Question3

Solution :

Assume the net as an array B with length m. And it's reverse array is B' .

Then we can compute the convolution $C = A * B'$ in time $O(nlogn)$:

First, compute $P_A(x) = A_k + A_{k+1}x + \cdots + A_{k+n-1}x^{k+n-1}$ and $P_{B'}(x) = B_0 + B'_1x + \cdots + B'_{n-1}x^{n-1}$ in time $O(n)$.

Then compute the DFT in time $O(logn)$ and do multiplication in time $O(n)$

We can get $\{ P_A(1)P_{B'}(1), P_A(w_{2n-1}), P_{B'}(w_{2n-1}), \ldots, P_A(w_{2n-1}^{2n-1}), P_{B'}(w_{2n-1}^{2n-1})\}$.

At last do IDFT we can get $C = \langle \sum_{i=0}^{j} A_{k+i}B'_{j-i}\rangle_{j=0}^{j=2n-2}$ in time $O(logn)$.

After that, check the maximum in C. If C_t is the maximum, the spot where you should place the left end of your net in order to catch the largest possible number of fish is:

$$\begin{cases} 0; & if\ t-n \le 0 \\ t-n; & if\ t-n > 0 \end{cases}$$

Proof :

$C_t = A_{t-n}B'_n + A_{t-n+1}B'_{n-1} + \cdots + A_tB'_0$

$If\ there\ is\ a\ hole, B_i = 0, then\ \ A_{t-i}B'_i = 0.$

$If\ there\ is\ a\ hole, B_i = 1, then\ \ A_{t-i}B'_i = A_{t-i}.$

$So, C_t\ means\ how\ may\ fishes\ the\ net\ can\ catch\ if\ we\ put\ it\ left\ end\ in\ A_{t-n}.$