

Interpretation and top-down and bottom-up ground proofs

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Representation and Reasoning Systems

The use of knowledge in AI is done with a Representation and Reasoning System (*RRS*).

$\text{RRS} = \text{syntax} + \text{semantics} + \text{reasoning}$

$\text{RRS} = \text{formal language} + \text{precise semantics} + \text{proof procedure}$

Note: the semantics aren't reflected in the implementation.

Properties of an RRS

- The *proof procedure* uses a nondeterministic *search strategy*
- It offers two functions: *tell* and *ask*

Simplifying assumptions in a RRS

We can define the following simplifying assumptions for a RRS:
Individuals and Relations (IR), *Definite Knowledge* (DK), *Static Environment* (SE) and *Finite Domain* (FD).

$$\text{RRS} + \text{IR} + \text{DK} + \text{SE} + \text{FD} \implies \text{Datalog}$$

The syntax of Datalog

- Variables** a sequence of alphanumeric characters and '_', beginning with an upper-case letter
- Constants** a sequence of letters starting with a lower-case letter or a sequence of digits (a number)
- Predicates** a sequence of alphanumeric characters and '_', beginning with a lower-case letter
- Terms** variables or constants
- Atoms** are of the form p or $p(t_1, t_2, \dots, t_n)$, where p is a predicate symbol and t_i are terms

The syntax of Datalog (cont'd)

Definite clauses are atomic symbols (facts) or have the form:

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1 \wedge \cdots \wedge b_n}_{\text{body}}$$

where a and b_i are atomic symbols.

Queries of the form $?b_1 \wedge \cdots \wedge b_n$

Knowledge base a set of definite clauses

Example knowledge base

Example

```
depozitare(X, container_metalic) ←  
    consistenta(X, solid) ∧  
    proprietate(X, toxic).  
depozitare(X, container_metalic) ←  
    consistenta(X, solid) ∧  
    proprietate(X, inflamabil).  
depozitare(X, rezervor_presurizat) ←  
    consistenta(X, gaz) ∧  
    proprietate(X, inflamabil).
```

Tarskian semantics

Semantics: specifies the meaning of sentences in a language. It specifies:

- a set of objects in the world (individuals); and
- a correspondence between the symbols in the language and objects (constants) and relations in the world (predicate symbols). **interpretation function**

Formal semantics

Interpretation: a triple $I = \langle D, \phi, \pi \rangle$, where:

- D is a nonempty set called the *domain* (D contains *real* objects);
- ϕ is a mapping that assigns to each constant c an element $\phi(c) \in D$; and
- π is a mapping that assigns to each n -ary predicate symbol p a function from D^n into $\{true, false\}$ (specifies whether the relation denoted by p is true or false for each n -tuple of individuals).

Example interpretation

- $D = \{\text{the person named Alan, room 123, room 023, the CS department's building}\}$.
- The symbolic constants: *alan*, *r123*, *r023*, *cs_building*.
- The ϕ function:
 - $\phi(\text{alan}) = \text{the person named Alan}$
 - $\phi(\text{r123}) = \text{room 123}$
 - $\phi(\text{r023}) = \text{the room 023}$
 - $\phi(\text{cs_building}) = \text{the CS department's building}$
- The predicate symbols: *person*/1, *in*/2, *part_of*/2.
- The π function:
 - $\pi(\text{person}) = \{(\text{the person named Alan})\}$
 - $\pi(\text{in}) = \{(\text{the person named Alan, room 123}), (\text{the person named Alan, the CS department's building})\}$
 - $\pi(\text{part_of}) = \{(\text{the room 123, the CS department's building}), (\text{the room 023, the CS department's building})\}$

Truth in an interpretation

- The symbolic constant c denotes in I the individual $c^I = \phi(c) \in D$.
- The ground atom $p(t_1, \dots, t_n)$ is true in interpretation I if $\pi(p)(t'_1, \dots, t'_n) = \text{true}$, where t_i denotes t'_i in interpretation I and false otherwise.
- The ground clause $h \leftarrow b_1 \wedge \dots \wedge b_n$ is false in interpretation I if h is false and every b_i is true, and true otherwise.

Models and logical consequences

- A knowledge base Δ is true in an interpretation I iff every clause in Δ is true in I .
- A **model** of a knowledge base is an interpretation in which all clauses are true.
- If Δ is a knowledge base and g is a conjunction of atoms, g is a *logical consequence* of Δ if g is true in every model of Δ , written as $\Delta \models g$.
- That is, $\Delta \models g$ if there is no interpretation in which Δ is true and g is false.

Example

$$\Delta = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

x	$\pi(p)$	$\pi(q)$	$\pi(r)$	$\pi(s)$
l_1	true	true	true	true
l_2	false	false	false	false
l_3	true	true	false	false
l_4	true	true	true	false
l_5	true	true	false	true

$$\Delta \models p, \Delta \models q, \Delta \not\models r, \Delta \not\models s$$

Proofs

Definition

A **proof** is a mechanically derivable demonstration that a formula derives from a knowledge base.

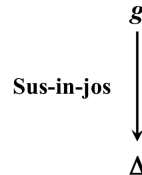
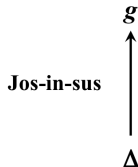
$\Delta \vdash g$ means that g can be derived from Δ

A proof can be **sound** ($\Delta \vdash g \Rightarrow \Delta \models g$) and **complete** ($\Delta \models g \Rightarrow \Delta \vdash g$)

Bottom-up and top-down proofs

Bottom-up proofs starting from the knowledge base and from the already proven facts new facts are obtained at every step.

Top-down proofs starting from the query each step works towards the knowledge base.



Bottom-up ground proofs

Only one rule of inference, a generalized form of *modus ponens*:

Definition

If $h \leftarrow b_1 \wedge \dots \wedge b_n$ in Δ , and each b_i has been derived, then h can be derived.

This is using *forward chaining*.

The bottom-up proof procedure

$\Delta \vdash g$ if $g \in C$ at the end of this procedure:

Proof procedure

```
C := {};  
repeat  
  select clause " $h \leftarrow b_1 \wedge \dots \wedge b_m$ "  $\in \Delta$ :  
     $\forall i, b_i \in C$ , and  
     $h \notin C$ ;  
   $C := C \cup \{h\}$   
until no more clauses can be selected.
```

Bottom-up proof example

Example

```
1.  $a \leftarrow b \wedge c.$   
2.  $b \leftarrow d \wedge e.$   
3.  $b \leftarrow g \wedge e.$   
4.  $c \leftarrow e.$   
5.  $d.$   
6.  $e.$   
7.  $f \leftarrow a \wedge g.$ 
```

```
 $C := \emptyset$   
5:  $C := \{d\}$   
6:  $C := \{d, e\}$   
2:  $C := \{d, e, b\}$   
4:  $C := \{d, e, b, c\}$   
1:  $C := \{a, d, e, b, c\}$   
STOP
```

Top-down ground proofs

- Start from an *answer clause* of the form:
 $yes \leftarrow a_1 \wedge \dots \wedge a_n.$
- Select an atom from the body of the answer clause, a_i , and a clause from Δ :
 $a_i \leftarrow b_1 \wedge \dots \wedge b_m.$
- Using SLD resolution on the clause and a_i the new answer clause becomes:
 $yes \leftarrow a_1 \wedge \dots \wedge a_{i-1} \wedge b_1 \wedge \dots \wedge b_m \wedge a_{i+1} \wedge \dots \wedge a_n.$

Proofs through SLD resolution

Resolution with a Linear selection function for Definite clauses (no disjunctions) – in Romanian: S - selectie; L - liniara; D - clauze precise

- A clause with $n = 0$ is an *answer* (i.e. $yes \leftarrow .$)
- A proof through SLD resolution for the $?q_1 \wedge \dots \wedge q_k$ query from a knowledge base Δ is a sequence of answer clauses $\gamma_0, \gamma_1, \dots, \gamma_n$ such that:
 - γ_0 is the initial answer clause: $yes \leftarrow q_1 \wedge \dots \wedge q_k$,
 - γ_i is obtained by resolving γ_{i-1} with clauses in Δ , and
 - γ_n is an answer

The top-down proof procedure

To solve the query $?q_1 \wedge \dots \wedge q_k$:

Proof procedure

```
ac := "yes  $\leftarrow q_1 \wedge \dots \wedge q_k$ "
```

```
repeat
```

```
    select a conjunct  $a_i$  from the body of  $ac$ ;
```

```
    choose clause  $C$  from  $\Delta$  with  $a_i$  as head;
```

```
    replace  $a_i$  in the body of  $ac$  by the body of  $C$ 
```

```
until  $ac$  is an answer.
```

Nondeterminism

- **Don't care nondeterminism** If one selection doesn't lead to a solution there's no point trying other alternatives. **select**
- **Don't know nondeterminism** If one choice doesn't lead to a solution other choices may. **choose**

Top-down proof example (successful)

Example

1. $a \leftarrow b \wedge c.$
2. $a \leftarrow e \wedge f.$
3. $b \leftarrow f \wedge k.$
4. $c \leftarrow e.$
5. $d \leftarrow k.$
6. $e.$
7. $f \leftarrow j \wedge e.$
8. $f \leftarrow c.$
9. $j \leftarrow c.$

Query: $?a$

Selection: the leftmost atom from the answer clause body.

$\gamma_0:$ $yes \leftarrow a$
 $\gamma_1(2): yes \leftarrow e \wedge f$
 $\gamma_2(6): yes \leftarrow f$
 $\gamma_3(8): yes \leftarrow c$
 $\gamma_4(4): yes \leftarrow e$
 $\gamma_5(6): yes \leftarrow$

Top-down proof example (failing)

Example

1. $a \leftarrow b \wedge c.$
2. $a \leftarrow e \wedge f.$
3. $b \leftarrow f \wedge k.$
4. $c \leftarrow e.$
5. $d \leftarrow k.$
6. $e.$
7. $f \leftarrow j \wedge e.$
8. $f \leftarrow c.$
9. $j \leftarrow c.$

Query: $?a$

Selection: the leftmost atom from the answer clause body.

γ_0 : $\text{yes} \leftarrow a$

$\gamma_1(1)$: $\text{yes} \leftarrow b \wedge c$

$\gamma_2(3)$: $\text{yes} \leftarrow f \wedge k \wedge c$

$\gamma_3(8)$: $\text{yes} \leftarrow c \wedge k \wedge c$

$\gamma_4(4)$: $\text{yes} \leftarrow e \wedge k \wedge c$

$\gamma_5(6)$: $\text{yes} \leftarrow k \wedge c$