# Interpretation and top-down and bottom-up ground proofs

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# Representation and Reasoning Systems

The use of knowledge in AI is done with a Representation and Reasoning Systems (*RRS*).

RRS = syntax + semantics + reasoning

RRS = formal language + precise semantics + proof procedure

**Note:** the semantics aren't reflected in the implementation.

# Properties of an RRS

- The *proof procedure* uses a nondeterministic *search strategy*
- It offers two functions: *tell* and *ask*

# Simplifying assumptions in a RRS

We can define the following Simplifying assumptions for a RRS: *Individuals and Relations* (IR), *Definite Knowledge* (DK), *Static Environment* (SE) and *Finite Domain* (FD).

$$\text{RRS} + \text{IR} + \text{DK} + \text{SE} + \text{FD} \Longrightarrow \textit{Datalog}$$

# The syntax of Datalog

**Variables** a sequence of alphanumeric characters and '_', beginning with an upper-case letter

**Constants** a sequence of letters starting with a lower-case letter or a sequence of digits (a number)

**Predicate symbols** a sequence of alphanumeric characters and '_', beginning with an lower-case letter

**Terms** variables or constants

**Atomic symbols** are of the form $p$ or $p(t_1, t_2, \ldots, t_n)$, where $p$ is a predicate symbol and $t_i$ are terms

# The syntax of Datalog (cont'd)

Definite clauses are atomic symbols (a fact) or have the form:

$$\underbrace{a}_{head} \leftarrow \underbrace{b_1 \wedge \cdots \wedge b_n}_{body}$$

where $a$ and $b_i$ are atomic symbols.

Queries of the form $?b_1 \wedge \cdots \wedge b_n$

Knowledge base a set of definite clauses

# Example knowledge base

## Example

```
depozitare(X, container_metalic) ←
    consistenta(X, solid) ∧
    proprietate(X, toxic).
depozitare(X, container_metalic) ←
    consistenta(X, solid) ∧
    proprietate(X, inflamabil).
depozitare(X, rezervor_presurizat) ←
    consistenta(X, gaz) ∧
    proprietate(X, inflamabil).
```

# Tarskian semantics

Semantics: specifies the meaning of sentences in a language. It specifies:

- a set of objects in the world (individuals); and
- a correspondence between the symbols in the language and objects (constants) and relations in the world (predicate symbols). interpretation function

# Formal semantics

Interpretation: a triple $I = \langle D, \phi, \pi \rangle$, where:

- $D$ is a nonempty set called the *domain* ($D$ contains *real* objects);
- $\phi$ is a mapping that assigns to each constant $c$ an element $\phi(c) \in D$; and
- $\pi$ is a mapping that assigns to each n-ary predicate symbol $p$ a function from $D^n$ into $\{true, false\}$ (specifies wether the relation denoted by $p$ is true or false for each n-tuple of individuals).

# Example interpretation

- $D = \frac{\{\text{the person named Alan,room 123,}}{\text{room 023,the CS department's building}\}}$.
- The symbolic constants: $alan$, $r123$, $r023$, $cs\_building$.
- The $\phi$ function:
  - $\phi(alan)$ = the person named Alan
  - $\phi(r123)$ = room 123
  - $\phi(r023)$ = the room 023
  - $\phi(cs\_building)$ = the CS department's building
- The predicate symbols: $person/1$, $in/2$, $part\_of/2$.
- The $\pi$ function:
  - $\pi(person) = \{(\text{the person named Alan})\}$
  - $\pi(in) = \frac{\{(\text{the person named Alan,room 123}),}{(\text{the person named Alan,the CS department's building})\}}$
  - $\pi(part\_of) = \frac{\{(\text{the room 123,the CS department's building}),}{(\text{the room 023,the CS department's building})\}}$

# Truth in an interpretation

- The symbolic constant $c$ denotes in $I$ the individual $c^I = \phi(c) \in D$.
- The ground atom $p(t_1, \ldots, t_n)$ is true in interpretation $I$ if $\pi(p)(t'_1, \ldots, t'_n) = true$, where $t_i$ denotes $t'_i$ in interpretation $I$ and false otherwise.
- The ground clause $h \leftarrow b_1 \wedge \ldots \wedge b_n$ is false in interpretation $I$ if $h$ is false and every $b_i$ is true, and true otherwise.

# Models and logical consequences

- A knowledge base $\Delta$ is true in an interpretation $I$ iff every clause in $\Delta$ is true in $I$.
- A model of a knowledge base is an interpretation in which all clauses are true.
- If $\Delta$ is a knowledge base and $g$ is a conjunction of atoms, $g$ is a *logical consequence* of $\Delta$ if $g$ is true in every model of $\Delta$, written as $\Delta \models g$.
- That is, $\Delta \models g$ if there is no interpretation in which $\Delta$ is true and $g$ is false.

# Example

$$\Delta = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

| x | $\pi(p)$ | $\pi(q)$ | $\pi(r)$ | $\pi(s)$ |
|---|---|---|---|---|
| $I_1$ | true | true | true | true |
| $I_2$ | false | false | false | false |
| $I_3$ | true | true | false | false |
| $I_4$ | true | true | true | false |
| $I_5$ | true | true | false | true |

$$\Delta \models p,\ \Delta \models q,\ \Delta \not\models r,\ \Delta \not\models s$$

# Proofs

## Definition

A **proof** is a mechanically derivable demonstration that a formula derives from a knowledge base.

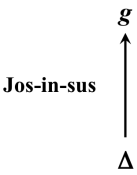
$\Delta \vdash g$ means that $g$ can be derived from $\Delta$

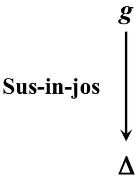
A proof can be **sound** ($\Delta \vdash g \Rightarrow \Delta \models g$) and **complete** ($\Delta \models g \Rightarrow \Delta \vdash g$)

# Bottom-up and top-down proofs

**Bottom-up proofs** starting from the knowledge base and from the already proven facts new facts are obtained at every step.

**Top-down proofs** starting from the query each step works towards the knowledge base.



# Bottom-up ground proofs

Only one rule of inference, a generalized form of *modus ponens*:

**Definition**

If $h \leftarrow b_1 \wedge \ldots \wedge b_n in \Delta$, and each $b_i$ has been derived, then $h$ can be derived.

This is using *forward chaining*.

# The bottom-up proof procedure

$\Delta \vdash g$ if $g \in C$ at the end of this procedure:

## Proof procedure

```
C := {};
repeat
    select clause "h ← b_1 ∧ ... ∧ b_m" ∈ Δ:
        ∀ i, b_i ∈ C, and
        h ∉ C;
    C := C ∪ {h}
until no more clauses can be selected.
```

# Bottom-up proof example

## Example

1. a ← b ∧ c.
2. b ← d ∧ e.
3. b ← g ∧ e.
4. c ← e.
5. d.
6. e.
7. f ← a ∧ g.

$C := \emptyset$

5: $C := \{d\}$

6: $C := \{d, e\}$

2: $C := \{d, e, b\}$

4: $C := \{d, e, b, c\}$

1: $C := \{a, d, e, b, c\}$

STOP

# Top-down ground proofs

- Start from an *answer clause* of the form:
  $yes \leftarrow a_1 \wedge \ldots \wedge a_n$.
- Select an atom from the body of the answer clause, $a_i$, and a clause from $\Delta$:
  $a_i \leftarrow b_1 \wedge \ldots \wedge b_m$.
- Using SLD resolution on the clause and $a_i$ the new answer clause becomes:
  $yes \leftarrow a_1 \wedge \ldots \wedge a_{i-1} \wedge b_1 \wedge \ldots \wedge b_m \wedge a_{i+1} \wedge \ldots \wedge a_n$.

# Proofs through SLD resolution

Resolution with a Linear selection function for Definite clauses (no disjunctions) – in Romanian: S - selectie; L - liniara; D - clauze precise

- A clause with $n = 0$ is an *answer* (i.e. $yes \leftarrow .$)
- A proof though SLD resolution for the $?q_1 \wedge \ldots \wedge q_k$ query from a knowledge base $\Delta$ is a sequence of answer clauses $\gamma_0, \gamma_1, \ldots, \gamma_n$ such that:
  - $\gamma_0$ is the initial answer clause: $yes \leftarrow q_1 \wedge \ldots \wedge q_k$,
  - $\gamma_i$ is obtained by resolving $\gamma_{i-1}$ with clauses in $\Delta$, and
  - $\gamma_n$ is an answer

# The top-down proof procedure

To solve the query $?q_1 \wedge \ldots \wedge q_k$:

## Proof procedure

$ac$ := "$yes \leftarrow q_1 \wedge \ldots \wedge q_k$$"
repeat
    select a conjunct $a_i$ from the body of $ac$;
    choose clause $C$ from $\Delta$ with $a_i$ as head;
    replace $a_i$ in the body of ac by the body of $C$
until $ac$ is an answer.

# Nondeterminism

- Don't care nondeterminism If one selection doesn't lead to a solution there's no point trying other alternatives. select
- Don't know nondeterminism If one choice doesn't lead to a solution other choices may. choose

# Top-down proof example (successful)

## Example

1. a ← b ∧ c.
2. a ← e ∧ f.
3. b ← f ∧ k.
4. c ← e.
5. d ← k.
6. e.
7. f ← j ∧ e.
8. f ← c.
9. j ← c.

Query: $?a$

Selection: the leftmost atom from the answer clause body.

$$
\begin{array}{llllll}
\gamma_0 : & yes & \leftarrow & a & & \\
\gamma_1(2) : & yes & \leftarrow & e & \wedge & f \\
\gamma_2(6) : & yes & \leftarrow & f & & \\
\gamma_3(8) : & yes & \leftarrow & c & & \\
\gamma_4(4) : & yes & \leftarrow & e & & \\
\gamma_5(6) : & yes & \leftarrow & & &
\end{array}
$$

# Top-down proof example (failing)

## Example

1. a ← b ∧ c.
2. a ← e ∧ f.
3. b ← f ∧ k.
4. c ← e.
5. d ← k.
6. e.
7. f ← j ∧ e.
8. f ← c.
9. j ← c.

Query: $?a$

Selection: the leftmost atom from the answer clause body.

$\gamma_0:\quad yes \leftarrow a$

$\gamma_1(1):\quad yes \leftarrow b \wedge c$

$\gamma_2(3):\quad yes \leftarrow f \wedge k \wedge c$

$\gamma_3(8):\quad yes \leftarrow c \wedge k \wedge c$

$\gamma_4(4):\quad yes \leftarrow e \wedge k \wedge c$

$\gamma_5(6):\quad yes \leftarrow k \wedge c$