Question2

Solution :

Use substitution $y = x^{100}$, we can get $P(x) = A_0 + A_1 + A_2y^2$.

So, $P(x)^2 = (A_0 + A_1 + A_2y^2)^2$

$$= A_0^2 + A_1^2y^2 + A_2^2y^4 + 2A_1A_2y^3 + 2A_0A_1y + 2A_0A_2y^2$$

$$= A_0^2 + 2A_0A_1y + (A_1^2 + 2A_0A_2)y^2 + 2A_1A_2y^3 + A_2^2y^4$$

Use substitution $A_0^2 = C_0, 2A_0A_1 = C_1, A_1^2 + 2A_0A_2 = C_3, 2A_1A_2 = C_4, A_2^2 = C_5$.

So, $P(x)^2 = C_0 + C_1y + C_3y^2 + C_4y^3 + A_2^2y^4$

We can compute these 5 large integer multiplications:

$$\begin{cases} (A_1 + 2A_0) * (A_1 + A_2) = A_1^2 + 2A_0A_2 + 2A_0A_1 + A_1A_2 \ \ (1) \\ A_0A_1 \ \ (2) \\ A_1A_2 \ \ (3) \\ A_0^2 \ \ (4) \\ A_2^2 \ \ (5) \end{cases}$$

Then we can get that:

$C_0 = (4)$

$C_1 = (2) + (2)$

$C_3 = (1) - (2) - (2) - (3)$

$C_4 = (3) + (3)$

$C_5 = (5)$

Because the addition and subtraction are cheap, we squares P(x) using only 5 large integer multiplications.