The Four Ways of Counting

Statistics for Data Science

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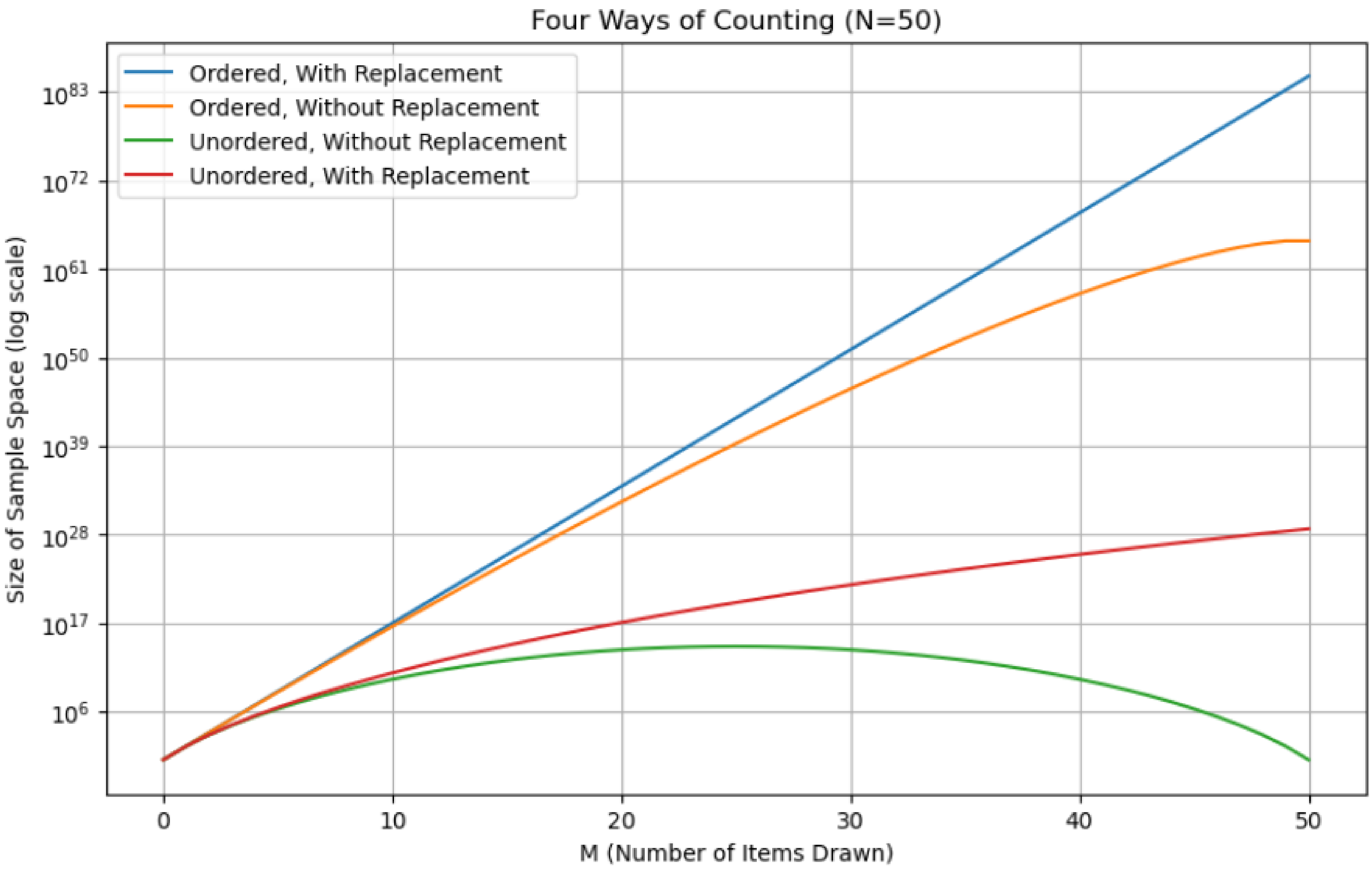
Four Ways of Counting (N=50)
Ordered, With Replacement
Ordered, Without Replacement
Unordered, Without Replacement
Unordered, With Replacement
Size of Sample Space (log scale)
10^{83}
10^{72}
10^{61}
10^{50}
10^{39}
10^{28}
10^{17}
10^{6}
0
10
20
30
40
50
M (Number of Items Drawn)

Order of the Curves (Largest → Smallest)

Ordered, With Replacement — largest :

- ❖ Because every draw can be any of the N objects and order matters.
- ❖ Total possibilities: N^M (exponential growth).

Ordered, Without Replacement — second largest:

- ❖ Order matters, but each selection removes an option.
- ❖ Total possibilities: N(N−1)(N-2) . . . .(N−M+1) = N!/(N-M)!
- ❖ Starts large, but decreases as M approaches N.

Unordered, With Replacement — third largest:

- ❖ Order doesn't matter, but repetition is allowed.
- ❖ The Combination formula will be used to determine the total number of positions.
- ❖ Growth is slower than exponential but still significant.

Unordered, Without Replacement — smallest:

- ❖ Order doesn't matter **and** you can't repeat selections.
- ❖ Again we will use Combination technique and out of N possible way we have to use M objects.
- ❖ Produces the smallest number of outcomes.

Thumb rule:

- ✓ Order multiplies possibilities because each arrangement counts separately.
- ✓ Replacement increases possibilities because items can reappear.
- ✓ So, allowing both (ordered + replacement) gives the largest sample space; disallowing both gives the smallest.

Explanation & Real-World Examples:

- The curve for **“Unordered, Without Replacement”** has a unique, symmetrical, bell-shaped (parabolic on the log-scale) appearance, while the other three curves show continuous growth.
- Ordered, With Replacement and Unordered, With Replacement are continuously increasing because the "With Replacement" rule allows M to theoretically grow indefinitely.
- Ordered, Without Replacement stops at M=N. Since no item can be repeated, once M exceeds N=50, you run out of unique items, making the number of arrangements 0.

Examples:

- **Ordered, With Replacement:** Secure PIN Generation: Counting the total number of possible M-digit ATM PINs where any of the N=10 digits can be used and repeated.
- **Ordered, Without Replacement :** Ranking top M sales employees from a group of N.
- **Unordered, Without Replacement:** Project Team Selection: Calculating how many different ways a manager can select a unique team of M employees from a department of N employees. The internal order of the team doesn't matter.
- **Unordered, With Replacement:** Choosing combinations of toppings for an ice cream order.

PYTHON File:

```
import math
import matplotlib.pyplot as plt
N = 50
M_values = range(0, 51)

ordered_with = [N**M for M in M_values]
ordered_without = [math.perm(N, M) for M in M_values]
unordered_without = [math.comb(N, M) for M in M_values]
unordered_with = [math.comb(N + M - 1, M) for M in M_values]

plt.figure(figsize=(10,6))
plt.plot(M_values, ordered_with, label='Ordered, With Replacement')
plt.plot(M_values, ordered_without, label='Ordered, Without Replacement')
plt.plot(M_values, unordered_without, label='Unordered, Without Replacement')
plt.plot(M_values, unordered_with, label='Unordered, With Replacement')

plt.yscale('log')
plt.title('Four Ways of Counting (N=50)')
plt.xlabel('M (Number of Items Drawn)')
plt.ylabel('Size of Sample Space (log scale)')
plt.legend()
plt.grid(True)
plt.show()
```