# Question5

***Solution :***

(a)

$$g(n) = log_2^{(n^{log_2^n})^2} = 2log_2^{n^{log_2^n}} = 2 * log_2^n * log_2^n = 2(log_2^n)^2$$

$$\therefore for\ c = 1\ and\ n_0 = 1, 0 \le f(n) \le cg(n)\ for\ all\ n \ge n_0$$

$$for\ c = 9\ and\ n_0 = 1, 0 \le g(n) \le cf(n)\ for\ all\ n \ge n_0$$

$$\therefore f(n) = O\big(g(n)\big)\ and\ g(n) = O\big(f(n)\big)$$

$$\therefore f(n) = \Theta\big(g(n)\big)$$

(b)

$$If\ \log f(n) \le \log C + \log g(n)\ \ then\ f(n) \le C\ g(n).$$

$$Then\ \lim_{n\to\infty}\left(\frac{\log C + \log g(n)}{\log f(n)}\right) = \lim_{n\to\infty}\frac{(\log C + \log g(n))\,'}{(\log f(n))'}$$

$$= \lim_{n\to\infty}\frac{(n^{\frac{1}{10}})'}{(10 * log_2^n)'} = \lim_{n\to\infty}\frac{\frac{1}{10} * n^{-\frac{9}{10}}}{\frac{1}{n * ln2}} = \lim_{n\to\infty} ln2 * \frac{1}{10} * n^{\frac{1}{10}}$$

$$= \infty$$

$$So\ for\ sufficiently\ large\ n,\ \log f(n)$$

$$\le \log C + \log g(n)\ \ and\ we\ can\ get\ f(n) \le C\ g(n).$$

$$Hence, f(n) = O(g(n)).$$

(c)

$$\because when\ n = 2k\ that\ f(n) = n^2\ and\ g(n) = n, we\ can\ get\ that$$

$$f(n) > cg(n) for\ c\ is\ a\ constant\ number.$$

$$when\ n = 2k + 1\ that\ f(n) = n^0 = 1\ and\ g(n) = n, we\ can\ get\ that$$

$$f(n) < cg(n) for\ c\ is\ a\ constant\ number.$$

$$\therefore Either\ f(n) = O(g(n))\ or\ g(n) = O(f(n))$$