FinMath Assignment 1

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Question 1

The maturity is given by $n = 1 \times 4$, r is given by (risk free rate)/n $\equiv \frac{5\%}{4} = 0.0125$ and we have a face value of 10000.Now to calculate the present value as define below:

$$\begin{aligned}\text{Present value} &= \frac{face\ value}{(1+r)^n} \\ &= \frac{10000}{(1+0.0125)^4} \\ &= \ 9515.242752\end{aligned}$$

First we need to have payment made over time and present value to calculate $P_{MT} = present\ face\ \times\ r = 10000 \times 6\% = 600$.Now to calculate the present value of the coupon payment as define below:

$$\begin{aligned}PV &= P_{MT}\frac{1 - \frac{1}{(1+r)^n}}{r} \\ &= 600\frac{1 - \frac{1}{(1+0.125)^4}}{0.125} \\ &= \ 600(3.8780579840) \\ &= 2326.83479\end{aligned}$$

Finally the present value of B is given by

$$\begin{aligned}\text{Present value} &= \ 9515.242752 + 2326.83479 \\ &= 11842.07754\end{aligned}$$

QUESTION 3

We need to calculate the value of $u = 1 + 7\% = 1.07$, the value of $d = 1 - 5\% = 0.95$ and the value of $\Delta t = \frac{3}{12} = 0.25$. All these variables will lead us to calculate the probability of the price going up as the following:

$$\begin{aligned} p &= \frac{e^{\Delta t} - d}{u - d} \\ &= \frac{e^{0.0125} - 0.95}{1.07 - 0.95} \\ p &= 0.5215 \end{aligned}$$

Now to do the opposite calculating the probability of a price going down is the following:

$$\begin{aligned} 1 - p &= \ 1 - 0.5215 \\ 1 - p &= \ 0.4785 \end{aligned}$$

Calculating the price of going up and down for the middle node is simple as:

$$\begin{aligned} &\text{For price going up with } 7\% \\ &\text{price going up} = R60 + R(60 \times 7\%) = R64.2 \\ &\text{For price going down with } 5\% \\ &\text{price going down} = R60 - R(60 \times 5\%) = R57 \end{aligned}$$

Calculating the price of going up and down for the last node which has 3 output(going up , middle and going down) as:

$$\begin{aligned} &\text{For price going up with } 7\% \\ &\text{price going up } = R64.2 + R(64.2 \times 7\%) = R68.694 \\ &\text{Price for the middle value with } 7\% \\ &\text{price for the middle value } = R64.2 - R(64.2 \times 7\%) = R60.99 \\ &\text{For price going down with } 5\% \\ &\text{price going down } = R57 - R(57 \times 5\%) = R54.15 \end{aligned}$$

The pay-off of the final node using a European call given the strike price K is $R64$

$$\begin{aligned} P &= \ S - K \\ P &= \ R68.694 - R64 \\ P &= R4.694 \end{aligned}$$

Pay-off P for the value of going down and the middle part is zero because $K > S$

Calculating the pay-off P for the middle node when we have $S = R64.2$

$$\begin{aligned} P &= P_0(p_{up})e^{r\Delta} \\ P &= \ 4.694(0.5215) \times \ e^{-0.05 \times 0.25} \\ P &= \ R2.4175 \end{aligned}$$

Now when we have $S = R57$

$$P = 0(0.4785) \times e^{-0.05 \times 0.25}$$
$$P = R0$$

The value of the European call option for the first node is:

$$P = 4.694(0.5215)^2 \times e^{-0.05 \times 0.25}$$
$$P = R1.2451$$

The Binomial tree is:

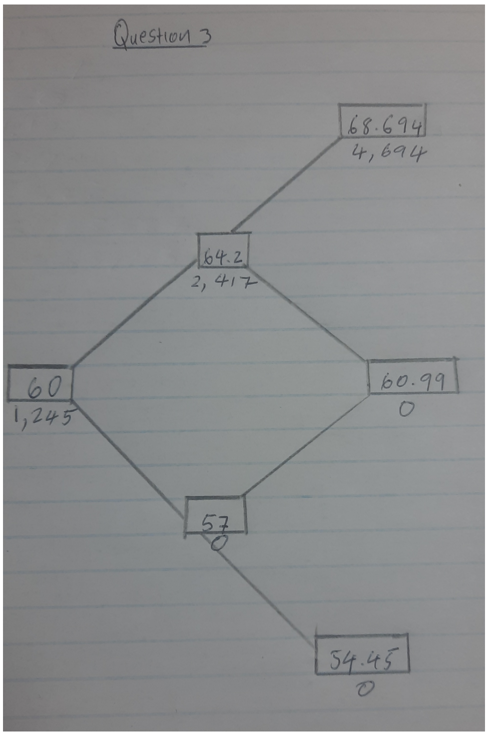

Figure 1: Binomial tree

QUESTION 4

The value of u , d and the probability of a price going up and down has not change. This time around we are going to use the strike price $K = R62$.The pay-off P is zero when $K < S$ for going up, now calculating pay-off P for the middle part

$$P = K - S$$
$$P = R62 - R60.99$$
$$P = R1.01$$

Calculating the pay-off for the value of going down

$$
\begin{aligned}
P =&\ K - S \\
P =&\ R62 - R54.15 \\
P =&\ R7.85
\end{aligned}
$$

Calculating the pay-off for the middle node where $S = R64.2$

$$
\begin{aligned}
P =&\ 1.01(0.4785) \times\ e^{-0.05\times 3/12} \\
P =&\ R0.4773
\end{aligned}
$$

Now when we have $S = R57$

$$
\begin{aligned}
P =&\ (1.01(0.5215) + 7.85(0.4785)) \times\ e^{-0.05\times 3/12} \\
P =&\ R4.2297
\end{aligned}
$$

The value European put option in six month:

$$
\begin{aligned}
P =&\ 1.014(2 \times 0.5215 \times 0.4785 + 7.85 \times 0.4785^2) \times\ e^{-0.05\times 6/12} \\
P =&\ R2.245
\end{aligned}
$$

The Binomial tree for European put option in six month is:

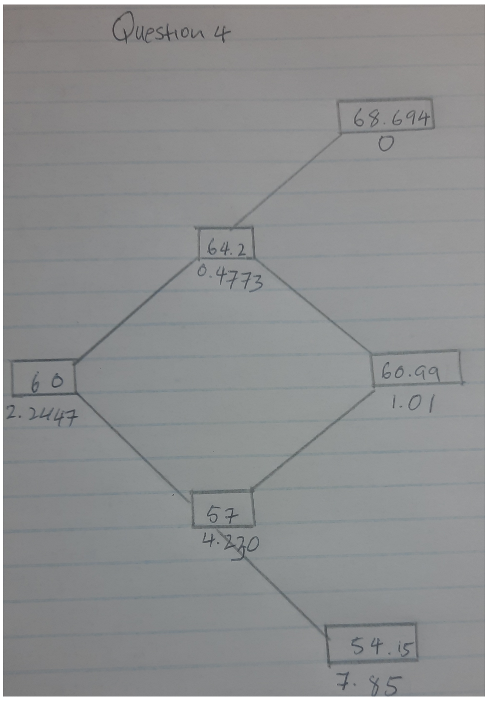

Figure 2: Binomial tree for European put option in six month

European call option, $S = R60 \times (1.07)^2$ and $K = R62$

$$
\begin{aligned}
P &= S - K \\
&= R68.694 - 62 \\
&= R6.694
\end{aligned}
$$

Now we have

$$\begin{aligned} P &= (0.5215 \times 6.694 + 0.4785 \times 0)e^{-0.05\times0.25} \\ &= R3.448 \end{aligned}$$

Now the six month European call option:

$$\begin{aligned} P &= (0.5215 \times 3.404 + 0.4785 \times 0)e^{-0.05\times0.25} \\ &= R1.776 \end{aligned}$$

The Binomial tree for the six month European call option is:

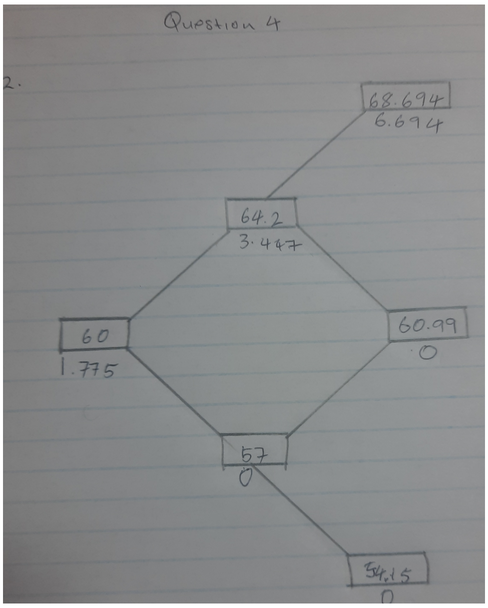

Figure 3: Binomial tree for European put option in six month

Put-Call purity we have R60 (Stock price),R2.245 (Six month European put option),R1.775 (Six month European call option) and K = R62 (Strike price)

Stock price + Six month European put option =R60+R2.245=E62.245

Six month European call option+Strike price($e^{-2r\Delta t}$) = R1.775 +R62($e^{-2\times0.05\times0.25}$)=62.245

Stock price + Six month European put option = Six month European call option+Strike price($e^{-2r\Delta t}$) $\implies put - callparity$

American put option:
This time around we are going to use the strike price $K = R62$.The pay-off P is zero when $K < S$ for going up, now calculating pay-off P for the middle part

$$\begin{aligned} P &= \ K - S \\ P &= \ R62 - R60.99 \\ P &= R1.01 \end{aligned}$$

Calculating the pay-off for the value of going down

$$\begin{aligned} P =&\ K - S \\ P =&\ R62 - R54.15 \\ P =&\ R7.85 \end{aligned}$$

Calculating the pay-off for the middle node where $S = R64.2$

$$\begin{aligned} P =&\ 1.01(0.4785) \times\ e^{-0.05\times 3/12} \\ P =&\ R0.4773 \end{aligned}$$

Now when we have $S = R57$

$$\begin{aligned} P =&\ (1.01(0.5215) + 7.85(0.4785)) \times\ e^{-0.05\times 3/12} \\ P =&\ R4.2297 \end{aligned}$$

The value American put option in six month:

$$\begin{aligned} P =&\ 1.014(2 \times 0.5215 \times 0.4785 + 7.85 \times 0.4785^2) \times\ e^{-0.05\times 6/12} \\ P =&\ R2.245 \end{aligned}$$

The Binomial tree for American put option in six month is:

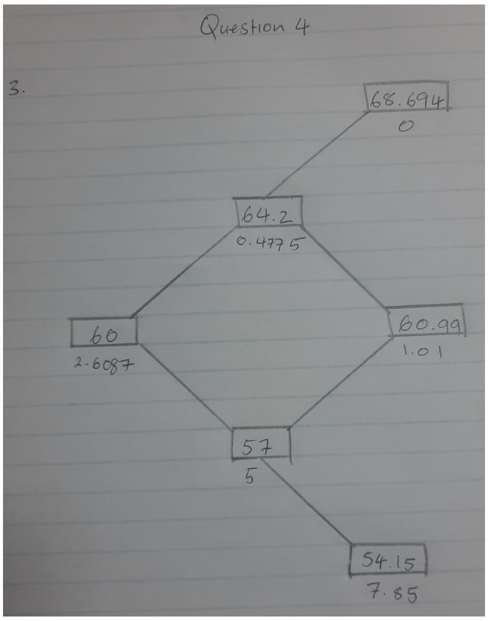

Figure 4: Binomial tree for American put option in six month

QUESTION 5

(a)We have the stock price (S) = R60, risk free rate(r)=0.12, strike price (K) = R64, the time step $\Delta t = 0.25$, the up jump ($u = 100 + 10 = 1.10$) and down jump ($d = 100 - 10 =$

0.90)

The probability of the price going up as the following:

$$\begin{aligned} p &= \frac{e^{\Delta t} - d}{u - d} \\ &= \frac{e^{0.25} - 0.90}{1.10 - 0.90} \\ p &= 0.65227 \end{aligned}$$

Now to do the opposite calculating the probability of a price going down is the following:

$$\begin{aligned} 1 - p =&\ 1 - 0.65227 \\ 1 - p =&\ 0.34773 \end{aligned}$$

Put pay-offs and we know where $S > K$ we have zero pay-off:

Calculating the price of going up and down for the middle node is simple as:

$$\begin{aligned} \text{For price going up with 7\%} \\ \text{price going up} &= R60 + R(60 \times 10\%) = R66 \\ \text{For price going down with 5\%} \\ \text{price going down} &= R60 - R(60 \times 10\%) = R54 \end{aligned}$$

The six-month European put option with a strike price of R64

Where $S = 59.4$

$$\begin{aligned} P_1 &= K - S \\ P_1 &= R64 - (59.4) = R4.6 \end{aligned}$$

Where $S = 48.6$

$$\begin{aligned} P_2 &= K - S \\ P_2 &= R64 - (R48.6) = R15.4 \end{aligned}$$

Using our probabilities we are going to have

$$\begin{aligned} P_3 &= (p_{up} \times P_1 + p_{down} \times P_2)e^{-r\Delta t} \\ P_3 = (0.65227 \times R4.6 &+ 0.34773 \times R15.4)e^{-0.12 \times 0.25} = R8.109 \end{aligned}$$

$$P_4 = (0.65227 \times R0 + 0.34773 \times R4.6)e^{-0.12 \times 0.25} = R1.55$$

$$P_4 = (0.65227 \times R1.55 + 0.34773 \times R8.109)e^{-0.12 \times 0.25} = R3.72$$

The six month European put option is $R3.72$.

Binomial tree for six month European put option:

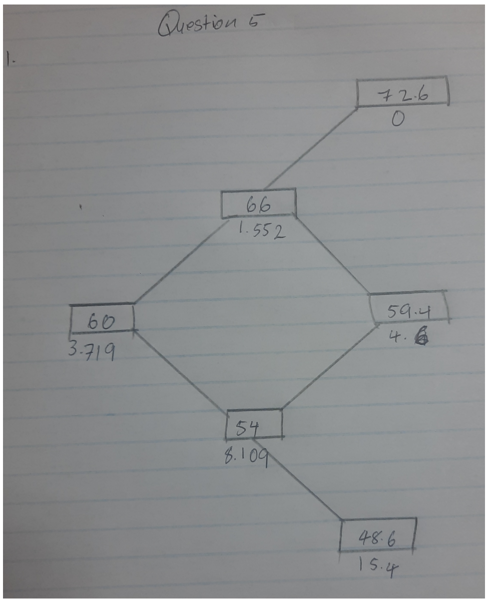

Figure 5: Binomial tree for six month European put option

(b)We have the stock price (S) = R60, risk free rate(r)=0.12, strike price (K) = R64, the time step $\Delta t = 0.25$, the up jump ($u = 100 + 10 = 1.10$) and down jump ($d = 100 - 10 = 0.90$)

The probability of the price going up as the following:

$$\begin{aligned} p &= \frac{e^{\Delta t} - d}{u - d} \\ &= \frac{e^{0.25} - 0.90}{1.10 - 0.90} \\ p &= 0.65227 \end{aligned}$$

Now to do the opposite calculating the probability of a price going down is the following:

$$\begin{aligned} 1 - p = &\ 1 - 0.65227 \\ 1 - p = &\ 0.34773 \end{aligned}$$

Put pay-offs and we know where $S > K$ we have zero pay-off:

Where $S = 59.4$

$$\begin{aligned} P_1 &= K - S \\ P_1 &= R64 - (59.4) = R4.6 \end{aligned}$$

Where $S = 48.6$

$$P_2 = K - S$$
$$P_2 = R64 - (R48.6) = R15.4$$

Where $S = 54$

$$P_3 = K - S$$
$$P_3 = R64 - (R54) = R10$$

$$P_4 = (0.65227 \times R0 + 0.34773 \times R4.6)e^{-0.12\times0.25} = R1.55$$

$$P_3 = (p_{up} \times P_4 + p_{down} \times P_3)e^{-r\Delta t}$$
$$P_3 = (0.65227 \times R1.55 + 0.34773 \times R10)e^{-r\Delta t} = R4.357$$

The six month American put option is $R4.357$.

Binomial tree for six month American put option:

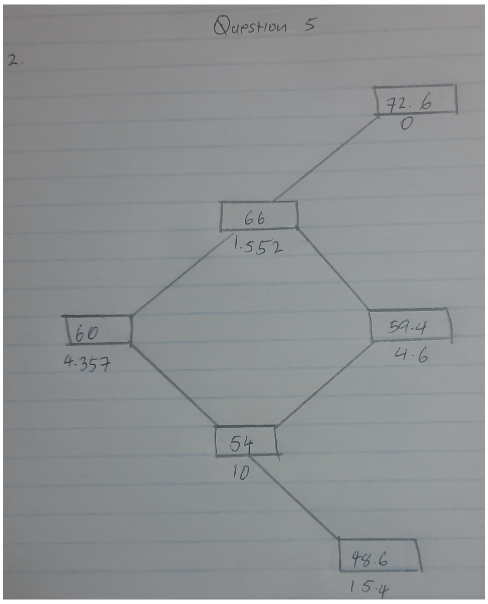

Figure 6: Binomial tree for six month American put option