Statistics: Assignment 2

Musawenkosi Khulu (musawenkosi@aims.ac.za)

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Question 2

1.
Let population mean of senior citizens patients = μ_1
Let population mean of other patients = μ_2

$H_0 : \mu_1 = \mu_2$ {The response time for the senior patients takes the same time as other patients }
$H_0 : \mu_1 > \mu_2$ {The response to call from senior patients takes longer on average than that from other patients}

We are going to use the 1 tailed upper tail test at 1% level. We are going to use the t-test because the population is less than 30 and the variance is unknown.

Right tailed test Statistics

$$
\begin{aligned}
t &= \frac{(\bar{x_1} - \bar{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \\
&= \frac{5.60 - 5.30}{\sqrt{\frac{(0.25)^2}{18} + \frac{0.21^2}{18}}} \\
&= 3.62
\end{aligned}
$$

Critical value

$$
\begin{aligned}
v &= \frac{\left[\frac{s^2}{n_1} + \frac{s_2 2}{n_2}\right]^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} \\
&= \frac{\left[\frac{0.25^2}{18} + \frac{0.21^2}{13}\right]^2}{\frac{\left(\frac{0.25^2}{18}\right)^2}{18-1} + \frac{\left(\frac{0.21^2}{13}\right)^2}{13-1}} \\
&= 28.24 \\
&\approx 28.
\end{aligned}
$$

$$
\begin{aligned}
\text{We know that } 1 - \alpha &= 1 - 0.01 \\
1 - \alpha &= 0.990 \\
\text{from the table we get } t_{1-\alpha}, v &= 2.467
\end{aligned}
$$

$\therefore$ The critical value($t_{1-\alpha}, v$) is less than the right tailed test statistics t , we reject the null hypothesis at 1% level of significant and conclude that calls from senior citizens is slower (takes longer time on average) than that to calls from other calls patients.

2.
a

$$H_0 : \mu_1 = \mu_2 \text{ \{There is no difference in patient stay regardless of gender\}}$$
$$H_0 : \mu_1 > \mu_2 \text{ \{The male patients stay longer than females\}}$$

$$\begin{aligned} z &= \frac{(\bar{x_1} - \bar{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \\ &= \frac{9 - 7.2}{\sqrt{\frac{55}{40} + \frac{47}{35}}} \\ &= 1.09 \\ \text{We know that } 1 - \alpha &= 1 - 0.05 \\ 1 - \alpha &= 0.950 \\ \text{from the table we get } z_{1-\alpha}, v &= 1.645 \end{aligned}$$

$\therefore$ The critical value($z_{1-\alpha}$) is more than the statistic value(z) , we fail to reject the null hypothesis at 5% level of significant and conclude that male patients stay longer on average than female patients.
b

$$\begin{aligned} \bar{x_1} - \bar{x_2} &\pm z_{1-\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \\ \text{We know that } \ 1 - \alpha/2 &= 1 - 0.05/2 \\ \text{we get } \ 1 - \alpha/2 &= 0.975 \\ z_{1-\alpha/2} &= 1.960 \\ 9 - 7.2 &\pm 1.960\sqrt{\frac{55}{40} + \frac{47}{35}} \\ 1.8 &\pm 3.23 \\ \text{Confidence Interval is } &(-1.43, 5.03). \end{aligned}$$

$\therefore$ We are 95% confidence that the mean difference lies within the interval $(-1.43, 5.03)$.
3.
In this question we are going to use the test of 1 proportion.

$H_0 : p = 85\%$
$H_{1a} : p > 85\%$
In this problem we are going to use the right tailed test and we apply the $z - test$

$$\begin{aligned} Z_0 &= \frac{P - P_0}{\sqrt{\frac{p_0 q_0}{n}}} \\ &= \frac{0.88 - 0.85}{\sqrt{\frac{0.85(1-0.85)}{200}}} \\ &= 1.188 \\ \text{The critical value will give us} & \\ Z_\alpha &= Z_{0.01} \\ &= 2.326 \\ Z_\alpha &> Z_0 \end{aligned}$$

$\therefore$ Since $Z_\alpha > Z_0$ We do not reject H_0, we have enough evidence with 99% confidence level.