample of size 10,000 for the lifetime of System 1

i) Draw a histogram representing the probability density of the sample. On top of the histogram, draw the pdf calculated in Question 1. Does the probability density of the sample follow similar pattern as the pdf?

ii) Estimate the probability that the system fails before 70 hours using the sampled data. Is the result close to the true probability value?

i)

In [3]:
```
n = 10000

A = rexp(n, rate = 1/80)
B = rexp(n, rate = 1/80)
C = rexp(n, rate = 1/80)

#I copy it 3 times with the expectation that the random element will make them different
#Despite the same parameters

Y = cbind(A,B,C)
head(Y)

Y1 = apply(Y,1,min)
head(Y1)
```

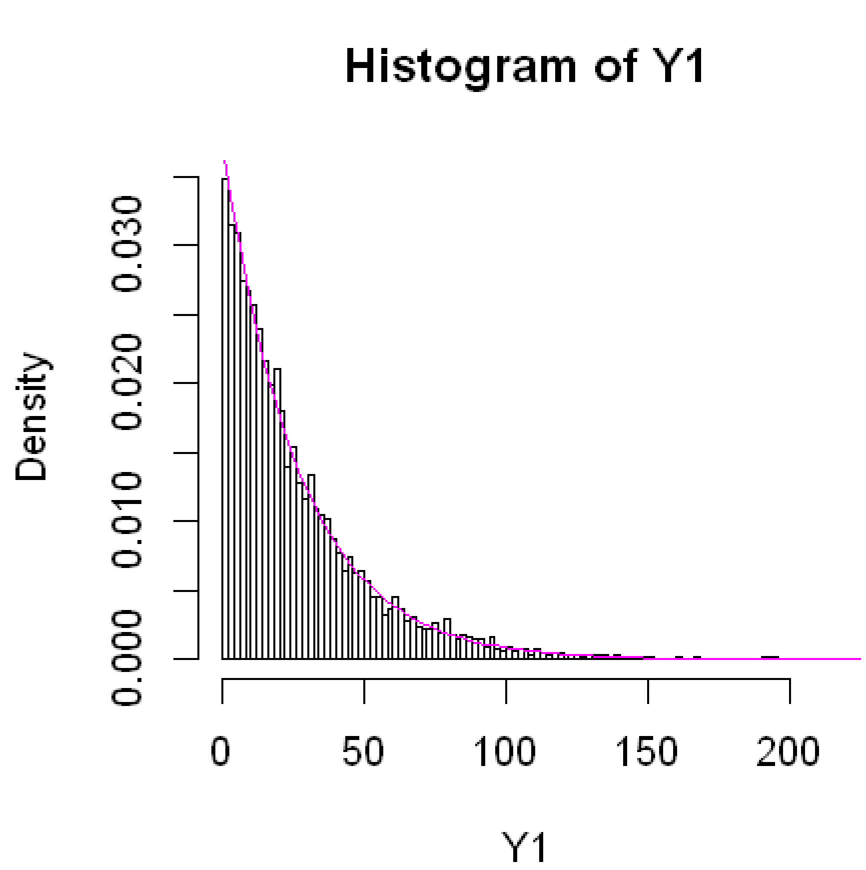
A	B	C
56.97403	162.030840	11.463573
64.45959	3.443281	7.140519
21.21353	61.099096	45.794849
138.54030	30.080904	79.072207
105.47026	46.590513	6.061617
16.62177	11.285458	58.171659

1. 11.4635729789734
2. 3.44328138977289
3. 21.2135274831955
4. 30.0809041038156
5. 6.06161680072546
6. 11.285457611084

In [4]:
```
#First, let's get the PDF function up and running
pdf = function(y){(3/80) * exp(-3*y/80)}

#Now the histogram
hist(Y1, prob=TRUE, breaks = 100)

#Now the curve
curve(pdf, from = 0, to = 300, add=TRUE, col = "Magenta")
```



In [5]:
```
cat(round(sum(Y1<70)/n,4)*100,"%")
```

```
92.47 %
```

And yes, it fits beautifully!