Question 4: The National Test

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Entry to a certificate program is determined by a national test comprised of two subjects A and B. The scores on subject A are normally distributed with a mean of 80 and a standard deviation of 3, and on subject B are normally distributed with a mean of 70 and a standard deviation of 4. The scores on subject A and B are assumed to be independent.

A) What proportion of students score between 145 and 160 points on the test? (round to the nearest 2 decimal places) *(3 Marks)*

$$A : \mu_A = 80, \sigma_A = 3$$
$$B : \mu_B = 70, \sigma_B = 4$$

Answer

From the question, I gather that the test is out of 200 points, where 100 possible points from part A, and 100 possible points from part B. These tests are:

- Independent of eachother
- Normally Distributed
- Added together for their final score

We will create a new normal distribution with a new random variable which I'll call T for test. For this new distribution, we need only know the mean and standard deviation, which comes as a result of part A and B.

The mean

The means can be combined via simple addition.

$$\mu_T = \mu_A + \mu_B$$
$$\mu_T = 80 + 70 = 150$$

The Standard deviation

Because these 2 tests (which are also random variables) are independent, we can take the sum of the **variance** (not the standard deviation) of these 2 random variables A and B, and use it to get to the standard deviation:

$$\sigma_T^2 = \sigma_A^2 + \sigma_B^2$$
$$\sigma_T^2 = (3)^2 + (4)^2 = 9 + 16$$
$$\sigma_T^2 = 25$$
$$\sigma_T = 5$$

Now that we have this information, we can use the Z-Score to see what proportion of students are going to score between the 2 values 145 and 160:

In [1]:
```
mean = 150
stdd = 5

z1 = (145-mean)/(stdd)
z2 = (160-mean)/(stdd)

prop = round(pnorm(z2)-pnorm(z1),3)*100

cat(prop,"% of students scored between 145 and 160 on the national test")
```

```
81.9 % of students scored between 145 and 160 on the national test
```

B) If a student needs to score more than 155 points to be admitted to the program. For a random student, what is the probability of getting admitted? (round to the nearest 2 decimal places) *(2 Marks)*

First, we need the probability that people get a score of 155 or LESS:

In [2]:
```
z1 = (155-mean)/stdd

#Now, we calculate the probability of getting a 155 or LESS:
prob = pnorm(z1)

#Now we can see the pronability of getting 155 or MORE by simplying taking 1-prob:
cat(round(1-prob,4)*100,"% is the probability of scoring above a 155 or more and being admitted")
```

```
15.87 % is the probability of scoring above a 155 or more and being admitted
```

C) If the program admits students that score better than at least 70% of the students who took the test, will a student who scored 153 points be admitted to the program?

Let's see what z-score that student got:

In [3]:
```
z1 = (153-mean)/stdd

#And that corresponds to a probability of:

cat(round(pnorm(z1),4)*100,"%")
```

```
72.57 %
```

So this student scored better than 73% of candidates, which is bigger than 70%, so yes he will be admitted.

D) If two students are randomly selected, calculate the probability that the difference between their subject A scores is less than 10 points. (round to the nearest 2 decimal places)

As a reminder:

The scores on subject A are normally distributed with a mean of 80 and a standard deviation of 3

So we have to find the probability that a student would score 10 points or less than another student. Well let's see what happens with some sample z-scores that are 10 points apart:

In [14]:
```
mean = 80
stdd = 3

z1 = (82-mean)/stdd
z2 = (92-mean)/stdd
z1
z2
```

0.666666666666667

4

In [15]:
```
z1 = (76-mean)/stdd
z2 = (86-mean)/stdd
z1
z2
```

-1.33333333333333

2

In [16]:
```
z1 = (44-mean)/stdd
z2 = (54-mean)/stdd
z1
z2
```

-12

-8.66666666666667

Their z-scores and probabilities vary wildley with a 10 point difference, so we have to make a whole new distribution, one in which we have 2 students scores A_1 and A_2.

Let $D = A_1 - A_2$ where A_1, A_2 and D are all distributions. D is the difference between these 2 distributions, and as we learned from part A, combining them should be easy!

$$\mu_D = E(A_1) - E(A_2) = 80 - 80 = 0$$
$$\sigma^2 = Var(A_1 - A_2) = Var(A_1) + Var(A_2) = 3^2 + 3^2 = 18$$
$$D\text{-}N(0, 4.2426^2)$$

Now the probability of there being a difference of 10 points, is the same as the probability of there being a 10 point difference from the mean:

$$z = \frac{x - \mu}{\sigma} = \frac{10 - 0}{\sqrt{18}} = \frac{10}{\sqrt{18}} \approx 2.357$$

A z-score of 2.357 corresponds to a probability of:

In [18]:
```
z = 10/(18)^0.5
pnorm(z)
```

0.99078893727295

But this is 10 points above, what about 10 points below? Well:

$$P(10below < z < 10above) = P(10above) - P(10below)$$

In [24]:
```
above = pnorm(z)
below = 1-above
prob = above-below

cat("The probability that 2 students score within 10 points of eachother in Part A of the test is ",round(prob,2),"or 98%")
```

```
The probability that 2 students score within 10 points of eachother in Part A of the test is  0.98 or 98%
```