

Quick sort Algorithm

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$A[f] \gg A[i] \quad i \rightarrow$
 $A[f] < A[j] \quad \leftarrow j$

Mechanism:

27 8 9 32 29 18 15 26 55 14
 $\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \quad \quad \quad \quad \uparrow \quad \uparrow$
 $f=1 \quad j=2 \quad i=3 \quad i=4 \quad \quad \quad \quad \quad j, l=10$

27 8 9 14 29 18 15 26 55 32
 $\uparrow \quad \quad \quad \quad \quad \uparrow \quad \quad \quad \quad \quad \uparrow \quad \quad \quad \uparrow$
 $f=1 \quad \quad \quad \quad \quad i=5 \quad \quad \quad \quad \quad j=9 \quad \quad \quad l=10$

27 8 9 14 29 18 15 26 55 32
 $\uparrow \quad \quad \quad \quad \quad \uparrow \quad \quad \quad \quad \quad \uparrow \quad \quad \quad \uparrow$
 $f=1 \quad \quad \quad \quad \quad i=5 \quad \quad \quad \quad \quad j=8 \quad \quad \quad l=10$

27 8 9 14 26 18 15 29 55 32
 $\uparrow \quad \quad \quad \quad \quad \uparrow \quad \quad \quad \uparrow \quad \quad \quad \uparrow \quad \quad \quad \uparrow$
 $f=1 \quad \quad \quad \quad \quad i=6 \quad \quad \quad j=7 \quad \quad \quad i=8 \quad \quad \quad l=10$

15 8 9 14 26 18 27 29 55 32
 $\uparrow \quad \quad \quad \quad \quad \quad \quad \quad \uparrow \quad \quad \quad \uparrow \quad \quad \quad \uparrow$
 $f=1 \quad \quad \quad \quad \quad j-1 \quad \quad \quad j=7 \quad \quad \quad j+1 \quad \quad \quad l=10$

First partitioning element

```

Quick sort Algorithm
void Qsort(int A[], int f, int l)
{
    if (f > l) {
        i = f + 1;
        j = l;
        while (i <= j) {
            while ((i <= l) && (A[f] >= A[i]))
                i = i + 1;
            while ((j >= f) && (A[f] < A[j]))
                j = j - 1;
            if (i > j) break;
            A[i] <-> A[j];
            i = i + 1;
            j = j - 1;
        }
        A[f] <-> A[j];
        Qsort(A, f, j - 1);
        Qsort(A, j + 1, l);
    }
}
    
```

Time Complexity Analysis

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$c(n)$ = element comparison over n elements (Average)

$$c(n) = \frac{1}{n} \left[\sum_{j=1}^n \{ (n+1) + c(j-1) + c(n-j) \} \right]$$

$$\begin{array}{c} j-1-1+1=j-1 \\ \text{---} \\ f=1 \quad j-1 \end{array}$$

$$= \frac{1}{n} \left[\sum_{j=1}^n (n+1) + \sum_{j=1}^n \{ c(j-1) + c(n-j) \} \right]$$

$$\begin{array}{c} n-(j+1)+1=n-j \\ \text{---} \\ j+1 \quad l=n \end{array}$$

$$= \frac{1}{n} \left[n(n+1) + 2 \{ c(0) + c(1) + \dots + c(n-1) \} \right]$$

$$\begin{array}{c} 1 \leftarrow \\ 2 \\ 3 \\ 4 \leftarrow \end{array} \left\{ \begin{array}{l} 4-1+1 \\ =4 \end{array} \right.$$

$$\Rightarrow c(n) = (n+1) + \frac{2}{n} \{ c(0) + c(1) + \dots + c(n-1) \} \quad [\text{Multiply by } n]$$

$$nc(n) = n(n+1) + 2 \{ c(0) + c(1) + \dots + c(n-1) \} \quad \dots \text{eq (1)}$$

Replace n by $(n-1)$

$$(n-1)c(n-1) = (n-1)n + 2 \{ c(0) + c(1) + \dots + c(n-2) \} \quad \dots \text{eq (2)}$$

$$\text{eq (1)} - \text{eq (2)} \Rightarrow$$

$$nc(n) - (n-1)c(n-1) = \underline{\underline{n(n+1) - (n-1)n + 2c(n-1)}}$$

$$\Rightarrow nc(n) - (n-1)c(n-1) = 2n + 2c(n-1)$$

$$\Rightarrow nc(n) = 2n + (n-1)c(n-1) + 2c(n-1)$$

$$\Rightarrow nc(n) = 2n + \{ (n-1) + 2 \} c(n-1)$$

$$\Rightarrow nc(n) = 2n + (n+1)c(n-1)$$

$$\Rightarrow \frac{c(n)}{n+1} = \frac{2}{n+1} + \frac{c(n-1)}{n} \quad [\text{Divide by } n(n+1)]$$

[Recurrence Relation]

By solving the Recurrence

Replace n by $n-1$

$$\frac{c(n)}{n+1} = \frac{2}{n+1} + \frac{c(n-1)}{n}$$

$$\therefore \frac{c(n-1)}{n} = \frac{2}{n} + \frac{c(n-2)}{n-1}$$

$$= \frac{2}{n+1} + \left[\frac{2}{n} + \frac{c(n-2)}{n-1} \right]$$

$$= \frac{2}{n+1} + \frac{2}{n} + \frac{c(n-2)}{n-1}$$

$$= \frac{2}{n+1} + \frac{2}{n} + \frac{2}{n-1} + \frac{c(n-3)}{n-2}$$

$$= \frac{2}{n+1} + \frac{2}{n} + \frac{2}{n-1} + \dots + \frac{2}{3} + \frac{c(1)}{2}$$

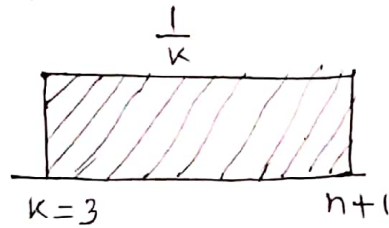
$$= \frac{2}{n+1} + \frac{2}{n} + \frac{2}{n-1} + \dots + \frac{2}{3} + \frac{0}{2}$$

$$\therefore c(1) = 0$$

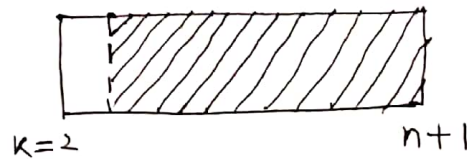
$$= 2 \left(\frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n+1} \right)$$

$$= 2 \sum_{k=3}^{n+1} \frac{1}{k}$$

$$= 2 \int_3^{n+1} \frac{1}{k} \cdot dk$$



$$\leq 2 \int_2^{n+1} \frac{1}{k} \cdot dk$$



$$= 2 \left[\log_e k \right]_2^{n+1}$$

$$= 2 \left[\log_e (n+1) - \log_e 2 \right]$$

$$\therefore c(n) \leq 2(n+1) \left[\log_e (n+1) - \log_e 2 \right]$$

$$c(n) \leq O(n \log_e n)$$