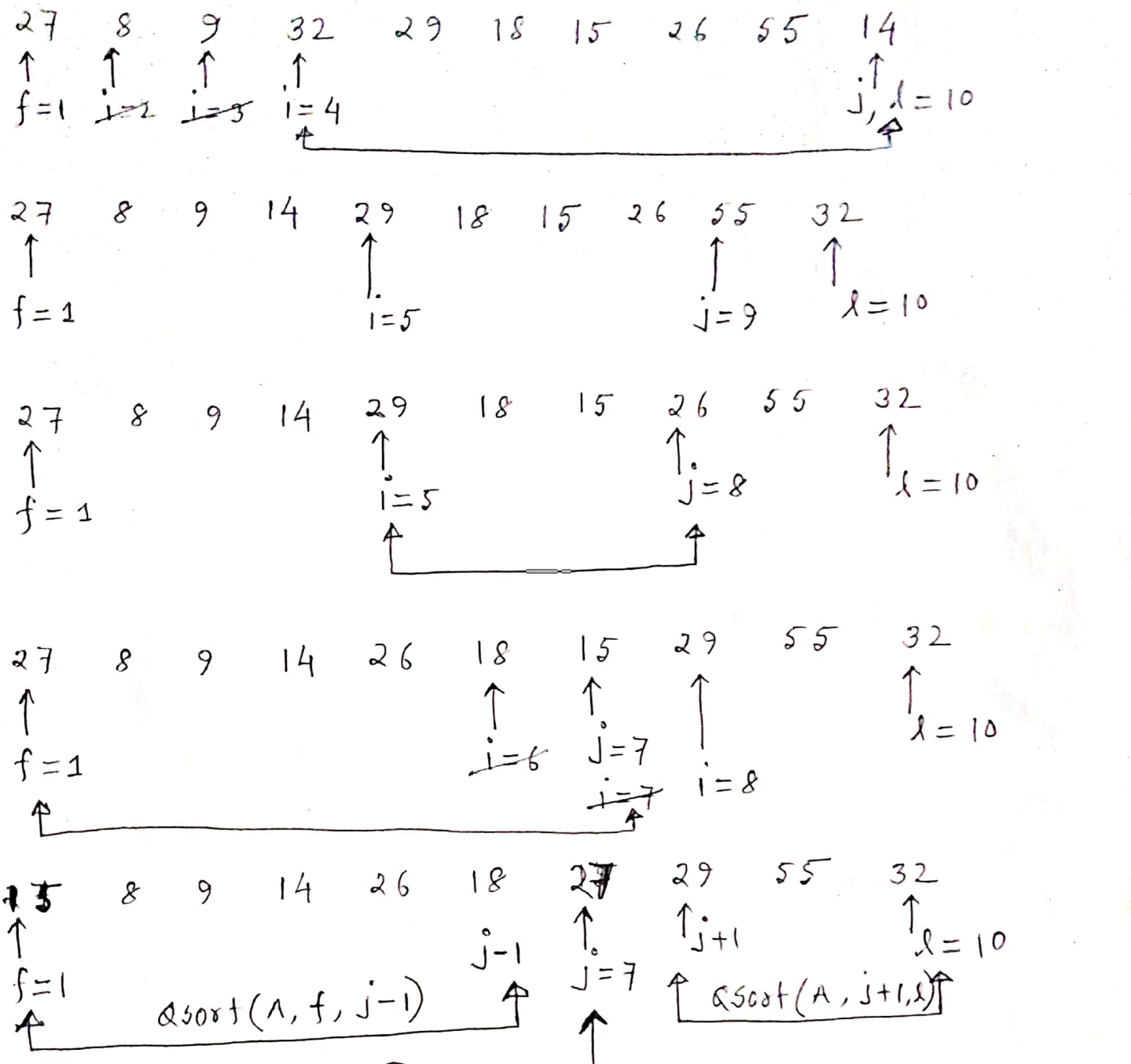
Quick sort Algorithm

A[f] >= A[i] i →
A[f] < A[j] ← j

Mechanism:




```
Qsort Algorithm
void Qsort(int A[], int f, int l)
{ if (l>f){
   i = f+1
   j = l
   while (i<=j) {
   while((i<=l)&&(A[f] >= A[i]))
   { i = i+1
   }
   while((j>=f)&&(A[f] < A[j]))
   { j = j-1
   }
   if (i>j) break;
   A[i] <-> A[j]
   i = i+1
   j = j-1
   }
   A[f] <-> A[j]
   Qsort(A, f, j-1)
   Qsort(A, j+1, l)
}
}
```

Time Complexity Analysis

$c(n)$ = element comparison over n elements (Average)

$$c(n) = \frac{1}{n}\left[\sum_{j=1}^{n}\{(n+1) + c(j-1) + c(n-j)\}\right]$$

$j-1-1+1 = j-1$ (from $f=1$ to $j-1$)

$n-(j+1)+1 = n-j$ (from $j+1$ to $l=n$)

$$= \frac{1}{n}\left[\sum_{j=1}^{n}(n+1) + \sum_{j=1}^{n}\{c(j-1)+c(n-j)\}\right]$$

$$= \frac{1}{n}\left[n(n+1) + 2\{c(0)+c(1)+\cdots+c(n-1)\}\right]$$

$1, 2, 3, 4$: $4-1+1 = 4$

$$\Rightarrow c(n) = (n+1) + \frac{2}{n}\{c(0)+c(1)+\cdots+c(n-1)\}$$ [Multiply by n]

$$nc(n) = n(n+1) + 2\{c(0)+c(1)+\cdots+c(n-1)\} \quad \text{eq (1)}$$

Replace n by (n-1)

$$(n-1)c(n-1) = (n-1)n + 2\{c(0)+c(1)+\cdots+c(n-2)\} \quad \text{eq (2)}$$

eq (1) − eq (2) $\Rightarrow$

$$nc(n) - (n-1)c(n-1) = n(n+1) - (n-1)n + 2c(n-1)$$

$$\Rightarrow nc(n) - (n-1)c(n-1) = 2n + 2c(n-1)$$

$$\Rightarrow nc(n) = 2n + (n-1)c(n-1) + 2c(n-1)$$

$$\Rightarrow nc(n) = 2n + \{(n-1)+2\}c(n-1)$$

$$\Rightarrow nc(n) = 2n + (n+1)c(n-1)$$

$$\Rightarrow \frac{c(n)}{n+1} = \frac{2}{n+1} + \frac{c(n-1)}{n}$$ [Divide by n(n+1)]

[Recurrence Relation]

By Solving the Recurrence

$$\frac{c(n)}{n+1} = \frac{2}{n+1} + \frac{c(n-1)}{n}$$

$$= \frac{2}{n+1} + \left[\frac{2}{n} + \frac{c(n-2)}{n-1}\right]$$

Replace n by n-1

$\because \frac{c(n-1)}{n} = \frac{2}{n} + \frac{c(n-2)}{n-1}$

$$= \frac{2}{n+1} + \frac{2}{n} + \frac{c(n-2)}{n-1}$$

$$= \frac{2}{n+1} + \frac{2}{n} + \frac{2}{n-1} + \frac{c(n-3)}{n-2}$$

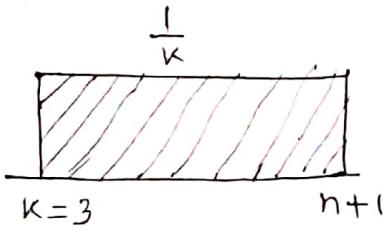
$$= \frac{2}{n+1} + \frac{2}{n} + \frac{2}{n-1} + \cdots\cdots + \frac{2}{3} + \frac{c(1)}{2}$$

$\because c(1) = 0$

$$= \frac{2}{n+1} + \frac{2}{n} + \frac{2}{n-1} + \cdots + \frac{2}{3} + \frac{0}{2}$$

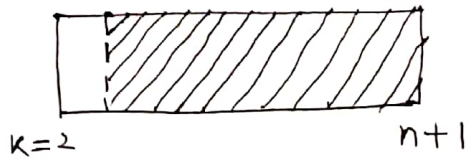
$$= 2\left(\frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n+1}\right)$$

$$= 2 \sum_{k=3}^{n+1} \frac{1}{k}$$



$$= 2 \int_{3}^{n+1} \frac{1}{k} \cdot dk$$

$$\Rightarrow \frac{c(n)}{n+1} \leq 2 \int_{2}^{n+1} \frac{1}{k} \cdot dk$$



$$= 2 \left[\log_e k\right]_2^{n+1}$$

$$= 2\left[\log_e (n+1) - \log_e 2\right]$$

$$\therefore c(n) \leq 2(n+1)\left[\log_e (n+1) - \log_e 2\right]$$

$$c(n) \leq O(n \log_e n)$$