
MODULE 4**QUANTUM MECHANICS AND LASERS****Dual nature of matter (de-Broglie Hypothesis)****Dual nature of light:**

The concept of photoelectric effect and Compton Effect gives the evidence for particle nature of light. Where as in physical optics the phenomenon like interference, diffraction, superposition was explained by considering wave nature of light. This is wave particle duality of light.

Dual nature of matter:

On the basis of above concept (dual nature of light), in 1923, Louis de Broglie gave a hypothesis

“Since nature loves symmetry, if the radiation behaves as particles under certain conditions and as waves under certain conditions, then one can expect that, the entities which ordinarily behaves as particles (ex. Like electrons, protons, neutrons) must also exhibit properties attributable to waves under appropriate circumstances”. This is known as **deBroglie hypothesis**

Matter is made up of discrete constituent particles like atoms, molecules, protons, neutrons and electrons, hence matter has particle nature. Wave nature of matter is explained by Davisson and Germer experiment. Hence matter also exhibit wave particle duality.

The waves associated with the moving particles are called de Broglie waves or matter waves or pilot waves.

Characteristics of matter waves:

1. Waves associated with moving particles are called matter waves. The wavelength ‘ λ ’ of a de-Broglie wave associated with particle of mass ‘ m ’ moving with velocity ‘ v ’ is

$$\lambda = h/(mv)$$

2. Matter waves are not electromagnetic waves because the de Broglie wavelength is independent of charge of the moving particle.
3. The amplitude of the matter wave depends on the probability of finding the particle in that position.
4. The speed of matter waves depends on the mass and velocity of the particle associated with the wave.

Debroglie’s Wavelength:

A particle of mass ‘ m ’ moving with velocity ‘ c ’ possess energy given by

$$E = mc^2 \quad \rightarrow \text{(Einstein’s Equation) (1)}$$

According to Planck's quantum theory the energy of quantum of frequency 'v' is

$$E = h\nu \rightarrow (2)$$

From (1) & (2)

$$mc^2 = h\nu = hc / \lambda \quad \text{since } \nu = c/\lambda$$

$$\lambda = hc / mc^2 = h/mc$$

$$\lambda = h/mv \quad \text{since } v \approx c$$

de-Broglie wavelength of a free particle in terms of its kinetic energy

Consider a particle, since the particle is free, the total energy is same as

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Where 'm' is the mass, 'v' is the velocity and 'p' is the momentum of the particle.

$$p = \sqrt{2mE}$$

The expression for de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

Debroglie Wavelength of an Accelerated Electron:

If an electron accelerated with potential difference 'V' the work done on the 'eV', which is converted to kinetic energy.

$$\text{Then } eV = \frac{1}{2}mv^2 \rightarrow (1)$$

If 'p' is the momentum of the electron, then $p=mv$

Squaring on both sides, we have

$$p^2 = m^2v^2$$

$$mv^2 = p^2/m$$

Using in equation (1) we have

$$eV = p^2/(2m)$$

$$\text{or } p = \sqrt{2meV}$$

According to de-Broglie $\lambda = h/p$

$$\text{Therefore } \lambda = \left[\frac{h}{\sqrt{2meV}} \right]$$

$$\lambda = \frac{1}{\sqrt{V}} \left[\frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.602 \times 10^{-19}}} \right] = \frac{1.226 \times 10^{-9}}{\sqrt{V}} \text{ m}, \quad \lambda = \frac{1.226}{\sqrt{V}} \text{ nm}$$

Heisenberg's Uncertainty Principle:

According to classical mechanics a particle occupies a definite place in space and possesses a definite momentum. If the position and momentum of a particle is known at any instant of time, it is possible to calculate its position and momentum at any later instant of time. The path of the particle could be traced. This concept breaks down in quantum mechanics leading to Heisenberg's Uncertainty Principle.

Heisenberg's Uncertainty Principle states that "It is impossible to measure simultaneously both the position and momentum of a particle accurately. If we make an effort to measure very accurately the position of a particle, it leads to large uncertainty in the measurement of momentum and vice versa".

If Δx and ΔP_x are the uncertainties in the measurement of position and momentum of the particle then the uncertainty can be written as

$$\Delta x \cdot \Delta P_x \geq (h/4\pi)$$

In any simultaneous determination of the position and momentum of the particle, the product of the corresponding uncertainties inherently present in the measurement is equal to or greater than $h/4\pi$.

Similarly 1) $\Delta E \cdot \Delta t \geq h/4\pi$

2) $\Delta L \cdot \Delta \theta \geq h/4\pi$

Significance of Heisenberg's Uncertainty Principle:

Heisenberg's Uncertainty Principle asserts that it is impossible to measure simultaneously both the position and momentum of a particle accurately. If we make an effort to measure very accurately the position of a particle, it leads to large uncertainty in the measurement of momentum and vice versa. Therefore one should think only of the probability of finding the particle at a certain position or of the probable value for the momentum of the particle.

Application of Uncertainty Principle:**Impossibility of existence of electrons in the atomic nucleus:**

The energy of a particle is given by

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \quad (1)$$

Heisenberg's uncertainty principle states that

$$\Delta x \cdot \Delta P_x \geq \frac{h}{4\pi} \rightarrow (4)$$

The diameter of the nucleus is of the order 10^{-14}m . If an electron is to exist inside the nucleus, the uncertainty in its position Δx must not exceed 10^{-14}m .

$$\text{i.e. } \Delta x \leq 10^{-14}\text{m}$$

The minimum uncertainty in the momentum

$$(\Delta P_x)_{\min} \geq \frac{h}{4\pi (\Delta x)_{\max}} \geq \frac{6.63 \times 10^{-34}}{4\pi \times 10^{-14}} \geq 0.5 \times 10^{-20} \text{ kg. m/s}$$

By considering minimum uncertainty in the momentum of the electron

$$\text{i.e., } (\Delta P_x)_{\min} \geq 0.5 \times 10^{-20} \text{ kg.m/s} = p \rightarrow (2)$$

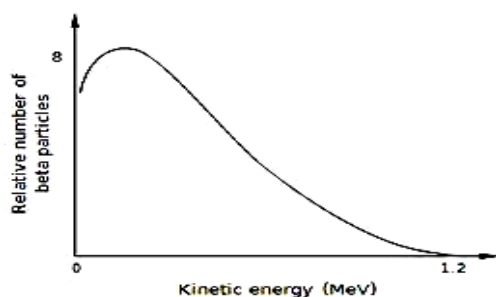
Consider eqn (1)

$$E = \frac{p^2}{2m} = \frac{(0.5 \times 10^{-20})^2}{2 \times 9.1 \times 10^{-31}} = 1.373 \times 10^{-11} = 85 \text{ MeV}$$

$$\text{Where } m_0 = 9.11 \times 10^{-31} \text{ kg}$$

If an electron exists in the nucleus its energy must be greater than or equal to 85 MeV.. It is experimentally measured that the beta particles ejected from the nucleus during beta decay have energies of about 3 to 4 MeV. This shows that electrons cannot exist in the nucleus.

[Beta decay: In beta decay process, from the nucleus of an atom, when neutrons are converting into protons in releasing an electron (beta particle) and an antineutrino. When proton is converted into a neutron in releasing a positron (beta particle) and a neutrino. In both the processes energy sharing is statistical in nature. When beta particles carry maximum energy neutrino's carries minimum energy and vice-versa. In all other processes energy sharing is in between maximum and minimum energies. The maximum energy carried by the beta particle is called as the end point energy (E_{\max}).

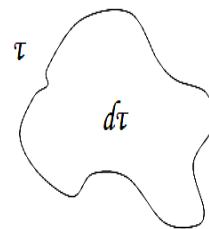


Wave Function:

A physical situation in quantum mechanics is represented by a function called wave function. It is denoted by ' ψ '. It accounts for the wave like properties of particles. Wave function is obtained by solving Schrodinger equation.

Physical significance of wave function:

Probability density: If ψ is the wave function associated with a particle, then $|\psi|^2$ is the probability of finding a particle in unit volume. If ' τ ' is the volume in which the particle is present but where it is exactly present is not known. Then the probability of finding a particle in certain elemental volume $d\tau$ is given by $|\psi|^2 d\tau$. Thus $|\psi|^2$ is called probability density. The probability of finding an event is real and positive quantity. In the case of complex wave functions, the probability density is $|\psi|^2 = \psi * \psi$ where ψ^* is Complex conjugate of ψ .



Normalization:

The probability of finding a particle having wave function ' ψ ' in a volume ' $d\tau$ ' is ' $|\psi|^2 d\tau$ '. If it is certain that the particle is present in finite volume ' τ ', then

$$\int_0^{\tau} |\psi|^2 d\tau = 1$$

If we are not certain that the particle is present in finite volume, then

$$\int_{-\infty}^{\infty} |\psi|^2 d\tau = 1$$

In some cases $\int |\psi|^2 d\tau \neq 1$ and involves constant.

The process of integrating the square of the wave function within a suitable limits and equating it to unity the value of the constant involved in the wave function is estimated. The constant value is substituted in the wave function. This process is called as normalization. The wave function with constant value included is called as the normalized wave function and the value of constant is called normalization factor.

Time independent Schrodinger wave equation

Consider a particle of mass 'm' moving with velocity 'v'. The de-Broglie wavelength 'λ' is

$$\lambda = \frac{h}{mv} = \frac{h}{P} \rightarrow (1) \quad \text{Where 'mv' is the momentum of the particle.}$$

The wave eqn is

$$\psi = A e^{i(kx - \omega t)} \rightarrow (2)$$

Where 'A' is a constant and 'ω' is the angular frequency of the wave.

Differentiating equation (2) with respect to 't' twice

$$\frac{d^2 \psi}{dt^2} = -A \omega^2 e^{i(kx - \omega t)} = -\omega^2 \psi \rightarrow (3)$$

The equation of a travelling wave is

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2}$$

Where 'y' is the displacement and 'v' is the velocity.

Similarly for the de-Broglie wave associated with the particle

$$\frac{d^2 \psi}{dx^2} = \frac{1}{v^2} \frac{d^2 \psi}{dt^2} \rightarrow (4)$$

where ' ψ ' is the displacement at time ' t '.

From eqns (3) & (4)

$$\frac{d^2\psi}{dx^2} = -\frac{\omega^2}{v^2}\psi$$

But $\omega = 2\pi\nu$ and $v = \nu\lambda$ where ' ν ' is the frequency and ' λ ' is the wavelength.

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi^2}{\lambda^2}\psi \text{ or } \frac{1}{\lambda^2} = -\frac{1}{4\pi^2\nu} \frac{d^2\psi}{dx^2} \rightarrow (5)$$

$$K.E = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{P^2}{2m} \rightarrow (6)$$

$$= \frac{h^2}{2m\lambda^2} \rightarrow (7)$$

Using eqn (5)

$$K.E = \frac{h^2}{2m} \left(-\frac{1}{4\pi^2\nu} \right) \frac{d^2\psi}{dx^2} = -\frac{h^2}{8\pi^2m\nu} \frac{d^2\psi}{dx^2} \rightarrow (8)$$

Total Energy $E = K.E + P.E$

$$E = -\frac{h^2}{8\pi^2m\nu} \frac{d^2\psi}{dx^2} + V$$

$$E - V = -\frac{h^2}{8\pi^2m\nu} \frac{d^2\psi}{dx^2}$$

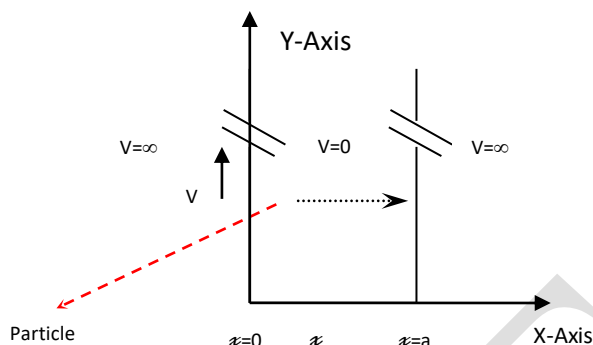
$$\frac{d^2\psi}{dx^2} = -\frac{8\pi^2m}{h^2}(E - V)\psi$$

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}(E - V)\psi = 0}$$

This is the time independent Schrodinger wave equation.

Application of Schrodinger wave equation:

Energy Eigen values of a particle in one dimensional, infinite potential well (potential well of infinite depth) or of a particle in a box



Consider a particle of a mass 'm' free to move in one dimension along positive x -direction between $x=0$ to $x=a$. The potential energy outside this region is infinite and within the region is zero. The particle is in bound state. Such a configuration of potential in space is called infinite potential well. It is also called particle in a box. The Schrödinger equation outside the well is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}(E - \infty)\psi = 0 \rightarrow (1) \quad \because V = \infty$$

For outside, the equation holds good if $\psi = 0$ & $|\psi|^2 = 0$. That is particle cannot be found outside the well and also at the walls

The Schrodinger's equation inside the well is:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}E\psi = 0 \rightarrow (2) \quad \because V = 0$$

$$\text{Let } \frac{8\pi^2m}{h^2}E = k^2 \rightarrow (3)$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

The solution of this equation is:

$$\psi = C \cos kx + D \sin kx \rightarrow (4)$$

$$\text{at } x = 0 \rightarrow \psi = 0$$

$$0 = C \cos 0 + D \sin 0$$

$$\therefore C = 0$$

$$\text{Also } x = a \rightarrow \psi = 0$$

$$0 = C \cos ka + D \sin ka$$

$$\text{But } C = 0$$

$$\therefore D \sin ka = 0 \quad \longrightarrow (5)$$

$$D \neq 0 \quad (\text{because the wave concept vanishes})$$

$$\text{i.e. } ka = n\pi \quad \text{where } n = 0, 1, 2, 3, 4 \dots (\text{Quantum number})$$

$$k = \frac{n\pi}{a} \rightarrow (6)$$

sub eqn (5) and (6) in (4)

$$\psi_n = D \sin \frac{n\pi}{a} x \rightarrow (7)$$

This gives permitted wave functions.

The Energy Eigen value given by

Substitute equation (6) in (3)

$$\frac{8\pi^2 m}{h^2} E = k^2 = \frac{n^2 \pi^2}{a^2}$$
$$E = \frac{n^2 h^2}{8ma^2}$$

For $n = 0$ is not acceptable inside the well because $\psi_n = 0$. It means that the electron is not present inside the well which is not true. Thus the lowest energy value for $n = 1$ is called zero point energy value or ground state energy.

$$\text{i.e. } E_{\text{zero-point}} = \frac{h^2}{8ma^2}$$

The states for which $n > 1$ are called excited states.

To find out the value of D, normalization of the wave function is to be done.

$$\text{i.e. } \int_0^a |\psi_n|^2 dx = 1 \rightarrow (8)$$

using the values of ψ_n from eqn (7)

$$\begin{aligned} \int_0^a D^2 \sin^2 \frac{n\pi}{a} x dx &= 1 \\ D^2 \int_0^a \left[\frac{1 - \cos(2n\pi/a)x}{2} \right] dx &= 1 \quad \because \sin^2 \theta = \left(\frac{1 - \cos 2\theta}{2} \right) \\ \frac{D^2}{2} \left[\int_0^a dx - \int_0^a \cos \frac{2n\pi}{a} x dx \right] &= 1 \\ \frac{D^2}{2} \left[x - \frac{a}{2n\pi} \sin \frac{2n\pi}{a} x \right]_0^a &= 1 \\ \frac{D^2}{2} [a - 0] &= 1 \\ \frac{D^2}{2} a &= 1 \\ D &= \sqrt{\frac{2}{a}} \end{aligned}$$

Substitute D in equation (7)

the normalized wave functions of a particle in one dimensional infinite potential well is:

$$\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x \rightarrow (9)$$

Wave functions, probability densities and energy levels for particle in an infinite potential well:

Let us consider the most probable location of the particle in the well and its energies for first three cases.

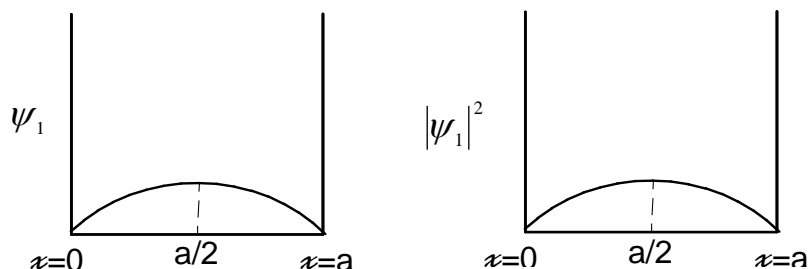
Case I $\rightarrow n=1$ It is the ground state and the particle is normally present in this state.

The Eigen function is

$$\psi_1 = \sqrt{\frac{2}{a}} \sin \frac{\pi}{a} x \quad \because \text{from eqn (7)}$$

$$\psi_1 = 0 \text{ for } x = 0 \text{ and } x = a$$

But ψ_1 is maximum when $x = a/2$.



The plots of ψ_1 versus x and $|\psi_1|^2$ versus x are shown in the above figure.

$|\psi_1|^2 = 0$ for $x = 0$ and $x = a$ and it is maximum for $x = a/2$. i.e. in ground state the particle cannot be found at the walls, but the probability of finding it is maximum in the middle.

The energy of the particle at the ground state is

$$E_1 = \frac{h^2}{8ma^2} = E_0$$

Case II $\rightarrow n=2$

In the first excited state the Eigen function of this state is

$$\psi_2 = \sqrt{\frac{2}{a}} \sin \frac{2\pi}{a} x$$

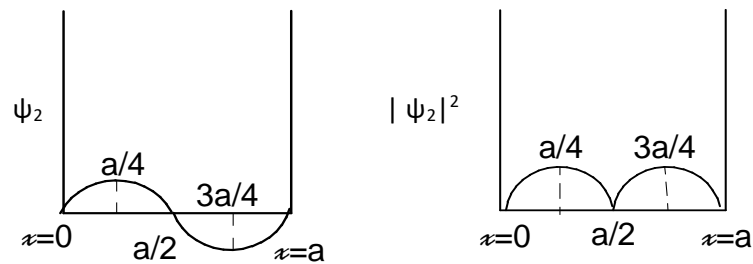
$$\psi_2 = 0 \text{ for the values } x = 0, a/2, a.$$

Also ψ_2 is maximum for the values $x = a/4$ and $3a/4$.

These are represented in the graphs.

$|\psi_2|^2 = 0$ at $x = 0, a/2, a$, i.e. particle cannot be found either at the walls or at the centre.

$$|\psi_2|^2 = \text{maximum for } x = \frac{a}{4}, x = \frac{3a}{4}$$



The energy of the particle in the first excited state is $E_2 = 4E_0$.

Case III $\rightarrow n=3$

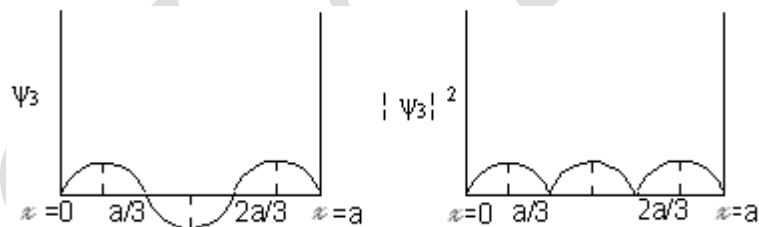
In the second excited state,

$$\psi_3 = \sqrt{\frac{2}{a}} \sin \frac{3\pi}{a} x$$

$\psi_3 = 0$, for $x = 0, a/3, 2a/3$ and a .

ψ_3 is maximum for $x = a/6, a/2, 5a/6$.

These are represented in the graphs.



$|\psi_3|^2 = 0$ for $x = 0, a/3, 2a/3$ and a . $|\psi_3|^2 = \text{maximum}$ for $x = \frac{a}{6}, x = \frac{a}{2}, x = \frac{5a}{6}$

The energy of the particle in the second excited state is $E_3 = 9 E_0$.

LASERS

The word Laser stands for **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation. It is a device which amplifies light. It has properties like Coherence, Unidirectional, Monochromatic, Focus ability, etc.

Interaction of an electromagnetic wave with matter leads to transition of an atom or a molecule from one energy state to another. If the transition is from lower state to higher state it absorbs the incident energy. If the transition is from higher state to lower state it emits a part of its energy.

Emission or Absorption takes through quantum of energy called photons. $h\nu$ is called quantum energy or photon energy.

$h = 6.626 \times 10^{-34}$ Joules Second is Planck's constant and ' ν ' is the frequency.

If ΔE is the difference between the two energy levels,

$$\text{Then } \Delta E = (E_2 - E_1) \text{ Joule}$$

According to Max Planck, $\Delta E = h\nu = (E_2 - E_1)$

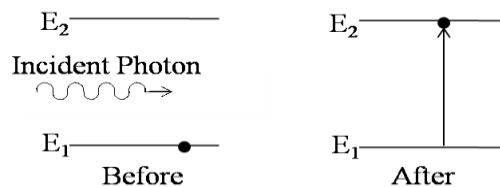
$$\nu = (E_2 - E_1)/h \quad \text{Hz.}$$

Three types of interactions, which are possible, are as follows:

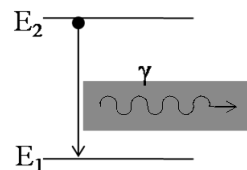
1) Induced Absorption:

Induced absorption is the absorption of an incident photon by system as a result of which the system is elevated from a lower energy state to a higher state, wherein the difference in energy of the two states is the energy of the photon.

Consider the system having two energy states E_1 and E_2 , $E_2 > E_1$. When a photon of energy $h\nu$ is incident on an atom at level E_1 , the atom goes to a higher energy level by absorbing the energy.



When an atom is at ground level (E_1), if an electromagnetic wave of frequency ν is applied to the atom, there is possibility of getting excited to higher level (E_2). The incident photon is absorbed. It is represented as



2) Spontaneous Emission: The emission of a photon by the transition of a system from a higher energy state to a lower energy state without the aid of an external energy is called spontaneous emission.

Let ' E_1 ' and ' E_2 ' be two energy levels in a material, such that $E_2 > E_1$. E_1 is ground level and E_2 is the higher level. $h\nu = E_2 - E_1$ is the difference in the energy. The atom at higher level (E_2) is more unstable as compared to that at lower level (E_1).

The life time of an atom is less in the excited state, In spontaneous emission atom emits the photon without the aid of any external energy. It is called spontaneous emission. The process is represented as



The photons emitted in spontaneous emission may not have same direction and phase similarities. It is incoherent.

Ex: Glowing electric bulbs, Candle flame etc.

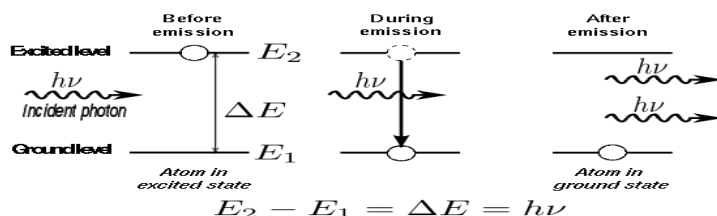
3) Stimulated Emission:

Stimulated emission is the emission of a photon by a system under the influence of a passing photon of just the right energy due to which the system transits from a higher energy state to a lower energy state.

The photon thus emitted is called stimulated photon and will have the same phase, energy and direction of movement as that of the passing photon called the stimulation photon.

Initially the atom is at higher level E_2 . The incident photon of energy $h\nu$ forces the atom to get de-excited from higher level E_2 to lower level E_1 .

i.e. $h\nu = E_2 - E_1$ is the change in energy.



The incident photon stimulates the excited atom to emit a photon of exactly the same energy as that of the incident photons. The emitted two photons have same phase, frequency, direction and polarization with the incident photon and results in coherent beam of radiation. This kind of action is responsible for lasing action.



Expression for energy density in terms of Einstein's Coefficients

Consider two energy levels E_1 and E_2 of a system of atoms with N_1 and N_2 are population of energy levels respectively.

Let U_ν be the energy density of incident beam of radiation of frequency γ . Let us consider the absorption and two emission process

1) Induced absorption:

Induced absorption is the absorption of an incident photon by system as a result of which the system is elevated from a lower energy state to a higher state.

The rate of absorption is proportional to $N_1 U_\nu$

$$\text{Rate of absorption} = B_{12} N_1 U_\nu \dots\dots\dots (1)$$

Where ' B_{12} ' is the proportionality constant called Einstein Coefficient of induced absorption.

2) Spontaneous emission:

The emission of a photon by the transition of a system from a higher energy state to a lower energy state without the aid of an external energy is called spontaneous emission.

Spontaneous emission depends on N_2 and independent of energy density.

$$\text{The rate of spontaneous emission} = A_{21} N_2 \dots\dots\dots (2)$$

Where ' A_{21} ' is called proportionality constant called Einstein coefficient of spontaneous emission.

3) Stimulated emission:

Stimulated emission is the emission of a photon by a system under the influence of a passing photon of just the right energy due to which the system transits from a higher energy state to a lower energy state

The rate of stimulated emission is directly proportional to $N_2 U_\nu$.

The rate of stimulated emission = $B_{21} N_2 U_\nu$ (3)

Where 'B₂₁' is the proportionality constant called Einstein's Coefficient of stimulated emission.

At thermal equilibrium,

Rate of absorption = (Rate of spontaneous emission + Rate of stimulated emission)

$$B_{12} N_1 U_\nu = A_{21} N_2 + B_{21} N_2 U_\nu$$

$$U_\nu (B_{12} N_1 - B_{21} N_2) = A_{21} N_2$$

$$U_\nu = \frac{A_{21} N_2}{(B_{12} N_1 - B_{21} N_2)}$$

$$\text{i.e. } U_\nu = \frac{A_{21}}{B_{21}} \left[\frac{N_2}{\left(\frac{B_{12}}{B_{21}} N_1 - N_2 \right)} \right]$$

$$U_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\left(\frac{B_{12} N_1}{B_{21} N_2} \right) - 1} \right] \rightarrow (4)$$

By Boltzmann's law, $N_2 = N_1 e^{-\left(\frac{E_2 - E_1}{KT}\right)} = N_1 e^{-h\nu/KT}$

$$\text{i.e., } N_1/N_2 = e^{h\nu/KT}$$

Eqn. (4) becomes

$$U_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\left(\frac{B_{12}}{B_{21}} e^{\left(\frac{h\nu}{kT}\right)} - 1 \right)} \right] \rightarrow (5)$$

By Planck's law, $U_\nu = \frac{8\pi h \nu^3}{c^3} \left[\frac{1}{\left(e^{\left(\frac{h\nu}{kT}\right)} - 1 \right)} \right] \rightarrow (6)$

Comparing equation (5) & (6)

$$\frac{A_{21}}{B_{21}} = 8\pi h\nu^3/c^3 \quad \& \quad \frac{B_{12}}{B_{21}} = 1 \quad \text{i.e. } B_{12} = B_{21}$$

The probability of induced adsorption is equal to the stimulated emission.

Therefore A_{12} is written as A and B_{12} , B_{21} written as B.

Equation (5) becomes

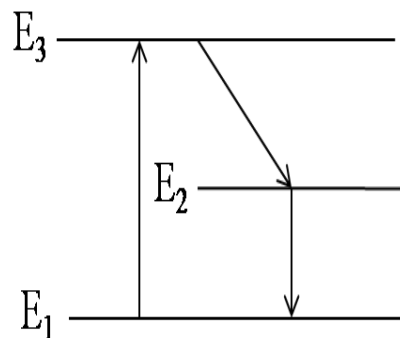
$$U_\nu = \frac{A}{B} \left[\frac{1}{\left(e^{\left(\frac{h\nu}{kT} \right)} - 1 \right)} \right]$$

Above equation is the expression for energy density

Condition for laser action:

1) Meta Stable State:

It is the special type of excited state where in the life time of atom is more than the normal excited state. This state plays an important role in lasing action. In metastable state, atoms stay of the order of 10^{-3} to 10^{-2} second. In normal excited state other than metastable atom stay of order of 10^{-8} to 10^{-9} seconds. It is possible to achieve population inversion condition in certain system which possesses a metastable state.



- 2) Population Inversion:** It is the state of the system at which the population of a higher energy level is greater than that of the lower energy level.

Let E_1 , E_2 , E_3 be the energy levels of the system $E_3 > E_2 > E_1$. E_2 is the metastable state of the system. Atoms get excited from the state E_1 to E_3 by means of external source and stay there for short time. These atoms undergo spontaneous transitions to E_2 and E_1 . The atoms at the state E_2 stay for longer time. A stage is reached in which the number of atoms at state E_2 is more than the number of atoms at E_1 which is known as population inversion.

Requisites of a Laser System:

1) The pumping process:

It is the process of supplying energy to the medium in order to transfer it to the state of population inversion is known as pumping process

Optical Pumping: It is the process of exciting atoms from lower energy level to higher energy level by using high intensity light or by operating flash tube as an external source called optical pumping.

Electrical pumping: It is the process of exciting atoms from lower energy level to higher energy level by using dc power supply as an external source called electrical pumping.

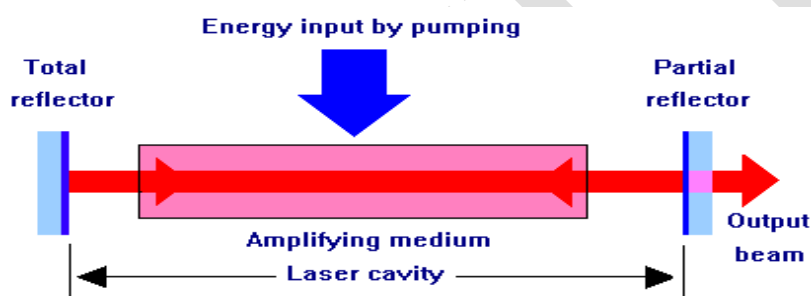
2) Active medium:

It is a medium which supports population inversion and promotes stimulated emission leading to light amplification

Active centers: In a medium consisting of different species of atoms only small fraction of the atoms of a particular type are responsible for stimulated emission and consequent light amplification they are known as Active centers

3) Laser cavity.

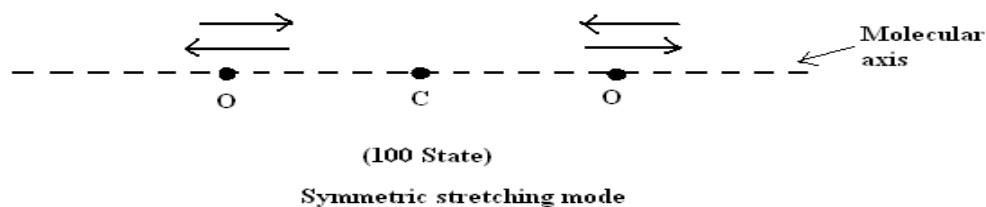
An active medium bounded between two mirrors is called as a laser cavity.



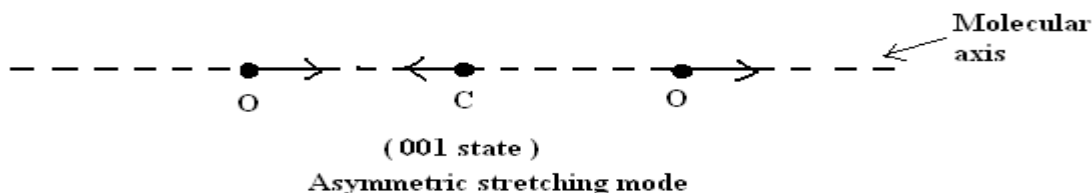
Vibrational modes of CO₂ molecule:

A carbon dioxide molecule has two oxygen atoms between which there is a carbon atom. It has 3 different modes of vibration.

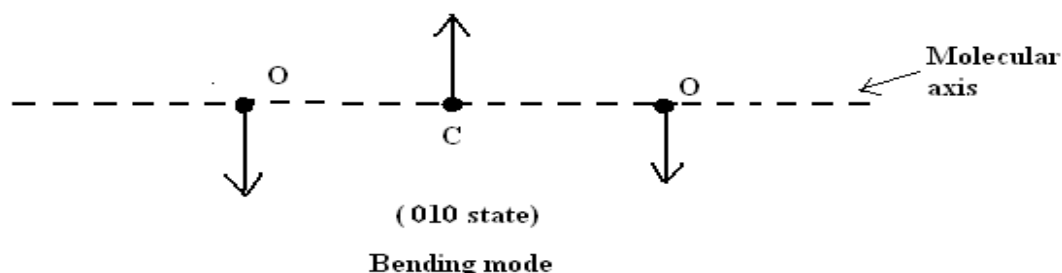
1. **Symmetric stretching mode :** In this mode, carbon atom is stationary and the oxygen atoms oscillate to and fro along the molecular axis. This state is referred as (100) state.



2. **Asymmetric stretching mode:** In this mode, both the oxygen atoms moves in one direction while the carbon atom moves in opposite direction along the molecular axis. This state is referred as (001) state.



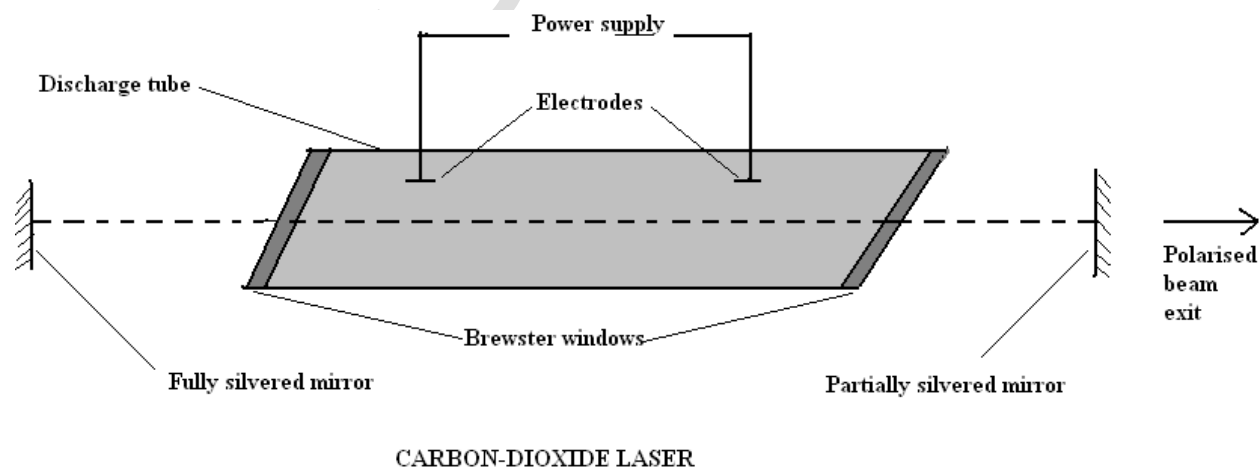
3. **Bending mode:** In this mode, the carbon atom and oxygen atoms moves perpendicular to molecular axis in the opposite direction. This state is referred to as (010) state.



CO₂ LASER:

It was devised by C.K.N Patel in 1924. CO₂ laser is molecular gas laser which operates in the IR region involving a set of vibrational – rotational transitions. It is a four level laser producing both continuous and pulsed laser.

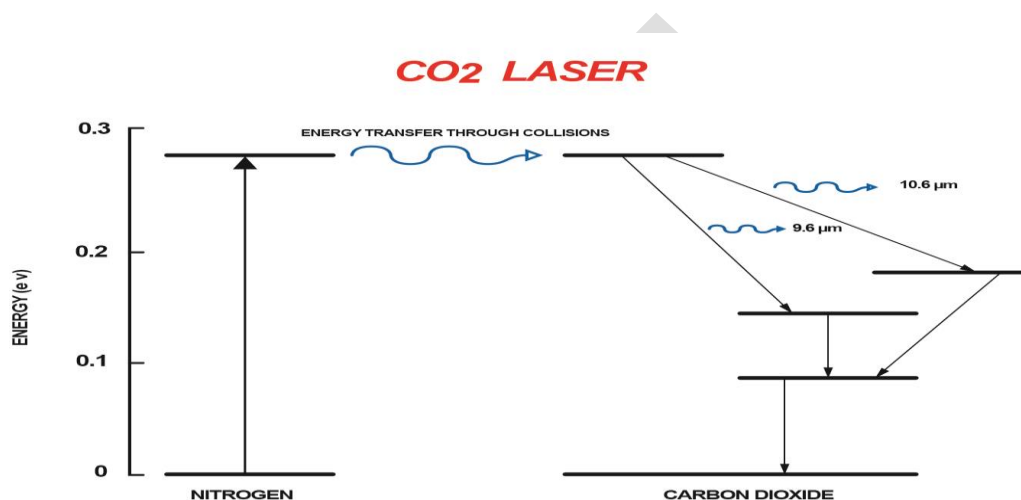
Construction:



- It consists of discharge tube (quartz) of diameter 2.5cm and length of 5m.

- The tube is filled with a mixture of CO₂, N₂ and He gas in the ratio 1:2:3.
- Sometimes water vapour is added because during discharge CO₂ molecule breaks up into CO and O. The water vapour additives help in deoxidizing CO to CO₂.
- Brewster window made up of flat quartz are sealed to the tube at both of its ends to give polarized light.
- The tube has got two parallel mirrors. One is partially silvered and the other is fully silvered to function as laser cavity

Working:



CO₂ laser Energy level diagram

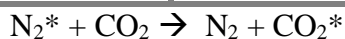
- When An electric field is applied electrons rendered free from atoms, collide with N₂ & CO₂ molecules in their path towards positive electrodes due to which N₂ atoms are excited to the higher energy level v_1 .
- Likewise it happens to the CO₂ molecule. This is collision of first kind

$$e_1 + N_2 \rightarrow e_2 + N_2^*$$

$$e_1 + CO_2 \rightarrow e_2 + CO_2^*$$

Where, e_1 and e_2 are the energies of electrons before and after collisions.

- Let the ground state, (010) state, (020) state, (100) state and (001) are represented as E_1, E_2, E_3, E_4 and E_5 levels respectively
- Because of matching energy levels, $v = 1$ state of N₂ is equal to (001) state of CO₂, N₂ molecule in the metastable state collide with the CO₂ in the ground state and transfer of energy takes place from N₂ to CO₂. As a result of which CO₂ molecule moved to (001) state where as the N₂ molecule moved to ground state. This is the collision of second kind.



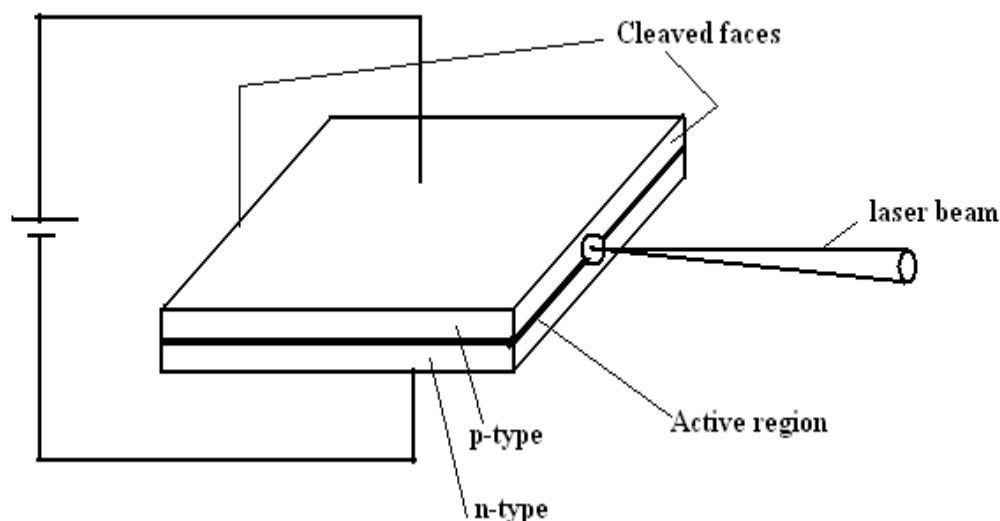
Where, CO_2 and CO_2^* are the energies of CO_2 in ground state and excited states.

- Because of the resonant transfer of energy, the population inversion is achieved in (001) state with respect to (100) and (020)
- The transition from E_5 to E_4 levels gives wavelength of $10.6\mu\text{m}$ (in far IR region)
- The transitions from E_5 to E_3 level gives wavelength of $9.6\mu\text{m}$ (in far IR region)
- Following these transitions the CO_2 molecules in E_4 and E_3 collide with the ground state CO_2 molecules (because of the matching energy levels) and arrive at E_2 state.
- The molecules in the E_2 state collide with He and water vapour molecules, so that come down to the ground state.
- The cycle of operation gives both continuous and pulsed laser.

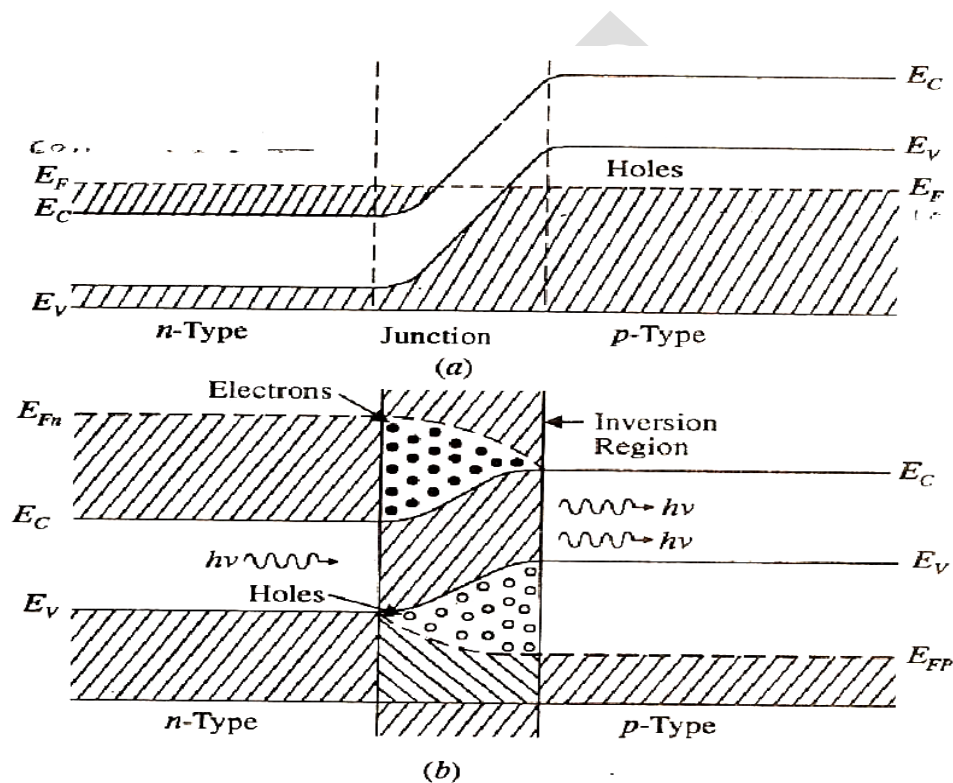
Gallium-Arsenide Laser Semiconductor laser:

A Semiconductor diode laser is one in which the active medium is formulated by semiconducting materials.

Construction: Gallium-Arsenide Laser is a single crystal of GaAs consists of heavily doped n-type and p-type. The diode is very small size with sides of the order of 1mm. The width of the junction varies from 1-100 μm . The top and bottom surfaces are metalized and Ohmic contacts are provided for external connection. The front and rear faces are polished. The polished faces functions as the resonant cavity. The other two faces are roughened to prevent lasing action in that direction.



Working:



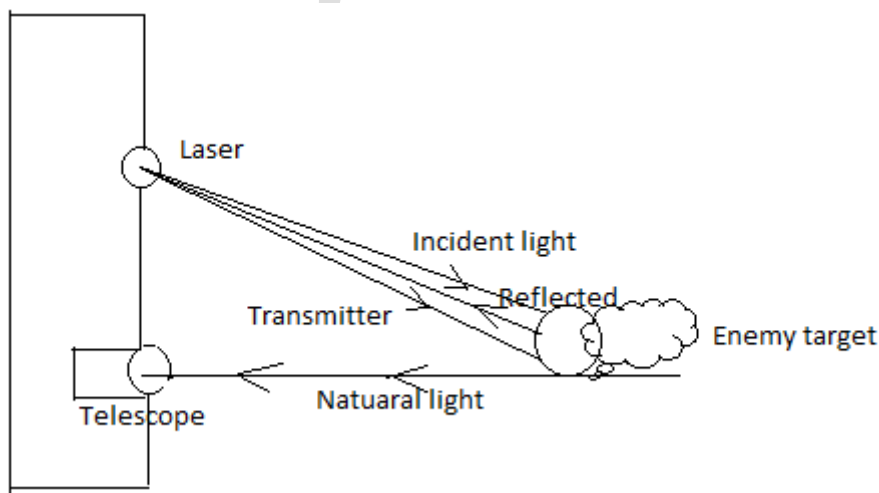
Energy level diagram of p-n junction Ga-As semi conductor diode laser
(a) Before biasing (b) After biasing

- The energy band diagram of heavily doped p-n junction is as shown. At thermal equilibrium the Fermi level is uniform.
- Because of very high doping on **n- side**, the Fermi level is pushed in to the conduction band and electrons occupy the portions of the conduction band that lies below the Fermi level and on **p-side**, the Fermi level lies within the valence band and holes occupy the portions of the valence band that lies above the Fermi level.

- A suitable forward bias is applied to overcome the potential barrier. As a result electrons from n-region and holes from p-region injected into the junction.
- The current begins to flow following which there will be a region in junction in which the population inversion can be achieved.
- Initially concentration of electrons in the energy levels at the bottom of the conduction band will be less than that of energy levels at top of valence band. So that the recombination of electrons and holes result only in spontaneous emission.
- When the current exceeds the threshold value, population inversion is achieved in the active region which is formulated in the junction.
- At this stage the photons emitted by spontaneous emission triggers stimulated emission, over a large number of recombination leading to build up laser.
- Since the energy gap of GaAs is 1.4eV, the wavelength of emitted light is 8400 Å.

Properties of laser:

1. **Coherence:** The emitted radiation after getting triggered is in phase with the incident radiation.
2. **Monochromaticity:** The laser beam is highly monochromatic than any other radiations.
3. **Unidirectionality:** Laser beam travels in only one direction. It can travel long distance without spreading.
4. **Focusability:** A laser beam can be focused to an extremely fine spot.

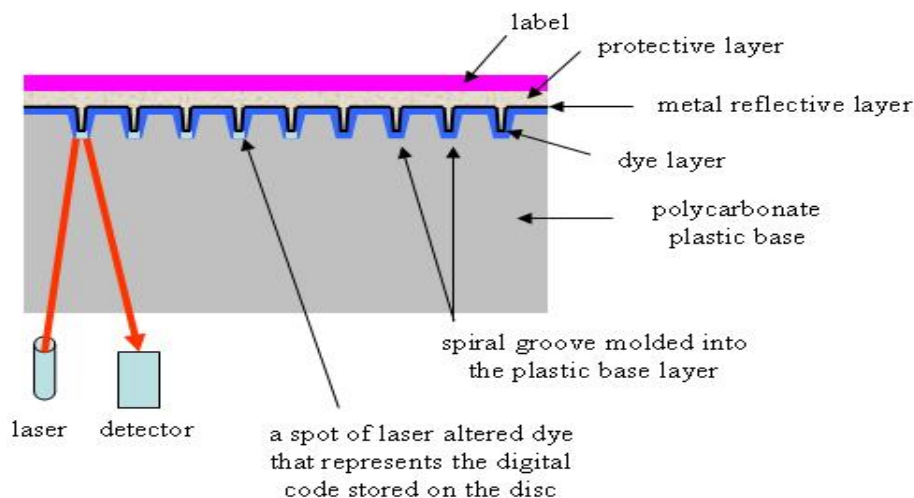
Applications of laser:**1) Laser range finder in defense**

A high power pulsed laser (Nd-YAG) beam is directed towards the enemy target from the transmitter. The beam bounces back from the surface of the target as a reflection. A part of the reflected beam is received as a signal by the receiver. The unwanted noise signal will be filtered by the optical filter and pure signal is amplified by the photomultiplier in the receiver. The range finder's high speed clock measures the exact time of incident and reflection of the pulse and then convert it into distance.

2) Laser in compact disc for data storage

A compact disc (CD) is a thin circular disc of about 12cm diameter and its shining side is made of metal and plastic consisting of 3 layers.

At the bottom is a layer of polycarbonate which is tough and brittle plastic. Above that is a layer of aluminum coated with plastic and lacquer (dye layer). The information is created in digital form in the CD by using laser.



The laser beam burns and etches bumps on its surface at certain specific intervals on a track. These bumps are called pits. The presence of a bump in a fixed length in the track indicates a zero. An unburnt space in a specific length of the track remains flat on the length called land and represents one. Thus, the laser beam can store the information by burning some lengths and leaving some lengths unburned in the binary language.

While reading a CD, the laser beam scans the track. As it is bounced, it follows the pattern of pits and lands. A photocell converts these into electric pulses in the same order. In turn, an electronic circuit generates zeros and ones. A decoder converts these binary numbers into a

changing pattern of electric currents and the analog form which is made of for required application. A CD can store a data around 700 MB.

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