contain atleast 3 defective parts.

Sol.ⁿ Mean no. of defectives, $m = 2 = np$

Here, $n = 20$. $\Rightarrow$ $p = \frac{2}{20} = 0.1$

The probability of a defective part, $p = 0.1$

The probability of non defective part, $q = 1 - p = 0.9$.

Probability of atleast 3 defective parts in a sample of 20 $= P(3) + P(4) + P(5) + \cdots + P(20)$

$= 1 - [P(0) + P(1) + P(2)]$

of 20 $= P(3) + P(4) + P(5) + \cdots + P(20)$

$$= 1 - \left[P(0) + P(1) + P(2) \right]$$

$$= 1 - \left[{}^{20}C_0 (0.1)^0 (0.9)^{20} + {}^{20}C_1 (0.1)^1 (0.9)^{19} + {}^{20}C_2 (0.1)^2 (0.9)^{18} \right]$$

$$= 1 - \left[1 \times 1 \times (0.9)^{20} + 20 \times (0.1)(0.9)^{19} + \frac{20!}{2!\,18!} (0.1)^2 (0.9)^{18} \right]$$

$$= 1 - \left[(0.9)^{20} + 2 \times (0.9)^{19} + \frac{20 \times 19 \times \cancel{18!}}{2 \times \cancel{18!}} (0.1)^2 (0.9)^{18} \right]$$

$$= 0.323.$$

$+ {}^{20}C_2$

$$= 1 - \left[1 \times 1 \times (0.9)^{20} + 20 \times (0.1)(0.9)^{19} + \frac{\lfloor 20}{\lfloor 2 \lfloor 18} (0.1)^2 \right.$$

$$= 1 - \left[(0.9)^{20} + 2 \times (0.9)^{19} + \frac{20 \times 19 \times \cancel{\lfloor 18}}{2 \times \cancel{\lfloor 18}} (0.1)^2 \right.$$

$$= 0.323.$$

Hence, the no. of samples having atleast three defectives out of 1000 samples $= 1000 \times 0.323$

$= 323$

Ans

Binomial Distribution

Problem#3

In 256 sets of 12 tosses of a coin, in how many cases one can expect 8 heads and 4 tails.

Que (3) In 256 sets of 12 tosses of a coi[n] how many cases one can expect 8 heads and 4 tails.

Sol.ⁿ Here, $P(H) = 0.5$, $P(T) = 0.5$

By Binomial distribution, probability of 8H and 4T in 12 trials is

$$P(8) = {}^{12}C_8 (0.5)^8 (0.5)^4 = \frac{\lfloor 12}{\lfloor 8 \ \lfloor 4} (0.5)^{12}$$

Here, $P(H) = 0.5$, $P(T) = 0.5$

By Binomial distribution, probability of 8H and 4T in 12 trials is

$$P(8) = {}^{12}C_8 (0.5)^8 (0.5)^4 = \frac{\lfloor 12}{\lfloor 8 \, \lfloor 4} (0.5)^{12}$$

$$= \frac{\cancel{12} \times 11 \times \overset{5}{\cancel{10}} \times 9 \times \cancel{\lfloor 8}}{\cancel{\lfloor 8} \times \cancel{4} \times \cancel{3} \times \cancel{2}} (0.5)^{12} = 0.1208$$

$\therefore$ The expected no. of such cases in 256 sets

$$= 256 \times P(8) = 256 \times 0.1208 = 30.9 \approx 31$$

Ans

Binomial Distribution

Problem#4

If the mean and variance of binomial distribution are 4 and 2 respectively, find the probability of

i. exactly 2 success

ii. less than 2 success

iii. atleast 2 success

Que (4) If the mean and variance of bi

distribution are 4 and 2 respectively, find the probability of

(a) Exactly 2 success

(b) Less than 2 success

(c) Atleast 2 success.

(c) Atleast 2 success.

Sol^n Given: $np = 4$ and $npq = 2$.

$$\frac{\cancel{n}\cancel{p}q}{\cancel{n}\cancel{p}} = \boxed{q = \frac{2}{4} = 0.5} \Rightarrow \boxed{p = 1 - q = 0.5}$$

Also, $np = 4 \Rightarrow n = \frac{4}{1/2} = 8 \Rightarrow \boxed{n = 8}$

(i) Probability of exactly two success:

$$P(2) = {}^8C_2 (0.5)^2 (0.5)^6 = \frac{7}{64}.$$

(ii)

$$P(2) = {}^8C_2 (0.5)^2 (0.5)^6 = \frac{7}{64}.$$

(ii) Probability of less than 2 success:

$$P(0) + P(1) = {}^8C_0 (0.5)^0 (0.5)^8 + {}^8C_1 (0.5)^1 (0.5)^8$$

$$= \frac{9}{256}$$

(iii) Probability of atleast 2 success:

$$P(2) + P(3) + P(4) + \cdots + P(8) = 1 - [P(0) + P(1)]$$

$$= 1 - \frac{9}{256} = \frac{256 - 9}{256} = \frac{247}{256} \quad \text{Ans}$$

Binomial Distribution

Problem#5

The probability that a bomb dropped from a plane will strike the target is 1/5. If six bombs are dropped, find the probability that

i. exactly 2 will strike the target

ii. atleast 2 will strike the target

Que (5) The probability that a bomb droppe

a plane will strike the target is $1/5$. If six bombs are dropped, find the probability that

(a) Exactly two strike the target.

(b) Atleast two will strike the target.

Sol^n Probability of striking the target, $p = 0.2 = 1/5$

Probability of non-striking the target, $q = 1 - p = 0.8$

Here, $n = 6$

Sol: Probability of striking the target, $p = 0.2$

Probability of non-striking the target, $q = 1 - p = 0.8$

Here, $n = 6$

(a) Probability that exactly two will strike the target

$$P(2) = {}^6C_2 (0.2)^2 (0.8)^4 = 0.245$$

(b) Probability that atleast two will strike the target

$$P(2) + P(3) + \cdots P(6) = 1 - [P(0) + P(1)]$$

$$= 1 - \left[{}^6C_0 (0.2)^0 (0.8)^6 + {}^6C_1 (0.2)^1 (0.8)^5 \right]$$

$$= 0.345 \text{ Ans}$$

Binomial Distribution

Problem#6

If the chance that one of the ten telephone lines is busy at an instant is 0.2,

i. What is the chance that 5 of the lines are busy?

ii. What is the probability that all the lines are busy?

Que(6) If the chance that one of the ten

lines is busy at an instant is 0.2,

(a) what is the chance that 5 of the lines are busy?

(b) what is the probability that all the lines are busy?

Sol^n Probability of one telephone busy out of 10, $p = 0.2$.

Probability of no telephone busy out of 10, $q = 1 - p = 0.8$

are busy?

Sol^n Probability of one telephone busy out of 10, $p = 0.2$.

Probability of no telephone busy out of 10, $q = 1 - p = 0.8$

(a) Probability that 5 of the lines are busy:

$$P(5) = {}^{10}C_5 (0.2)^5 (0.8)^5 = 0.0265$$

(b) Probability that all the lines are busy:

$$P(10) = {}^{10}C_{10} (0.2)^{10} (0.8)^{10-10}$$

$$= 1 \times (0.2)^{10} \times 1 = 1.024 \times 10^{-7}.$$

Binomial Distribution

Problem#7

Out of 800 families with 5 children each, how many would you expect to have

i. 3 boys

ii. 5 girls

iii. either 2 or 3 boys

Que 7 Out of 800 families with 5 children ea[ch]
how many would you expect to have
(a) 3 boys (b) 5 girls (c) Either 2 or 3 boys.
Assume equal probabilities for boys and girls.

Sol.ⁿ $p = 0.5$, $q = 0.5$ $p \rightarrow$ probability of a boy.
$n = 5$ $q \rightarrow$ probability of a girl.

(a) Probability of 3 boys in 800 families.

$$= {}^5C_3 (0.5)^3 (0.5)^2 \times 800 = 250$$

(b) Probability of 5 girls in 800 families.

$$= {}^5C_5 (0.5)^5 (0.5)^0$$

Sol^n $p = 0.5$, $q = 0.5$

$n = 5$

$p \to$ probability of a boy.

$q \to$ probability of a girl.

(a) Probability of 3 boys in 800 families.

$$= {}^5C_3 (0.5)^3 (0.5)^2 \times 800 = 250$$

(b) Probability of 5 girls in 800 families.

$$= {}^5C_5 (0.5)^5 (0.5)^0 \times 800 = 25$$

(c) Probability of either 2 or 3 boys in 800 families:

$$= \left[{}^5C_2 (0.5)^2 (0.5)^3 + {}^5C_3 (0.5)^3 (0.5)^2 \right] \times 800$$

$$= 500$$

Binomial Distribution

Problem#8

The following data are the number of seeds germinating out of 10 on damp filter paper for 80 sets of seeds, fit a binomial distribution of these data and compare the theoretical frequencies with the actual ones:

x:	0	1	2	3	4	5	6	7	8	9	10
f:	6	20	28	12	8	6	0	0	0	0	0

Que ⑧ The following data are the no. of seeds germinating out of 10 on damp filter paper for 80 sets of seeds, fit a binomial distribution of these data and compare the theoritical frequencies with the actual ones:

x :	0	1	2	3	4	5	6	7	8	9	10
f :	6	20	28	12	8	6	0	0	0	0	0

of these data and compare the theoretical [illegible]
with the actual ones:

x:	0	1	2	3	4	5	6	7	8	9	10
f:	6	20	28	12	8	6	0	0	0	0	0

Sol^n Here, $\boxed{n = 10}$, $\boxed{\sum f_i = 80 = N}$

Mean for grouped data,

$$m = np = \frac{\sum f_i x_i}{\sum f_i} = \frac{0 + 20 + 56 + 36 + 32 + 30 + 0}{80} = 2.$$

$$\boxed{p = 0.2175} \Rightarrow \boxed{q = 1 - p = 0.7825}$$

$\boxed{p = 0.2175} \Rightarrow \boxed{q = 1 - p = 0.7825}$

Hence, the binomial distribution to be fitted is

$\boxed{N(q+p)^n} = 80(0.7825 + 0.2175)^{10}$. Ans

Use formula: $P(r) = {}^nC_r \, p^r \, q^{n-r} \times N$.

For $r=0$, $P(0) = {}^{10}C_0 \, (0.2175)^0 \, (0.7825)^{10} \times 80 =$

For $r=1$, $P(1) = {}^{10}C_1 \, (0.2175)^1 \, (0.7825)^9 \times 80 =$

For $r=2$, $P(2) = {}^{10}C_2 \, (0.2175)^2 \, (0.7825)^8$

Hence, the binomial distribution to be ...

$$\boxed{N(q+p)^n} = 80(0.7825 + 0.2175)^{10}. \quad \text{Ans}$$

Use formula: $P(r) = {}^nC_r \, p^r \, q^{n-r} \times N.$

For $r=0$, $P(0) = {}^{10}C_0 (0.2175)^0 (0.7825)^{10} \times 80 =$

For $r=1$, $P(1) = {}^{10}C_1 (0.2175)^1 (0.7825)^9 \times 80 =$

For $r=2$, $P(2) = {}^{10}C_2 (0.2175)^2 (0.7825)^8 \times 80 =$

Binomial Distribution

Problem#9

Fit a binomial distribution for the following data and compare the theoretical frequencies with the actual ones:

x:	0	1	2	3	4	5
f:	2	14	20	34	22	8

Ex ⑨ Fit a binomial distribution for the [illegible]
and compare the theoritical frequencies with
actual ones:

$x \rightarrow$	0	1	2	3	4	5
$f \rightarrow$	2	14	20	34	22	8

Sol.^{n} Here, $\boxed{n = 5}$, $\Sigma f_i = \boxed{100 = N}$

$\therefore$ Mean for grouped data

$$m = np = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{0 + 14 + 40 + 102 + 88 + 40}{100} = 2.84$$

$\longrightarrow \quad 2 \quad 14 \quad 20 \quad 34 \quad 22 \quad 8$

Sol.ⁿ Here, $\boxed{n = 5}$, $\sum f_i = \boxed{100 = N}$

$\therefore$ Mean for grouped data

$$m = np = \frac{\sum f_i x_i}{\sum f_i} = \frac{0 + 14 + 40 + 102 + 88 + 40}{100} = 2.8$$

$$\Rightarrow \boxed{p = 0.57} \Rightarrow q = 1 - p = \boxed{0.43 = q}$$

$\therefore$ The binomial distribution to be fitted is

$$N(q+p)^n = 100\,(0.43 + 0.57)^5$$

Ans.

$\Rightarrow \boxed{p = 0.57} \Rightarrow q = 1 - p = \boxed{0.43 = q}$

$\therefore$ The binomial distribution to be fitted is

$$N(q+p)^n = 100\,(0.43 + 0.57)^5 \quad \text{Ans.}$$

<u>Use formula</u>:

$$P(r) = N \cdot {}^nC_r \, p^r \, q^{n-r}$$

For $r = 0$, $P(0) = 100 \times {}^5C_0 \, (0.57)^0 (0.43)^5 =$

For $r = 1$, $P(1) = 100 \times {}^5C_1 \, (0.57)^1 (0.43)^4 =$

For $r = 2$, $P(2) = 100 \times$

Use formula:

$$P(r) = N \cdot {}^{n}C_{r}\, p^{r} q^{n-r}$$

For $r=0$, $P(0) = 100 \times {}^5C_0 (0.57)^0 (0.43)^5 =$

For $r=1$, $P(1) = 100 \times {}^5C_1 (0.57)^1 (0.43)^4 =$

For $r=2$, $P(2) = 100 \times {}^5C_2 (0.57)^2 (0.43)^3 =$

For $r=3$, $P(3) = 100 \times {}^5C_3 (0.57)^3 (0.43)^2 =$

For $r=4$, $P(4) = 100 \times {}^5C_4 (0.57)^4 (0.43)^1 =$

For $r=5$, $P(5) = 100 \times {}^5C_5 (0.57)^5 (0.43)^0 =$