

MODULE 1**OSCILLATIONS AND SHOCK WAVES**

Oscillations and vibrations are most frequently occurring phenomena. Oscillation is a repeating motion that occurs when a time varying force acts on the system. Oscillations are periodic motions. If the oscillations occur in the system without the action of an external force, then the oscillation is said to be “free oscillation”. Simple harmonic motion is a special kind of periodic motion.

IMPORTANT TERMINOLOGIES TO BE KNOWN TO UNDERSTAND SHM OF A BODY:

- **Displacement (x):** The distance of location of a body from its mean position at a particular instant of time.
- **Amplitude (a):** The maximum value of displacement that a body can undergo on either side from its mean position during the oscillation.
- **Frequency (v):** The number of oscillations executed by an oscillating body in unit time ($v = \frac{1}{T}$)
- **Angular frequency (ω) (or) angular velocity:** It is the angle covered in unit time by a body moving in circular motion at that instant of time. $\omega = 2\pi v$
- **Period (T):** It is the time taken by the body to complete one oscillation. $T = \frac{2\pi}{\omega}$

Restoring force: When a body is oscillating in a medium, the action of force whose magnitude is proportional to displacement and acting in a direction opposite to displacement w.r.t equilibrium position. This force is called restoring force and is basically responsible for the oscillation of the body.

If F is the restoring force, and “ x ” is the displacement then ,

$$F \propto -x,$$

$$\mathbf{F} = -\mathbf{k x}$$

Where, k is force constant.

The above equation is called Hooke’s law. This law states that, “the restoring force in oscillating body is directly proportional to displacement and acting in a direction opposite to displacement”.

Simple Harmonic Motion (SHM): The motion of a body is said to be SHM if the (restoring force) acceleration is directly proportional to the displacement and acts in a direction opposite to that of motion from the equilibrium position”.

(OR)

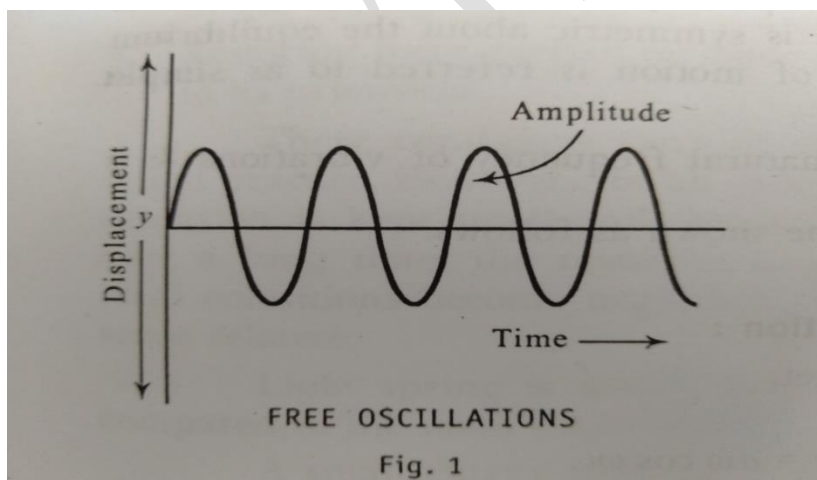
It is a periodic motion executed by a body such that its acceleration is proportional to its displacement from the equilibrium position and always directed towards it under the action of restoring force.

Characteristics of SHM:

- ❖ It is a particular type of periodic motion.
- ❖ There is a constant restoring force continuously acting on the body.
- ❖ The acceleration developed during the motion is directly proportional to displacement.
- ❖ Restoring force acts in a direction opposite to displacement. i.e., $F \propto -x$
- ❖ It is represented by sine or cosine functions. i.e. $x = a \sin \omega t$

Examples of SHM:

1. A pendulum set for oscillation.
2. Excited tuning fork.
3. A shock absorber after being bumped.
4. A mass suspended to a spring and left free to oscillate.
5. Oscillations of LC circuits.

Differential equation of motion of SHM:

Consider a body of mass “m” executing SHM. Let “x” be the displacement of the body under the action of restoring force.

For an oscillating body, from Hooke’s law

$$F = -k x \quad \dots (1)$$

From Newton’s second law of motion, the force experienced by the body under motion is given by

$$F = m a = m \frac{d^2 x}{dt^2} \quad \dots (2) \quad \text{Where ‘a’ is acceleration}$$

Equating equation (1) and (2),

$$m \frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

OR

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \dots(3) \quad \text{where } \omega = \sqrt{\frac{k}{m}} \text{ (angular frequency)}$$

This equation is called, equation of motion of SHM

The solution of equation (3) is given by

$$x = a \sin \omega t$$

Differentiate w.r.t “t” twice

$$\frac{dx}{dt} = \omega a \cos \omega t$$

$$\frac{d^2x}{dt^2} = -\omega^2 a \sin \omega t$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \dots(4)$$

Comparing equations (3) and (4)

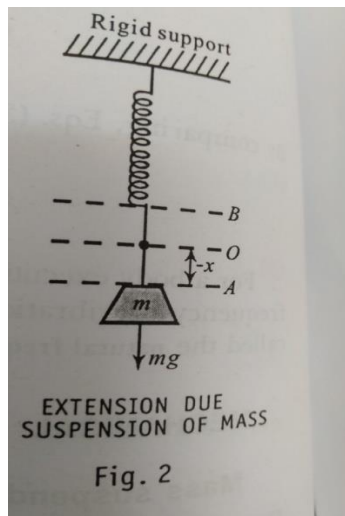
$$\omega^2 = \frac{k}{m}$$

The angular frequency is given by $\omega = \sqrt{\frac{k}{m}}$

Mechanical Simple Harmonic Oscillator:

Mass suspended to a spring [vertical vibrations]

Consider a spring fixed at one end by the rigid support. Let a body of mass “m” is suspended at the lower end of the spring due to which force “mg” acts on the spring vertically downwards. Let “O” be equilibrium position of the spring with the application of mass “m”. The body of mass “m” is pulled down and left free to oscillate with respect to “O” and “x” be the displacement of the body under the action of restoring force.



Therefore, from Hooke's law,

$$F = -kx$$

$$k = -\frac{F}{x}$$

Consider the magnitude of force. $|F| = |-F|$

Then for $x = 1$, $k = F$

Thus force constant k is defined as, "the magnitude of the applied force that produces unit extension in the spring while it is loaded within the elastic limit."

Physical significance of force constant (k):

Force constant is a measure of stiffness of the material. In case of spring it represents the amount of force required to stretch the spring by unit length. The springs with larger value of force constant will be stiffer. It is also called spring constant (or) stiffness factor.

Period of oscillation (T): The period of oscillation of mass spring system is given by

$$T = 2\pi \sqrt{\frac{m}{k}} \text{ second}$$

Where, m is suspended mass and k is force constant.

Frequency of oscillation (v): The frequency is the reciprocal of time period and is given by

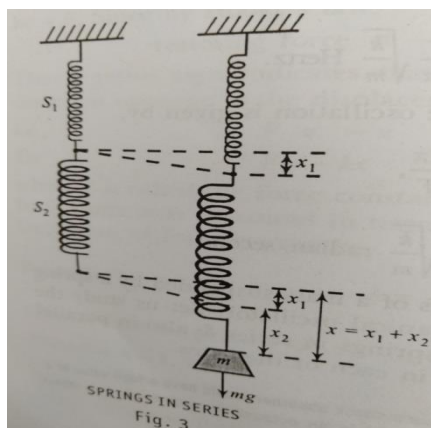
$$v = \frac{1}{T} \quad , \quad v = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \text{ Hz}$$

Angular frequency (ω): Angular frequency of oscillation is given by

$$\omega = 2\pi \nu = \frac{2\pi}{T}$$

$$\omega = \sqrt{\frac{k}{m}} \text{ radian/second}$$

Expression for force constant (Spring constant) for series combination of springs:



Consider the spring “S₁” with force constant “k₁” suspended by mass “m” and displaced through “x₁” under the action of restoring force.

Therefore, from Hooke’s law,

$$F = -k_1 x_1$$

$$x_1 = -\frac{F}{k_1} \quad \dots\dots (1)$$

Similarly for the spring s₂

$$F = -k_2 x_2$$

$$x_2 = -\frac{F}{k_2} \quad \dots\dots(2)$$

x₂ is the displacement of s₂, k₂ is the force constant of S₂. When the springs are connected in series, k_s is the force constant and displaced through “x” suspended by a same mass “m”.

From Hooke’s law,

$$F = -k_s x$$

$$x = -\frac{F}{k_s} \quad \dots\dots(3)$$

The displacement for series combination is $x = x_1 + x_2 \quad \dots\dots\dots(4)$

Substituting eqn (1), (2) and (3) in equation (4)

$$-\frac{F}{k_s} = -\frac{F}{k_1} - \frac{F}{k_2}$$

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k_s = \frac{k_1 k_2}{k_1 + k_2}$$

This is an expression for spring constant of two springs connected in series combination. If “n” number of springs connected in series, then

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots + \frac{1}{k_n}$$

$$\text{Time period, } T = 2\pi \sqrt{\frac{m}{k_s}}$$

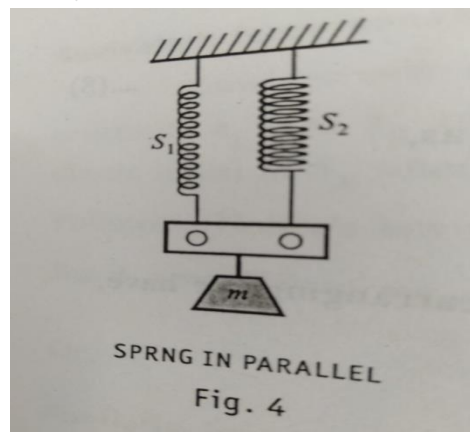
Expression for force constant (spring constant) for parallel combination of two springs:

Consider two springs S_1 and S_2 with spring constants k_1 and k_2 respectively. Let x_1 and x_2 be displacement (extension) of the individual springs when they suspended by a mass “m”.

$$\text{Hence, } F_1 = -k_1 x_1 \quad \dots(1)$$

$$\text{Similarly for spring } S_2 \quad F_2 = -k_2 x_2 \quad \dots(2)$$

where F_1 and F_2 are restoring force of the springs S_1 and S_2 respectively.



When the springs are connected in parallel and suspended by the same load, with force constant (K_p) and displaced through distance “x” under the action of restoring force (F_p).

$$\text{From Hooke's law, } F_p = k_p x \quad \dots(3)$$

The restoring force (F_p) is equally shared by two springs when they are connected in parallel with same extension $x_1 = x_2 = x$.

$$\text{Then } F_p = F_1 + F_2$$

From (1), (2) and (3)

$$- k_p x = -k_1 x_1 - k_2 x_2 \quad (\text{since } x_1 = x_2 = x.)$$

$$k_p = k_1 + k_2$$

This is the expression for equivalent force constant for springs in parallel combination.

If 'n' number of spring are connected in parallel then,

$$K_p = k_1 + k_2 + k_3 + \dots k_n$$

The time period for parallel combination of springs is given by

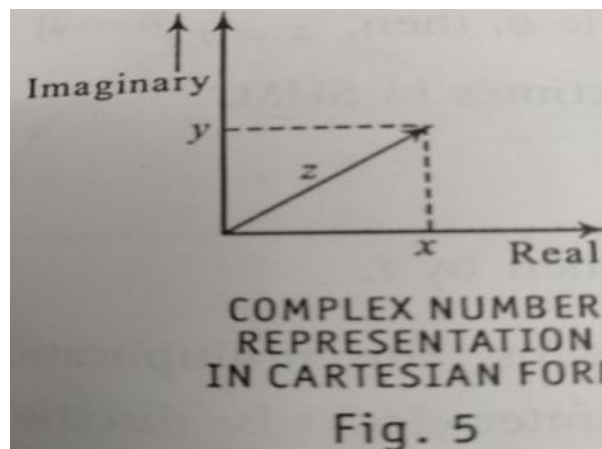
$$T = 2\pi \sqrt{\frac{m}{k_p}}$$

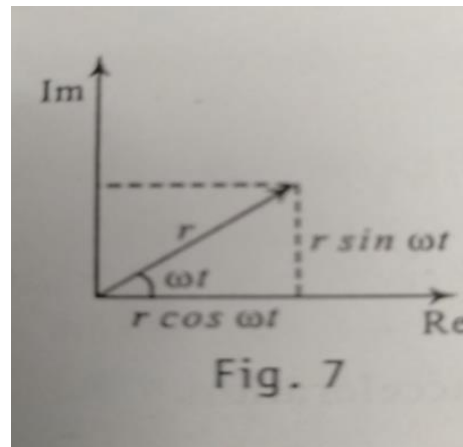
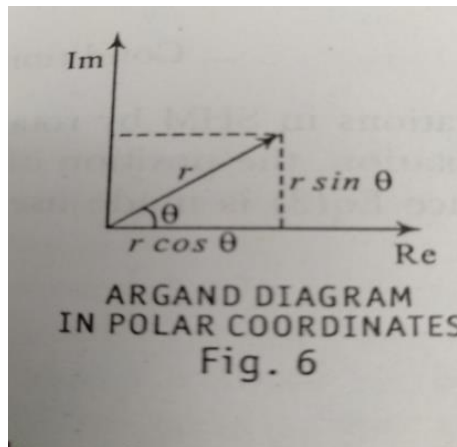
Complex Notation of SHM:

Complex numbers are a convenient tool to mathematically analyze sinusoidal functions. It can be used to represent amplitude and phase of a periodically varying function.

A complex number Z is of the form $Z = x + iy$, where x is the real part, which is the projection of Z on x -axis and y is imaginary which is the projection of Z on y -axis.

The method of representation of complex number in a co-ordinate system called Argand diagram.





In polar coordinates, $Z = r \cos \theta + i \sin \theta$, where 'r' is magnitude of Z.

since, $e^{i\theta} = \cos \theta + i \sin \theta$

$$Z = r e^{i\theta}$$

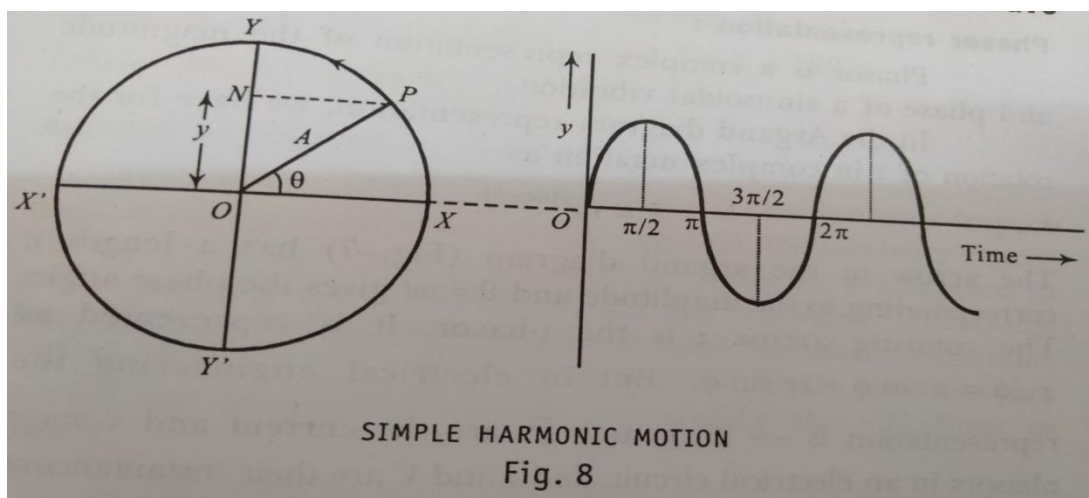
If 'θ' changes W.r.t. time, then $\theta = \omega t$

$$\text{Therefore, } Z = r e^{i\omega t}$$

At $t = 0$, if already Z makes an angle ϕ , then

$Z = r e^{i(\omega t + \phi)}$, is the **complex notation of SHM**.

Phasor representation of SHM:



Phasor is a complex representation of the magnitude and phase of sinusoidal vibrations (SHM).

Consider a particle moving on a circular path of radius “R” in the anticlockwise direction with angular velocity ω .

Let “P” be the position of the particle at particular instant of time “t”. Draw perpendicular PN to the diameter. As the particle move in circular path , the point ‘N’ moves to and fro motion on the diameter W.r.t origin ‘O’ executing SHM.

Where, O N → Displacement, OY → maximum displacement = amplitude.

The angle $\angle POX = \theta$ gives the phase,

Therefore, we have $Y = A \sin \theta$

Phasor representation:

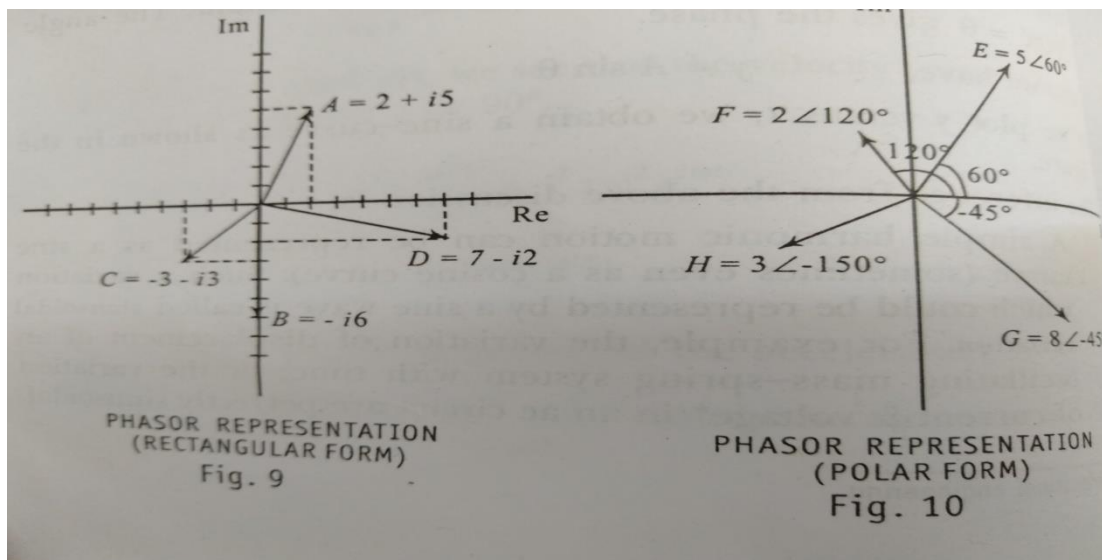
$$Z = r e^{i(wt+\phi)}$$

$$r \angle \phi = r \cos \phi + i r \sin \phi$$

the current and voltage phasor in electrical circuit is

$$I = I \angle \phi = I \cos \phi + j I \sin \phi$$

$$V = V \angle \phi = V \cos \phi + j V \sin \phi$$



Free oscillations: “The oscillatory body oscillates with undiminished amplitude with its own natural frequency of vibrations for infinite length of time under the action of restoring force, until an external force affects its motion” are called free oscillations.

Examples:

- Oscillation of mass suspended to spring with negligible damping and small displacement.
- Oscillations of simple pendulum.
- LC oscillations.

Equation of Motion of Free Oscillations:

If “m” is the mass of oscillating body with a force constant “k” and ‘x’ is the displacement at the instant “t” of the oscillating body, then

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Where ω is the angular frequency, $\omega = \sqrt{\frac{k}{m}}$

Natural frequency of oscillations: It is a characteristic with which a body (system) body oscillates under the action of restoring force after it gets displaced from its equilibrium position and left force.

Damped oscillations: An oscillatory body oscillates such that its amplitude gradually decreases and comes to rest at equilibrium position in a finite interval of time due to the action of resistive force.

Examples:

- Mechanical oscillations of simple pendulum.
- Electrical oscillations of LC circuit
- A swing left free to oscillate after being pushed once.

Theory of Damped Oscillations:

Consider a body of mass “m” executing damped oscillations in a resistive medium. The resistive force is proportional to velocity of the body and acting in opposite directions.

$$\text{Therefore, Resistive force} = -r \frac{dx}{dt} \quad \dots (1)$$

Where, $r \rightarrow$ damping constant, $\frac{dx}{dt} \rightarrow$ velocity.

The restoring force acting on the body set for oscillations given by

$$\text{Restoring force} = -k x \quad \dots (2)$$

Where, $k \rightarrow$ force constant, $x \rightarrow$ displacement

The resultant force acting on the body is,

$$\text{Resultant force} = \text{resistive force} + \text{restoring force}$$

From (1) and (2)

$$\text{Resultant force} = -r \frac{dx}{dt} - kx \quad \dots (3)$$

From Newton's second law of motion, the resultant force experienced by the body under motion is given by

$$\text{Resultant force} = m a = m \frac{d^2x}{dt^2} \quad \dots (4)$$

where $a \rightarrow$ acceleration

Equating equations (3) and (4)

$$m \frac{d^2x}{dt^2} = -r \frac{dx}{dt} - kx$$

$$\frac{d^2x}{dt^2} = -\frac{r}{m} \frac{dx}{dt} + \frac{k}{m} x = 0$$

$$\frac{d^2x}{dt^2} + 2b \frac{dx}{dt} + \omega^2 x = 0 \quad \dots (5) \quad \text{where, } 2b = \frac{r}{m}, \omega = \sqrt{\frac{k}{m}}$$

this is the equation of motion for damped oscillations

the solution of equation (5) is given by

$$x = C e^{(-b + \sqrt{b^2 - \omega^2})t} + D e^{(-b - \sqrt{b^2 - \omega^2})t} \quad \dots (6)$$

where C and D are constants given by,

$$C = \frac{x_0}{2} \left[1 + \frac{b}{\sqrt{b^2 - \omega^2}} \right], \quad D = \frac{x_0}{2} \left[1 - \frac{b}{\sqrt{b^2 - \omega^2}} \right]$$

where, x is displacement at $t = 0$

the general solution of equation (5) is given by substituting the values of C and D in equation (6)

$$x = \frac{x_0}{2} \left\{ \left(1 + \frac{b}{\sqrt{b^2 - \omega^2}}\right) e^{(-b + \sqrt{b^2 - \omega^2})t} + \left(1 - \frac{b}{\sqrt{b^2 - \omega^2}}\right) e^{(-b - \sqrt{b^2 - \omega^2})t} \right\} \dots\dots(7)$$

This is the General solution of damped oscillations.

In the above equation, as “t” varies, x also varies. But the nature of variation depends on $\sqrt{b^2 - \omega^2}$.

Case (i): If $b^2 > \omega^2$, over damping (or) dead beat case:

When $b^2 > \omega^2$, $b^2 - \omega^2$ is positive, but $\sqrt{b^2 - \omega^2} < b$

Therefore, the coefficient of “t” in both terms of equation (7) is negative, this indicates exponential decay of displacement W.r.t time, and the body after maximum displacement comes to rest at equilibrium position with the decay of time.

Over damping is the condition such that the body comes to rest at equilibrium position under the action of restoring force and resistive force with long interval.

Example: motion of pendulum in highly viscous liquid.

Case (ii): when $b^2 = \omega^2$, critical damping:

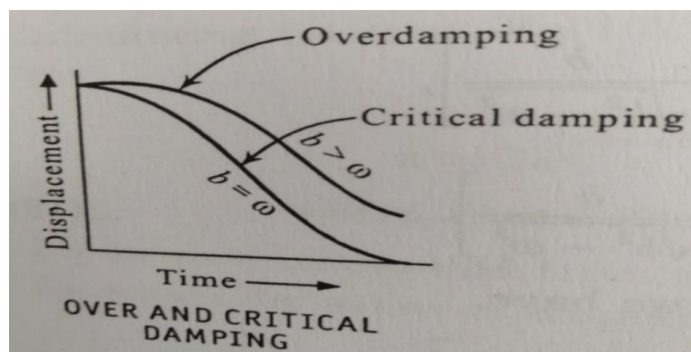
If $b^2 = \omega^2$, then $\sqrt{b^2 - \omega^2} \approx \xi$ (small quantity),

Then equation (6) reduces to $x = e^{-bt} [(C+D) + (C-D) \xi t]$

As “t” varies, the displacement decreases exponentially and become zero for short interval of time.

“It is a condition such that the body comes to half at equilibrium position under the action of restoring and resistive force in a short interval of time.”

Example: Pointer galvanometer, voltmeter, current meter. Shock absorbers designed with springs.



Case(iii): when $b^2 < \omega^2$, under damping:

When $b^2 < \omega^2$, $b^2 - \omega^2$ is negative. But $\omega^2 - b^2$ is positive

Therefore, $\sqrt{b^2 - \omega^2} = \sqrt{-(\omega^2 - b^2)} = i\sqrt{\omega^2 - b^2} \quad (i = \sqrt{-1})$

Let $\omega^2 - b^2 = n$

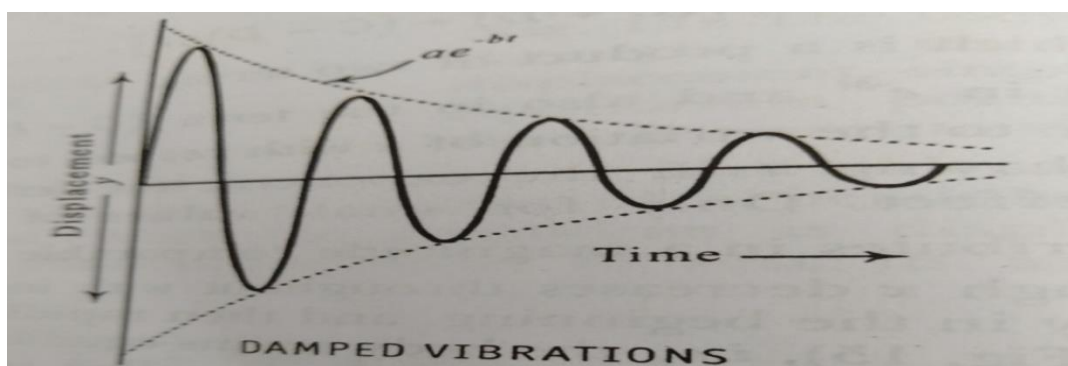
Then, equation (7) reduces to,

$$x = a e^{-bt} \sin(nt + \phi), \quad \text{where } a = x_0 (\alpha/n), \quad \phi = \tan^{-1}(n/b)$$

As time “t” increases, the displacement ‘x’ decreases exponentially.

“It is a condition such that, the body vibrates with diminishing amplitude with the decay of time under the action of restoring and resistive forces and comes to rest at equilibrium position”.

Examples: Any real physical system like oscillations of simple pendulum, vibrations of tuning fork, mass suspended to spring.



Quality factor (Q):

The amount of damping is described by the quantity called quality factor and is given by,

$$Q = \frac{\omega}{2b}, \quad \omega \rightarrow \text{angular frequency}, \quad 2b \rightarrow \frac{r}{m}, \quad r \rightarrow \text{damping factor}.$$

“Quality factor is the number of cycles required for the energy to fall off by a factor $e^{2\pi}$ (≈ 535).

If Q value is more indicated the sustained oscillations overcoming the resistive forces.

Q factor describes how much under damped is the oscillatory system.

Forced oscillations (or) forced vibrations:

“It is a steady state sustained vibrations of a body vibrating in a resistive medium under the action of external periodic force which acts independently of the restoring force.”

Examples:

- Oscillations of swing pushed periodically by a person
- oscillations of LC circuit with the applied ac source
- Motion of diaphragm in a telephone receiver.

Theory of forced oscillations (OR) Expression for amplitude and phase of the forced vibrations:

Consider a body of mass “m” executing vibrations in a damping medium under the application of external periodic force. $F \sin p t$,

If ‘x’ is the displacement of the body at any instant of time “t”.

The restoring force = $-k x$ (1), where $k \rightarrow$ force constant.

The resistive force acts opposite to velocity given by

Resistive force = $-r \frac{dx}{dt}$ (2), where $r \rightarrow$ damping constant, $\frac{dx}{dt} \rightarrow$ velocity

The applied periodic force = $F \sin p t$... (3), where $p \rightarrow$ frequency of external force.

Therefore, the resultant force experienced by the body is given by

From equations (1), (2) & (3)

$$\text{Resultant force} = -r \frac{dx}{dt} - kx + F \sin p t \quad \text{.....(4)}$$

From Newton second law of motion, the resultant force experienced by moving body is given by

$$\text{Resultant force} = ma = m \frac{d^2x}{dt^2} \quad \text{.....(5)}, \quad \text{where } a \rightarrow \text{acceleration.}$$

Equating (4) and (5)

$$m \frac{d^2x}{dt^2} = -r \frac{dx}{dt} - kx + F \sin p t$$

$$\frac{d^2x}{dt^2} + \frac{r}{m} \frac{dx}{dt} + \frac{k}{m} x = \frac{F}{m} \sin p t \quad \text{.....(6)}$$

This is the equation of motion of forced oscillations.

$$\frac{d^2x}{dt^2} + 2b \frac{dx}{dt} + \omega^2 x = \frac{F}{m} \sin p t \quad \text{.....(7)} \quad \text{where, } 2b = \frac{r}{m}, \omega = \sqrt{\frac{k}{m}}$$

The solution of equation (7) is given by

$$x = a \sin(pt - \alpha)$$

Differentiate W.r.t “t” twice,

$$\frac{dx}{dt} = p a \cos(pt - \alpha)$$

$$\frac{d^2x}{dt^2} = -p^2 a \sin(pt - \alpha)$$

equation (7) becomes,

$$-p^2 a \sin(pt - \alpha) + 2b p a \cos(pt - \alpha) + \omega^2 a \sin(pt - \alpha) = \frac{F}{m} \sin[(pt - \alpha) + \alpha]$$

$$-p^2 a \sin(pt - \alpha) + 2b p a \cos(pt - \alpha) + \omega^2 a \sin(pt - \alpha) = \frac{F}{m} [\sin(pt - \alpha) \cos \alpha + \cos(pt - \alpha) \sin \alpha]$$

Equating the coefficients of $\cos(pt - \alpha)$ & $\sin(pt - \alpha)$ on both sides of the equation

$$2b p a = \frac{F}{m} \sin \alpha \quad \dots\dots(8)$$

$$a(\omega^2 - p^2) = \frac{F}{m} \cos \alpha \quad \dots\dots(9)$$

squaring and adding equations (8) and (9)

$$a^2 [4b^2 p^2 + (\omega^2 - p^2)^2] = \left(\frac{F}{m}\right)^2 (\sin^2 \alpha + \cos^2 \alpha)$$

$$a^2 = \frac{\left(\frac{F}{m}\right)^2}{(4b^2 p^2 + (\omega^2 - p^2)^2)}$$

$$a = \frac{\frac{F}{m}}{\sqrt{4b^2 p^2 + (\omega^2 - p^2)^2}}$$

This is the equation for amplitude of the forced oscillations.

Equation (8) divided by (9) gives,

$$\frac{2bp}{\omega^2 - p^2} = \frac{\sin \alpha}{\cos \alpha} = \tan \alpha$$

$$\tan \alpha = \left(\frac{2bp}{\omega^2 - p^2} \right)$$

$$\alpha = \tan^{-1} \left(\frac{2bp}{\omega^2 - p^2} \right)$$

this is the equation for phase of forced oscillations.

Dependence of amplitude and phase on the frequency of the applied force:

Case (i): $p \ll \omega$, $p \rightarrow$ angular frequency of applied force, $\omega \rightarrow$ natural frequency of the vibrating body. When $p \ll \omega$, p^2 is very small.

Thus, $\omega^2 - p^2 = \omega^2$ and $2bp = 0$. Since b is small,

$$\text{therefore the amplitude } a = \frac{F}{m\omega^2}$$

The phase, $\alpha = \tan^{-1}\left(\frac{0}{\omega^2}\right) = 0$. The displacement and force will be in the same phase.

Case(ii): $p = \omega$, (resonance)

$$\omega^2 - p^2 = 0$$

Amplitude $a = \frac{F/m}{2bp} = \frac{F/r\omega}{r}$ the amplitude have highest value. Since $a \propto \left(\frac{1}{\omega}\right)$, phase

$$a = \tan^{-1}(2bp / 0) = \tan^{-1}(\infty) = \frac{\pi}{2}$$

The displacement has a phase lag of $\left(\frac{\pi}{2}\right)$ w.r.t phase of the applied force.

Case(iii): $p \gg \omega$, it is significant when damping forces are small.

When $p \gg \omega$, $(\omega^2 - p^2) \approx -p^2$

$$\text{Amplitude } a = \frac{\frac{F}{m}}{\sqrt{4b^2 p^2 + p^4}}$$

As ' p ' increases ' a ' becomes smaller. Since b is small

$$4b^2 p^2 \ll p^4$$

$$\text{Amplitude } a = \frac{F/m}{p^4} = \frac{F/m}{p^2}$$

$$\text{Phase } \alpha = \tan^{-1}\left[\frac{2bp}{\omega^2 - p^2}\right] = \tan^{-1}\left(\frac{2bp}{-p^2}\right) = \tan^{-1}\left(\frac{2bp}{p^2}\right) = \pi$$

The displacement has a phase lag of ' π ' w.r.t phase of the applied force.

Resonance: The expression for amplitude of the forced oscillations given by

$$a = \frac{F/m}{\sqrt{4b^2 p^2 + (\omega^2 - p^2)^2}}$$

the amplitude of vibration is maximum, this state of vibration called resonance.

Condition for resonance:

In the above equation, “a” is maximum when, “b” is minimum (or) damping caused by the medium is made minimum (since $b = \frac{r}{2m}$),

$p = \omega$, i.e., frequency of applied force (p) is equal to natural frequency of vibration (ω) of the body.

$$\text{Therefore, } a_{\max} = \frac{F/m}{2bp} = \frac{F/m}{2b\omega}$$

Significance of resonance:

At resonance, the vibrating body will have the ability to receive completely the energy delivered by the periodic force. Hence amplitude of vibrations are maximum. “When the frequency of a periodic force acting on a vibrating body is equal to natural frequency of vibrations of the body, the energy transfer from the periodic force to the body becomes maximum because of which the body vibrates with maximum amplitude”, this phenomenon is called resonance.

Sharpness of resonance:

“sharpness of resonance is the rate at which the amplitude changes with respect to change in the frequency of the applied force at the stage of resonance”.

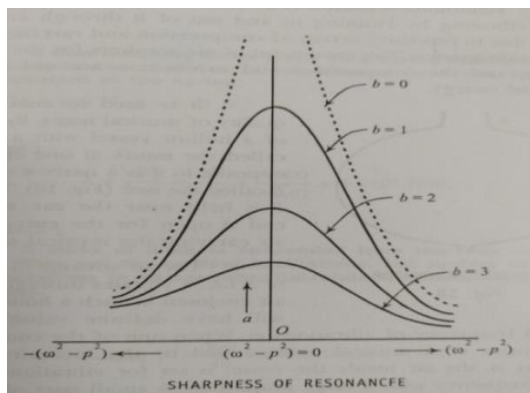
$$\text{Therefore, sharpness of resonance} = \frac{\text{change in amplitude}}{\text{change in frequency}}$$

$$\text{Sharpness of resonance} = \frac{F/m}{2b\omega p}$$

Effect of damping on sharpness of resonance:

In the above equation, sharpness of resonance $\propto \frac{1}{b}$

(since $b = \frac{r}{2m}$, $r \rightarrow$ damping constant)

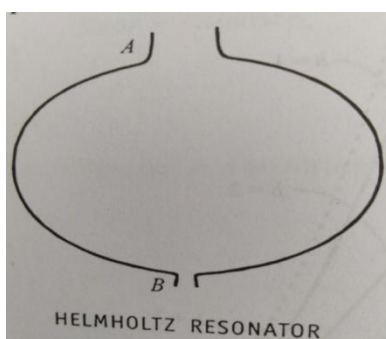


From the graph, for small value of b exhibits higher peaks refer to sharpness of resonance, on the other for higher value of b the resonance is flat. For $b = 0$, the amplitude is ∞ , can not exist in reality.

Significance of sharpness of resonance:

The rise of amplitude of amplitude will be very sharp when the damping is very small at resonance.

Examples: Helmholtz resonator, a radio receiver tuned to frequency of transmitting station, vibrations of excited tuning fork, LC oscillations with applied ac source.

Example of mechanical resonance:**Helmholtz Resonator:**

Helmholtz resonator is used to analyze the quality of musical notes. It consists of a hollow vessel with a hole (A) called mouth and a narrow opening on opposite side (B) called neck. The end (A) open for the entry of the air carrying the musical note and end (B) is held near the ear. The air inside the resonator vibrates with its natural frequency of vibration (ω). When the frequency of a musical note carrying by the incoming air from the mouth "A" is equal to the natural frequency of the air enclosed in the resonator, the amplitude of vibration of air is maximum and a part of the air leaks through (neck) B and is heard. Thus the resonator is said to resonate for the note of that particular frequency.

For Helmholtz resonator it can be shown that the square of natural frequency (ω) of oscillation is inversely proportional to the volume (V) of the resonator.

$$\text{i.e., } \omega^2 \propto \frac{1}{V}$$

$$\omega^2 = \frac{k}{V}$$

$$\omega^2 V = k \quad \text{where } k \rightarrow \text{proportionality constant}$$

SHOCK WAVES

Mach number:

The ratio of speed of the object to the speed of sound in the given medium is called as a mach number.

$$\text{Mach number} = \frac{\text{Object speed}}{\text{Speed of sound in the medium}}$$

$$M = \frac{v}{a}$$

Mach number is a pure number.

Distinctions between –acoustic, ultrasonic, subsonic & supersonic waves:

Acoustic waves:

- An acoustic wave is simply a sound wave which travels with the speed of 333 m/s in air at STP.
- Amplitude of acoustic wave is very small.
- Acoustic waves are a type of longitudinal waves that propagate by means of adiabatic compression and decompression.
- Important quantities for describing acoustic waves are sound pressure, particle velocity, particle displacement and sound intensity.
- Acoustic waves travel with the speed of sound which depends on the medium they're passing through.

Ultrasonic waves:

- Ultrasonic waves are pressure waves having frequencies beyond 20000 Hz.
- At higher power levels, ultrasonic's is useful for changing the chemical properties of substances.
- Amplitude of the ultrasonic wave is also small.

Subsonic waves:

- If the speed of mechanical wave or body moving in the fluid is lesser than that of sound, then such a speed is referred to as subsonic & the wave is a subsonic wave.
- All subsonic waves have **a mach number < 1**.
- E.g., Speed of flight of birds is subsonic, vehicles such as motor cars or trains move with subsonic speed.

Supersonic waves:

- Supersonic waves are mechanical waves which travel with speed greater than that of sound.
- They have a **mach number** > 1
- E.g.:- Fighter planes fly with supersonic speed.
- Amplitude of supersonic waves will be high & it affects the medium in which it is travelling.

SHOCK WAVES:

Any fluid that propagates at supersonic speeds, gives rise to a shock wave.

Shock waves can be produced by a sudden dissipation of mechanical energy in a medium enclosed in a small space.

A shock wave is a surface that manifests as a discontinuity in a fluid medium in which it is propagating with supersonic speed.

Shock waves are produced in nature during earth quakes (as seismic waves which travel with speed ranging from 2 km/s to 8 km/s) & when lightning strikes.

Shock waves are characterized by sudden increase in pressure, temperature & density of the gas through which it propagates.

When the speed of a source exceeds the speed of sound ($v > c$) the wave fronts lag behind the source in a cone –shaped region with the source at the vertex. The edge of the cone forms a supersonic wave front with unusually large amplitude called a “Shock wave”. When a shock wave reaches an observer a “sonic boom” is heard.

Properties of shock waves:

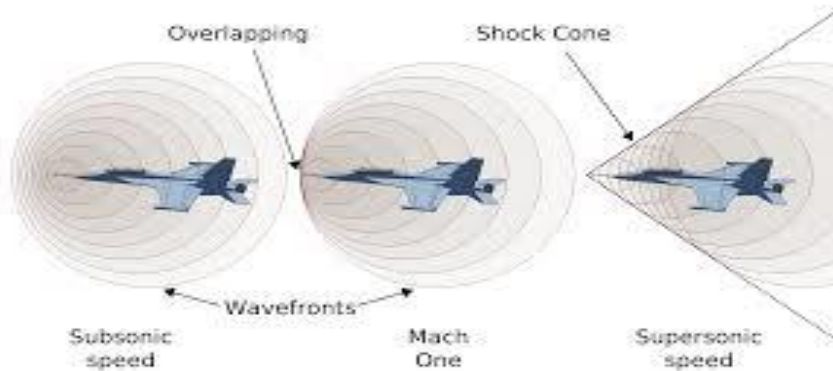
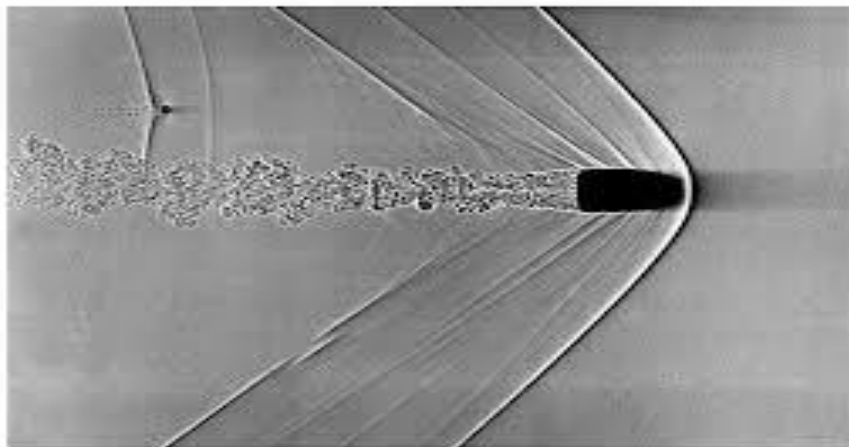
- Shock waves travel in a medium with mach number exceeds 1.
- Shock waves obey the laws of fluid dynamics.
- The change in entropy of the medium due to the generation of shock waves.
- Shock waves are confined in a medium of very thin space of thickness about $1\mu\text{m}$.
- Shock waves are not electromagnetic in nature.
- Shock waves are not physical waves.
- Shock waves are produced in medium where the object speed is more than the speed of sound.

NOTE: pictures of shockwaves formed

1) When fighter plane moving at supersonic speed



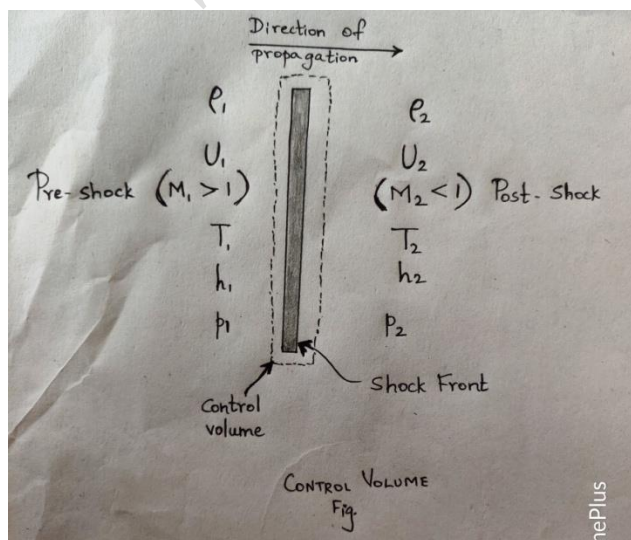
2) when bullet moving at supersonic speed



- When a shock wave passes through a matter, the total energy is preserved but the energy which can be extracted as work decreases & the entropy increases. This for example, creates additional drag force on aircraft with shocks.
- Like an ordinary wave, it carries energy & can propagate through a medium (solid, liquid, gas or plasma) or in some cases in the absence of a material medium, through a field such as an electromagnetic field.
- Contact front: in a shock wave caused by a driver gas (for example the “impact” of a high explosive on the surrounding air), the boundary between the driver (explosive products) & the driven (air) gases. The contact front trails the shock wave.
- Measurements of the thickness of shock waves in air have resulted in values around 200nm (about 10^{-5} in), which is on the same order of magnitude as the mean free gas molecule path.
- Shock waves are identified as strong or weak depending on the magnitude of the instantaneous changes in pressure & temperature in the medium that is held pressed in the space bound within the thickness of the shock front.
- Shock wave created by the Bursting of tyre & crackers is weak. Whereas those created during lightning thunder or bombing are strong. The shock waves produced during nuclear explosion are strongest ever witnessed on earth.

Control volume:

- Shock waves are analyzed by a model called “control volume”. It is an imaginary thin envelope that surrounds the shock front within which; there is a sharp increase in the pressure, temperature and density in the compressed medium.
- It is a one-dimensional confinement in the medium with two surfaces. One on the pre-shock side and the other on the post-shock side. Their inter-separation is too small. The entire shock is bound between these two surfaces. Density, flow velocity, internal energy, temperature, specific enthalpy and pressure are $\rho_1, U_1, T_1, h_1, & p_1$ respectively on the pre-shock side. and $\rho_2, U_2, T_2, h_2, & p_2$ for the post-shock side.
- It is assumed that, the heat energy is constant i.e., no heat leaves or gets into this volume.
- The energy transfer is adiabatic. The equations for the conservation of mass, momentum and energy are the governing equations for the control volume.



Basics of conservation of mass, momentum & energy:**Law of conservation of mass :**

“The total mass of any isolated system remains unchanged & is independent of any chemical & physical changes that could occur within the system”

$$\rho_1 u_1 = \rho_2 u_2$$

Law of conservation of momentum :

“In a closed system, the total momentum remains a constant”.

Or

“When two objects collide in an isolated system, the total momentum of the objects before collision is equal to the total momentum of the two objects after the collision”

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

Law of conservation of energy :

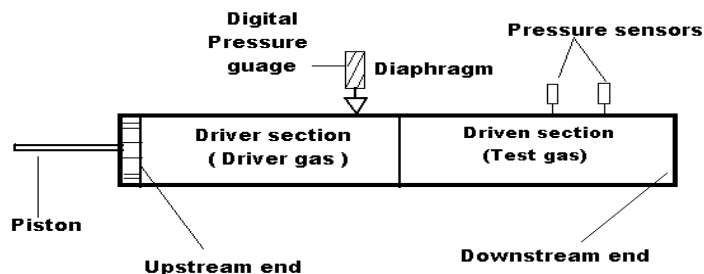
“The total energy of a closed system remains constant & is independent of any changes occurring within the system”

$$h_1 + u_1^2/2 = h_2 + u_2^2/2$$

Construction and working of Reddy Shock tube.

Reddy tube is a hand operated shock tube capable of producing shock waves by using human energy.

It is long cylindrical tube with two sections separated by a diaphragm. It's one end is fitted with a piston & the other end is closed or open to the surrounding.

**Construction:**

- Reddy tube consists of a cylindrical stainless steel tube of about 30mm diameter & of length nearly 1m.

- It is divided into two sections one is **driver section** & the other is **driven section** separated by a thick aluminium or Mylar or paper diaphragm of thickness 0.1mm.
- Far end of driver tube is fitted with a piston & the far end of driven tube is closed.
- A digital pressure gauge is mounted in the driver section next to the diaphragm.
- Two piezoelectric sensors S_1 and S_2 are mounted 70mm apart towards the closed end of the shock tube.
- A port is provided at the closed end of the driven section for filling the test gas to the required pressure.
- The driver section is filled with driver gas which is held at a relatively high pressure due to the compressing action of the piston. The gas in the driven section is called driven gas (test gas).

Working:

- The driver gas is compressed by pushing the piston hard into the driver tube until the diaphragm ruptures.
- Due to rupture of diaphragm the driver gas moves to the driven section, & pushes the driven gas towards the far downstream end. Hence there is sudden increase of pressure, temperature and density of driven section near the downstream end. This generates a moving shock wave that traverses the length of the driven section.
- The propagating primary shock wave is reflected back from the downstream end. After reflection, the test gas further undergoes compression produces the secondary shock waves, which increases the pressure & temperature to still higher values.
- This state of high values of pressure & temperature is sustained at the downstream end until an expansion waves reflected from the downstream end of the driver tube arrives there & neutralizes the compression partially
- The period over which the extreme temperature & pressure conditions at the downstream end is sustained, is typically in the order of milliseconds.
- The pressure rise caused by the primary & also the reflected shock wave are sensed as signals by the sensors S_1 and S_2 respectively & are recorded in a digital cathode ray oscilloscope.
- From the recording in the CRO, the shock arrival times are found out by the associated time base calculations. Using the data so obtained, Mach number, pressure & temperatures can be calculated.
-

Characteristics of Reddy tube:

- a. The Reddy tube operates on the principle of free piston driven shock tube.
- b. It is a hand operated shock producing device.
- c. It is capable of producing mach number exceeding 1.5
- d. The rupture pressure is a function of the thickness of the diaphragm.
- e. Temperature exceeding 900K can be easily obtained by the Reddy tube using helium as driver gas & argon as the driven gas. This temperature is useful in chemical kinetic studies.

Applications of shock waves:**Cell information (biological application) :**

By passing shock wave of appropriate strength, DNA can be pushed inside a cell.
(Functionality of DNA will not be affected by the impact of the shock wave)

Wood preservation:

By using shock waves, chemical preservatives in the form of solutions could be pushed into the interior of wood samples which helps in withstanding the microbial attacks. The actual set-up used for this purpose is named as “shockwave reactor”.

This method helps in substantial increase in the life of ordinary bamboo & soft wood.

Use in pencil industry:

In the manufacture of pencil, the wood need to be softened by soaking it in a polymer for about 3 hours at 70⁰ c & then dried which is time consuming.

In modern technique, the wood is placed in the liquid & a shock wave is sent through. The liquid gets into the wood almost instantaneously & takes no longer time to dry. The wood treated with shock wave is now ready for the further processing.

Kidney stone treatment:

Shock wave is used in a therapy called ‘extra–corporal lithotripsy’ to shatter the kidney stones into smaller fragments after which they are passed out of the body smoothly through the urinary tracts.

Gas dynamic studies:

The extreme conditions of pressure & temperature that can be produced in the shock tube, enables the study of high temperature gas dynamics. This knowledge is crucial in the study of supersonic motion of bodies & hypersonic re-entry of space vehicles into the atmosphere.

Shock wave assisted needleless drug delivery:

By using shock waves, drugs can be injected into the body without using needles.

The drug is filled in the cartridge which is kept pressed on the skin & the shock wave is sent into the body using high pressure. The drug enters the body directly through the porosity of the skin. Typical depth of penetration is about 100 microns.

Treatment of dry bore wells:

Water will be available in the bore wells when water from the feeder sources accumulates in the bore well through a number of seepage points which are porous. Sometimes such seepage routes are blocked by sand particles clogging the pores. It has been observed that, a shockwave sent through such dry bore well, clears the blockages & rejuvenates the bore well into a water source.