

DSA-II - L-1 - 23 SEP

Before MID

Time complexity

Divide and Conquer. ($O(n^c)$)

Greedy

D.P. → (Maximization/minimization) Optimization

After MID:

Graph → Traversal

→ Sorting

→ Bipartite

→ MST

→ SSSP

Hashing

String

NFC

Data Structure & Algorithm

Linear

Non - Linear

- ① 1st element define
- ② Last ..
- ③ every element except 1st one must have a previous element
- ④ every element ex. last one must have a next element.

ste set of steps to figure out results from given inputs.

Algorithm analysis steps

- Time complexity*
- space/memory (C.A. complexity. O.S.)
- correctness*
 - ↳ Halt
 - ↳ correct output.
- Readability
- Optimality

Readability (Documentation) \rightarrow S.E.

Exm 1: $n = 100$

1. int sumN (int n) { $\rightarrow 1$

2. int sum = 0 $\rightarrow \textcircled{1}$

3. for(int i = $\textcircled{1}$; i <= n; i++) { $\rightarrow n+1$
 $= 101$

4. sum += 1; $\rightarrow n = 100$

5. } $\textcircled{1}$

6. return sum; $\rightarrow 1$

7. }

$$T(n) = 1 + (n+1) + n+1 \\ = 2n+3 = O(n)$$

~~O \rightarrow asymptotic notation~~
 ~~$\Omega \rightarrow$ best~~

Asymptotic notation

$O \rightarrow$ worst case

$\Omega \rightarrow$ Best case.

$\Theta \rightarrow$ average case.

$$T(n) = 5n^4 + 2n + 3$$

$$n=1 \Rightarrow 5 + 5 = 10$$

$$n=2 \Rightarrow \cancel{80} + 7 = \cancel{27} 87$$

$$n=5 \Rightarrow 3125 + 13 = 3138$$

$$n=10 \Rightarrow 50000 + 23 = 50023$$

$$n=100 \Rightarrow 50000000 + 203 = 500000203$$

$$\therefore T(n) = O(n^4)$$

$$1 < \log_2(n) < \sqrt{n} < n < n \log_2 n < n\sqrt{n}$$

$$< n^2 < \dots < 2^n < n!$$

```
int search(int arr[]; int n; int key){
```

```
    for(int i=0; i<n; i++) { } → O(n+1)
```

```
        if (arr[i] == key) { } → n
```

```
            return i; → 1/0
```

```
        }
```

```
    }
```

```
    return -1; → 0/1
```

```
}
```

2	5	0	7	8	3	1
---	---	---	---	---	---	---

Best case: $O(1)$

worst case: $O(n)$

Best $T(n) = n + 1 + n + 1$ worst $T(n) = 1 + 1 + 1 = 3$

$$\approx 2n + 2$$

$$= \Omega(n)$$

$$= O(n)$$

$$= \Omega(1)$$

Exm: 3

```
1 int function(int n){  
2     int wordSum = 0; → 1  
3     for(int i=1; i<=n; i++) { → n+1  
4         wordSum += i; → n  
5     }  
6     if(wordSum % 2 == 0) → 1  
7     return wordSum; → 1  
8 } → 0 or 1  
9 for(int i=1; i<=n; i++) { → n+1  
10    wordSum *= i; → n  
11 } → 0 or 1  
12 return wordSum; → 0 or 1  
13 }
```

worst case:

$$\begin{aligned}T(n) &= 1 + n + 1 + n + 1 + n + 1 \\&= \Theta(n+5) \\&= O(n)\end{aligned}$$

Best case:

$$\begin{aligned}T(n) &= 1 + n + 1 + n + 1 + 1 \\&= 2n + 4 \\&= \Theta(n^2)\end{aligned}$$

~~Upper bound~~

Tight " "
Lower "

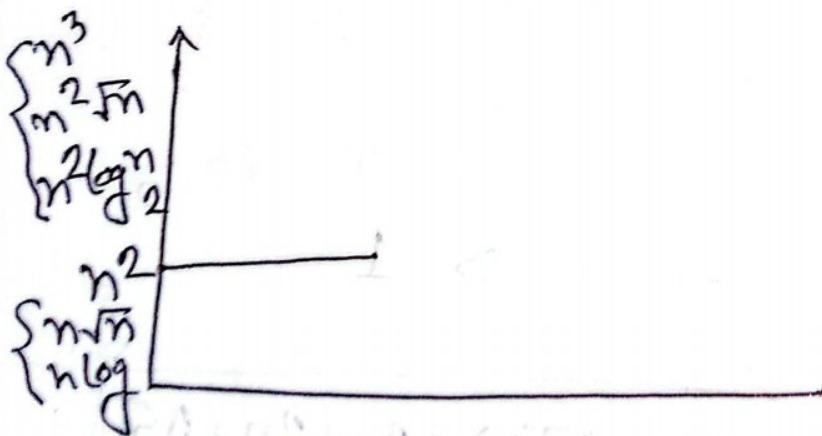
$$T(n) = 2n^5 + 3n^2 + 9n + 2$$

$$= 2n^5 + 3n^2 + 9n + 2 \leq (2+3+9)n^5 + 2 \leq 16n^5$$

for all n , $2n^5 + 3n^2 + 9n + 2 \leq 16n^5$

$O(n^5)$

$$T(n) = 2n^2 + 3n + 5 = O(n^2)$$



average case
close to
worst case

$$LB \leq TB \leq UB$$

Exm 9:

W

B

1. int function(int n; int m) {
2. int sum = 0;
3. for(int i=1; i<=n; i++) { → n+1
4. sum += i; n
5. } n
6. if (sum > 2) 1
7. return sum; 0
8. else { 0
9. for(int i=1; i<=m; i++) { m+1
10. sum += i; m
11. } m
12. } 0
13. return sum; 1
14. } 0

$$T(n, m) = 2m + 2n + 85 \\ = O(m+n)$$

$$T(n) = 2n + 85 \\ = \Theta(n)$$

$$\# T(n, m, 0) = 3n^5 + 4n^3 + 9n^6 + 3m^6 + 3n^7$$

exact $+ 8m + 500^3 + 80^2 + 0$

time
function.
 $= O(n^7 + m^6 + 0^3)$

Exm 5:

```
int func(int n){
```

 int prod = 1. $\longrightarrow 1$

 for(int i=1; i<=n; i+=2){ } $\rightarrow \frac{n}{2} + 1$

 prod + 2^i; $\longrightarrow \frac{n}{2}$

}

 return prod; $\longrightarrow 1$

}

$$T(n) = 1 + \frac{n}{2} + 1 + \frac{n}{2} + 1$$

$$= 2 + \frac{n}{2} + 3$$

$$= n + 3 = O(n).$$

題

for(int i=1; i≤n; i++) {
 ^{i=i+k}
 P Statement.
}

for(int i=n; i>=1; i=i-k) {
 ^{i=i-k}
 P Statement
}

Exm 6:

int &function(int n) {

 int prod = 0;
 → 1

 for(int i=n; i>=1; i=i-4) {
 ⁱ⁼ⁱ⁻⁴
 prod += i;
 }

ⁱ⁼ⁱ⁻⁴
 prod += i;
 → $\frac{n}{4} + 1$

 return prod;
}

?

$$T(n) = \frac{n}{2} + \frac{n}{4} + 1 + 1 + 1$$

$$= 2 \cdot \frac{n}{4} + 3 = \frac{n}{2} + 3,$$

$$= O(n)$$

Binary Search Time Complexity.

$$n \quad \frac{n}{2} \quad \frac{n}{4} \quad \frac{n}{8} \quad \frac{n}{16} \dots 1$$

$$\frac{n}{2^0} \quad \frac{n}{2^1} \quad \frac{n}{2^2} \quad \frac{n}{2^3} \quad \frac{n}{2^4} \dots \frac{n}{2^i}$$

$$\frac{n}{2^i} = 1$$

$$\Rightarrow n = 2^i$$

$$\Rightarrow \log_2 n = \log_2 2^i$$

$$\therefore i = \log_2 n.$$

Exm 7

```
int func(int n){
```

```
    int sum = 0;           → 1
```

```
    for(int i=1; i<=n; i=i*2) { → log2(n)+1
```

```
        sum += i; → log2(n)
```

```
    return sum; → 1
```

```
}
```

$$T(n) = 1 + \log_2(n) + 1 + \log_2(n) + 1$$

$$= 2\log_2(n) + 2$$

$$= O(\log_2(n))$$

i = 1	2	4	8	16	...	n
2^0	2^1	2^2	2^3	2^4		$2^{\frac{n}{2}}$

$$2^x = n$$

$$x = \log_2 n.$$

Ques

for(int i=1; i<=n; i*=k) $\rightarrow \log_k(n) + 1$
 if statement. $\rightarrow \log_k(n)$

for(int i=n; i>=1; i/=k) $\rightarrow \log_k(n) + 1$
 if statement. $\rightarrow \log_k(n)$

Exm 8:

```
int func(int n){  
    int sum = 0;  
    int k = n;  
    for(int i =  
        while(k >= 1) {  $\rightarrow \log_{15}(n) + 1$   
            sum += k;  $\rightarrow \log_5(n)$   
            k = k/5;  $\rightarrow \log_5(n)$   
        }  
    return sum;  $\rightarrow 1$ 
```

$$T(n) = 1 + 1 + \log_5(n) + 1 + \log_5^{(n)} \\ + \log_5(n) + 1$$

$$= 3\log_5(n) + 4$$

$$= O(\log_5^{(n)})$$

~~400~~

DSA II - L-3-30 Sep

Exm 9:

bool is_prime(int n) {

 for (int i = 2; i <= n; i++) { } $\rightarrow \sqrt{n}$

 if (n % i == 0) : $\rightarrow \sqrt{n} - 1$

 return false; $\rightarrow O(1)$

}

 return true; $\rightarrow O(1)$

}

~~$$T(n) = \frac{\sqrt{n} + \sqrt{n} + 1 + 1}{2} = 2\sqrt{n} + 2 = O(\sqrt{n})$$~~

$$\sqrt{n} + \sqrt{n} - 1 + 1 = 2\sqrt{n} = O(\sqrt{n})$$

Ans

~~$$T(n) = 1 + 1 + 1 = O(1)$$~~

$$T(n) = 1 + 1 + 1 = O(1) \rightarrow O(1) = O(1)$$

Exm - 10

void func1(int n) {
 for (int i = 1; i <= n; i++) {
 func2();
 }
}

B W

func() { Best case: $\Omega(1)$
 worst case: $O(\log_2 n)$ }

worst $\rightarrow T(n) = n + 1 + (\log_2 n) n$.

$$\begin{aligned} &= n(\log_2 n + 1) + 1 \\ &= O(n \log_2 n) \end{aligned}$$

Best $\rightarrow T(n) = n + 1 + n = \Omega(n)$

Exm 11

int m,
void func(int n){
int sum = 0;
for(int i = 1; i <= n; i++) { } → n+1
for(int j = 1; j <= m; j++) { } → n(m+1)
sum += i+j; n × m
sum += i → n
}

$$T(n) = n+1 + nm + n + n \times m + n$$
$$= O(n \times m)$$

Exm 12:

```
void function(int n){
```

```
    for(int i=n; i>=1; i--) { } n+1
```

```
        for(int j=2; j<=n; j= j+1 ) { } n(log3 n)
```

print

$\rightarrow n(\log_3 n - 1)$

print

$\rightarrow n$

$$T(n) = n+1 + n(\log_3 n) + n(\log_3 n - 1) + n$$

$$= O(n \log_3 n)$$

$$\# f(n) = \frac{n(n+1)}{2}$$

$$f(n+2) = \frac{(n+2)(n+2+1)}{2}$$

$$= \frac{(n+2)(n+3)}{2}$$

$$\# 5 + 6 + 7 + \dots + (n+2)$$

$$\frac{(n+2)(n+3)}{2} - \frac{4(4+1)}{2}$$

Exm 13

```
void function(int n) {
    sum = 0;
    for(int i=1; i<=n; i++) {
        for(j=i; j<=n; j++)
            sum += j;
    }
}
```

$$\text{stat. } \rightarrow \frac{n(n+1)}{2}$$

}

st. $\rightarrow n$

return - $\rightarrow 1$

$$\left\{ T(n) = 1 + n + \frac{(n+1)(n+2)}{2} - 1 + \frac{n(n+1)}{2} + n + 1 \right.$$

$$= O(n^2)$$

DSA-2-L-9-30 ct

exm 14:

Best

worst

int function(int n, int m){

 int sum = 1;

 for(int i=0; i<n; i++) {

 for(int j=0; j<~~n~~=i; j++) {

 print

 sum+=i+j;

}

 if(sum % 2 == 0) {

 for(j=1; j<~~n~~*j <= m; j++) {

 sum

}

}

 return sum;

 → 1

}

$T(n)$

worst $T(n, m) = 1 + n + \frac{(n+1)(n+2)}{2} - 1 + \frac{n(n+1)}{2}$
 $+ \frac{n(n+1)}{2} + n + (\sqrt{m} + 1)n + n\sqrt{m}$

$$= O(n^2 + n\sqrt{m})$$

Best: $T(n) = \Omega(n^2)$

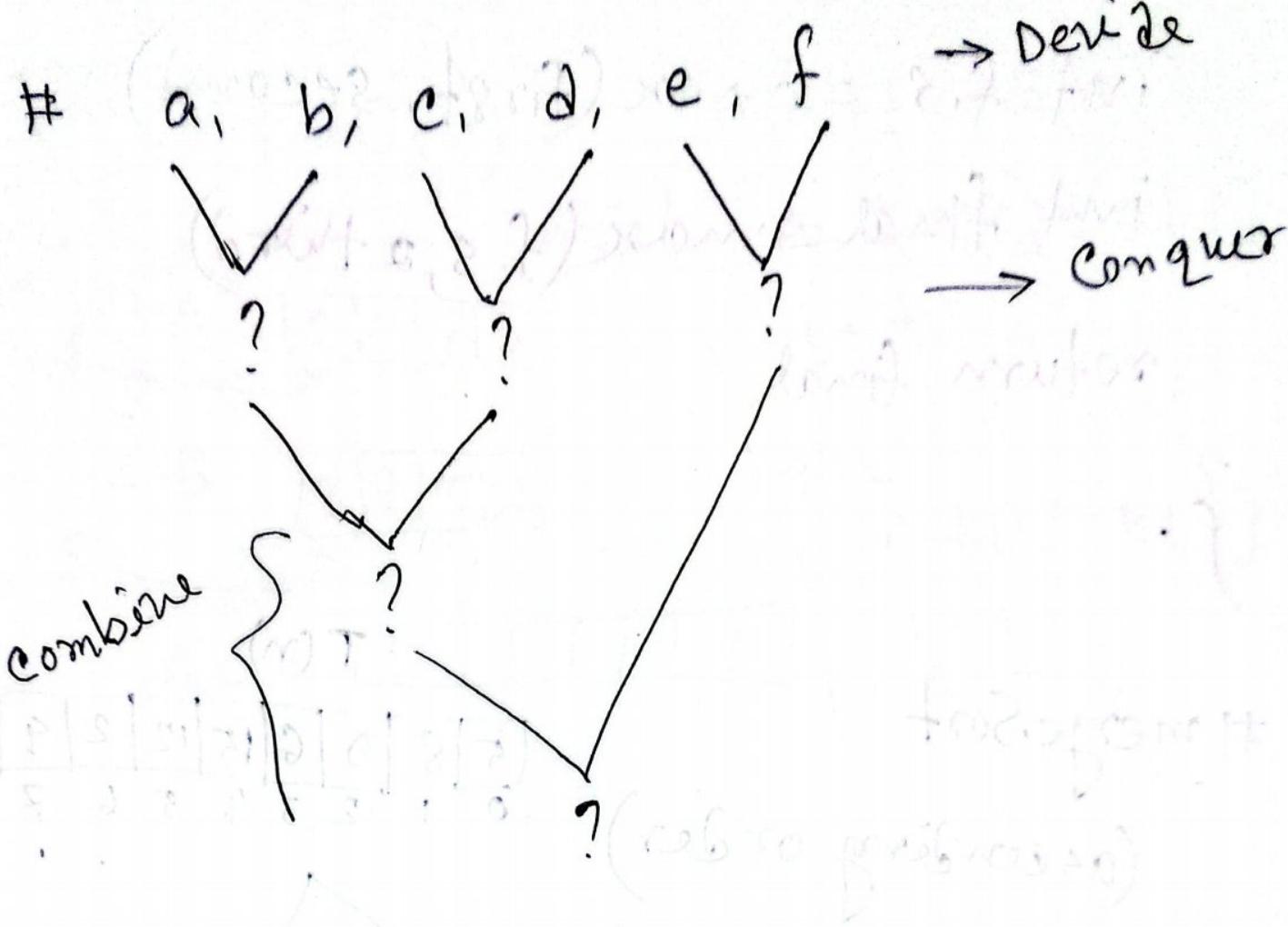
↗ Divide & Conquer (DnC)

3 Steps

↳ Divide

↳ Conquer

↳ Combine (Merge)



```

int max(int a, int b){
    if (a >= b) return a
    return b
}

int max6(a, b, c, d, e, f){
    int first = max(a, b)
    int second = max(c, d)
    int third = max(e, f)
}

```

```
int f.s = max(first, second);
```

```
int final = max(f.s, a.third)
```

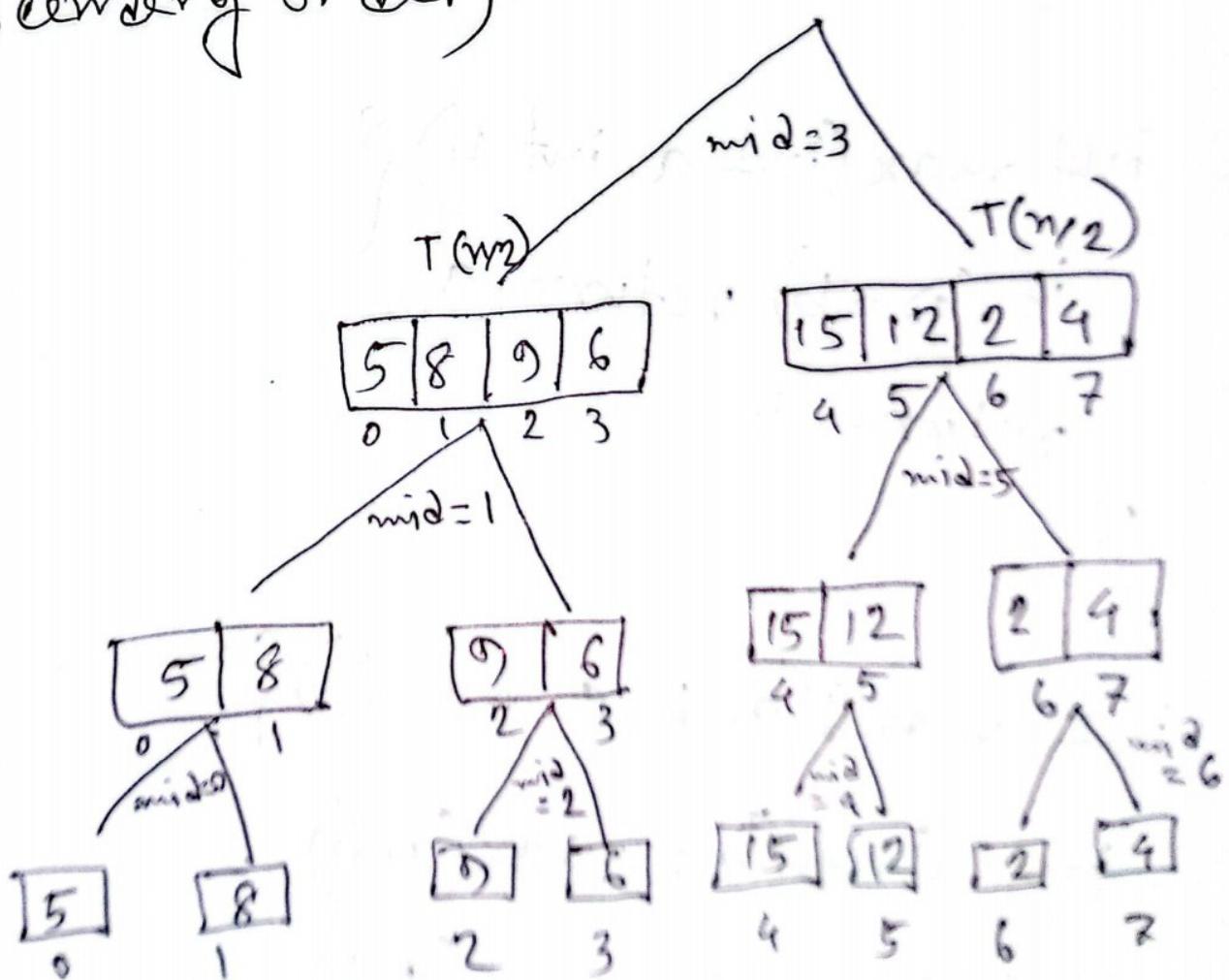
```
return final
```

```
}
```

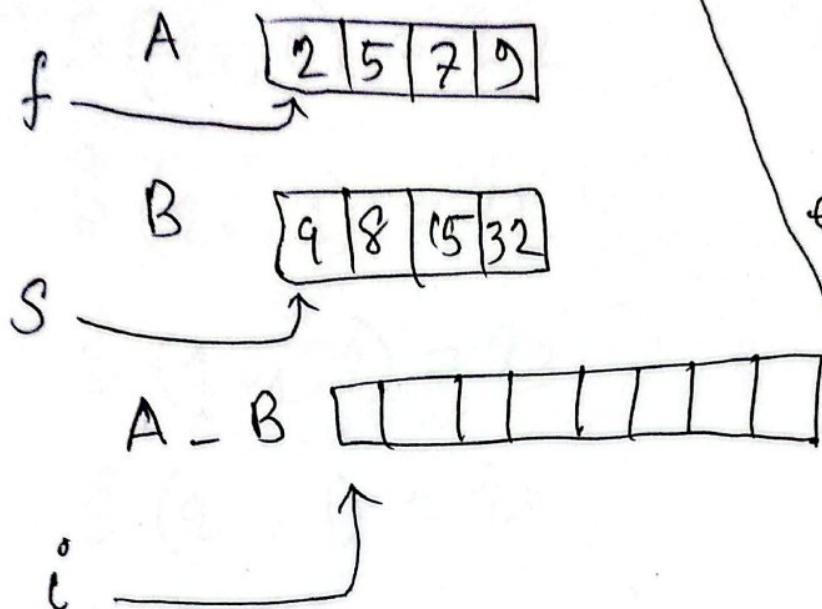
```
#mergeSort
```

```
(ascending order)
```

T(n)							
5	8	9	6	15	12	2	4



merge = $O(n)$



$\left\{ \begin{array}{l} \text{if } (A[f] \leq B[s]) \\ \quad A - B[i] = A[f] \\ \quad i++ \\ \quad f++ \\ \text{else} \\ \quad A - B[i] = B[s] \\ \quad i++ \\ \quad s++ \end{array} \right.$

$$\begin{aligned}
 T(n) &= T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + n \\
 &= 2T\left(\frac{n}{2}\right) + n
 \end{aligned}$$

$$T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + n & \text{when } n > 1 \\ 1 & \text{when } n = 1 \end{cases}$$

→ conquer time complexity.

DSA II - L-5 - 7 Oct

Min Max Problem

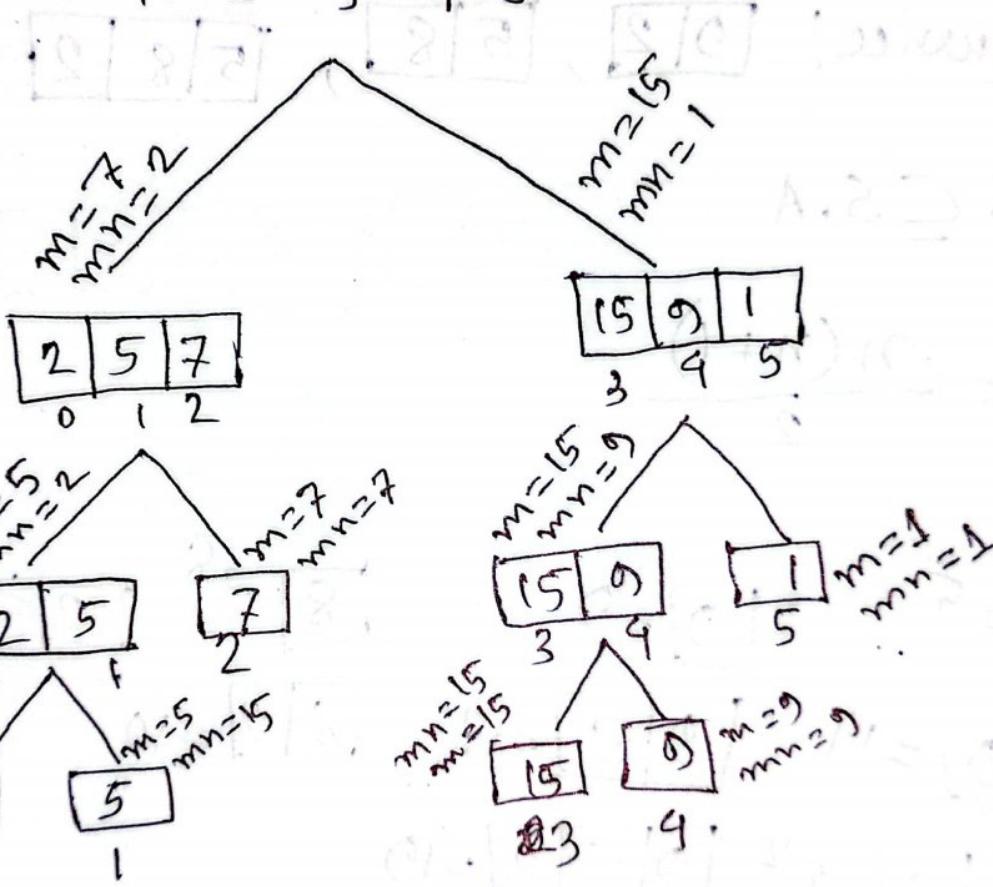
using DnC approach.

$m = \max$
 $mn = \min$

arr.

2	5	7	15	9	1
0	1	2	3	4	5

$$m = 15 \\ mn = 1$$



$$\begin{aligned} T(n) &= T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + C \\ &= 2T\left(\frac{n}{2}\right) + C \end{aligned}$$

$$\therefore T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + C ; n > 1 \\ 1 ; n = 1 \end{cases}$$

#Maximum Sum Subarray ($O(n^2)$)

$\boxed{5 \ 9 \ 8 \ 2}$

Subarray: $\boxed{5 \ 9}$, $\boxed{8 \ 2}$, $\boxed{5 \ 9 \ 8}$

~~Subsequence~~ vs

Subsequence: $\boxed{9 \ 2}$, $\boxed{5 \ 8}$, $\boxed{5 \ 8 \ 2}$

S.A. ~~AS~~ \subseteq S.A

$$S.A. = \frac{n(n+1)}{2}$$

$$\boxed{5} = 5$$

$$\boxed{9} = 9$$

$$\boxed{8} = 8 \quad \boxed{2} = 2$$

$$\boxed{5 \ 9} = 14$$

$$\boxed{9 \ 8} = 17$$

$$\boxed{8 \ 2} = 10$$

$$\boxed{5 \ 9 \ 8} = 22$$

$$\boxed{9 \ 8 \ 2} = 19$$

$$\boxed{5 \ 9 \ 8 \ 2} = 29$$

4

3

2

1

Kadane's Algo : $O(n)$

#	-10	17	-15	50
	0	1	2	3

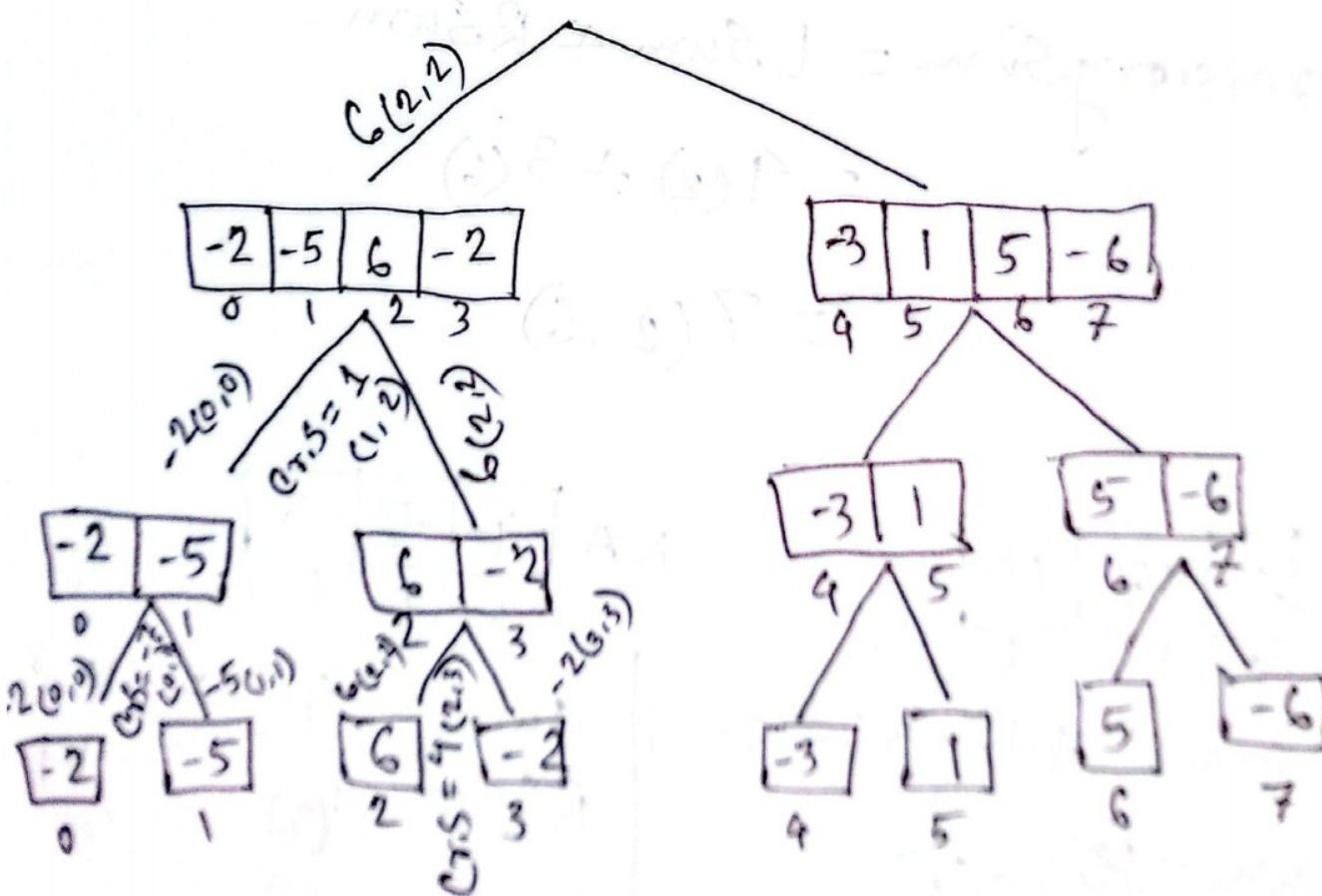
$$= 5^2$$

Maximum Sum Subarray =

17	-15	50
1	2	3

$O(n^3) \xrightarrow[\text{prefix sum}]{\text{opt}} O(n^2) \xrightarrow{\text{Div C}} O(n \log n)$

#	-2	5	6	-2	-3	1	5	-6
	0	1	2	3	4	5	6	7



Crossing Sum

LA	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>-2</td><td>-5</td><td>6</td><td>-2</td></tr> <tr><td>0</td><td>1</td><td>2</td><td>3</td></tr> </table>	-2	-5	6	-2	0	1	2	3
-2	-5	6	-2						
0	1	2	3						
	$\xleftarrow{\hspace{1cm}}$								

RA	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>3</td><td>1</td><td>5</td><td>-6</td></tr> <tr><td>4</td><td>5</td><td>6</td><td>7</td></tr> </table>	3	1	5	-6	4	5	6	7
3	1	5	-6						
4	5	6	7						
	$\xrightarrow{\hspace{1cm}}$								

$$\text{Sum} = -2 + 6 = 4 + (-5)$$

$$\text{L Sum} = \cancel{-2} 4 \underset{(2)}{=} -1 + (-2) \underset{(2)}{=} -3$$

$$\text{Sum} = -3 + 1 = \cancel{0} - 2 + 5 = 3 - 6 = -3$$

$$\text{R Sum} = \cancel{-3} \cancel{-2} 3 \underset{(6)}{=} 3$$

$$\text{Crossing Sum} = \text{L Sum} + \text{R Sum}$$

$$= 4(2) + 3(6)$$

$$= 7(2, 6)$$

LA	#	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>-15</td><td>17</td><td>19</td></tr> <tr><td>0</td><td>1</td><td>2</td></tr> </table>	-15	17	19	0	1	2
-15	17	19						
0	1	2						

RA	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>-4</td><td>-7</td><td>-3</td></tr> <tr><td>3</td><td>4</td><td>5</td></tr> </table>	-4	-7	-3	3	4	5
-4	-7	-3					
3	4	5					

$$\text{Sum} = 19 + 17 = 36 - 15 = 21$$

$$\text{L Sum} = \cancel{19} 36(1)$$

$$\text{Sum} = -4 - 7 = -11 - 3$$

$$\text{R Sum} = -4 \underset{(3)}{=} -12$$

$$\begin{aligned}\therefore \text{Cr. Sum} &= 36(1) + (-4(3)) \\ &= 32(1, 3)\end{aligned}$$

DSA II - H.W. (L-6-10 Out)

-2	-5	6	-2	-3	1	5	-6
0	1	2	3	4	5	6	7

-2	-5	6	-2
----	----	---	----

3	1	5	-6
4	5	6	7

6(5,6)

1(5,5)

5(6,6)

-3	1
4	5

Cr.S = -2
(4,5)

1(5,5)

1

5

5	-6
6	7

Cr.S = -1
(6,7)

-6

7

$$\text{Sum} = 1 + (-3) = -2$$

$$\rightarrow L.\text{Sum} = 1(5,5) = \text{cr}$$

$$\rightarrow \text{Sum} = 5 + (-6) = -1$$

$$R.\text{Sum} = 5(6,6)$$

$$(L.R.) \text{Cr.Sum} = L.\text{Sum} + R.\text{Sum}$$

$$1 \approx 6(5,6)$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + n$$

$$= 2T\left(\frac{n}{2}\right) + n$$

Merge sort:

$$T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + n; & n > 1 \\ 1; & n = 1 \end{cases}$$

MinMax Problem:

$$T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + 1; & n > 1 \\ 1; & n = 1 \end{cases}$$

MaximumSubArraySum:

$$T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + n; & n > 1 \\ 1; & n = 1 \end{cases}$$

BinarySearch:

$$T(n) = \begin{cases} T\left(\frac{n}{2}\right) + 1; & n > 1 \\ 1; & n = 1 \end{cases}$$

$\rightarrow T(n)$
 \downarrow
 $T(n_2)$
 \downarrow
 $T(n_4)$
 \downarrow
 X
 $A-B$

Recursion solving

→ Substitution

→ Master theorem

→ Recursion tree

Recursion tree:

exm 1:

$$T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + n; & n > 1 \\ , & n = 1 \end{cases}$$

$$\text{Divide } = \frac{n}{a} = \frac{n}{2}$$

$$\text{No. of child (each node)} = b = 2$$

$$\text{combine complexity} = \Theta(n)$$

$$\text{conquer } n = 1$$

$$\text{No. of leaf node} = b^{\log_a n} = 2^{\log_2 n}$$

$$T(n) = \text{internal layer complexity} = n \log_{\frac{n}{2}} n + n \\ + \text{leaf layer } n = O(n \log_{\frac{n}{2}} n)$$

$$\text{leaf layer complexity} = (\text{No. of leaf node} \times \text{conquer complexity})$$

internal layers complexity:

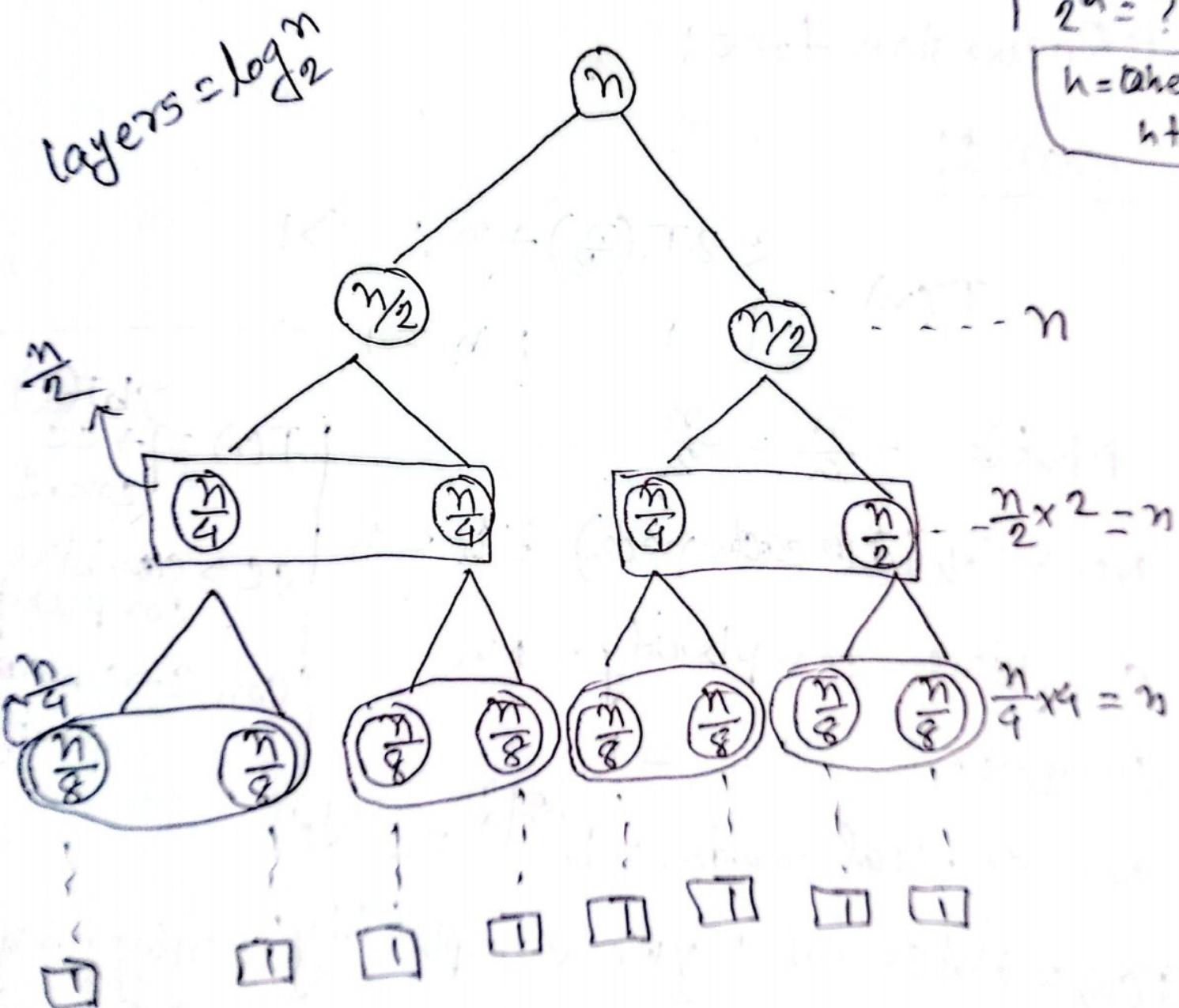
$$(\log_2 n) \times n = n \log_2 n$$

$$\rightarrow \text{leaf layer complexity} = 2^{\log_2 n} \times 1 \\ = n^{\log_2 2} \\ = n$$

$$\left| \begin{array}{l} 2^0 = 1 \\ 2^1 = 2 \\ 2^2 = 4 \\ 2^3 = 8 \\ \vdots \\ 2^{d+1} = ? \end{array} \right.$$

$$h = \Theta(\log n)$$

Tree:



$$a^{\log_b k} = b^{\log_a k}$$

$$\text{exam-2} \\ \# \text{ } T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + 1 & ; n > 1 \\ 1 & ; n = 1 \end{cases}$$

$$\text{Divide} = \frac{n}{a} = \frac{n}{2}$$

$$\text{No. of child (each node)} = b = 2$$

$$\text{combine complexity} = 1$$

$$\text{conquer } n = 1$$

$$\text{No. of leaf node} = b^{\log_a n} = 2^{\log_2 n} = n$$

internal layers complexity:

$$\cdot c + 2c + 4c + 8c + 16c + \dots \\ = c(1 + 2 + 4 + 8 + 16 + \dots)$$

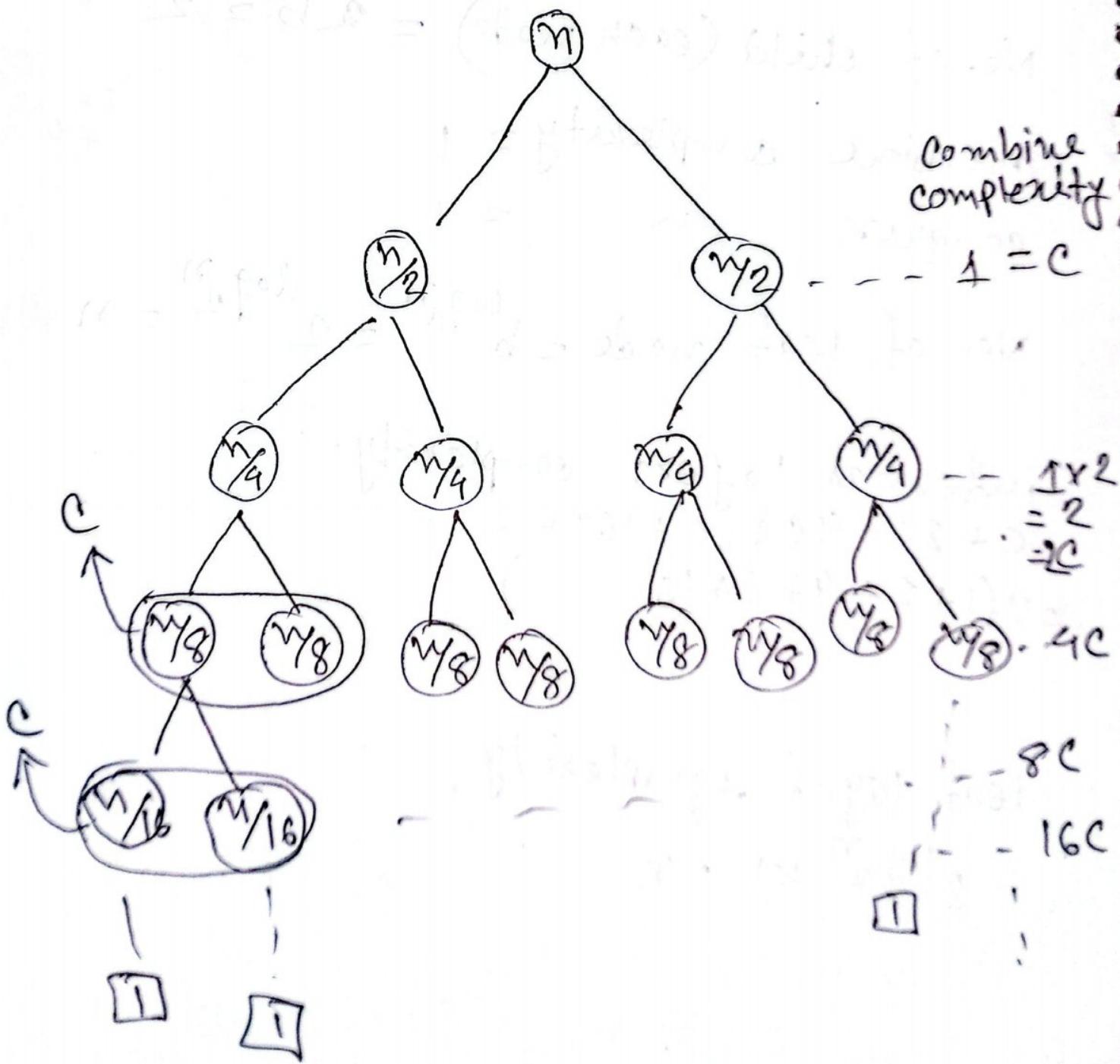
leaf layer complexity:

$$2^{\log_2 n} \times 1 = n$$

$T(n)$ = internal layer complexity
+ leaf layer complexity

$$= C(1 + 2 + 4 + 8 + \dots) + n$$

$$= O(n)$$



Exm: 3

$$T(n) = \begin{cases} T\left(\frac{n}{2}\right) + 1 & ; n > 1 \\ 1 & ; n = 1 \end{cases}$$

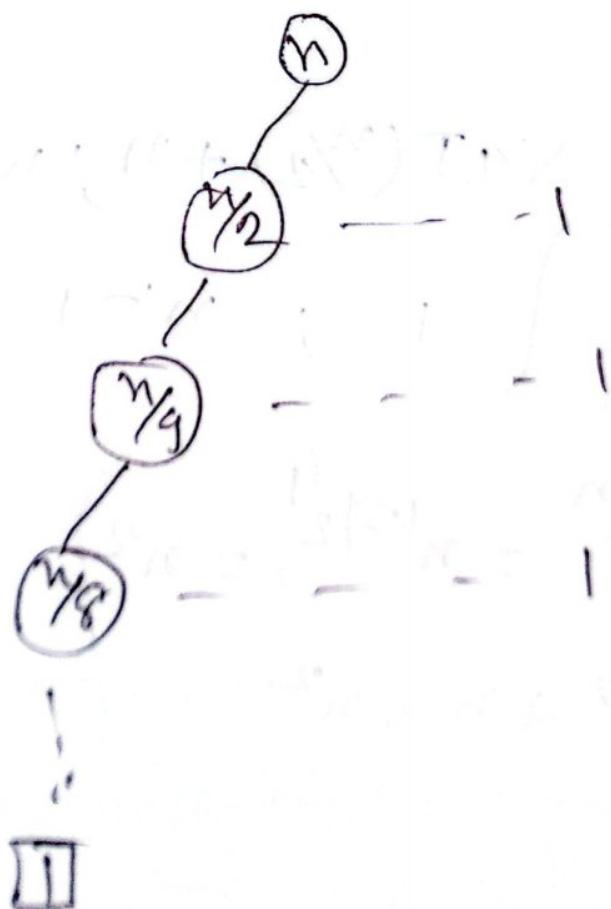
$$\text{Divide} = \frac{n}{a} = \frac{n}{2}$$

$$\text{No. of child (each node)} = b = 1$$

$$\text{combine complexity} = 1$$

$$\text{conquer } n = 1$$

$$\text{no. of leaf node} = b^{\log_a n} = n^{\log_2 1} = n^0 = 1$$



internal layers complexity?

$$\log_2 n \times 1 = \log_2 n$$

leaf layers

$$1^{\log_2 n} \times 1 = 1$$

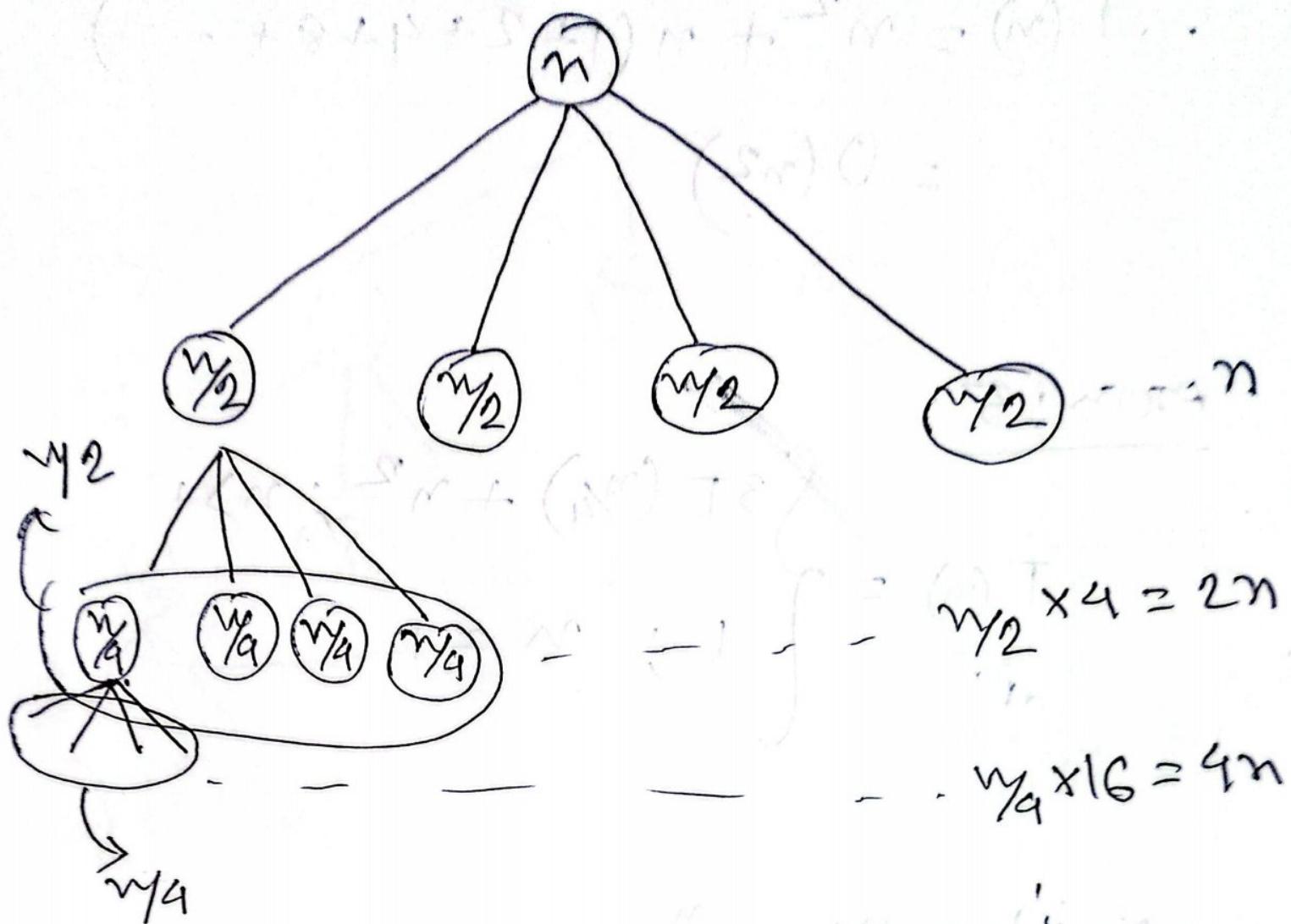
$$T(n) = \log_2 n + 1 = O(\log_2 n)$$

exm - 9

$$T(n) = \begin{cases} 4T(n/2) + n; & n \\ 1; & n=1 \end{cases}$$

$$4^{\log_2 n} = n^{\log_2 4} = n^2$$

$$n^2 + n = n^2$$



internal layers complexity:

$$n + 2n + 4n + 8n + \dots$$

$$n(1 + 2 + 4 + 8 + \dots)$$

leaf layer complexity:

$$n^2 \times 1 = n^2$$

$$\therefore T(n) = n^2 + n(1+2+4+8+\dots)$$

$$= O(n^2)$$

exm: 5:

$$T(n) = \begin{cases} 3T\left(\frac{n}{4}\right) + n^2; & n > 1 \\ 1; & n = 1 \end{cases}$$

a Divide = $\frac{n}{a} = \frac{n}{4}$

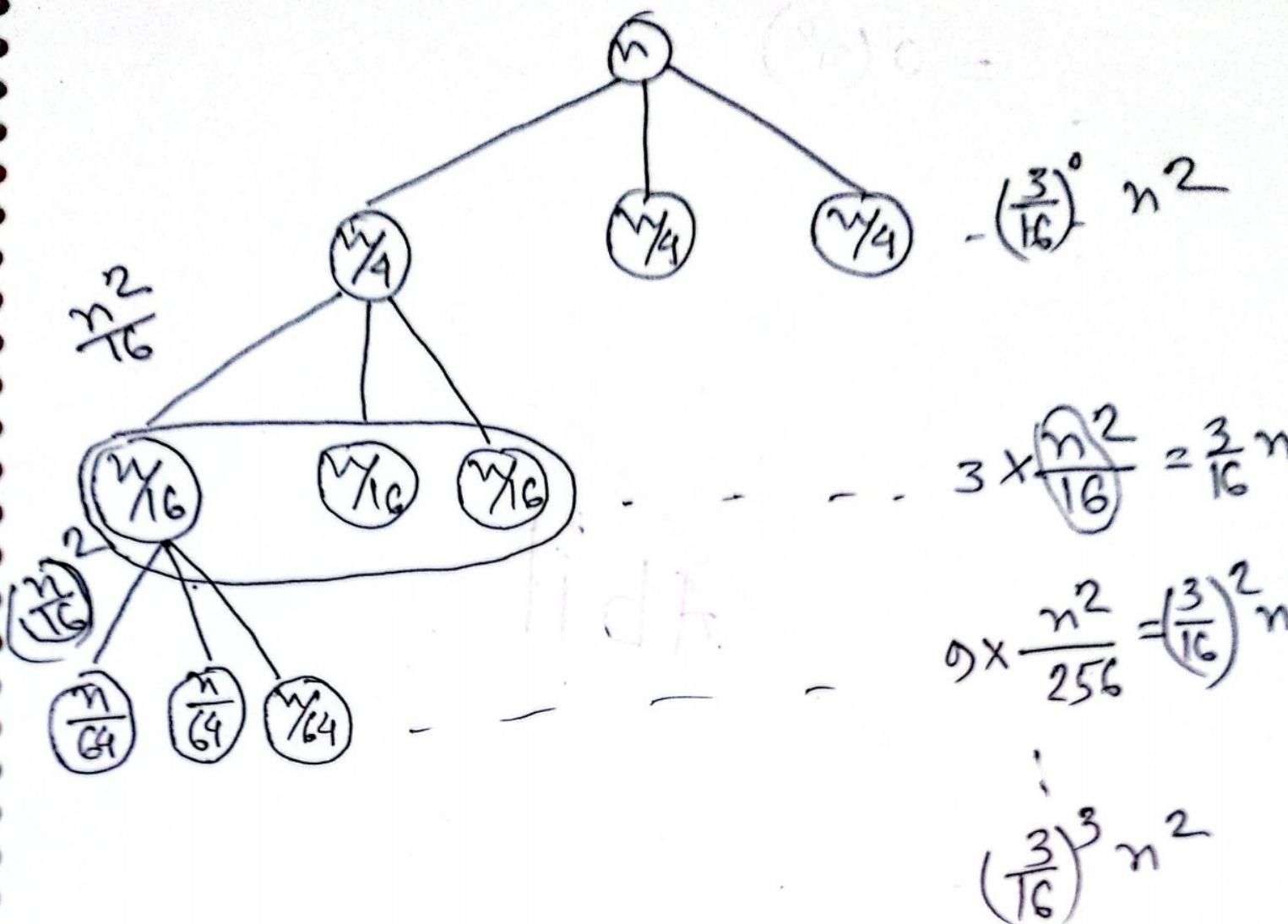
no. of child = b = 3

combine com. = n^2

conquer $n = 1$

No. of leaf node = $3^{\log_4 n} = n^{\log_4 3} = n^{0.79}$

tree:



internal layer comp.

$$n^2 + \frac{3}{16} n^2 + \left(\frac{3}{16}\right)^2 n^2 + \dots$$

$$= n^2 \times c = cn^2$$

leaf :

$$n^{0.79} \times 1 = n^{0.79}$$

$$T(n) = Cn^2 + n^{0.79}$$
$$= O(n^2)$$

DSA II - L-8 - 16 oct

Greedy Approach / Strategy

Coin change

Ex 1

Change Amount = 52727

bill = 47273

pay = 1000.00

coin / notes = $\left[\frac{1000}{100}, 50, 20, 10, 5, 2, 1 \right]$

minimum number of coins = 57

Exm- 2

Amount = 30

coins = [25, 10, 1]

Current amount = 30

$$\begin{array}{rcl} 25 \times 1 & = & 25 \\ \hline & n & = 5 \\ 1 \times 5 & = & 5 \end{array}$$

"

"

= 0

Total coins = 6

minimization ~~maximization~~

→ Greedy

optimal ~~not~~ times

Greedy, 2.7 ~~not~~ ~~optimal~~ worst solution
at target

amount = 127

coins = [50, 1, 20, 5, 10, 17]

→ sorted coins descending order

= [50, 20, 17, 10, 5, 1]

C.A. = 127

$50 \times 2 = 100$
 $\underline{R.A. = 27}$

C.A = Current Amount
R.A = Remaining

u

$$\begin{array}{r}
 20 \times 1 = 20 \\
 \hline
 R.A. = 7 \\
 \\
 5 \times 1 = 5 \\
 \hline
 R.A. = 2 \\
 \\
 2 \times 1 = 2 \\
 \hline
 0
 \end{array}$$

Total coins \geq

optimal $SOL^M = 9$

$$\begin{aligned}
 & [4, 9 \times 2.7] \quad ; \quad 4 \leq 6 \leq 10.8 \\
 & = [4, 10.8]
 \end{aligned}$$

Activity Selection (maximization)

always optimal solution $O(n^2)$

Greedy $O(2)$ ex. case - 1.

Activity	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
Start	1	0	1	4	2	5	3	04
Finish	3	4	2	6	9	8	5	8

→ Sort the tasks based on -

- Start time

- Finish " * (optimal solution)

- Duration

	S	F
T_1	1	8
T_2	4	7
T_3	2	9
T_4	3	0

sort → start

	S	F
T_1	1	8
T_3	2	4
T_4	3	0
T_2	4	7

sort → Finish

	S	F
T ₃	2	4

	S	F
T ₂	4	7

8

3 9

T₉

sort → Finish time

Activity	a ₃	a ₁	a ₂	a ₇	a ₄	a ₈	a ₆	a ₅
start	0	0	3	4	9	9	5	2
Finish	2	3	4	5	6	8	8	9

$$\text{picked} = 1 + 1 + 2 + 1 + 2 = 7$$

tasks = [a₃, a₇, a₆]

availability = 12 / 8

```
struct Task {
```

```
    int start;
```

```
    int finish;
```

```
};
```

```
int activitySelection (Task tasks[], int n) {
```

```
    sort (tasks); // based on finish time
```

```
    int picked = 1
```

```
    int availability = tasks[0].finish
```

```
    for (int i=1; i < n; i++) {
```

```
        if (tasks[i].start >= availability) {
```

```
            picked++;
```

```
            availability = tasks[i].finish;
```

```
}
```

```
    return picked;
```

```
}
```

Sort - এর optimal time complexity

$$O(n \log n)$$

DSA II - 1 - 10 - 21 Oct

Greedy -

knapsack

↳ 0/1 knapsack

↳ fractional "

~~0/1 knapsack~~

<u>Product</u>	<u>weight</u>	<u>value</u>	<u>value/weight</u>
1 $\leftarrow P_1$	2	10	5
5 $\leftarrow P_2$	5	15	3
4 $\leftarrow P_3$	0)	30	3.33
3 $\leftarrow P_4$	6	20	3.33
2 $\leftarrow P_5$	4	18	4.55

Capacity = 10 kg

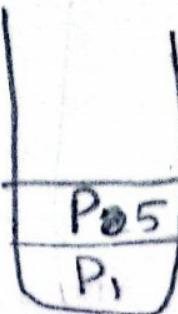
#



$$C = 10 \text{ kg}$$



$$C = 8 \text{ kg}$$



$$C = 9 \text{ kg}$$

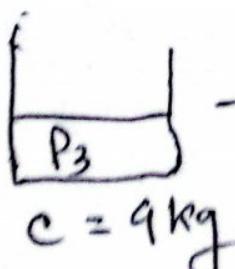
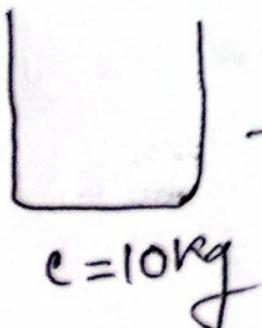
$$w: 2 + 4 = 6 \text{ kg}$$

$$v: 10 + 18 = 28 \text{ $}$$

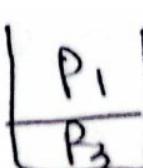
#

<u>Product</u>	<u>weight</u>	<u>value</u>	<u>value/weight</u>
$3 \leftarrow P_1$	4	20	5
$2 \leftarrow P_2$	5	30	6
$1 \leftarrow P_3$	6	40	6.67

$$\text{capacity} = 10 \text{ kg}$$



$$C = 9 \text{ kg}$$



$$C = 0 \text{ kg}$$

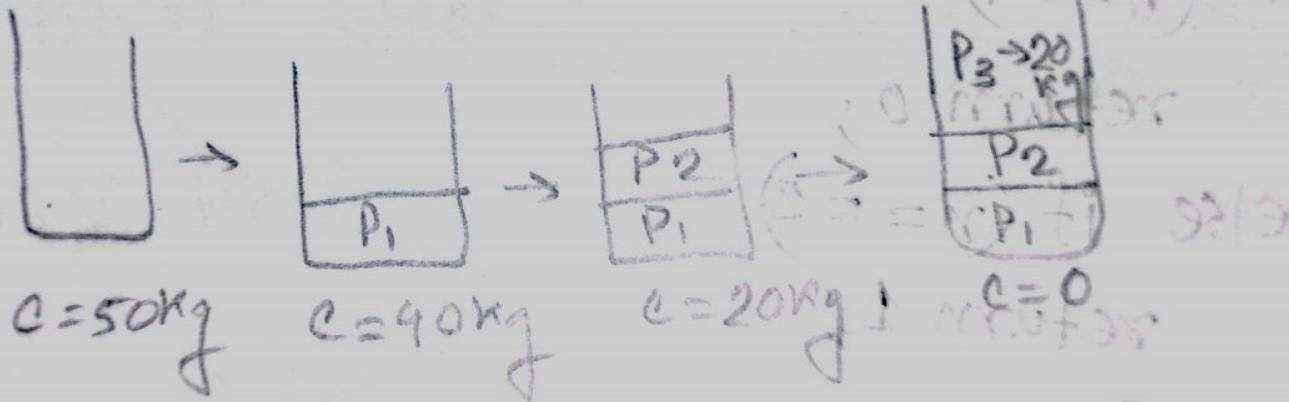
$$\therefore w: 6 + 4 = 10 \text{ kg}$$

$$v: 90 + 20 = 60 \text{ $}$$

fractional knapsack

<u>Product</u>	<u>weight</u>	<u>value</u>	<u>v/w</u>
$1 \leftarrow P_1$	10	60	6
$2 \leftarrow P_2$	20	100	5
$3 \leftarrow P_3$	30	120	4

capacity = 50 kg.



$$\therefore w: 10 + 20 + 20 = 50\text{kg}$$

$$v: 60 + 100 + 80 = 240\text{\$}$$

*Greedy never fails in fractional knapsack.

Dynamic Programming

fibonacci series:

0	1	1	2	3	5	8	13	21
1st	2nd	3rd	4th	5th	6th	7th	8th	9th
34	-	-	-	-	-	-	-	-

```
int fib(int n) {
```

```
    if (n == 1)
```

```
        return 0;
```

```
    else if (n == 2)
```

```
        return 1;
```

```
    else {
```

```
        int fwd1 = fib(n - 1);
```

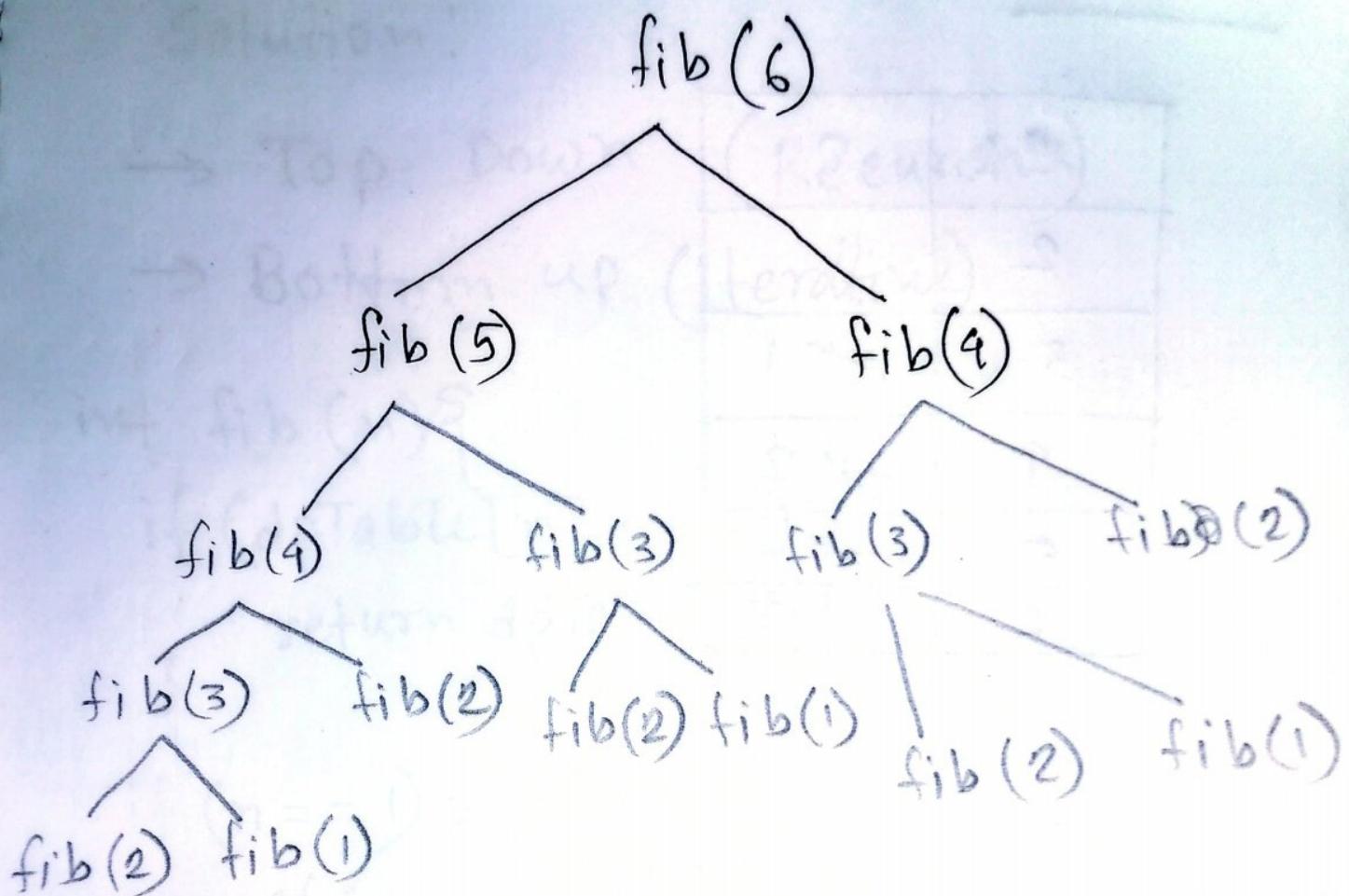
```
        int fwd2 = fib(n - 2);
```

```
        int result = fwd1 + fwd2;
```

```
        return result;
```

3 } 3

if $n = 6$



$$O(2^n)$$

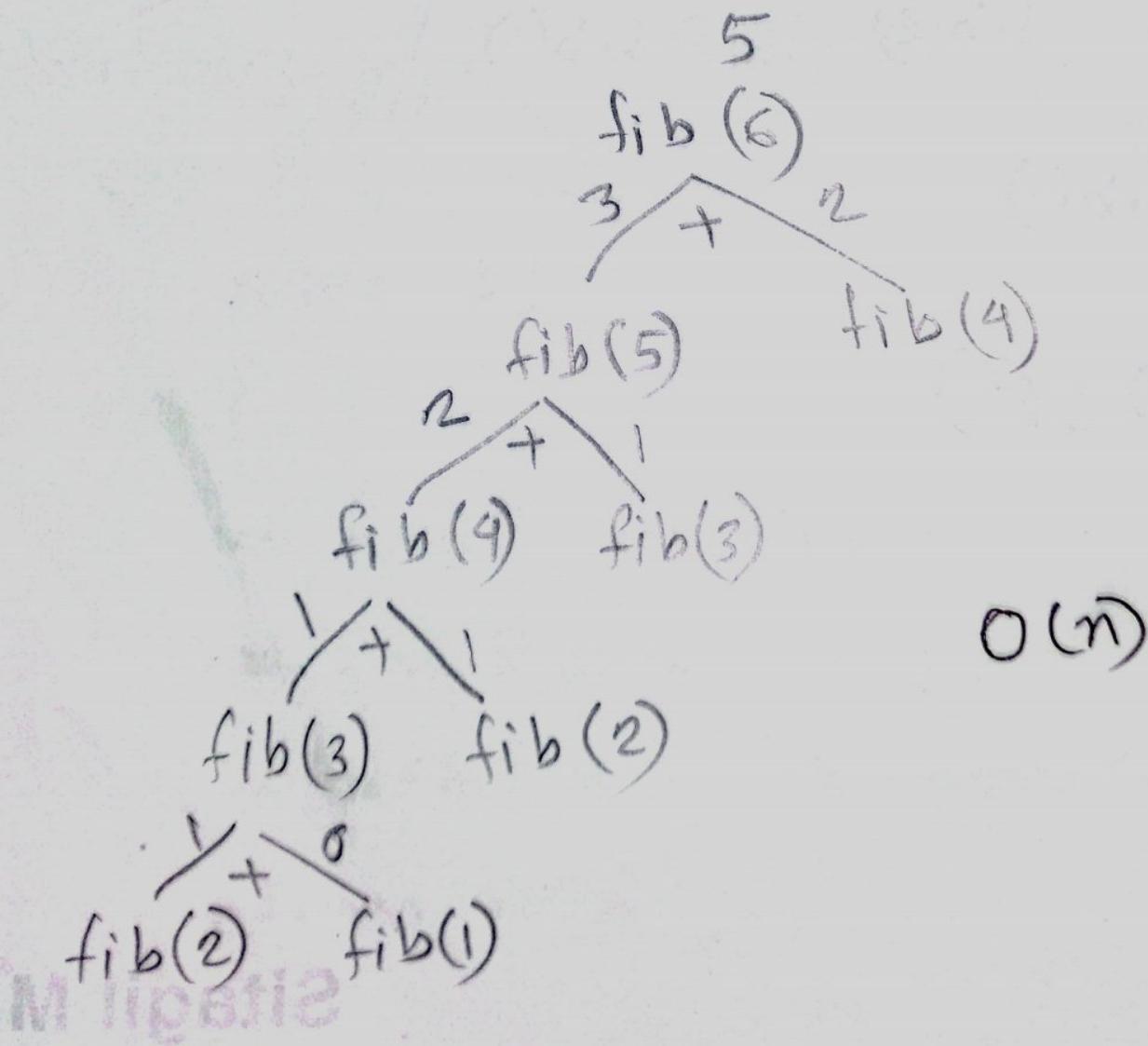
DPTable

TOP Down (Recursive)

1	0
2	
3	(A) fib 1
4	fib 2
5	fib 3
6	fib 5

(A) fib

(B) fib



Multiplication

2 ways to implement a DP

Solution:

→ Top Down (Recursive)

→ Bottom up (iterative)

```
int fib(n){
```

```
    if (dpTable[n] == -1) {
```

```
        return dpTable[n];
```

```
}
```

```
    if (n == 1) return 0;
```

```
    else if (n == 2) return 1;
```

```
    else {
```

```
        int fwd1 = fib(n-1)
```

```
        int fwd2 = fib(n-2)
```

```
        int result = fwd1 + fwd2
```

```
        dpTable[n] = result;
```

```
        return result;
```

```
}
```

DP applies for overlapping

sub-problem.

Dynamic Programming

A coin Change-

amount = 11

coins = [1, 5, 6, 8]

optimal substructural property

coins\amount	0	1	2	3	4	5	6	7	8	9	10	11
1	0	1	2	3	4	5	6	7	8	9	10	11
5	0	1	2	3	4	1	2	3	4	5	2	3
6	0	1	2	3	4	1	1	2	3	4	2	2
8	0	1	2	3	4	1	1	2	1	2	2	2

whether we'll take it or not:

$$\text{amount} = 5 - 5 = 0$$

$$\text{take: } 1 + 0 = 1$$

$$\text{not take: } 5$$

amount = 6

coins = [2, 3, 5]

+50 ♂ α = 0x7f7f

coin amount	0	1	2	3	4	5	6
2	0	α	1	α	1	2	α
3	0	α	1	1	2	2	2
5	0	α	1	1	2	1	2