Question4

Solution :

(a):

$Because\ A = B = \langle 1,0,0, \ldots ,0, 1\rangle, so\ P_A = P_B = 1 + x^{k+1}$

$Thus\ the\ convolution\ of\ A\ and\ B\ is\ the\ same\ as\ convolution\ of\ itself.$

$The\ convolution\ with\ itself\ is\ the\ sequence\ of\ the\ coefficients\ of\ the\ polynomial\ P_A^2$

$P_A^2 = (1 + x^{k+1})^2 = 1 + 2x^{k+1} + x^{2k+2}$

$A * B =$

$\langle 1,0, \ldots (n\ 0s\ between\ the\ two\ 1s) \ldots ,0,2,0, \ldots (n\ 0s\ between\ the\ two\ 1s) \ldots ,0,1\rangle$

(b):

$Because\ A = \langle 1,0,0, \ldots ,0, 1\rangle, so\ P_A = 1 + x^{k+1}$

$$Then\ DFT(B) = \langle P_A(w_{k+2}^0), P_A(w_{k+2}^1), \ldots , P_A\left(w_{k+2}^{k+1}\right)\rangle$$

$$= \langle 1 + w_{k+2}^{0\cdot(k+1)}, 1 + w_{k+2}^{1\cdot(k+1)}, \ldots ,1 + w_{k+2}^{(k+1)\cdot(k+1)}\rangle$$

$$= \langle 0,1 + w_{k+2}^{k+1}, 1 + w_{k+2}^{2(k+1)}, \ldots ,1 + w_{k+2}^{(k+1)^2}\rangle$$