Time Complexity Analysis of Merge sort

```
Procedure MSort (A, l, h) {
    if (h > l) {
        m = ⌊ (l+h)/2 ⌋          ⟶ D
        MSort (A, l, m)          ⟶ T(n/2)
        MSort (A, m+1, h)        ⟶ T(n/2)
        Merge (A, l, m, h)       ⟶ cn
    }
End Procedure
```

T(n)

```
Procedure Merge (A, l, m, h) {
    while loop ⟶ if..else ⟶ c1 n/2
    if..else   ⟶ for loop ⟶ c2 n/2
    for loop   ⟶ c3 n
}
```

Total = $c_1\frac{n}{2} + c_2\frac{n}{2} + c_3 n$

$= \left(\frac{c_1}{2} + \frac{c_2}{2} + c_3\right) n$

$= cn$

Recurrence Relation for Time

$$T(n) = \begin{cases} A & n = 1 \\ 2T\left(\frac{n}{2}\right) + cn + D & n > 1 \text{ and } n = 2^k \end{cases}$$

Solution:

$$T(n) = 2T\left(\frac{n}{2}\right) + cn + D$$

$$= 2\left[2T\left(\frac{n}{2^2}\right) + c\frac{n}{2} + D\right] + cn + D \qquad \left[T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + c\frac{n}{2} + D\right]$$

$$= 2^2 T\left(\frac{n}{2^2}\right) + cn + 2D + cn + D$$

$$= 2^2 T\left(\frac{n}{2^2}\right) + 2cn + 2D + D \qquad \left[T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + c\cdot\frac{n}{2^2} + D\right]$$

$$= 2^2 T\left(\frac{n}{2^2}\right) + 2cn + D\left[2^1 + 1\right]$$

$$= 2^2\left[2T\left(\frac{n}{2^3}\right) + c\cdot\frac{n}{2^2} + D\right] + 2cn + 2D + D$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + cn + 2D + 2cn + 2D + D$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + 3cn + D\left[2^2 + 2 + 1\right]$$

$$\vdots$$

$$= 2^k T\left(\frac{n}{2^k}\right) + kcn + D\left[2^{k-1} + 2^{k-2} + \cdots + 2 + 1\right]$$

$$= 2^k T\left(\frac{n}{2^k}\right) + kcn + D\left[1 + 2 + \cdots + 2^{k-1}\right]$$

$$= 2^k T\left(\frac{n}{2^k}\right) + kcn + D\,\frac{1(2^k - 1)}{(2-1)}$$

$$= 2^k T\left(\frac{n}{2^k}\right) + kcn + D(2^k - 1)$$

$$= nT(1) + cn\log_2 n + D(n-1)$$

$$= An + cn\log_2 n + Dn - D$$

$$= cn\log_2 n + (A+D)n - D$$

$$= O(n\log_2 n)$$

$$a + ar + \cdots + ar^{n-1} = \frac{a(r^n - 1)}{(r-1)}$$

$n = 2^k$

$\Rightarrow \log_2 n = \log_2 2^k$

$\Rightarrow \log_2 n = k\log_2 2$

$\therefore k = \log_2 n$

$T(n) = A$ when $n = 1$

$\Rightarrow T(1) = A$