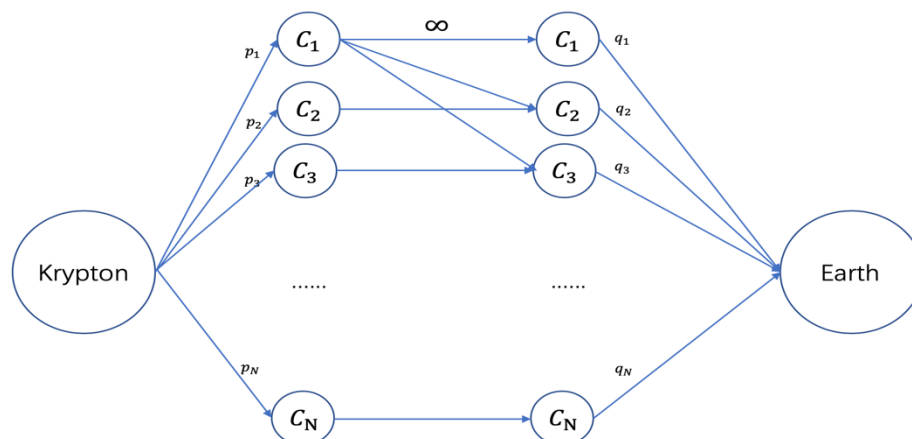


Question1

Solution :

Construct a flow network as a directed graph and it should be bipartite graph:

1. The Krypton is the super source and has directed edges to each city C_1, \dots, C_N at the left-side of the flow network/bipartite graph, the capacity of the link/edge is the p_i of the city C_i .
2. Each city is a vertex at the left-side and the right-side of the flow network/bipartite graph, and each city at the left-side has directed edges to those cities at the right-side which people can arrive there within X day which include itself, we can compute it by Dijkstra algorithm, and the capacity of the edge is infinity.
3. The Earth is the super source and each city C_1, \dots, C_N at the right-side of the flow network/bipartite graph has directed edge to the Earth, the capacity of the edge is the q_i of the city C_i .
4. For example, if people can only arrive C_1, C_2, C_3 within X days from C_1 by Dijkstra algorithm, so C_1 on the left-side have directed edge to C_1, C_2, C_3 and capacity of the edges are ∞ . Meanwhile, the capacity of the edge $[Krypton, C_1]$ is p_1 and the capacity of the edge $[C_1, Earth]$ is q_1 :



The other cities are similar to C_1 .

5. Compute the maximal flow by Ford-Fulkerson algorithm. The largest number of invaders the Earth will have to deal with is the max flow from *Krypton* to *Earth*.