Ensemble methods for Yelp review classification

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1 Introduction

In this report we discuss the five very popular classifiers for the task of yelp review classification. In particular we discuss the support vector machines, decision trees, bagged decision trees, random forests, and boosted decision trees.

The report is organised as follows. The next four sections deal with the Analysis 1, Analysis 2, Analysis 3, and Analysis 4 of the homework 4 handout. Finally in the last section we discuss the Boosted decision trees.

2 Analysis 1

2.1 Learning Curves

The plot of results is shown in Figure 1. On x-axis we vary the training set sizes. Instead of percents, we have plotted the actual number of training set examples used to learn the model. On y-axis is the zero one loss. We have three plots — one for each model. The different colored plots belong to different models as indicated in the legend. The vertical bars show the standard error calculated as described in the homework handout. The values of zero one loss for each of the six training set size averaged accross 10 fold are shown in the figure.

These average and standard error values are calculated from the following zero one loss values calculated for each model for training set size for each of the 10 folds. Notice that the horizontal = bar separated the results for each training set size. The zero one losses present within two horizontal = bars belong to the different folds for a specific training set size.

ZERO-ONE-LOSS-DT 0.385 ZERO-ONE-LOSS-BT 0.48 ZERO-ONE-LOSS-RF 0.54 ZERO-ONE-LOSS-BS 0.44 ZERO-ONE-LOSS-SVM 0.24 ZERO-ONE-LOSS-DT 0.485 ZERO-ONE-LOSS-BT 0.34 ZERO-ONE-LOSS-RF 0.34 ZERO-ONE-LOSS-BS 0.3 ZERO-ONE-LOSS-SVM 0.25 ZERO-ONE-LOSS-BT 0.4 ZERO-ONE-LOSS-BT 0.4 ZERO-ONE-LOSS-BS 0.3 ZERO-ONE-LOSS-SVM 0.22 ZERO-ONE-LOSS-DT 0.28 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-RF 0.28 ZERO-ONE-LOSS-BS 0.26 ZERO-ONE-LOSS-SVM 0.235 ZERO-ONE-LOSS-DT 0.31 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-RF 0.2 ZERO-ONE-LOSS-BS 0.2 ZERO-ONE-LOSS-SVM 0.13 ZERO-ONE-LOSS-DT 0.39 ZERO-ONE-LOSS-BT 0.32 ZERO-ONE-LOSS-RF 0.24 ZERO-ONE-LOSS-BS 0.34 ZERO-ONE-LOSS-SVM 0.18 ZERO-ONE-LOSS-BT 0.38 ZERO-ONE-LOSS-SVM 0.17 ZERO-ONE-LOSS-DT 0.405 ZERO-ONE-LOSS-BT 0.32 ZERO-ONE-LOSS-BT 0.32 ZERO-ONE-LOSS-BS 0.2 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-BT 0.26 ZERO-ONE-LOSS-SVM 0.19 ZERO-ONE-LOSS-DT 0.315 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-BT 0.26 ZERO-ONE-LOSS-BS 0.2 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-BT 0.38 ZERO-ONE-LOSS-BS 0.2 ZERO-ONE-LOSS-BS 0.2 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-BS 0.2 ZERO-ONE-LOSS-BS 0.24 ZERO-ONE-LOSS-DT 0.38 ZERO-ONE-LOSS-BS 0.2 ZERO-ONE-LOSS-BS 0.24 ZERO-ONE-LOSS-DT 0.38 ZERO-ONE-LOSS-BS 0.24 ZERO-ONE-LOSS-BS 0.24 ZERO-ONE-LOSS-SVM 0.255

ZERO-ONE-LOSS-DT 0.325 ZERO-ONE-LOSS-BT 0.36 ZERO-ONE-LOSS-RF 0.36 ZERO-ONE-LOSS-BS 0.26 ZERO-ONE-LOSS-SVM 0.15 ZERO-ONE-LOSS-DT 0.35 ZERO-ONE-LOSS-BT 0.28 ZERO-ONE-LOSS-RF 0.32 ZERO-ONE-LOSS-BS 0.26 ZERO-ONE-LOSS-SVM 0.14 ZERO-ONE-LOSS-DT 0.335 ZERO-ONE-LOSS-BT 0.14 ZERO-ONE-LOSS-RF 0.28 ZERO-ONE-LOSS-BS 0.14 ZERO-ONE-LOSS-SVM 0.11 ZERO-ONE-LOSS-DT 0.295 ZERO-ONE-LOSS-BT 0.22 ZERO-ONE-LOSS-RF 0.24 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.145 ZERO-ONE-LOSS-DT 0.325 ZERO-ONE-LOSS-BT 0.26 ZERO-ONE-LOSS-RF 0.26 ZERO-ONE-LOSS-BS 0.24 ZERO-ONE-LOSS-BS 0.26 ZERO-ONE-LOSS-BS 0.27 ZERO-ONE-LOSS-BS 0.28 ZERO-ONE-LOSS-

SVM 0.13 ZERO-ONE-LOSS-DT 0.29 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-RF 0.2 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-SVM 0.125 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-RF 0.26 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.11 ZERO-ONE-LOSS-BT 0.425 ZERO-ONE-LOSS-BT 0.1 ZERO-ONE-LOSS-RF 0.22 ZERO-ONE-LOSS-BS 0.18 ZERO-ONE-LOSS-SVM 0.16 ZERO-ONE-LOSS-DT 0.35 ZERO-ONE-LOSS-BT 0.26 ZERO-ONE-LOSS-BS 0.2 ZERO-ONE-LOSS-SVM 0.12 ZERO-ONE-LOSS-DT 0.29 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-RF 0.26 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-SVM 0.135

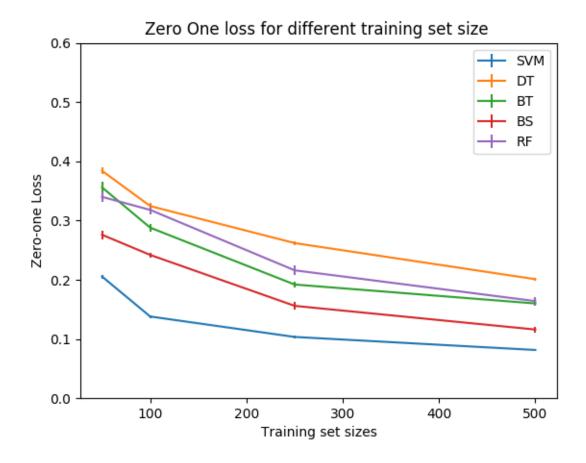
ZERO-ONE-LOSS-DT 0.26 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-RF 0.3 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-SVM 0.08 ZERO-ONE-LOSS-DT 0.295 ZERO-ONE-LOSS-BT 0.22 ZERO-ONE-LOSS-RF 0.06 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.07 ZERO-ONE-LOSS-DT 0.375 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-RF 0.32 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-SVM 0.105 ZERO-ONE-LOSS-DT 0.18 ZERO-ONE-LOSS-BT 0.22 ZERO-ONE-LOSS-RF 0.16 ZERO-ONE-LOSS-BS 0.2 ZERO-ONE-LOSS-SVM 0.125 ZERO-ONE-LOSS-DT 0.175 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-RF 0.04 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.095 ZERO-ONE-LOSS-DT 0.26 ZERO-ONE-LOSS-BT 0.26 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-SVM 0.095 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-BS 0.11 ZERO-ONE-LOSS-BS 0.11 ZERO-ONE-LOSS-BS 0.11 ZERO-ONE-LOSS-BS 0.11 ZERO-ONE-LOSS-BS 0.11 ZERO-ONE-LOSS-BT 0.11 ZERO-ONE-LOSS-BS 0.11 ZERO-ONE-LOSS-SVM 0.011 ZERO-ONE-LOSS

ZERO-ONE-LOSS-DT 0.28 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-RF 0.24 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-SVM 0.065 ZERO-ONE-LOSS-DT 0.205 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-RF 0.3 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.08 ZERO-ONE-LOSS-BT 0.13 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-RF 0.04 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.085 ZERO-ONE-LOSS-DT 0.19 ZERO-ONE-LOSS-BT 0.14 ZERO-ONE-LOSS-BT 0.14 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-SVM 0.09 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-SVM 0.055 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-SVM 0.06 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-BS 0.04 ZERO-ONE-LOSS-SVM 0.055 ZERO-ONE-LOSS-BS 0.04 ZERO-ONE-LOSS-SVM 0.055 ZERO-ON

2.2 Hypothesis

Let's compare the two algorithms SVM and Bagged Decision Trees (BT).

Null hypothesis is that the SVM and BT are equivalent in terms of performance. It is assumed to be true. So null hypothesis is the mean of SVM 0/1 losses is equivalent to mean of BT 0/1 losses. While the alternative hypothesis is that SVM and BT zero one losses are significantly different. Or, the mean of SVM 0/1 losses is lesser than BT 0/1 losses.



2.3 Discussion to test the hypothesis

It is obvious from the plot of Figure 1 of average zero one losses and standard errors of the two algorithms (SVM and BT) that their performances are very different from each other. This is because in none of the training set sizes the error bars in the plot for the two algorithms are overlapping each other as the plots are very far apart from each other.

To confirm this we perform paired t-test to compare their performances and to figure out whether the differences between performances are statistically significant. The t-test are performed using python statistics toolbox called scipy. Following are the results for each training set size.

For 2.5% training set size: The two-tailed P value equals 0.0010.

For 5% training set size: The two-tailed P value equals 0.0100.

For 12.5% training set size: The two-tailed P value equals 0.0002.

For 25% training set size: The two-tailed P value equals 0.0021.

The average of alpha = 0.05 is 0.05 / 4 = 0.0125.

The null hypothesis is rejected since all p is less than 0.0125. In all the cases null is rejected.

It can also be seen from the graph that SVM being a non parametric model is significantly outperforming the parametric models (DT and ensembles) when the size of training set is very low (100). While this difference is not that significant when the training set size is increased to 500. May be if the training set size is further improved the ensembles will outperform SVM.

3 Analysis 2

3.1 Learning Curves

In this analysis we vary the feature set size — the number of words to consider while building the model. The comparative result while varying number of features is shown in Figure 2.

On x-axis we vary the feature set sizes. On y-axis is the zero one loss. We have five plots — one for each model with different color. The vertical bars show the standard error calculated as described in the homework handout. The values of zero one loss for each of the four feature set size averaged across 10 fold for each of the five models along with the corresponding standard errors are given below. The first array shows the average 0/1 losses while the second array shows the corresponding standard errors.

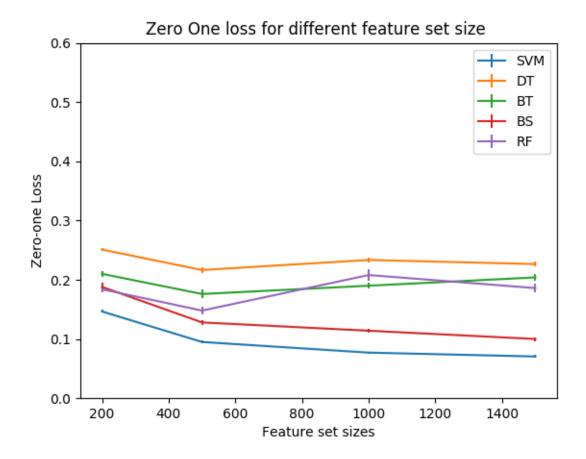
The above values are calculated from the following zero one loss values for varying training set size each having 10 folds:

ZERO-ONE-LOSS-DT 0.24 ZERO-ONE-LOSS-BT 0.28 ZERO-ONE-LOSS-RF 0.14 ZERO-ONE-LOSS-BS 0.32 ZERO-ONE-LOSS-SVM 0.105 ZERO-ONE-LOSS-DT 0.215 ZERO-ONE-LOSS-BT 0.14 ZERO-ONE-LOSS-RF 0.1 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.14 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.14 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-SVM 0.145 ZERO-ONE-LOSS-DT 0.225 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-SVM 0.145 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-SVM 0.11 ZERO-ONE-LOSS-DT 0.24 ZERO-ONE-LOSS-BT 0.26 ZERO-ONE-LOSS-RF 0.24 ZERO-ONE-LOSS-BS 0.22 ZERO-ONE-LOSS-DT 0.305 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-RF 0.18 ZERO-ONE-LOSS-BS 0.22 ZERO-ONE-LOSS-SVM 0.17 ZERO-ONE-LOSS-SVM 0.16 ZERO-ONE-LOSS-BT 0.22 ZERO-ONE-LOSS-BT 0.22 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-SVM 0.13 ZERO-ONE-LOSS-DT 0.29 ZERO-ONE-LOSS-BT 0.22 ZERO-ONE-LOSS-BT 0.22 ZERO-ONE-LOSS-BT 0.14 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-SVM 0.155 ZERO-ONE-LOSS-DT 0.215 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-BT 0.28 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-BT 0.28 ZERO-ONE-LOSS-BT 0.28 ZERO-ONE-LOSS-BS 0.28 ZERO-ONE-LOSS-BT 0.28 ZERO-ONE-LOSS-BT 0.28 ZERO-ONE-LOSS-BT 0.28 ZERO-ONE-LOSS-BS 0.28 ZERO-ONE-LOS

ZERO-ONE-LOSS-DT 0.18 ZERO-ONE-LOSS-BT 0.26 ZERO-ONE-LOSS-RF 0.16 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-SVM 0.075 ZERO-ONE-LOSS-DT 0.21 ZERO-ONE-LOSS-BT 0.06 ZERO-ONE-LOSS-RF 0.06 ZERO-ONE-LOSS-BS 0.14 ZERO-ONE-LOSS-SVM 0.105 ZERO-ONE-LOSS-DT 0.28 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-RF 0.24 ZERO-ONE-LOSS-BS 0.2 ZERO-ONE-LOSS-SVM 0.1 ZERO-ONE-LOSS-DT 0.265 ZERO-ONE-LOSS-BT 0.08 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-SVM 0.075 ZERO-ONE-LOSS-DT 0.17 ZERO-ONE-LOSS-BT 0.3 ZERO-ONE-LOSS-RF 0.1 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-SVM 0.105 ZERO-ONE-LOSS-DT 0.275 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-RF 0.26 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-SVM 0.09 ZERO-ONE-LOSS-DT 0.19 ZERO-ONE-LOSS-BT 0.2 ZERO-ONE-LOSS-BT 0.14 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-RF 0.18 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.13 ZERO-ONE-LOSS-DT 0.25 ZERO-ONE-LOSS-BT 0.14 ZERO-ONE-LOSS-BS 0.14 ZERO-ONE-LOSS-SVM 0.105

ZERO-ONE-LOSS-DT 0.2 ZERO-ONE-LOSS-BT 0.26 ZERO-ONE-LOSS-RF 0.2 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-SVM 0.035 ZERO-ONE-LOSS-DT 0.17 ZERO-ONE-LOSS-BT 0.1 ZERO-ONE-LOSS-RF 0.14 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.07 ZERO-ONE-LOSS-DT 0.255 ZERO-ONE-LOSS-BT 0.14 ZERO-ONE-LOSS-BT 0.14 ZERO-ONE-LOSS-RF 0.1 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.07 ZERO-ONE-LOSS-DT 0.22 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-RF 0.1 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-SVM 0.075 ZERO-ONE-LOSS-DT 0.215 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-SVM 0.105 ZERO-ONE-LOSS-DT 0.215 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-RF 0.08 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-SVM 0.1 ZERO-ONE-LOSS-DT 0.255 ZERO-ONE-LOSS-BT 0.22 ZERO-ONE-LOSS-RF 0.26 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.055 ZERO-ONE-LOSS-DT 0.325 ZERO-ONE-LOSS-BT 0.26 ZERO-ONE-LOSS-DT 0.235 ZERO-ONE-LOSS-BS 0.14 ZERO-ONE-LOSS-SVM 0.075 ZERO-ONE-LOSS-DT 0.235 ZERO-ONE-LOSS-BS 0.14 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-DT 0.245 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-DT 0.245 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-DT 0.245 ZERO-ONE-LOSS-BS 0.18 ZERO-ONE-

ZERO-ONE-LOSS-DT 0.23 ZERO-ONE-LOSS-BT 0.22 ZERO-ONE-LOSS-RF 0.22 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.04 ZERO-ONE-LOSS-DT 0.275 ZERO-ONE-LOSS-BT 0.22 ZERO-ONE-LOSS-RF 0.08 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.09 ZERO-ONE-LOSS-DT 0.305 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-RF 0.32 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.08 ZERO-ONE-LOSS-DT 0.19 ZERO-ONE-LOSS-BT 0.1 ZERO-ONE-LOSS-BT 0.1 ZERO-ONE-LOSS-BS 0.04 ZERO-ONE-LOSS-SVM 0.04 ZERO-ONE-LOSS-DT 0.215 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-RF 0.22 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-SVM 0.055 ZERO-ONE-LOSS-DT 0.27 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-RF 0.22 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-SVM 0.07 ZERO-ONE-LOSS-DT 0.14 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-SVM 0.07 ZERO-ONE-LOSS-DT 0.205 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE



3.2 Hypothesis

Let's compare SVM with RF.

Null hypothesis is that both are equivalent in terms of performance. This means that the mean of the SVM 0/1 losses is equivalent to the mean of the RF 0/1 losses. It is assumed to be true.

While the alternative hypothesis is that SVM and RF zero one losses are significantly different. Or, that the mean of the SVM zero one losses is lesser than the mean of the RF 0/1 losses.

3.3 Discussion to test the hypothesis

It can be seen from the average zero one loss and standard error values for the two algorithms that the differences are significant. The plots do not overlap each other.

We perform t-test to compare the performance. The results for different feature set sizes are given below:

For 200 feature set size: The two-tailed P value equals 0.0010

For 500 feature set size: The two-tailed P value equals 0.0035

For 1000 feature set size: The two-tailed P value equals 0.0019

For 1500 feature set size: The two-tailed P value equals 0.0012

Since p values are always lesser than 0.05 / 4 = 0.0125 therefore null is rejected and we conclude SVM mean losses are lesser than RF.

4 Analysis 3

4.1 Learning Curves

In this analysis we vary the depth limit of the trees in DT, and its ensembles. The comparative result while varying number of features is shown in Figure 3.

On x-axis we vary the depth of the tree. On y-axis is the zero one loss. We have five plots — one for each model with different color. The vertical bars show the standard error calculated as described in the homework handout. The values of zero one loss for each of the four depth limits averaged accross 10 fold for each of the five models along with the corresponding standard errors are given below. The first array shows the average 0/1 losses while the second array shows the corresponding standard errors.

=====SVM======

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 \begin{bmatrix} 0.07150000000000001, \ 0.078, \ 0.07100000000000001, \ 0.0750000000000000002] \\ [0.0015337861650177966, \ 0.001345362404707371, \ 0.0011789826122551598, \ 0.0014317821063276352] \\ ======DT===== \\ \\ [0.28450000000000003, \ 0.2350000000000001, \ 0.192, \ 0.2295000000000000004] \\ [0.0047457876058669126, \ 0.0048114446894877635, \ 0.00440000000000003, \ 0.0026781523481684165] \\ ======BT===== \\ \\ [0.238, \ 0.16, \ 0.16599999999999999, \ 0.158] \\ [0.0070682388188289171, \ 0.00800000000000002, \ 0.008002499609497022, \ 0.0060959002616512688] \\ ======RF===== \\ \\ [0.314, \ 0.182000000000000005, \ 0.15400000000000003, \ 0.12200000000000003] \\ [0.0071582120672693123, \ 0.0089643739324059887, \ 0.0072691127381544996, \ 0.0047707441767506258] \\ ======BS=====
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 $[0.094,\, 0.08199999999999999,\, 0.098,\, 0.101999999999999999]$

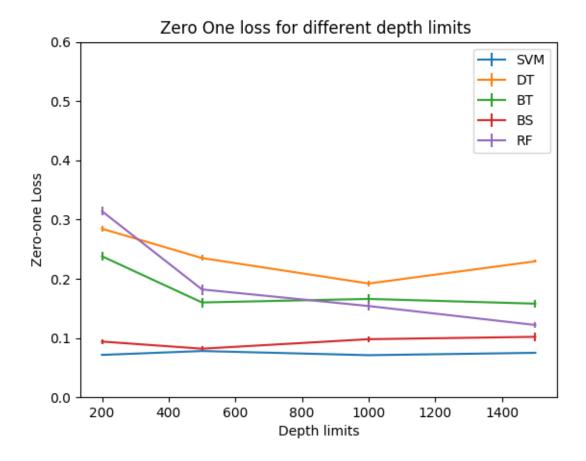
[0.0044766058571198784, 0.0036276714294434112, 0.0045122056690713921, 0.0070682388188289163] The above values are calculated from the following zero one loss values for varying training set size each having 10 folds:

 ZERO-ONE-LOSS-DT 0.26 ZERO-ONE-LOSS-BT 0.2 ZERO-ONE-LOSS-RF 0.12 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-SVM 0.085 ZERO-ONE-LOSS-DT 0.195 ZERO-ONE-LOSS-BT 0.2 ZERO-ONE-LOSS-RF 0.14 ZERO-ONE-LOSS-BS 0.04 ZERO-ONE-LOSS-SVM 0.09 ZERO-ONE-LOSS-DT 0.295 ZERO-ONE-LOSS-BT 0.22 ZERO-ONE-LOSS-RF 0.32 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.105 ZERO-ONE-LOSS-DT 0.31 ZERO-ONE-LOSS-BT 0.28 ZERO-ONE-LOSS-RF 0.14 ZERO-ONE-LOSS-BS 0.14 ZERO-ONE-LOSS-SVM 0.085 ZERO-ONE-LOSS-DT 0.225 ZERO-ONE-LOSS-BT 0.06 ZERO-ONE-LOSS-RF 0.2 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.06 ZERO-ONE-LOSS-DT 0.24 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-RF 0.26 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.08 ZERO-ONE-LOSS-DT 0.15 ZERO-ONE-LOSS-BT 0.06 ZERO-ONE-LOSS-RF 0.08 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.075 ZERO-ONE-LOSS-DT 0.18 ZERO-ONE-LOSS-BT 0.04 ZERO-ONE-LOSS-RF 0.08 ZERO-ONE-LOSS-BS 0.02 ZERO-ONE-LOSS-SVM 0.065 ZERO-ONE-LOSS-DT 0.225 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-RF 0.14 ZERO-ONE-LOSS-BS 0.04 ZERO-ONE-LOSS-SVM 0.06 ZERO-ONE-LOSS-DT 0.27 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-RF 0.34 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-DT 0.18 ZERO-ONE-LOSS-BT 0.22 ZERO-ONE-LOSS-RF 0.28 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-SVM 0.075 ZERO-ONE-LOSS-DT 0.22 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-RF 0.1 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.085 ZERO-ONE-LOSS-DT 0.26 ZERO-ONE-LOSS-BT 0.26 ZERO-ONE-LOSS-RF 0.2 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-SVM 0.065 ZERO-ONE-LOSS-DT 0.195 ZERO-ONE-LOSS-BT 0.22 ZERO-ONE-LOSS-RF 0.14 ZERO-ONE-LOSS-BS 0.14 ZERO-ONE-LOSS-SVM 0.085 ZERO-ONE-LOSS-DT 0.155 ZERO-ONE-LOSS-BT 0.02 ZERO-ONE-LOSS-RF 0.02 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.065 ZERO-ONE-LOSS-DT 0.22 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-RF 0.2 ZERO-ONE-LOSS-BS 0.14 ZERO-ONE-LOSS-SVM 0.08 ZERO-ONE-LOSS-DT 0.23 ZERO-ONE-LOSS-BT 0.2 ZERO-ONE-LOSS-RF 0.22 ZERO-ONE-LOSS-BS 0.04 ZERO-ONE-LOSS-SVM 0.08 ZERO-ONE-LOSS-DT 0.125 ZERO-ONE-LOSS-BT 0.02 ZERO-ONE-LOSS-RF 0.12 ZERO-ONE-LOSS-BS 0.02 ZERO-ONE-LOSS-SVM 0.065 ZERO-ONE-LOSS-DT 0.215 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-RF 0.18 ZERO-ONE-LOSS-BS 0.14 ZERO-ONE-LOSS-SVM 0.065 ZERO-ONE-LOSS-DT 0.12 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-RF 0.08 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-RF 0.14 ZERO-ONE-LOSS-BS 0.04 ZERO-ONE-LOSS-SVM 0.075 ZERO-ONE-LOSS-DT 0.21 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-RF 0.04 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-SVM 0.095 ZERO-ONE-LOSS-DT 0.265 ZERO-ONE-LOSS-BT 0.26 ZERO-ONE-LOSS-RF 0.16 ZERO-ONE-LOSS-BS 0.3 ZERO-ONE-LOSS-SVM 0.095 ZERO-ONE-LOS ONE-LOSS-DT 0.19 ZERO-ONE-LOSS-BT 0.28 ZERO-ONE-LOSS-RF 0.18 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.07 ZERO-ONE-LOSS-DT 0.26 ZERO-ONE-LOSS-BT 0.14 ZERO-ONE-LOSS-RF 0.12 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.09 ZERO-ONE-LOSS-DT 0.225 ZERO-ONE-LOSS-BT 0.14 ZERO-ONE-LOSS-RF 0.12 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.07 ZERO-ONE-LOSS-DT 0.265 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-RF 0.2 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.08 ZERO-ONE-LOSS-DT 0.21 ZERO-ONE-LOSS-BT 0.08 ZERO-ONE-LOSS-RF 0.1 ZERO-ONE-LOSS-BS 0.04 ZERO-ONE-LOSS-SVM 0.065 ZERO-ONE-LOSS-DT 0.2 ZERO-ONE-LOSS-BT 0.14 ZERO-ONE-LOSS-RF 0.1 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.06 ZERO-ONE-LOSS-DT 0.22 ZERO-ONE-LOSS-BT 0.1 ZERO-ONE-LOSS-RF 0.06 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-SVM 0.05

4.2 Hypothesis

Let's compare BT with DT.

Null hypothesis is that both are equivalent in terms of performance. That is the mean of the zero



one losses of BT is equivalent to the mean of the zero one losses of the DT. It is assumed to be true. While the alternative hypothesis is that BT and DT zero one losses are significantly different. Or, that the mean of the BT zero one losses is lesser than the mean of the DT 0/1 losses. This is because the ensemble BT reduces error due to variance.

4.3 Discussion to test the hypothesis

We perform t-test to compare the performance. The results for different depths are given below: For 5 depth: The two-tailed P value equals 0.0256

For 10 depth: The two-tailed P value equals 0.0017

For 15 depth: The two-tailed P value equals 0.0325

For 20 depth: The two-tailed P value equals 0.0019

From the above testing it can be seen that null is rejected only in two cases (10 and 20). While it is accepted for 5 and 15 depths. Null is only rejected when p values is less than 0.0125.

5 Analysis 4

5.1 Learning Curves

In this analysis we vary the number of trees in ensembles of DT. The comparative result while varying number of features is shown in Figure 4.

On x-axis we vary the number of trees considered. On y-axis is the zero one loss. We have five plots — one for each model with different color. The vertical bars show the standard error calculated as described in the homework handout. The values of zero one loss for each of the four different numbers of trees averaged across 10 fold for each of the five models along with the corresponding standard errors are given below. The first array shows the average 0/1 losses while the second array shows the corresponding standard errors.

The above values are calculated from the following zero one loss values for varying training set size each having 10 folds:

======== ZERO-ONE-LOSS-DT 0.315 ZERO-ONE-LOSS-BT 0.5 ZERO-ONE-LOSS-RF 0.5 ZERO-ONE-LOSS-BS 0.3 ZERO-ONE-LOSS-SVM 0.065 ZERO-ONE-LOSS-DT 0.235 ZERO-ONE-LOSS-BT 0.2 ZERO-ONE-LOSS-RF 0.4 ZERO-ONE-LOSS-BS 0.3 ZERO-ONE-LOSS-SVM 0.09 ZERO-ONE-LOSS-DT 0.18 ZERO-ONE-LOSS-BT 0.1 ZERO-ONE-LOSS-RF 0.1 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.07 ZERO-ONE-LOSS-DT 0.205 ZERO-ONE-LOSS-BT 0.3 ZERO-ONE-LOSS-RF 0.3 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.09 ZERO-ONE-LOSS-DT 0.245 ZERO-ONE-LOSS-BT 0.6 ZERO-ONE-LOSS-RF 0.1 ZERO-ONE-LOSS-BS 0.2 ZERO-ONE-LOSS-SVM 0.07 ZERO-ONE-LOSS-DT 0.23 ZERO-ONE-LOSS-BT 0.3 ZERO-ONE-LOSS-RF 0.1 ZERO-ONE-LOSS-BS 0.0 ZERO-ONE-LOSS-SVM 0.05 ZERO-ONE-LOSS-DT 0.18 ZERO-ONE-LOSS-BT 0.1 ZERO-ONE-LOSS-RF 0.1 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.085 ZERO-ONE-LOSS-DT 0.275 ZERO-ONE-LOSS-BT 0.0 ZERO-ONE-LOSS-RF 0.4 ZERO-ONE-LOSS-BS 0.0 ZERO-ONE-LOSS-SVM 0.08 ZERO-ONE-LOSS-DT 0.195 ZERO-ONE-LOSS-BT 0.2 ZERO-ONE-LOSS-RF 0.4 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.03 ZERO-ONE-LOSS-DT 0.255 ZERO-ONE-LOSS-BT 0.2 ZERO-ONE-LOSS-RF 0.4 ZERO-ONE-LOSS-BS 0.2 ZERO-ONE-LOSS-SVM 0.075 ============================ ZERO-ONE-LOSS-DT 0.185 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-RF 0.04 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-SVM 0.075 ZERO-ONE-LOSS-DT 0.225 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-RF 0.28 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.135 ZERO-

ONE-LOSS-DT 0.155 ZERO-ONE-LOSS-BT 0.2 ZERO-ONE-LOSS-RF 0.12 ZERO-ONE-LOSS-BS

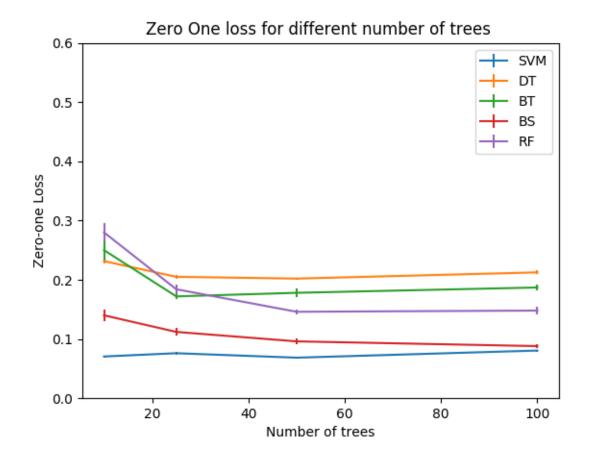
0.12 ZERO-ONE-LOSS-SVM 0.07 ZERO-ONE-LOSS-DT 0.21 ZERO-ONE-LOSS-BT 0.2 ZERO-ONE-LOSS-RF 0.36 ZERO-ONE-LOSS-BS 0.24 ZERO-ONE-LOSS-SVM 0.095 ZERO-ONE-LOSS-DT 0.25 ZERO-ONE-LOSS-BT 0.08 ZERO-ONE-LOSS-RF 0.16 ZERO-ONE-LOSS-BS 0.0 ZERO-ONE-LOSS-SVM 0.05 ZERO-ONE-LOSS-DT 0.225 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-RF 0.24 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-SVM 0.08 ZERO-ONE-LOSS-DT 0.18 ZERO-ONE-LOSS-BT 0.2 ZERO-ONE-LOSS-RF 0.2 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.075 ZERO-ONE-LOSS-DT 0.25 ZERO-ONE-LOSS-BT 0.2 ZERO-ONE-LOSS-RF 0.12 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-SVM 0.07 ZERO-ONE-LOSS-DT 0.17 ZERO-ONE-LOSS-BT 0.2 ZERO-ONE-LOSS-RF 0.16 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-SVM 0.05 ZERO-ONE-LOSS-DT 0.2 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-RF 0.16 ZERO-ONE-LOSS-BS 0.04 ONE-LOSS-DT 0.17 ZERO-ONE-LOSS-BT 0.14 ZERO-ONE-LOSS-RF 0.12 ZERO-ONE-LOSS-BS 0.14 ZERO-ONE-LOSS-SVM 0.065 ZERO-ONE-LOSS-DT 0.205 ZERO-ONE-LOSS-BT 0.2 ZERO-ONE-LOSS-RF 0.22 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.09 ZERO-ONE-LOSS-DT 0.19 ZERO-ONE-LOSS-BT 0.1 ZERO-ONE-LOSS-RF 0.14 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.055 ZERO-ONE-LOSS-DT 0.235 ZERO-ONE-LOSS-BT 0.22 ZERO-ONE-LOSS-RF 0.12 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.07 ZERO-ONE-LOSS-DT 0.205 ZERO-ONE-LOSS-BT 0.14 ZERO-ONE-LOSS-RF 0.22 ZERO-ONE-LOSS-BS 0.08 ZERO-ONE-LOSS-SVM 0.085 ZERO-ONE-LOSS-DT 0.205 ZERO-ONE-LOSS-BT 0.28 ZERO-ONE-LOSS-RF 0.16 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-SVM 0.07 ZERO-ONE-LOSS-DT 0.205 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-RF 0.14 ZERO-ONE-LOSS-BS 0.1 ZERO-ONE-LOSS-SVM 0.06 ZERO-ONE-LOSS-DT 0.205 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-RF 0.12 ZERO-ONE-LOSS-BS 0.04 ZERO-ONE-LOSS-SVM 0.075 ZERO-ONE-LOSS-DT 0.2 ZERO-ONE-LOSS-BT 0.28 ZERO-ONE-LOSS-RF 0.14 ZERO-ONE-LOSS-BS 0.16 ZERO-ONE-LOSS-SVM 0.055 ZERO-ONE-LOSS-DT 0.2 ZERO-ONE-LOSS-BT 0.06 ZERO-ONE-LOSS-RF 0.08 ZERO-ONE-LOSS-BS 0.04 ZERO-ONE-LOSS-DT 0.23 ZERO-ONE-LOSS-BT 0.18 ZERO-ONE-LOSS-RF 0.11 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-SVM 0.075 ZERO-ONE-LOSS-DT 0.16 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-RF 0.18 ZERO-ONE-LOSS-BS 0.11 ZERO-ONE-LOSS-SVM 0.1 ZERO-ONE-LOSS-DT 0.225 ZERO-ONE-LOSS-BT 0.12 ZERO-ONE-LOSS-RF 0.07 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-SVM 0.06 ZERO-ONE-LOSS-DT 0.205 ZERO-ONE-LOSS-BT 0.15 ZERO-ONE-LOSS-RF 0.28 ZERO-ONE-LOSS-BS 0.12 ZERO-ONE-LOSS-SVM 0.09 ZERO-ONE-LOSS-DT 0.255 ZERO-ONE-LOSS-BT 0.24 ZERO-ONE-LOSS-RF 0.16 ZERO-ONE-LOSS-BS 0.07 ZERO-ONE-LOSS-SVM 0.1 ZERO-ONE-LOSS-DT 0.185 ZERO-ONE-LOSS-BT 0.16 ZERO-ONE-LOSS-RF 0.19 ZERO-ONE-LOSS-BS~0.05~ZERO-ONE-LOSS-SVM~0.05~ZERO-ONE-LOSS-DT~0.165~ZERO-ONE-LOSS-BT~0.19ZERO-ONE-LOSS-RF 0.16 ZERO-ONE-LOSS-BS 0.11 ZERO-ONE-LOSS-SVM 0.075 ZERO-ONE-LOSS-DT 0.245 ZERO-ONE-LOSS-BT 0.21 ZERO-ONE-LOSS-RF 0.14 ZERO-ONE-LOSS-BS 0.11 ZERO-ONE-LOSS-SVM 0.085 ZERO-ONE-LOSS-DT 0.255 ZERO-ONE-LOSS-BT 0.25 ZERO-ONE-LOSS-RF 0.12 ZERO-ONE-LOSS-BS 0.07 ZERO-ONE-LOSS-SVM 0.08 ZERO-ONE-LOSS-DT 0.2 ZERO-ONE-LOSS-BT 0.13 ZERO-ONE-LOSS-RF 0.07 ZERO-ONE-LOSS-BS 0.06 ZERO-ONE-LOSS-SVM 0.09

5.2 Hypothesis

Let's compare RF with DT.

Null hypothesis is that both are equivalent in terms of performance. That is the mean of the zero one losses of RF is equivalent to the mean of the zero one losses of the DT. It is assumed to be true. While the alternative hypothesis is that RF and DT zero one losses are significantly different. Or,

that the mean of the RF zero one losses is lesser than the mean of the DT 0/1 losses. This is because



the ensemble RF reduces error due to variance.

5.3 Discussion to test the hypothesis

We perform t-test to compare the performance. The results for different number of trees are given below:

For 10: The two-tailed P value equals 0.3017

For 25: The two-tailed P value equals 0.4728

For 50: The two-tailed P value equals 0.3599

For 100: The two-tailed P value equals 0.0343

Since the p values are always greater than 0.0125 we conclude that null is accepted.

6 Bonus part

Boosted decision trees (BS) are also implemented using the tutorial mentioned at:

https://engineering.purdue.edu/kak/Tutorials/AdaBoost.pdf

The results are included in the figures mentioned above.

6.1 Hypothesis 1

Let's compare SVM with BS using cross validation on training set sizes.

Null hypothesis is that both are equivalent in terms of performance. That is the mean of the zero one losses of SVM is equivalent to the mean of the zero one losses of the BS. It is assumed to be true.

While the alternative hypothesis is that SVM and BS zero one losses are significantly different. Or, that the mean of the SVM zero one losses is lesser than the mean of the BS 0/1 losses.

6.2 Discussion to test the hypothesis 1

We perform t-test to compare the performance. The results for different training set sizes are given below: For training percents of [2.5, 5, 12.5, 25] p values are 0.0182, 0.8287, 0.0107, 0.435

Sometimes null is accepted and sometimes it is rejected. It depends on whether the p values is less than 0.0125 or greater than 0.0125. If p is less than 0.0125 null is rejected.

6.3 Hypothesis 1

Let's compare BS with RF while doing cross validation on training set size.

Null hypothesis is that both are equivalent in terms of performance. That is the mean of the zero one losses of RF is equivalent to the mean of the zero one losses of the BS. It is assumed to be true.

While the alternative hypothesis is that RF and BS zero one losses are significantly different. Or, that the mean of the BS zero one losses is lesser than the mean of the RF 0/1 losses.

6.4 Discussion to test the hypothesis 1

We perform t-test to compare the performance. The results for different training set sizes are given below:

For 2.5: The two-tailed P value equals 0.0292

For 5: The two-tailed P value equals 0.0002

For 12.5: The two-tailed P value equals 0.6899

For 25: The two-tailed P value equals 0.0343

Sometimes null is accepted and sometimes it is rejected. It depends on whether the p values is less than 0.0125 or greater than 0.0125. If p is less than 0.0125 null is rejected.

7 Decomposition

If h(x) is the prediction label of an example x and y is true label. Then expected mean squared error is:

$$E[MSE] = E[y - h(x)]^2 \tag{1}$$

We need to decompose this into noise, bias, and variance.

$$E[(h(x) - y)^{2}] = E[h(x)^{2} - 2h(x)y + y^{2}] = E[h(x)^{2}] - 2E[h(x)]E[y] + E[y]$$
(2)

We know that:

$$E[Z^{2}] = E[(Z - Z^{*})^{2}] + Z^{*2}$$
(3)

where Z is a random variable and Z^* is the mean of Z. Therefore,

$$E[(h(x) - y)^{2}] = E[h(x) - h(x)^{*}] + h(x)^{*2} - 2h(x)^{*}f(x) + E[(f(x) - y)^{2}] + f(x)^{2}$$
(4)

$$= E[(h(x) - h(x)^*)^2] + (h(x) - f(x))^2 + E[(f(x) - y)^2]$$
(5)

$$= Variance + bias^2 + noise (6)$$