Question5

Solution :

If the chemicals are produced in order $C_1, C_2, \dots, C_n$, then the total evaporation loss is $W_1(1-(1-p)^n) + W_2(1-(1-p)^{n-1}) + \cdots + W_n(1-(1-p)^0)$.

Because $(1-(1-p)^n)$ will increase when n is increased, we should arrange the production schedule according to the quality of the chemicals. The smaller the kilogram demand (W_i) is, the earlier the production is, and the larger the kilogram demand(W_i) is, the later the production.

So, we should produce these chemicals in increasing order of kilograms of chemicals.

Proof :

If we swap two chemicals $C_{k+i}\ and\ C_k$.

The total evaporation loss is $L' = W'_1(1-(1-p)^n) + W'_2(1-(1-p)^{n-1}) + \cdots + W'_{k+i}(1-(1-p)^{n-k}) + \cdots + W'_k\left(1-(1-p)^{n-k-i}\right) + \cdots + W'_n(1-(1-p)^0)$ and $W'_{k+i} > W'_k$.

The total evaporation loss before swaping is $L = W'_1(1-(1-p)^n) + W'_2(1-(1-p)^{n-1}) + \cdots + W'_k(1-(1-p)^{n-k}) + \cdots + W'_{k+i}\left(1-(1-p)^{n-k-i}\right) + \cdots + W'_n(1-(1-p)^0)$

Thus, $L' - L = ({W'}_{k+i} - W'_k)(1-(1-p)^{n-k}) +$

$$(W'_k - W'_{k+i})\left(1-(1-p)^{n-k-i}\right)$$

$$= ({W'}_{k+i} - W'_k)((1-(1-p)^{n-k}) - \left(1-(1-p)^{n-k-i}\right))$$

Because $W'_{k+i} > W'_k$ and $(1-(1-p)^{n-k}) > \left(1-(1-p)^{n-k-i}\right)$

So, $L' - L > 0$. Any swap in this schedule will increase the total evaporation loss.

Thus, this schedule is optimal.