Pergamon

Computers Math. Applic. Vol. 32, No. 1, pp. 145–173, 1996

0898-1221/96 $15.00 + 0.00

S0898-1221(96)00097-1

The Sounds of Science: II. Listening to Dynamical Systems— Towards a Musical Exploration of Complexity

M. WITTEN
Computational Medicine and Public Health Group, IOG
University of Michigan, Ann Arbor
300 North Ingalls, Ann Arbor, MI 48109-2007, U.S.A.
mwitten@umich.edu

Abstract—General philosophical aspects of the rendering of scientific data are discussed, with applications to auditory representations of scientific models. Scientific simulations or models of *real world* phenomena frequently lead to large quantities of complex numerical output. The generating mathematical/computer model is often realized through a set of *static* and/or (frequently) *dynamic* descriptors. The *rendering* of this descriptor set is formally considered in terms of *mappings* which result in visual, sonic, etc. *sensorial* stimulation of some sort. In this paper, we concern ourselves with the issue of rendering of complex descriptor sets. The emphasis is on sonic rendering as viewed from both an artistic and a scientific viewpoint. We examine the problem of finding and creating musical compositions based upon such scientific formulations.

Keywords—Scientific visualization, MIDI, Electronic music, Computer generated music, Dynamical systems.

1. PRINCIPIA

In a past panel discussion on scientific visualization, at Supercomputing 1991, the importance of terminology, as well as methodology, was pointed out by a number of the panel participants. Witten pointed out that we ought to drop the use of the term *visualization* in the phrase scientific visualization, as this has the implication of being only *visual* as opposed to using all of the senses. Witten argues for the use of the term *scientific realization*, while D. Cox at NCSA argued for the use of the term *viscerization*. Others have proposed using the terms *sonification*, *audification*, *numerosonics*, and *sonometrics* (the last two suggested by T. Mikiten) as identifiers for the field of *sonic visualization*, the idea of using sound to aid in our understanding of data and of what it is telling us about our model. However, the use of scientific data and models as a creative tools has been both little discussed and implemented. In order for us to understand this perspective, it behooves us to carefully examine the meaning of the various terms we will use in our discussion.

We begin our discussion by considering that an artist's creative capabilities derive, in part, from his understanding of space and time, as well as the inherent relationships therein contained.

I would like to thank P. Lansky for his discussions on my original work *BrainGlow* presented at the 1991 Nashville USENIX Conference. I would like to thank R. Pinkston for his discussions and his ongoing support of this research effort. I would like to thank T. Eakin for his discussion on the mathematical aspects of this paper, particularly for his unique insight into the unfolding mapping. I would like to thank G. Liu for her efforts in rendering the figures for this paper, R. Shouman for executing the three-dimensional graphics, V. Nair for executing the neural graphics, and J. Richardson of the UT System Center for High Performance Computing for executing the molecular modeling figures for me.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

Understanding is tied to an initial *perception*[1] of space and time. It is also tied to the artist's personal life history as perceptions result from the interaction of prior experience and current situations filtered through the artist's world metaphor. Awareness, however, implies only that. We must internalize that awareness as a mental image or sound. This *representation*[2] may be obtained by a *metaphorical representation*. That is, we use the concept of equivalence class or similarity class to allow us to represent one object or sound in terms of another. Artistically, we are creating a *metaphor*[3] of the system in that the mind, abstracting various aspects/properties about the physical object, selects an internal, previously defined representation and deems the new object *metaphorically equivalent*. We will discuss the formalization of the term *metaphorically equivalent* in an upcoming manuscript. Alternatively, the mind may have no *a priori* structure on which to map its perception of the object and it must, from first principles, create an internal representation.

Once we have defined an internal representation, either visually, sonically, or using a combined sensorial representation, we then wish to externalize what we have seen/heard in our mind's eye/ear. The process of externalizing requires that we first cause our mental concept to seem real. *Realization*[4] differs from *actualization*[5] in that actualization forces existence in fact or reality. When we attempt to represent and realize, we do this with our senses and our cognitive processes. When we do this visually, the process is called *visualization*;[6] when sound senses are involved, the terminology is a bit more ambiguous. The field has settled on *sonification*[7] as the word of choice. We will use that for the purposes of discussion. These definitions formalize the development of a concept and its transfer into fact or a seeming reality. They do not give leeway for artistic impression. We do this by a process of *rendering*.[8] These rather abstract/conceptual definitions may be formalized in a more rigorous fashion. We begin with the definition of a *model*.[9] In this context, we may refine our definition for a *realization* by using the following definition.

DEFINITION. **Realization**: *Let $f : \Omega \to \Gamma$, where both Ω and Γ are vector spaces, be an input/output relation. Let X be a state-space. Assume there exist mappings $g : \Omega \to X$ and $h : X \to \Gamma$ such that the diagram*

$$\begin{tikzcd} \Omega \arrow[r, "f"] \arrow[dr, "g"'] & \Gamma \\ & X \arrow[u, "h"'] \end{tikzcd} \tag{1.1}$$

commutes. Then, assuming $x_0 \in X$ is an initial state of the system, we have that

$$\Sigma \equiv (X, g, h, x_0)$$

[1]Perception: A mental image or awareness of elements of the physical environment.

[2]Representation: An image of an object or entity; an idea in the mind, distinct from the external object/entity which is the occasion of the perception.

[3]Metaphor: The result of denoting one kind of object or idea in place of another to suggest likeness or analogy between them.

[4]Realization: The result of bringing into concrete existence; causing to seem real.

[5]Actualization: The result of causing to exist in fact or reality.

[6]Visualization: The result of the formation of mental visual images; the result of the process of interpreting in visual terms or putting into visual form.

[7]Sonification: The result of the formation of mental sound images; the result of the process of interpreting in sonic terms or putting into sonic form.

[8]Rendering: The result of reproducing or representing by artistic or by verbal means.

[9]Model: A model $M_\oplus$ is a *realization* of a system $\mathbb{S}$ in which

(1) there exists a clear and direct correspondence between the states of the model and the states of the system, and
(2) the model $M_\oplus$ is simpler than the system $\mathbb{S}$ in that $\mathbb{S}$ is capable of more behaviors than $M_\oplus$.

is the realization of the input/output relation $\mathcal{I}$

$$\mathcal{I} \equiv (\Omega, \Gamma, f).$$

Building mathematical models of biological/medical systems is both an art as well as a science. It is interesting to observe that there is an *art* to this scientific endeavor. We do not know the real world dynamics $\mathbb{W}_R(\mathbb{S})$ of a particular system $\mathbb{S}$. At best, our *cognitive understanding* of the real world $\mathbb{S}$ is based upon our knowledge $\mathbb{W}_{KR}(\mathbb{S})$ of the system $\mathbb{S}$. This knowledge is gained by our observation $\mathcal{O}(t)$ of the real world system $\mathbb{S}$ through various experimental interventions. That is,

$$\mathcal{O}(t) : \mathbb{W}_R(\mathbb{S}) \longrightarrow \mathbb{W}_{KR}(\mathbb{S}). \tag{1.2}$$

We take this knowledge $\mathbb{W}_{KR}(\mathbb{S})$ and subsequently develop a cognitive understanding or model of $\mathbb{S}$, denoted $\mathbb{W}_{\mathrm{CU}}(\mathbb{S})$. This cognitive understanding is then turned into one or more metaphors $M_\oplus$ of that understanding.

$$\mathbb{W}_R(\mathbb{S}) \longrightarrow \mathbb{W}_{KR}(\mathbb{S}) \longrightarrow \mathbb{W}_{\mathrm{CU}}(\mathbb{S}). \tag{1.3}$$

In this case, I is a set of mappings called internalizers. These represent mappings/operations that we perform, internally, so as to obtain our cognitive understanding of things. Verbally, we might diagram this as follows:

$$\text{metaphor} \longrightarrow \text{representation} \longrightarrow \text{realization} \longrightarrow \text{model}. \tag{1.4}$$

However, the more accurate our metaphor, the closer it is to a representation, of which models are a special subcase, $M_\oplus$ of that understanding. It is important to be aware of the fact that representations of knowledge may be one-to-many in that a particular cognitive understanding may be mapped into numerous mathematical representations $M_\oplus$, the set of which we denote by $\mathcal{M}$ and $M_\oplus \in \mathcal{M}$.

A model $M_\oplus$ is actually an n-tuple $M_\oplus = \langle t, \mathcal{D} \rangle$ where $\mathcal{D} = \mathcal{D}_S \cup \mathcal{D}_D$, t is time, $\mathcal{D}_S$ is the set of *model descriptors* which are *static* in the representation (they do not change in time) and $\mathcal{D}_D$ is the set of *dynamic* model descriptors (they do change in time). We will discuss these in greater detail in a moment. For now, we shall define what we now call *visualization* as the act of transferring (transforming) our cognitive understanding of the real world and a subsequent representation $M_\oplus$ to a sensoral realization $\mathcal{V}$. This realization $\mathcal{V}$, when rendered, is a visualization. Be wary that we have blatantly misused the terminology here. Within this context, is clear that a visualization $\mathcal{V}$ is the action of a mapping $\mathcal{A}$ (the actualization mapping) such that the cognitive descriptors in the cognitive model are mapped into the visualization descriptors.

2. INTRODUCTORY THOUGHTS

Scientific data is a goldmine for the creative mind [1–12]. On the one hand, it is possible to envision how a scientific dataset could be examined using musical tools and techniques so as to enhance understanding of the complexities of the data, and through this process, potentially enhance our understanding of the basic structure of the system originating the data. Scientific datasets are often large, complex, and beyond the scope of simple visual understanding. The idea of a picture being worth a thousand words is still relevant but no longer feasible, as datasets contain on the order of megawords of information as well as visual complexity exceeding the four or five independent variables most people are capable of visualizing. Hence, the idea of using sound or *sonic representation* to further enhance our understanding of complex datasets and their internal relationships is one that is of growing interest. (Witten and Wyatt [13] provide a summary review of the literature of the sonification field at the time of publication. Readers of that article should be made aware that the reference section was severely cut, at publication time, at the insistence of the journal editorial board. Hence, a large number of articles were removed because they were not immediately cited in the text.)

Whether the data sonification effort will ultimately lead to tools allowing a deeper understanding of scientific datasets remains to be seen. The field of data sonification is still in its neonatal stages of development and, like any new tool, it is still being played with, explored, and refined. As we explore this new tool, we must always keep in mind the question of whether *sensory enhancement of data* is meant to make the scientific model more real or whether it is meant to help us, as scientists, to derive relationships about the real-world system not known when the process of model building was initiated. We can easily imagine that adding sound to data might help us to delve through the complexities of a multidimensional scientific dataset. Multisensory expansion using *computational reality* tools may afford us yet more ability, to investigate potential relationships in data [13]. How we will do this is still open for experimentation.

There is, on the other hand, another less obvious side to scientific datasets. It is an aspect that is artistic in nature. The artistic process of scientific data sonification attempts to discover the intrinsic musical beauty within a scientific model or dataset and attempts to use this newly gained understanding to create a *data symphony* in the same sense that we now use fractal mathematics to create fractal art. As early as 1970, Dodge [14] used the earth's magnetic field as a basis for a musical composition. Many artists have used taped or digitized sounds as part of their musical creations. For example, Lansky's work, in particular his piece "Quakerbridge" [15], attempts to find music in and to imbue music to the sounds of the daily goings on in an area shopping mall. Here, Lansky not only paints/sculpts with sound, but also forms a collage of sounds (real and synthesized) to create a picture that integrates the reality of everyday life and a musical experience that evokes yet more that either synthesized sound or real world sound would yield separately. Thus, the union of the two facets contains more than its parts. Lansky's concept of finding and using the music of life represents the artistic side of what we are attempting to do, as scientists, by *using sound to amplify and clarify the information contained within a complex set of scientific data*. It is both striking and reassuring to see that there is such an increased fusion of art and science.

More recently, numerous artists have made use of the inherent stochastic nature of chaotic dynamical processes to create what might be termed *chaos based* musical compositions. Musical pieces driven by the seemingly unstructured and/or structured portions of dynamically generated data afford a tempting variety for the musical ear while opening the door to various compositional tools and techniques which may prove to expand our musical capabilities and experience. It is only through transdisciplinary research that we can begin to solve the more complex real-world problems that we need to address in today's complex scientific world. And it is from these investigations that we gain a doorway into a new creative world. In this vein, it is easy to envision how we might create a Molecular Symphony based upon molecular drug/receptor docking (Figure 1). Almost any process that generates three or more dimensional datasets can be rendered symphonically: a spiral wave of *D. discoideum*, the scrolls of Bénard, or the Brusselator's rotating waves of chemical oscillation. The phrase *can you hear what I see* becomes entirely more relevant with the advent of our increased capabilities in the audio domain. However, it only begins to touch *the deeper question of seeking and finding the music in scientific phenomena*.

It is this fusion of science and of electronic music composition that I wish to explore in this series of articles. In particular, I will attempt to fuse my own scientific training with my background as a performing artist in order to examine the artistic and creative side of using scientific data, mathematical algorithms and equations, and formal scientific concepts to examine the issue of finding an artistic component to the *sounds of science*. In this, the second in a series of papers (see [13] for the initial discussion), I will examine the issue of using time series or dynamical systems algorithms (linear or nonlinear) to create musical structures as well as how we might actually listen to "the sounds of science."

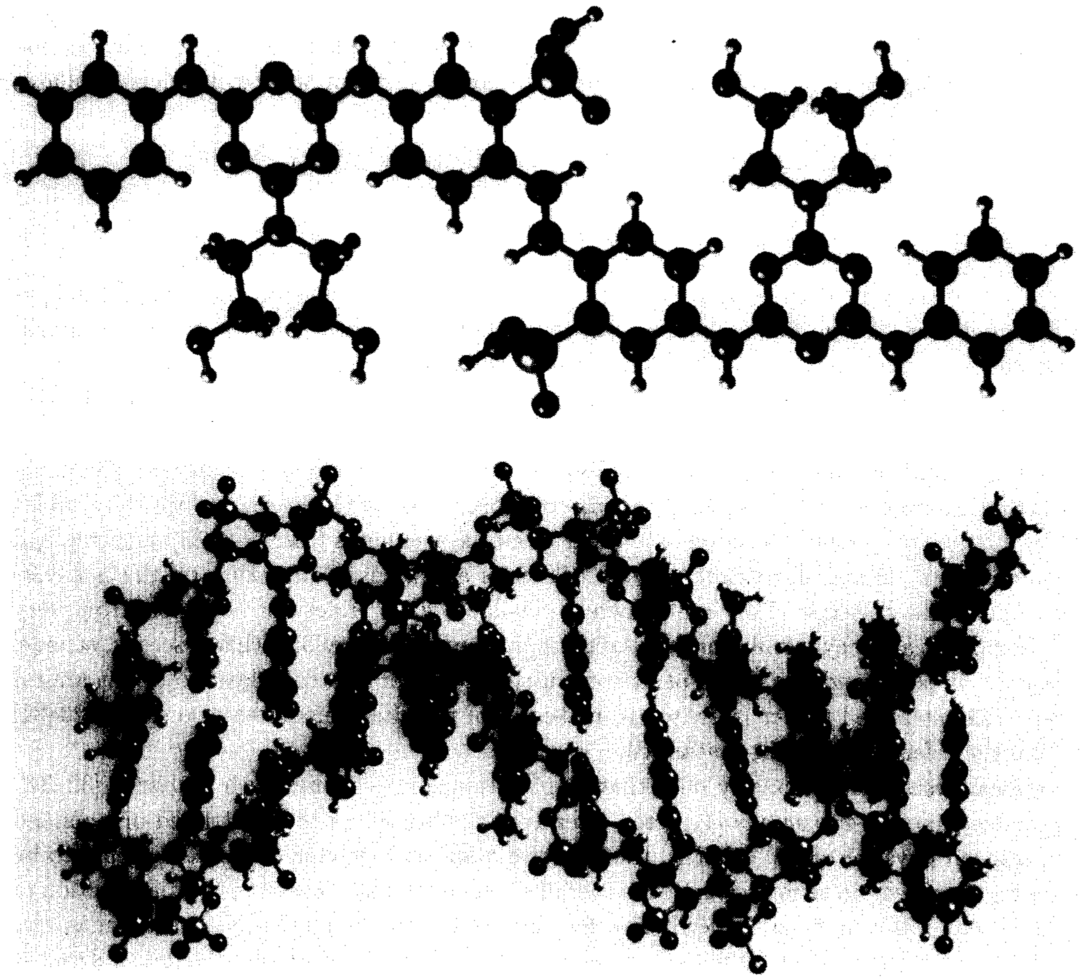


Figure 1. An illustration of two different molecular models of drug docking. The three-dimensional coordinates of each atom can be used, along with other properties of the structures (such as electric field intensities), to create a musical rendering of the molecules docking with each other.

3. THE SOUNDS OF SCIENCE

Mathematical models of naturally occurring systems often result in systems of differential and/or difference equations. Equation (3) illustrates that the simple difference equation given by

$$x_{n+1} = bx_n (1 - x_n) \tag{3.1}$$

is often called the "simplest dynamical system," and is well known to illustrate a variety of parameter dependent dynamics.

Differential and difference equations may be mono or multivariate. An interesting example of a system of two differential equations that exhibit a variety of dynamics is the hexapodal gait model [16,17]

$$\begin{aligned} \frac{dx_p(t)}{dt} &= -4x_p(t) + y_p(t) + \left(x_p(t)^2 + y_p(t)^2\right)\left(Px_p(t) - Qy_p(t)\right) \\ &\quad - 4\mu\left(x_{p-1}(t) + x_{p+1}(t)\right) + 2\mu\left(y_{p-1}(t) + y_{p+1}(t)\right) \\ \frac{dy_p(t)}{dt} &= -x_p(t) - 4y_p(t) + \left(x_p(t)^2 + y_p(t)^2\right)\left(Py_p(t) + Qx_p(t)\right) \\ &\quad - 2\mu\left(x_{p-1}(t) + x_{p+1}(t)\right) - 4\mu\left(y_{p-1}(t) + y_{p+1}(t)\right), \end{aligned} \tag{3.2}$$

where $0 \leq p \leq 5$ and the value of p is taken modulo 6. Witten [18] has used these equations to create a musical composition entitled "Bugged 1.0" which integrates the simulation results of equation (3.2) above, digitally sampled insect sounds, and live instrumental performance.

The multivariate, equations depending upon more than one independent variable, may depend upon both space and time—for example, the well-studied, very simple, biochemical oscillator equation given by

$$\begin{aligned}\frac{\partial A(t,x,y,z)}{\partial t} &= \nabla^2 A - A - B + (A \text{ if } A > 0.05)\\ \frac{\partial B(t,x,y,z)}{\partial t} &= \nabla^2 B + \frac{A}{2},\end{aligned} \tag{3.3}$$

where A is the concentration of species A at time t and B is the concentration of species B at time t. Under various conditions, we see spiral waves of chemical concentration in the concentration phase space. Multivariate equations may also occur singly or in systems.

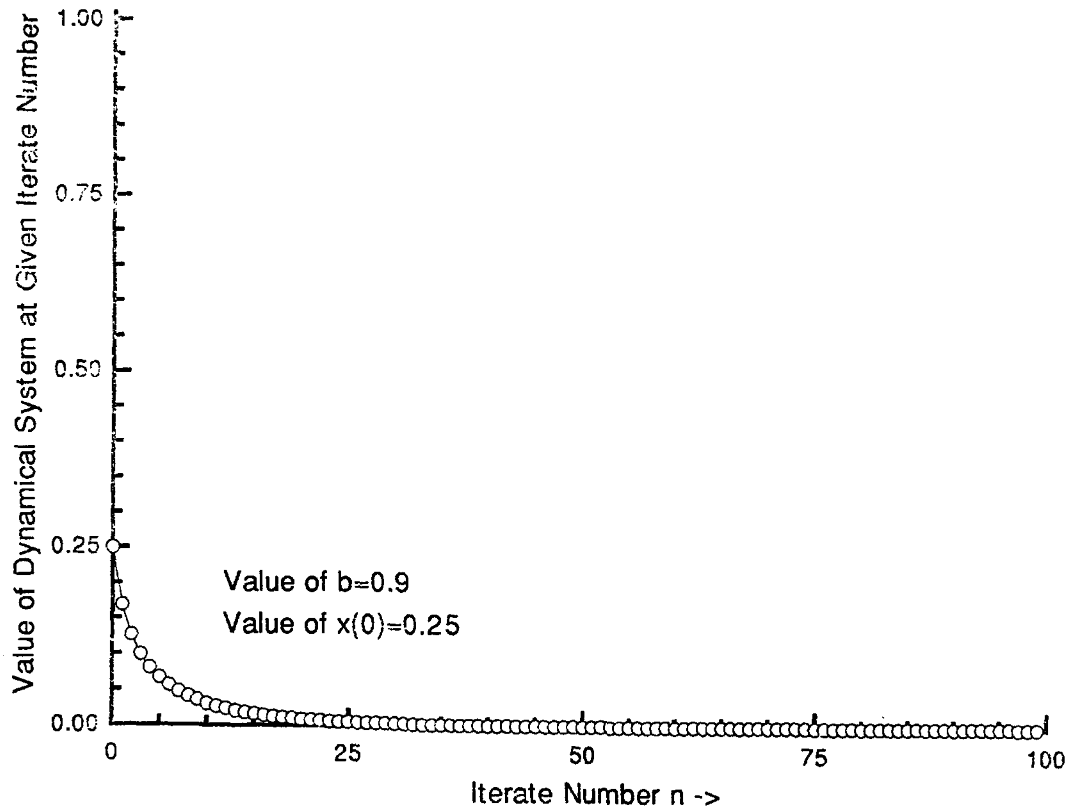
Dynamical systems may depend upon additional other variables such as pressure, velocity, age, particle number, etc. These systems are, more often than not, time dependent [19–21]. As such, their output could be considered a *temporal signal* in the sense that some dependent variable or variables are ordered with respect to a time variable t. And, within reason, these variables could be mapped onto some collection of musical variables/attributes [13,22]. Such mathematical systems are often termed *dynamical systems*. There is an extensive literature on the behavior of such dynamical systems. They are known to display a wide variety of behaviors from very simple (exponential decay, oscillations) to exceedingly complex behaviors (chaos, scrolls, strange attractors). It is beyond the scope of this discussion to cover the complexities of dynamical systems. The interested reader is invited to make use of the cited references as a start ([23–31] and all of the references therein contained).

Dynamical systems, particularly ones that display complex dynamics such as *chaos* [32–34], *strange attractors* [35–40], *scrolls* [41], etc., often confine themselves to a restricted or bounded area of a given physical space. That is, the overall dynamical behavior or *signal* appears to be contained in/localized in some region of the overall space of interest. The size and the location of this region may evolve in time. Nevertheless, for any fixed time $t > 0$, there is always a maximal bounding box for the orbit or signal of the system. We might say that the dynamical systems give a performance in a known auditorium size. The size of the performance auditorium may be governed by parameters in the model, length of time the model is examined, as well as other less obvious constraints.

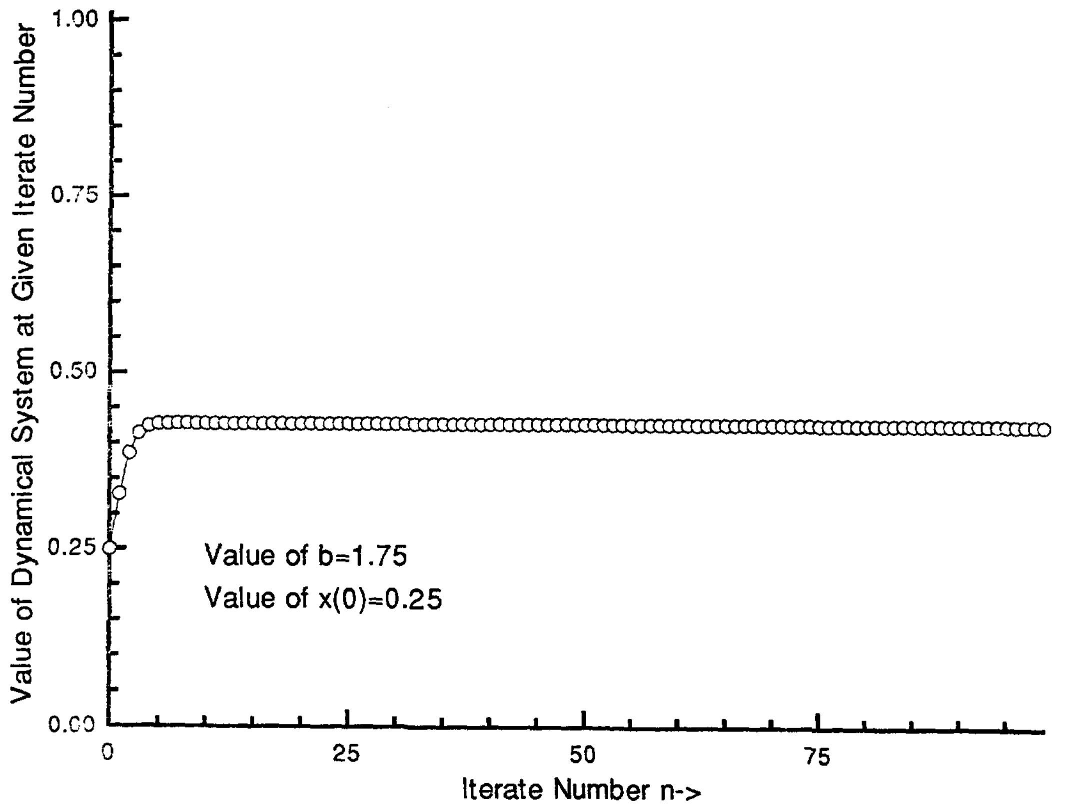
We begin our examination of the problem of listening to musical complexity of scientific data by considering an elementary example, the simple one-dimensional iterative dynamical system given by equation (3.1)

$$f_b(x_n) = x_{n+1} = bx_n(1 - x_n).$$

For $b \in [0,4]$, the mapping f_b maps the interval $[0,1] \to [0,1]$ where $x_n \in [0,1]$, $n = 0,1,2,\ldots$ [23,30,31,35,36,42–45]. Equation (3.1) can be used to generate a variety of complex dynamical behaviors depending upon the choice of the value of the parameter $b \in [0,4]$ [30,31,36,42–45]. For example, for b in the interval $(0,1)$, the sequence of values $x_1, x_2, \ldots$ tends to the value 0. However, for $b \in (1,3)$, the sequence of values $x_1, x_2, \ldots$ tends asymptotically to the value $(b-1)/b$. In Figure 2, we illustrate the varying dynamics for $b \in (0,3)$. Observe that the dynamics is singularly different when comparing the intervals $(0,1)$ (Figure 2a), $(1,2)$ (Figure 2b), and $(2,3)$ (Figure 2c). Within any of the aforementioned intervals, the dynamics is considered to be qualitatively equivalent or what is called *topologically conjugate*. When comparing across the intervals, the dynamics is considered to be topologically different. That is, we cannot map the dynamics of one b parameter interval uniquely onto the dynamics of another interval. Thus, if we were to use the dynamics of a series generated from a b parameter $b_1 \in (1,2)$ to generate a musical form, this form would be qualitatively different from a musical form generated by $b_2 \in (2,3)$. However, it would only be quantitatively different from a musical form generated by $b_3 \in (1,2)$ where $b_1 \neq b_3$. Witten [30,31] discusses the formal theory illustrating the dynamical equivalence classes. Thus, for any two values of b chosen *within* one of the intervals, it is possible to

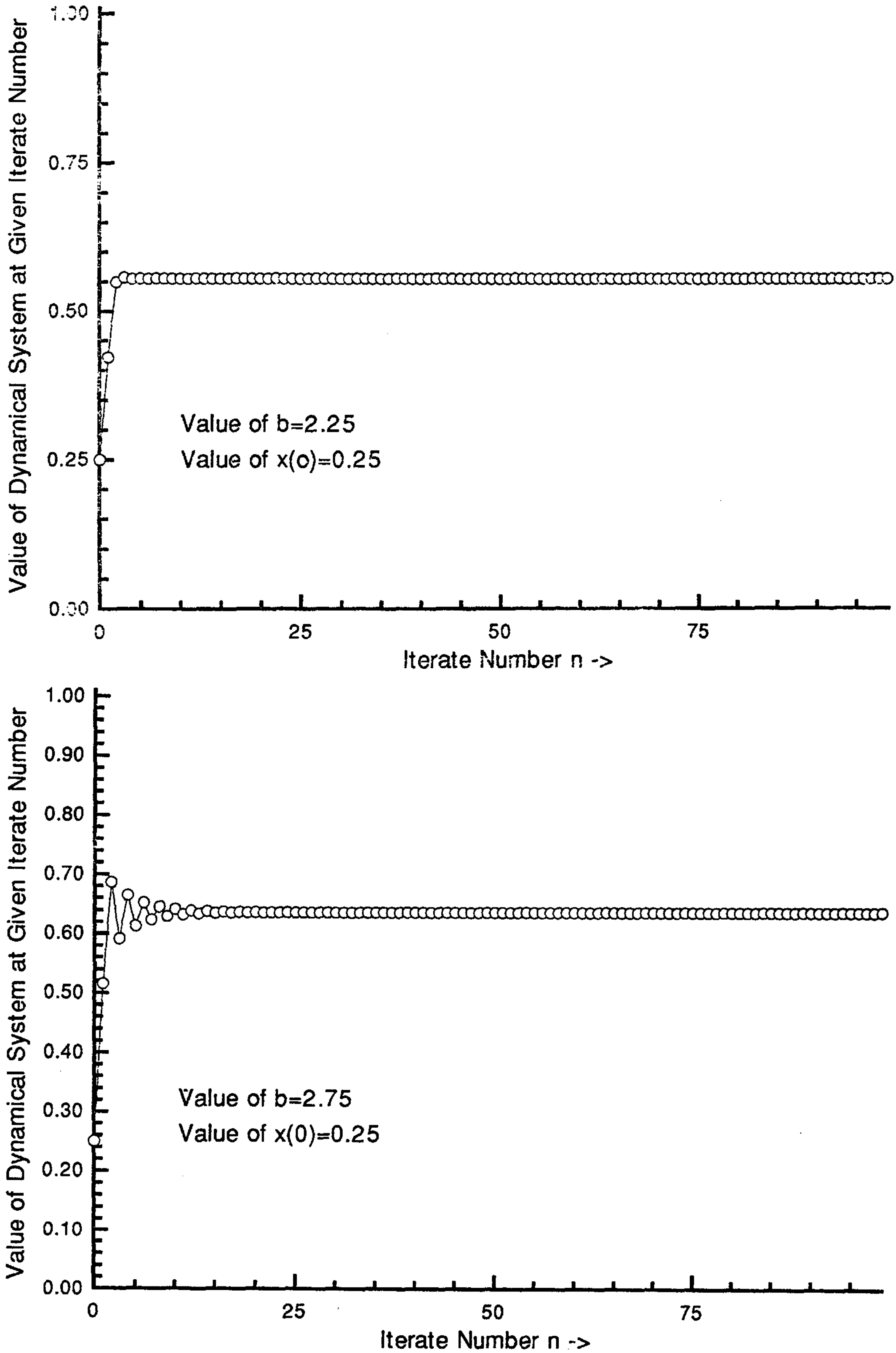


(a) An illustration of the dynamical systems behavior of equation (3.1) for $b \in (0, 1)$.



(b) An illustration of the dynamical systems behavior of equation (3.1) for $b \in (1, 2)$.

Figure 2. (continued on next page)



(c) An illustration of the dynamical systems behavior of equation (3.1) for $b \in (2, 3)$.

Figure 2 (continued).

demonstrate that there is a mapping between the two dynamical behaviors that will uniquely map one dynamical behavior onto the other, preserving all of the topological and temporal properties of the dynamics. Mathematically this process is illustrated by the following commutative diagram:

$$\begin{array}{ccc} [0,1] & \xrightarrow{f_{b_1}} & [0,1] \\ \phi\downarrow & & \phi\downarrow \\ [0,1] & \xrightarrow{f_{b_2}} & [0,1] \end{array} \tag{3.4}$$

The dynamics of equation (3.1), in the interval $(0,3)$, is known to behave in a either an exponential-like fashion or in a slightly more complex damped manner. However, for values of $b \in (3,4)$ (Figure 3), the dynamics of this model becomes exceedingly more complex—from simple oscillatory behavior to what is termed chaotic behavior. We summarize the conjugacy classes of the dynamics in Table 1.

Table 1. Conjugacy class dynamics for the simplest nonlinear interval mapping equation (3.1).

Conjugacy Class	Comments
0	Fixed point of mapping
$(0,1)$	Figure 2a
$(1,2)$	Figure 2b
$(2,3)$	Figure 2c
$B_0 = (3, 3.236)$	Figure 3a
$B_1 = (3.236, 3.8318)$	Figure 3b
$B_2 = (3.8318, 3.9605)$	Figure 3c
$B_3 = (3.9605, 3.9905)$	Figure 3d
$B_k = (\beta_k, \beta_{k+1}]\quad k = 4, 5\dots$	See note following table
All intervals with odd periods	
$b = 4$	Figure 4

NOTE. B_k is the interval $(\beta_k, \beta_{k+1}]$ and β_k is the first value of the parameter b in equation (3.1) at which a cycle of period 2^k appears. The intervals B_k are disjoint and have dynamics which is not conjugate. Figure 5 illustrates conjugacy class dynamics for equation (3.1). Most of the references cited in this discussion contain numerous illustrations of the variety of dynamical behaviors illustrated by this simple dynamical system.

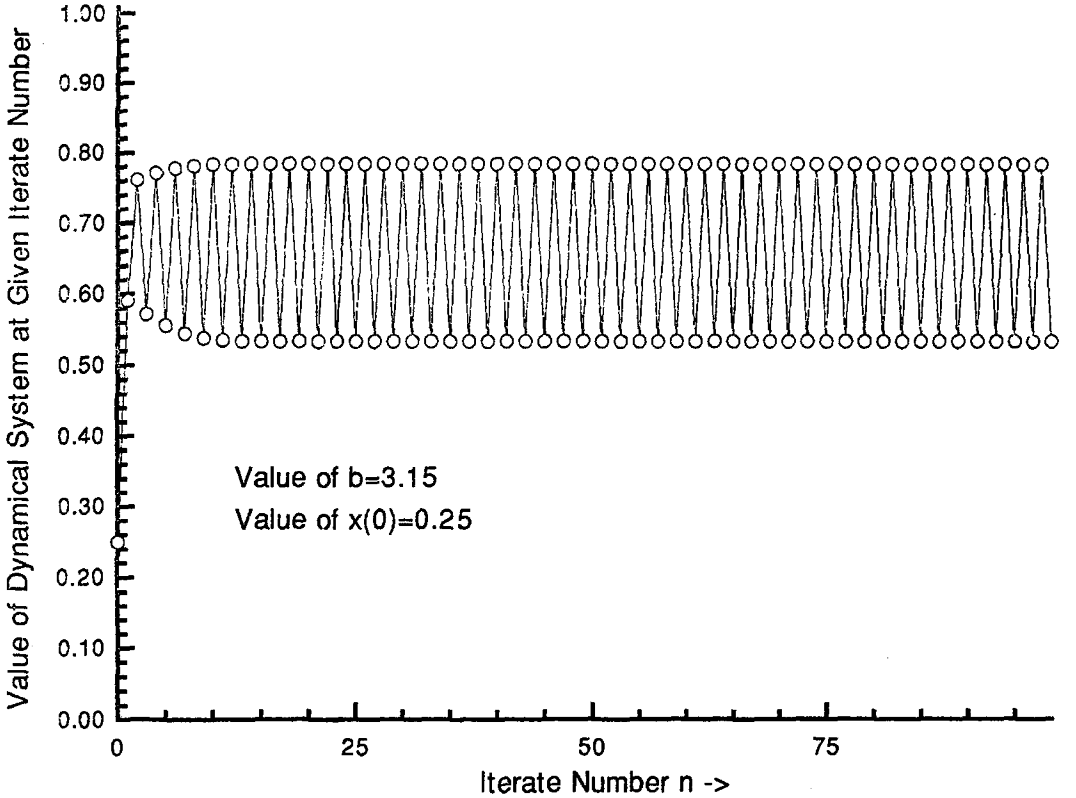
Similar arguments can be made for various mappings of the xy-plane (bivariate or two-dimensional mappings). For example, the Henon [37] mapping

$$\begin{aligned} x_{n+1} &= y_n + 1 - ax_n^2 \\ y_{n+1} &= bx_n \end{aligned} \tag{3.5}$$

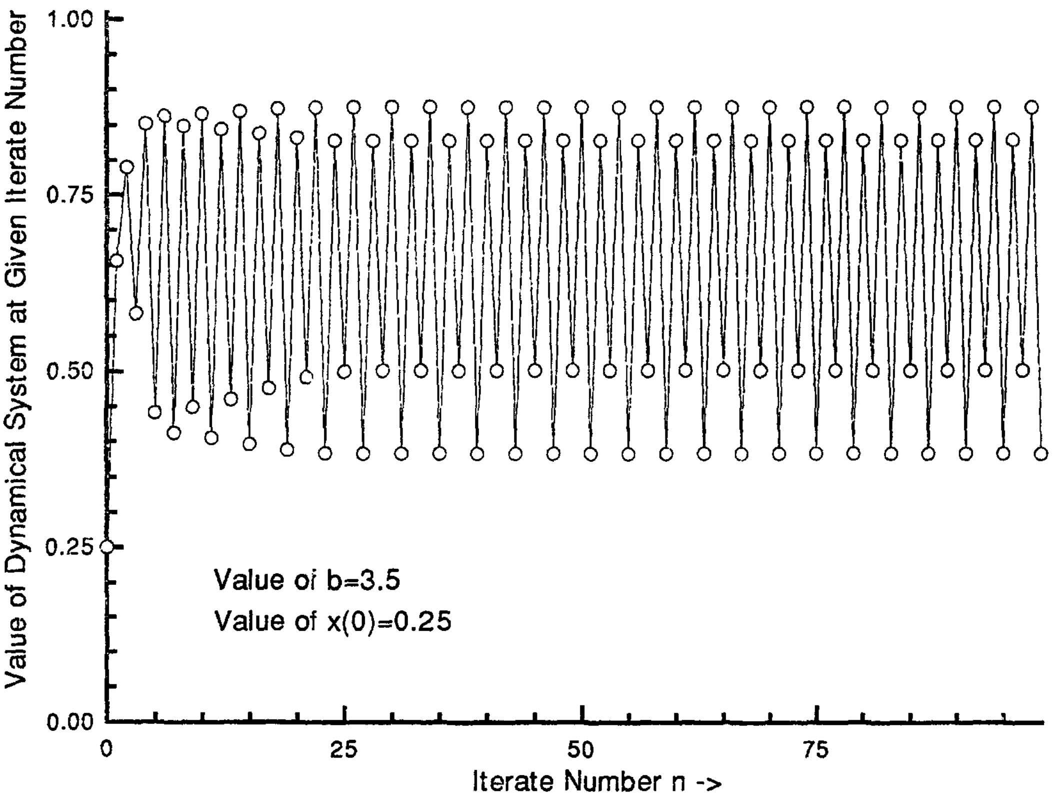
maps a bounded area of the plane into itself, where a, b are parameters of the model [24,29,36,37, 46,47]. Figure 6 illustrates sample phase and time plots of the Henon map. An example of an interesting three-dimensional dynamical system is the Lorentz system [38,39] given by

$$\begin{aligned} \frac{dx(t)}{dt} &= P(y - x) \\ \frac{dy(t)}{dt} &= -(xz + rx - y) \\ \frac{dz(t)}{dt} &= xy - bz, \end{aligned} \tag{3.6}$$

where P, r, and b are parameters of the model [24,28,29,32–35,38,39]. The Lorenz model was originally created as an attempt to model the dynamics of certain types of weather phenomena. Figure 7 illustrates a sample numerically computed nonperiodic orbit of the Lorentz equations as

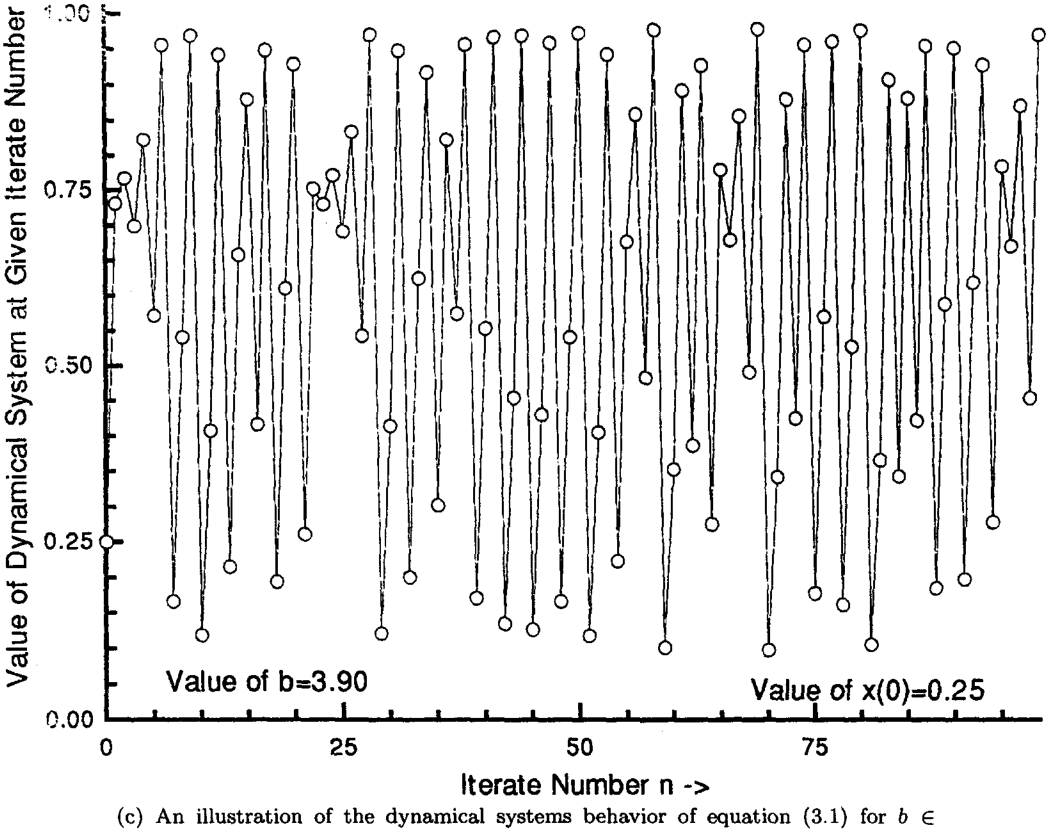


(a) An illustration of the dynamical systems behavior of equation (3.1) for $b \in (3, 3.236)$.

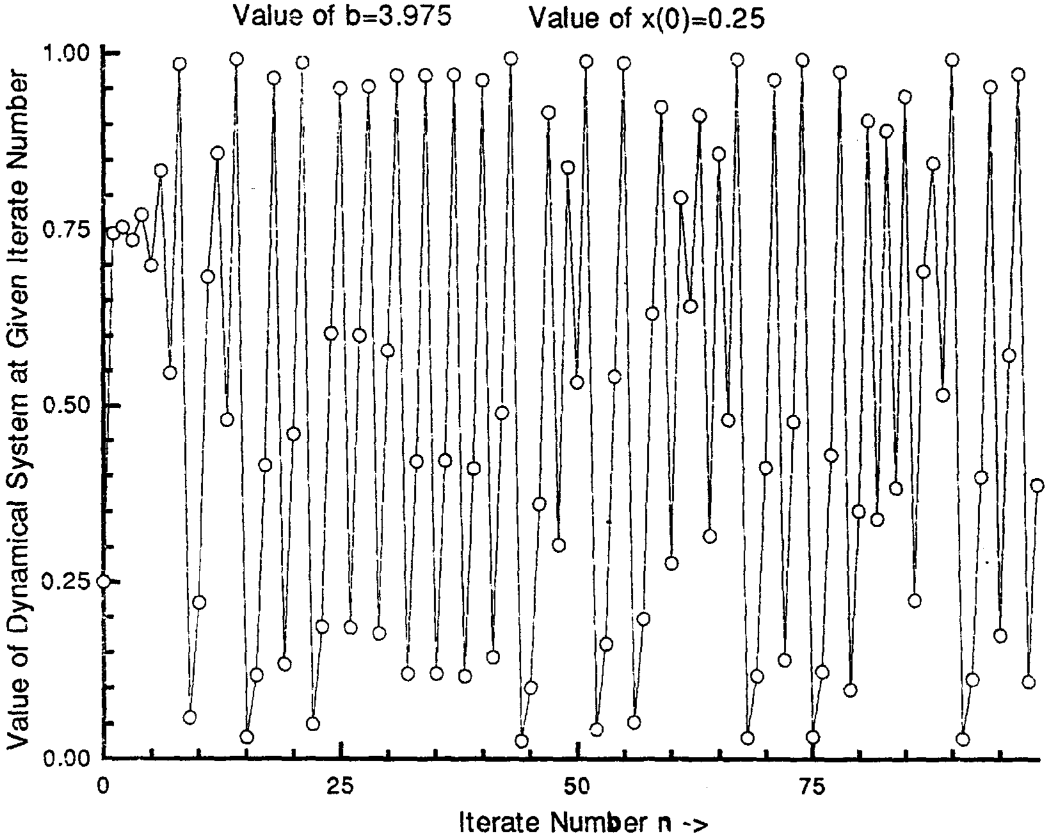


(b) An illustration of the dynamical systems behavior of equation (3.1) for $b \in (3.236, 3.8318)$.

Figure 3. (continued on next page)



(c) An illustration of the dynamical systems behavior of equation (3.1) for $b \in (3.8318, 3.9605)$.



(d) An illustration of the dynamical systems behavior of equation (3.1) for $b \in (3.9605, 3.9905)$.

Figure 3. (continued)

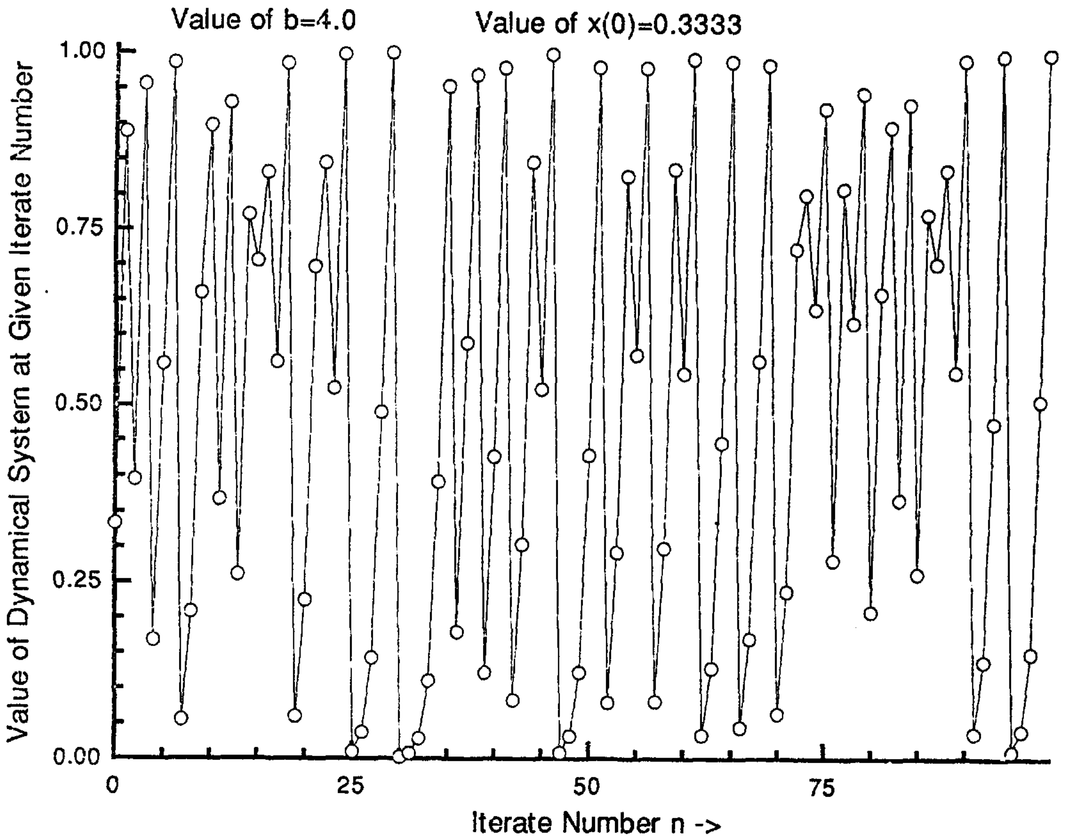


Figure 4. An illustration of the dynamical systems behavior of equation (3.1) for $b = 4$.

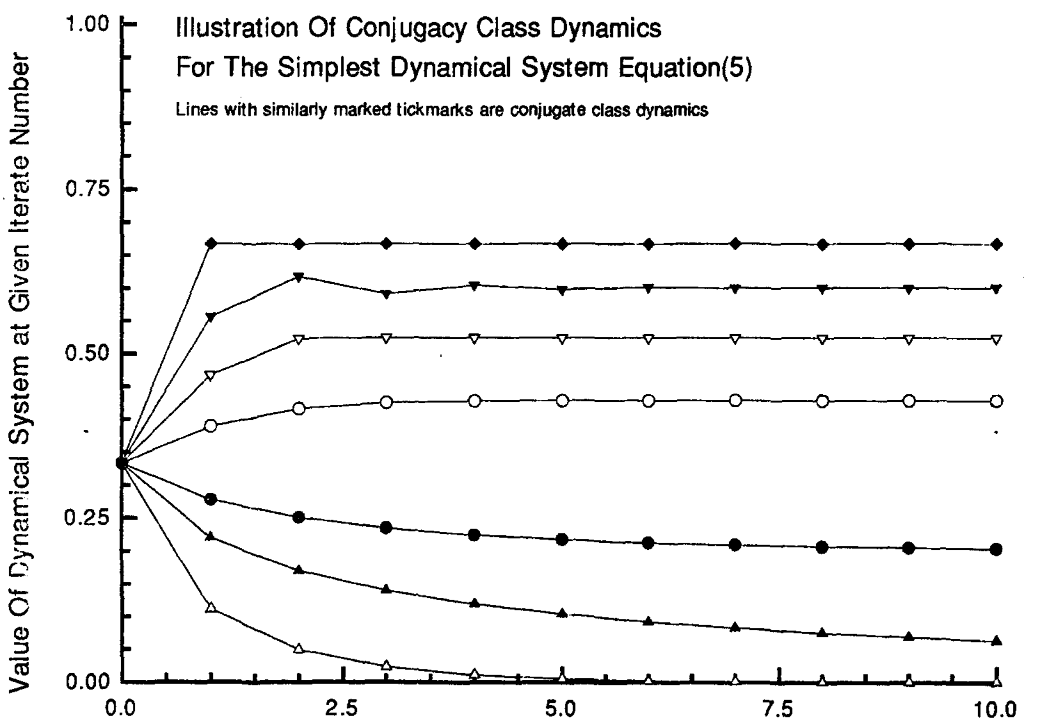
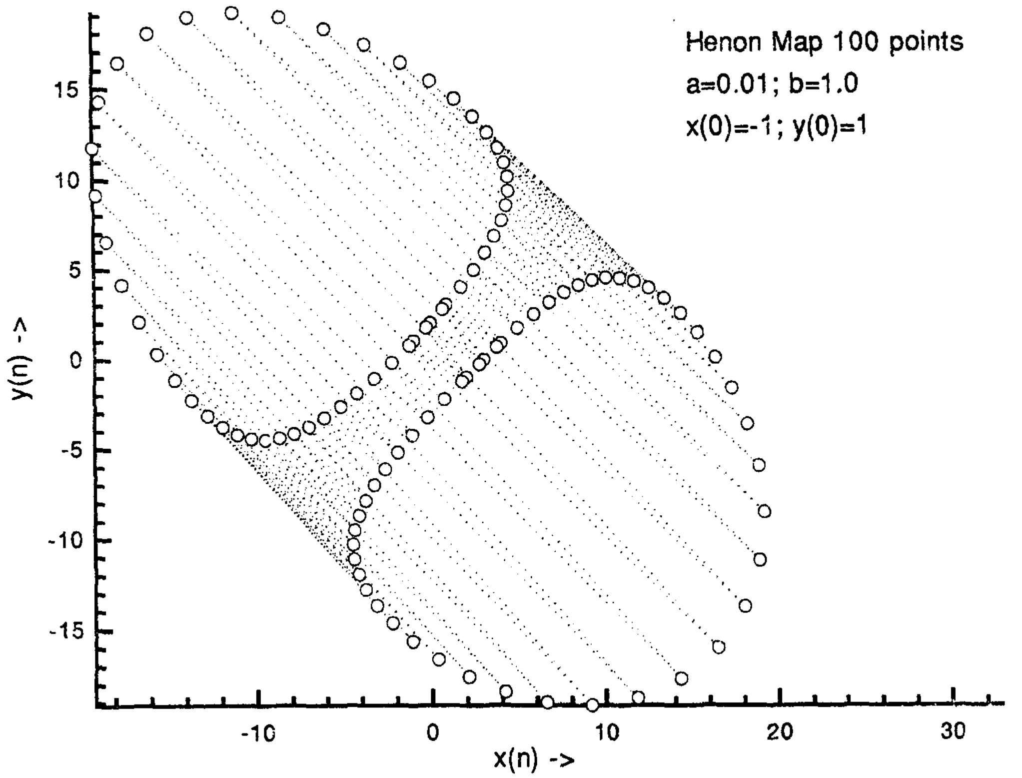
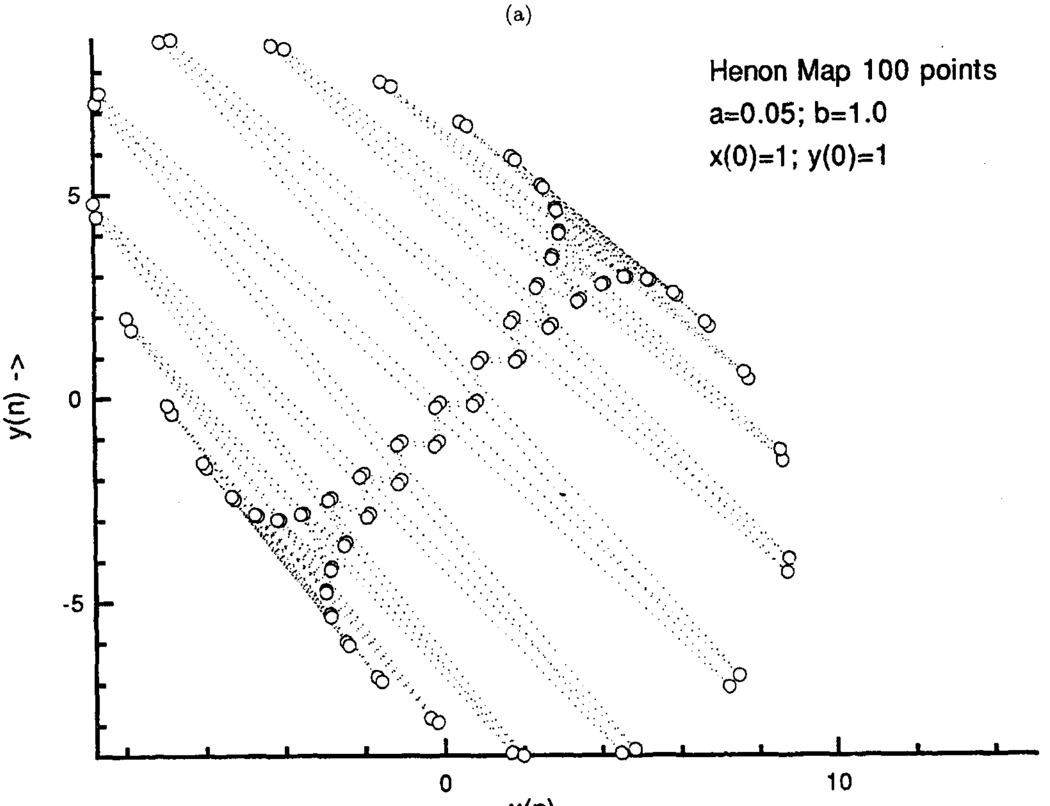


Figure 5. An illustration of the conjugacy classes for the dynamics of the dynamical system given in equation (3.1).

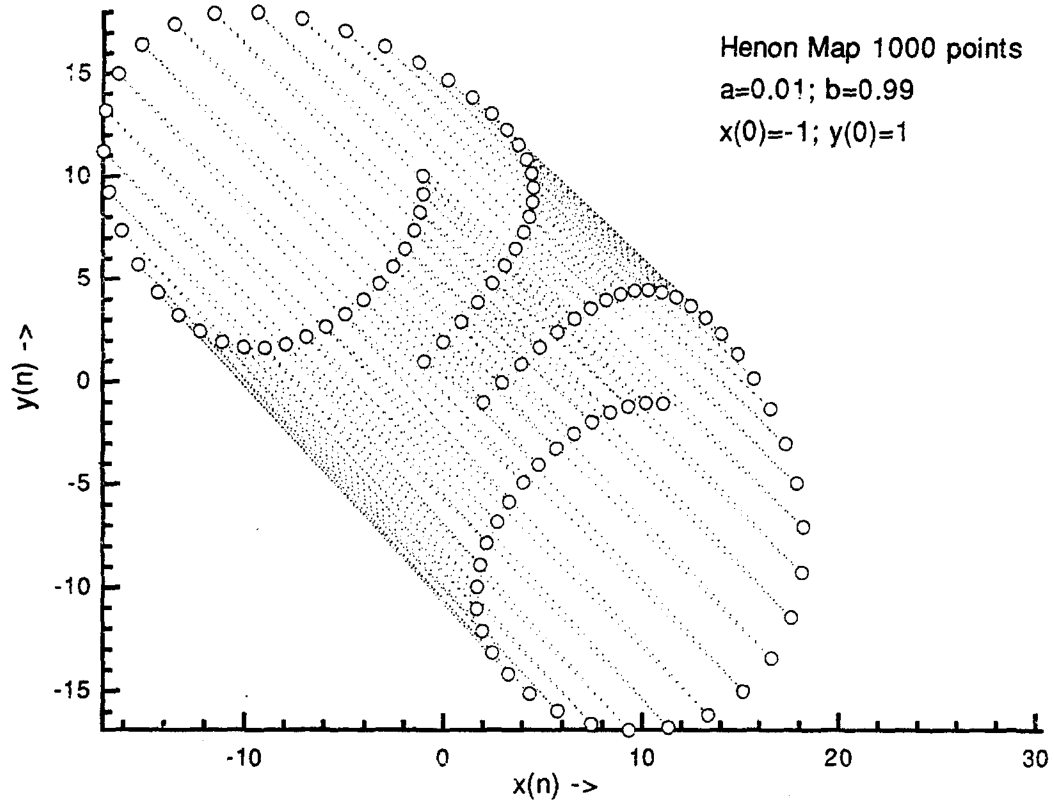


(a)

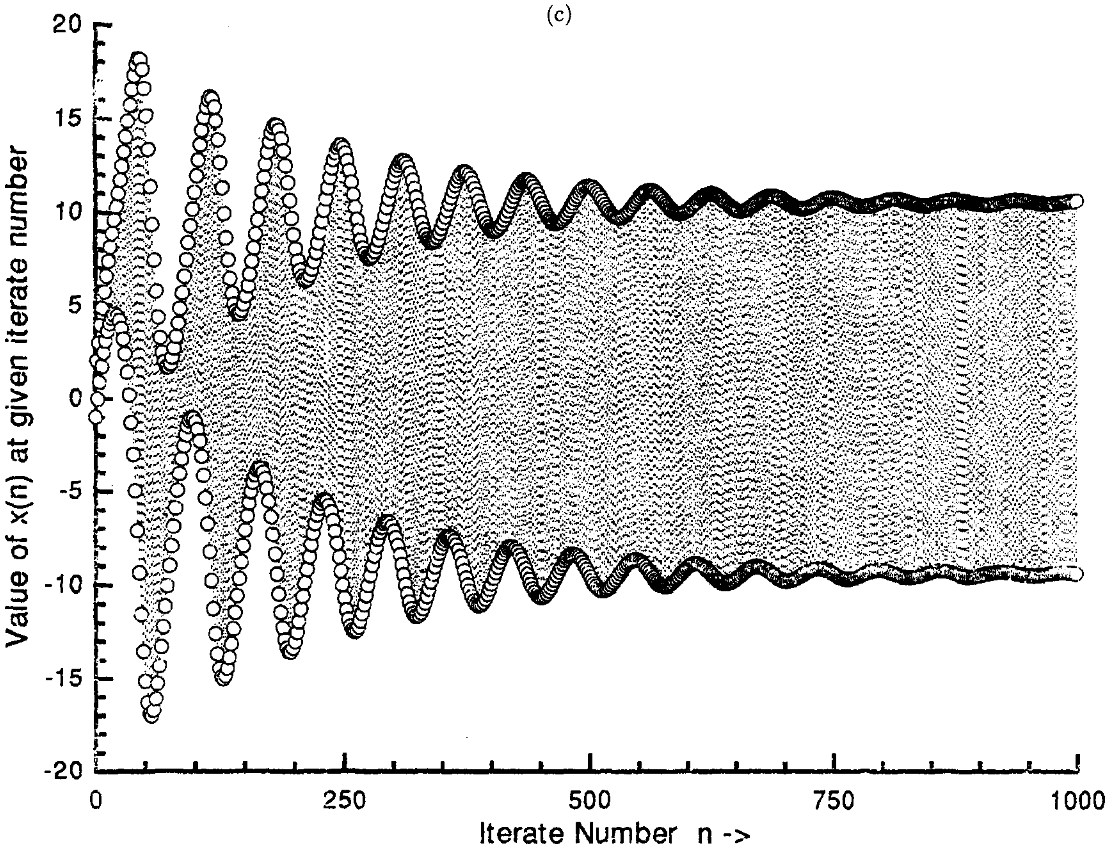


(b)

Figure 6. An illustration of the dynamics illustrated by the Henon mapping specified in equation (3.5). Figures 6a–6c illustrate the two-dimensional phase plane dynamics for different values of the two model parameters. Figures 6d,e illustrate the temporal dynamics of the two Henon variables. (continued on next page)

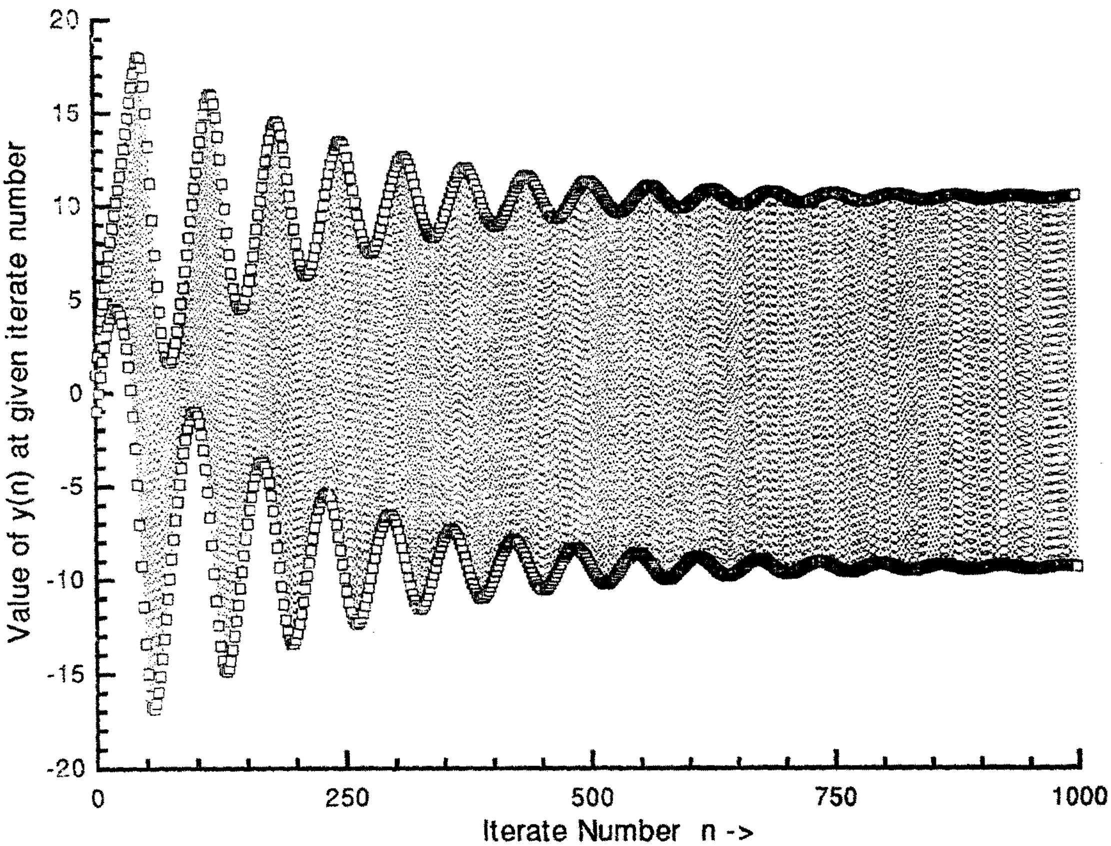


(c)



(d)

Figure 6. (continued on next page)



(e)

Figure 6. (continued)

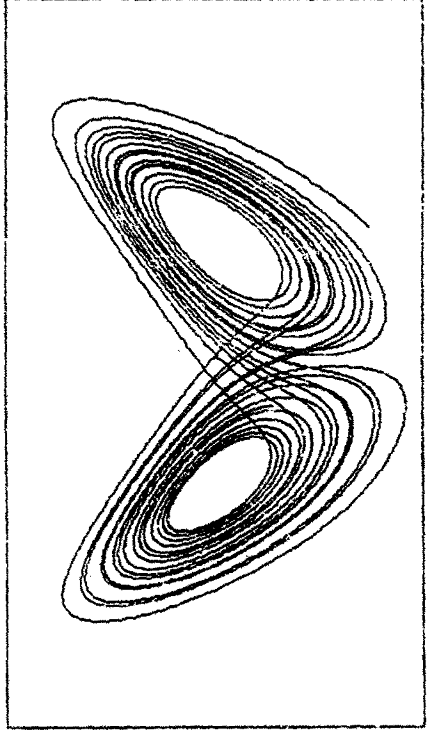


Figure 7. An illustration of the two-dimensional projection of a three-dimensional orbit of the Lorenz equation system.

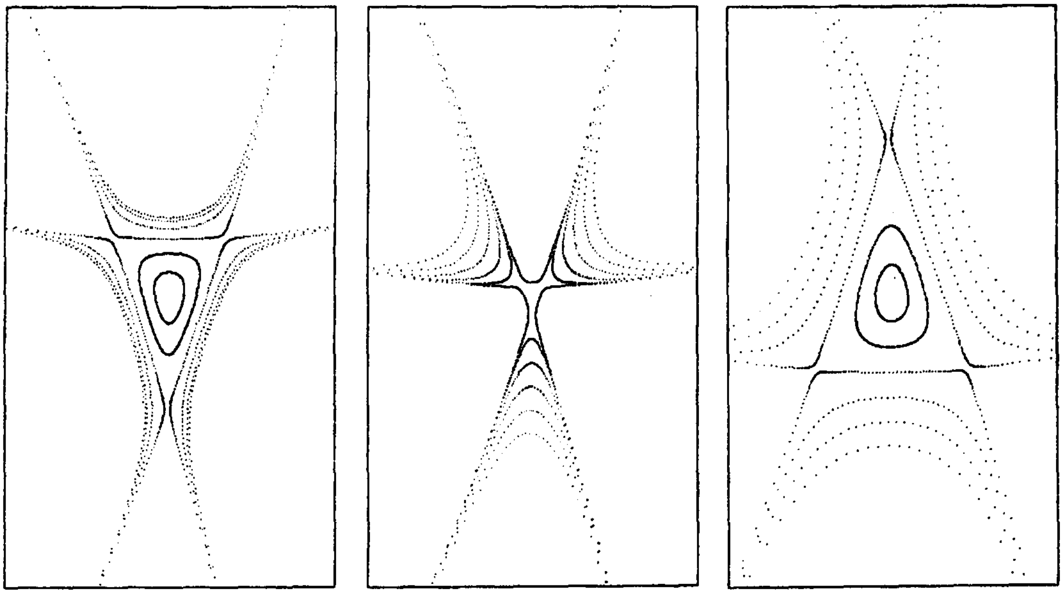


Figure 8. An illustration of the changing dynamics of the Cremona map specified by equation (3.7) as the parameter λ is varied.

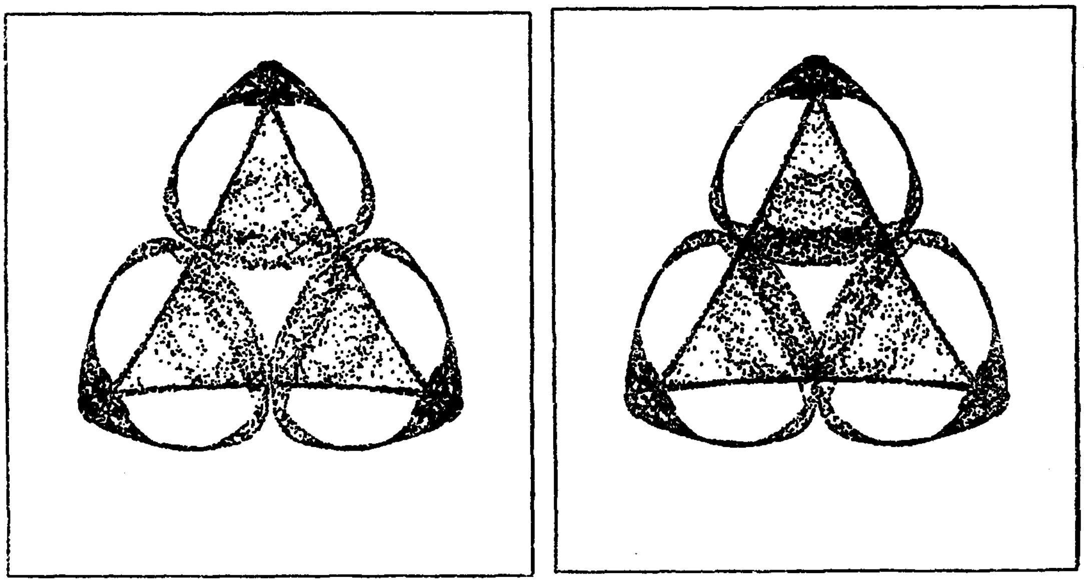


Figure 9. An illustration of the symmetry breaking dynamics associated with equation (3.8).

projected into the xy-plane. The Cremona map is a quadratic planar map given by the following system of discrete equations.

$$\begin{aligned} x_{n+1} &= x_n \cos(\lambda) - \left[y_n - x_n^2\right] \sin(\lambda) \\ y_{n+1} &= x_n \sin(\lambda) + \left[y_n - x_n^2\right] \cos(\lambda). \end{aligned} \tag{3.7}$$

The bifurcation of the dynamics about the origin, for value of $\lambda = 2\pi/3$, is illustrated in Figure 8.

Figure 9 illustrates symmetry increasing crisis in the complex variable mapping

$$f(z, \lambda) = (\alpha u + \beta v + \lambda)z + \gamma \bar{z}^{n-1} \tag{3.8}$$

for $z = u + iv$, $\alpha = -1.1$, $\beta = 0.212$, $\gamma = 0.6$, $\lambda = 1.89$, and $n = 3$ (see [48–50]). Another interesting dynamical system is given by the following differential equation system:

$$\begin{aligned}
\frac{dx(t)}{dt} &= y \\
\frac{dy(t)}{dt} &= z \\
\frac{dz(t)}{dt} &= -y - az + T \equiv \begin{cases} 1.0 - bx, & \text{for } x > 0.0 \\ 1.0 + cx, & \text{for } x \leq 0.0. \end{cases}
\end{aligned} \tag{3.9}$$

If we set $a = 0.3375$ and $c = 0.633625$ and subsequently vary the remaining parameter b, we obtain a variety of interesting 3D dynamics. This is illustrated in Figure 10.

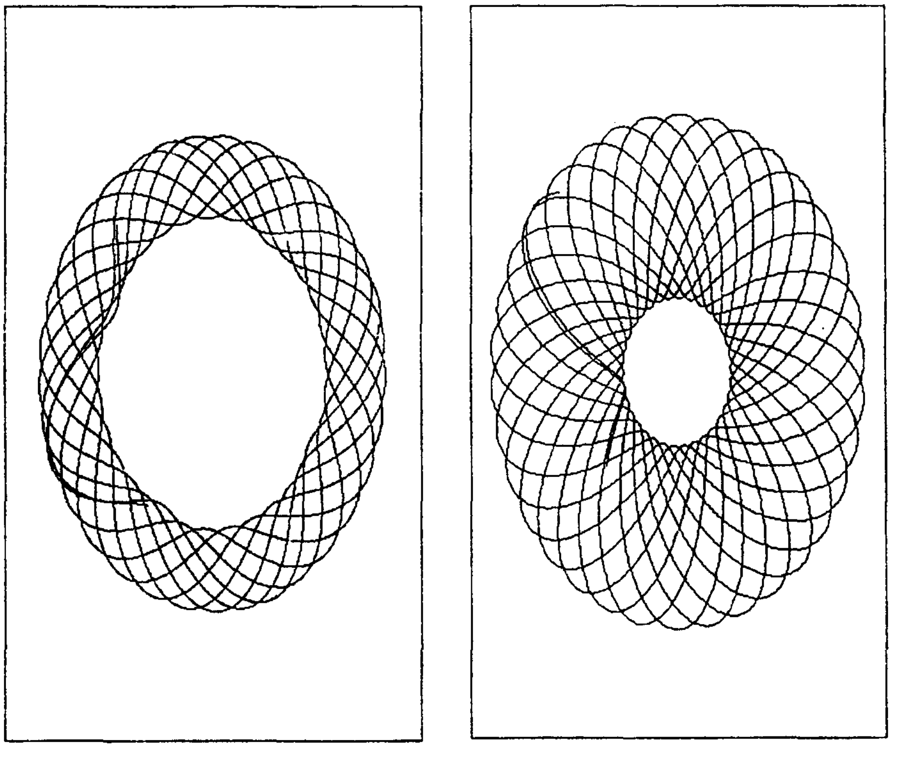


Figure 10. An illustration of the appearance of a three-dimensional invariant torus-associated with varying the parameters of equation (3.9).

There are a myriad of other examples which we can invoke as illustrations of how scientific models, when simulated, can give rise to a time series of data. Numerous books on mathematical modeling, in all disciplines, contain such models and we will not attempt to address this literature.

4. UNDERSTANDING DATA COMPLEXITY

We have already established that the study of scientific structures/models opens new doorways when it comes to the use of musical tools and techniques. We have seen that sound can be used to potentially enhance our understanding of complexity in data. And we have also observed that it can be used, in conjunction with the scientific models and data, to generate artistic creations that are based upon the aesthetics of the underlying dynamics of a scientific model combined with the personality of the composer/artist.

Let us begin our excursion into the sounds of science by asking the following simple question. How can we actually "hear" the dynamical behavior of various classes of mathematical/computer model? In particular, consider the following more detailed series of questions:

(1) How does *chaos* sound, if we were sitting in the middle of it?
(2) What does a *strange attractor* sound like?
(3) Can we hear the sound of *period doubling* or of a *scroll*?

Even if we are able to hear these sounds, will these sounds be artistically interesting or scientifically illuminating? Will they aid us in illuminating the complexity of a given dataset? Or, will they give us an unseen doorway to a new artistic creation?

For the purposes of discussion, we will address these problems in the world of two-dimensional models. That is, we will examine how to map the dynamics of a two-dimensional model onto the domain of a theatre in such a way as to allow an audience to hear the dynamics of these complex and interesting systems. The use of 3D sound is of increasing interest in a number of areas [51–56]. These relationships can be readily generalized to three dimensions.

5. CONSTRUCTING A TRANSFORMATION

We begin by assuming that our dynamical process takes place within a small rectangle in the xy-plane. This rectangle is dimensionless in the sense that it could be in feet, inches, number of animals, concentration of chemical species, or any other *abstract variable*. We will return to the dimensional issue in an upcoming section of this paper.

Let us call this rectangle R_D. Assume that the audience sits in a theatre that is bounded by a rectangle denoted by R_A. It is straightforward to show that there is a transformation $T : R_D \to R_A$ that allows us to map the dynamics of the process over the audience seating area. The transformed rectangle is denoted $T(R_D)$. This transformation process is illustrated in Figure 11.

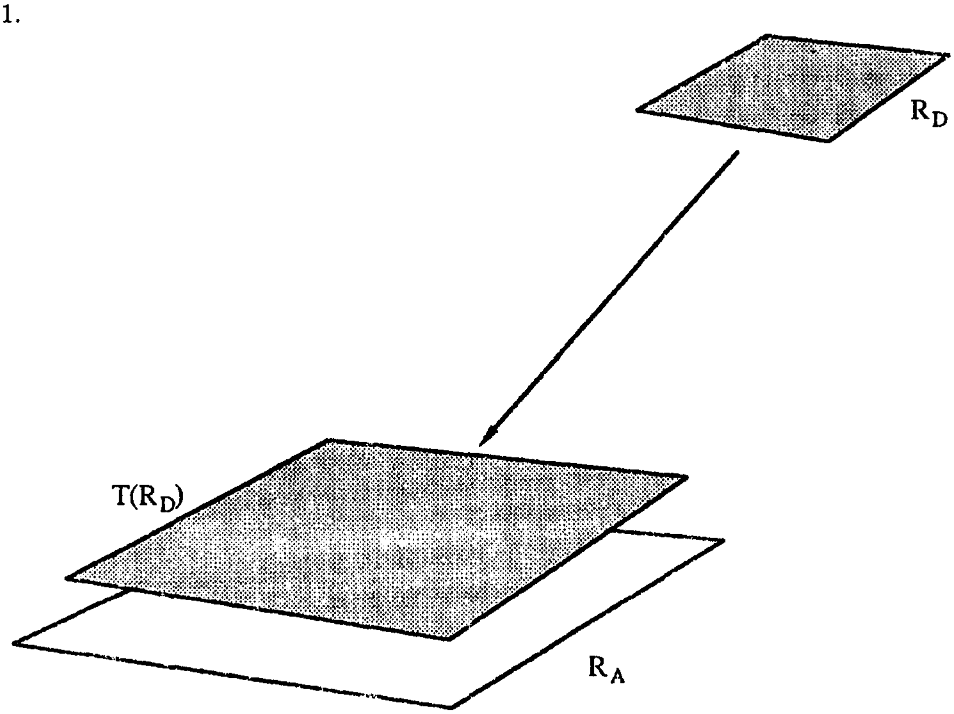


Figure 11. The hypothetical audience region given by R_A and how the region R_D of the dynamics of our dynamical system is mapped onto the audience.

It is also immediately clear, however, that the dynamics of our model process would collide, in spatial location, with the audience and its physical location. What we wish to do is to unfold the model dynamics so that it occurs around the audience, and allows the audience to participate in it by having the dynamics occur in an environment surrounding the audience. This requirement is illustrated in Figure 12, where $T' : R_D \to R_F$ and R_F is a rectangular frame around the region of the audience R_A.

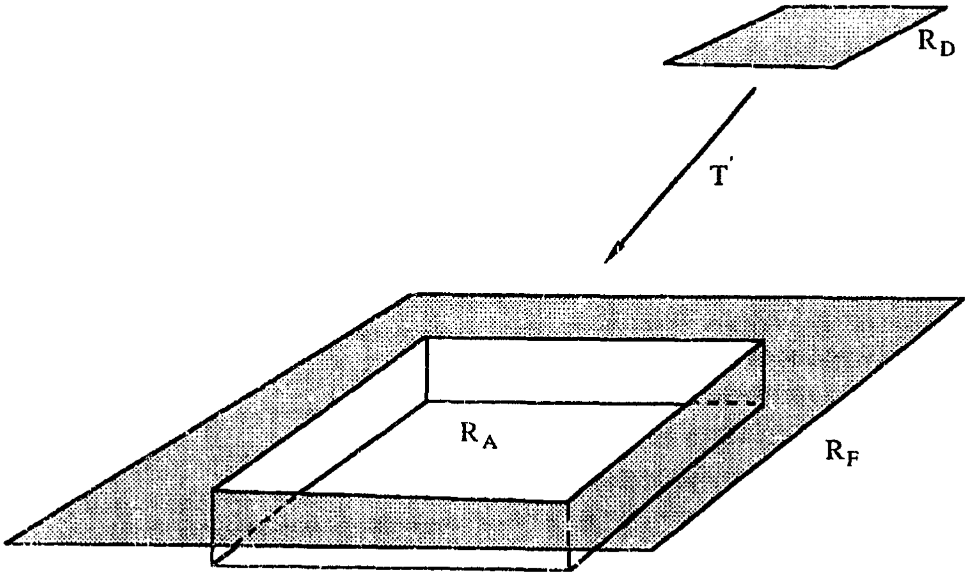


Figure 12. Details of the unfolding of the dynamics around the region of the audience.

For the purposes of discussion, we consider the rectangular region and frame shown in Figure 13a. We will assume that the shaded rectangle is R_D and that the frame surrounding it is of some predetermined size. We begin the transformation sequence by translating Figure 13a so that the center of R_D is now located at the origin of the figure (Figure 13b). The requisite transformation is given by

$$\begin{aligned} u &= x - \left(\frac{H+G}{2}\right) \\ v &= y - \left(\frac{E+F}{2}\right). \end{aligned} \tag{5.1}$$

If we assume that the original figure is symmetric about an axis drawn through the center of R_D, we can simplify our analysis (it will turn out that this symmetry assumption is not unreasonable for the class of problems which we are about to tackle). The assumption of symmetry allows us to relabel Figure 13b as illustrated in Figure 13c.

Table 2. Transformation of coordinate values of Figure 12a to coordinate values of Figure 13 under equation (5.1).

Figure 13b Label	Value in Figure 13a coordinates
I	$A - \left(\frac{E+F}{2}\right)$
J	$B - \left(\frac{E+F}{2}\right)$
K	$C - \left(\frac{H+G}{2}\right)$
L	$D - \left(\frac{H+G}{2}\right)$
M	$E - \left(\frac{E+F}{2}\right)$
N	$F - \left(\frac{E+F}{2}\right)$
O	$G - \left(\frac{H+G}{2}\right)$
P	$H - \left(\frac{H+G}{2}\right)$

Next, we scale the center box of Figure 13c so that it is a unit box. To do this, we introduce the new coordinate system

$$\begin{aligned} p' &= \frac{u}{P} \\ q' &= \frac{v}{N}. \end{aligned} \tag{5.2}$$

The result of this transformation is given in Figure 13d.

At this point, we are now prepared to unfold the model dynamics into the outer frame. To do this, we introduce the following mapping:

$$\begin{aligned} \alpha &= \frac{\frac{\pi}{2}p'}{\sin\frac{\pi}{2}p'}\left[1-\left(1-\frac{2L/P}{\pi}p'\right)\right] \\ \beta &= \frac{\frac{\pi}{2}q'}{\sin\frac{\pi}{2}q'}\left[1-\left(1-\frac{2J/N}{\pi}q'\right)\right]. \end{aligned} \tag{5.3}$$

This mapping is illustrated in Figure 13e.

It is straightforward to demonstrate that the transformation defined by equation (5.1) does, in point of fact, map the inner unit square onto the outer rectangular frame. The only problem is that the origin $(0,0)$ is a fixed point and will map onto itself. Thus, if the model system generates an (x,y) coordinate of value $(0,0)$, this point will map to the origin of the audience. However, this can be ignored as chances of the origin occurring will be negligible. And should $(0,0)$ occur in the analysis, there will most likely be no person sitting in the theatre at that exact location. Hence, for any point $(x,y) \in R_D$, there is a series of mappings that will map $(x,y) \to (\alpha,\beta)$ where (α,β) is a point in a well-defined frame about the unit square given in Figure 13e.

6. MAPPING ONTO AN AUDITORIUM

Suppose that we now wish to have the outer edges of the frame fit onto an auditorium whose width is w and whose length is l. This means that we wish to map $R \to (w/2)$ and $Q \to (l/2)$. To do this, define the following frame to auditorium transformation given by

$$\begin{aligned} x' &= \left(\frac{w}{2}\right)\frac{\alpha}{R} \\ y' &= \left(\frac{l}{2}\right)\frac{\beta}{Q}. \end{aligned} \tag{6.1}$$

Figure 13f illustrates the transformed dynamics region in terms of the auditorium space. Observe that in order for this method to work, the audience must be confined to a region within the inner box. That is, the seats must all be located within a region R_A defined as the set of all points satisfying

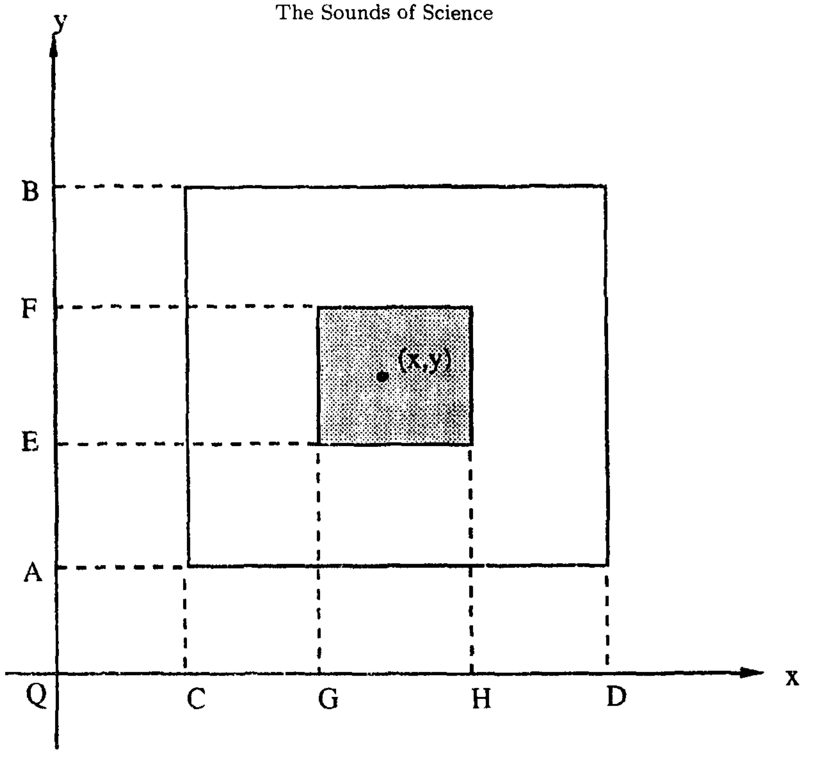
$$R_A = \left\{(x,y) \mid x \le \left|\frac{w}{R}\right| \text{ and } y \le \left|\frac{l}{Q}\right|\right\}. \tag{6.2}$$

The point (x,y) has now been mapped to a point (x',y') outside the audience regions.

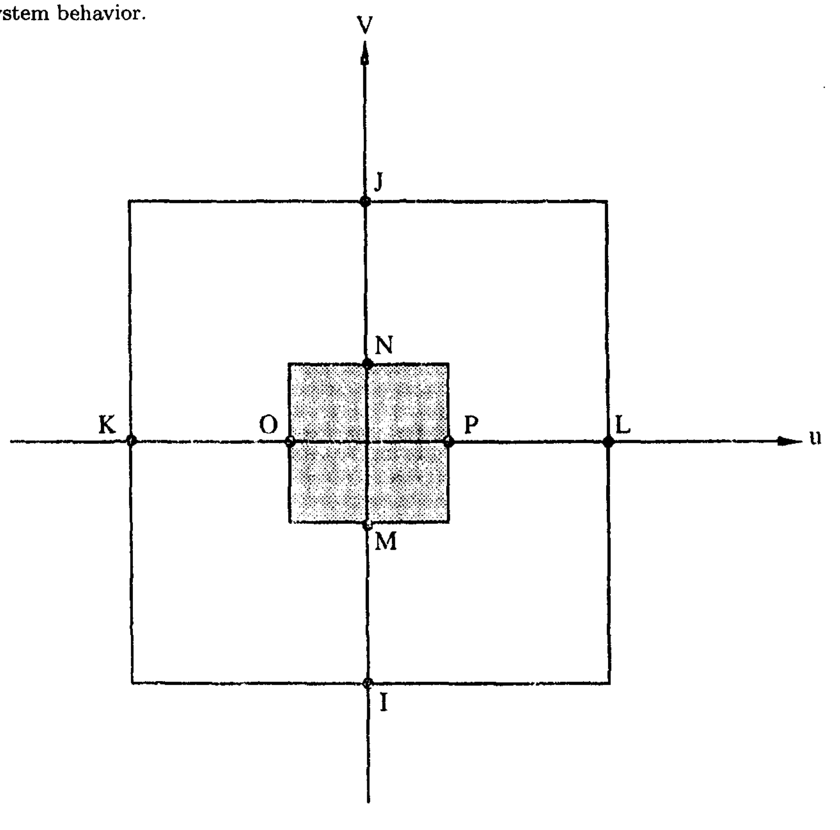
Similar arguments may be used to construct the transformation for three-dimensional regions. Before closing the discussion on mapping from dynamical system space to auditorium space, it is important to point out that the set of mappings, given by equation (5.3), is not unique. Other mappings may be constructed to map the unit square onto the surrounding frame.

7. ASSIGNING MUSICAL ATTRIBUTES

Having unfolded the model dynamics to a space around the audience, we now ask how we might assign musical/sonic attributes to the dynamical phenomena. To address this question, we consider the following initial approach. As before, let our point (x,y) reside inside the shaded rectangle illustrated in Figure 13a. Let us focus upon this rectangle alone, as is illustrated in

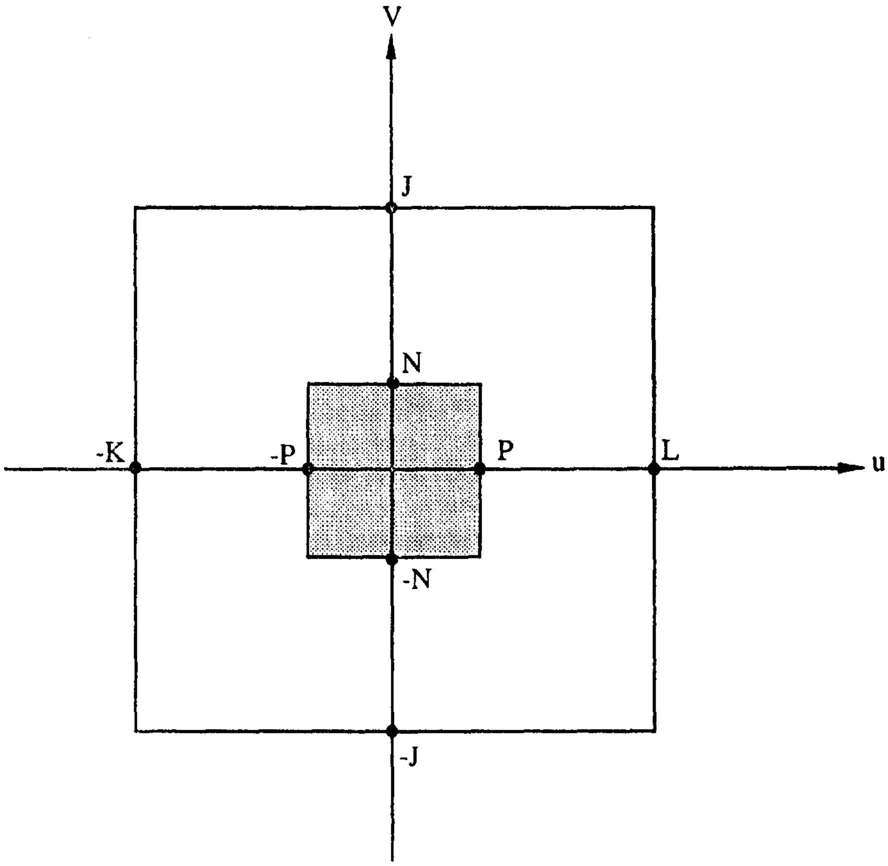


(a) An illustration of the hypothetical region in which the audience resides (the shaded box) and the frame around it into which we wish to map the dynamical system behavior.

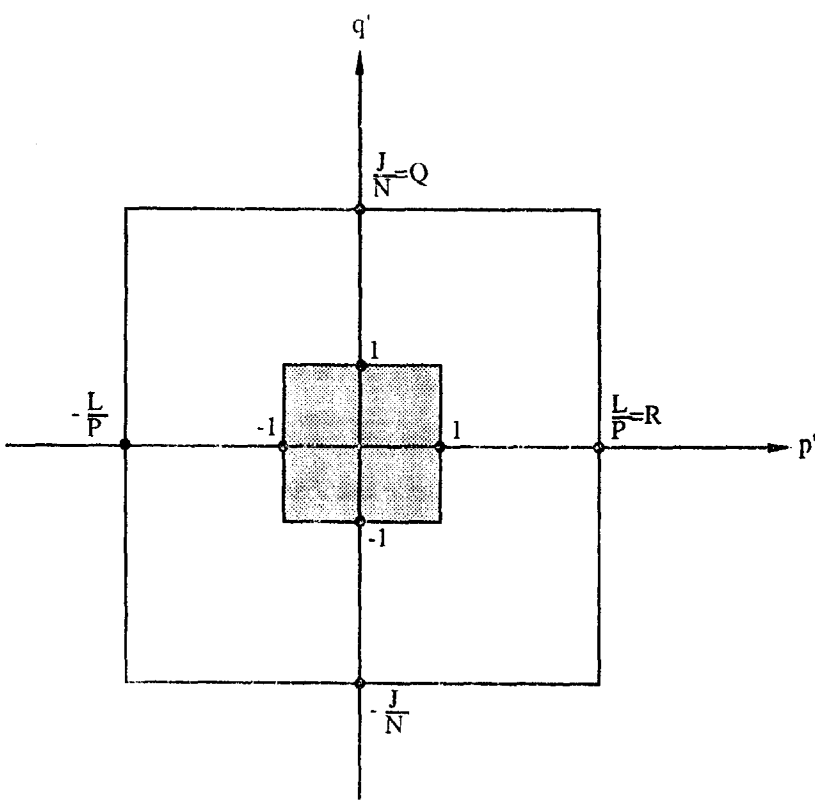


(b) Transformation of the region to an origin about (0, 0). Coordinate transformations are given in Table 2.

Figure 13. (continued on next page)

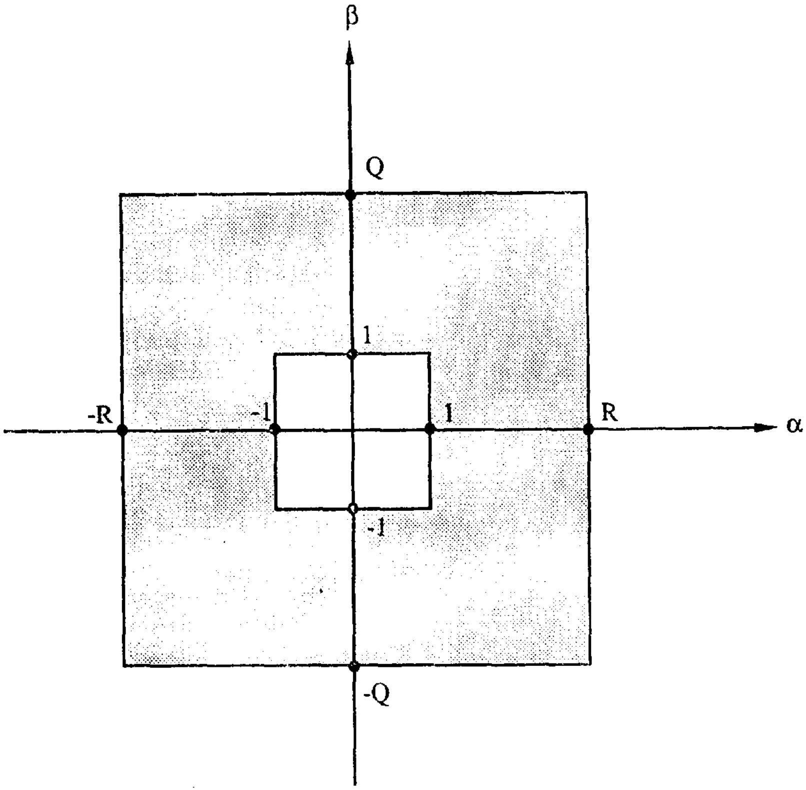


(c) Relabeling under the assumption of symmetry.

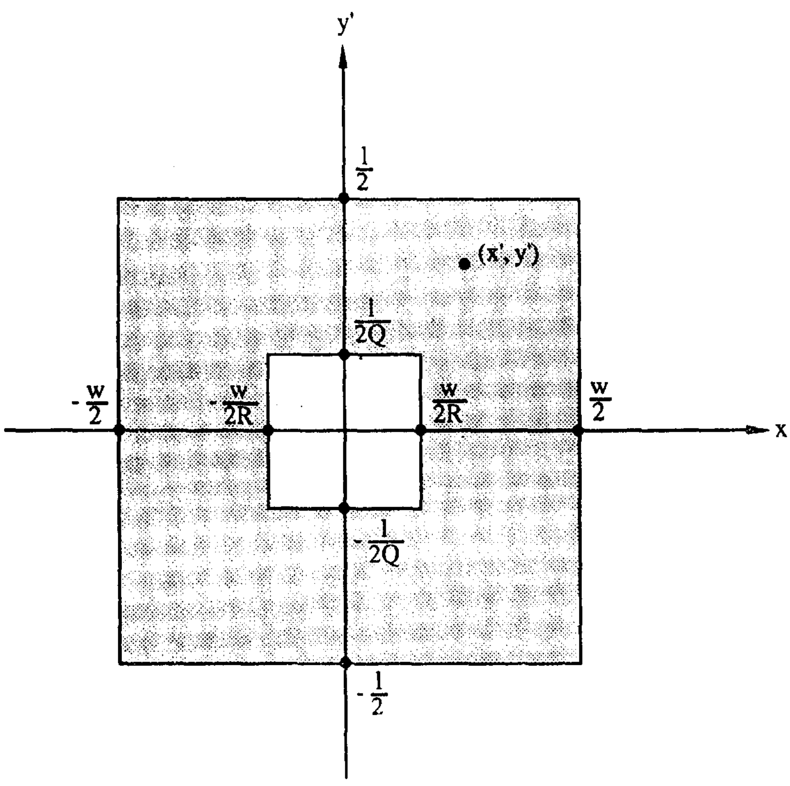


(d) Scaling of the region to a unit region.

Figure 13. (continued on next page)

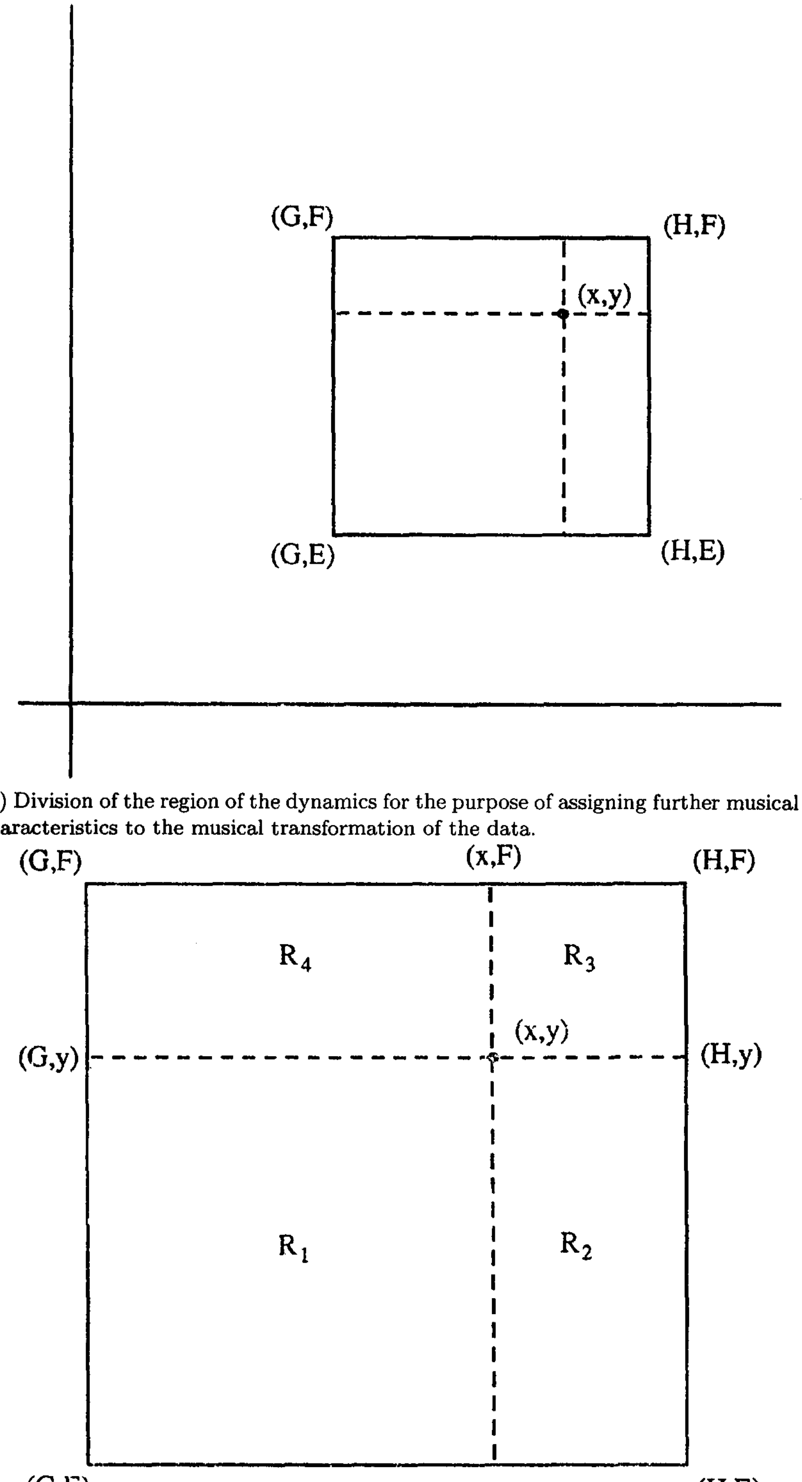


(e) Unfolding of the dynamics into the shaded region about the unit square as specified **by the transformation** given in equation (5.3).



(f) Mapping of the dynamics from the unit square and frame onto the audience **auditorium** as given by equation (6.1).

Figure 13. (continued)



(a) Division of the region of the dynamics for the purpose of assigning further musical characteristics to the musical transformation of the data.

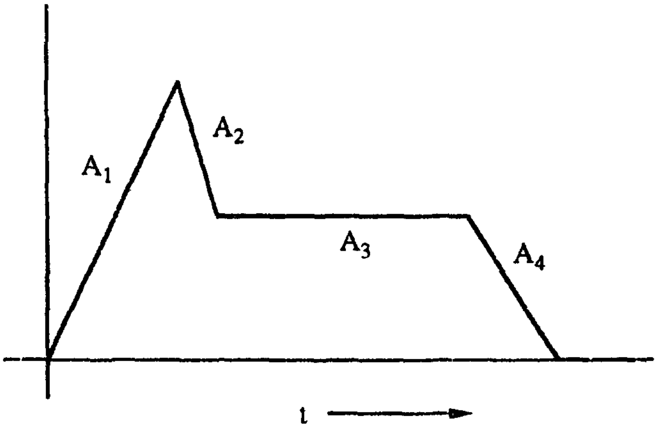
(b) Specification of the coordinate values for each of the area regions given in Table 3.

Figure 14.

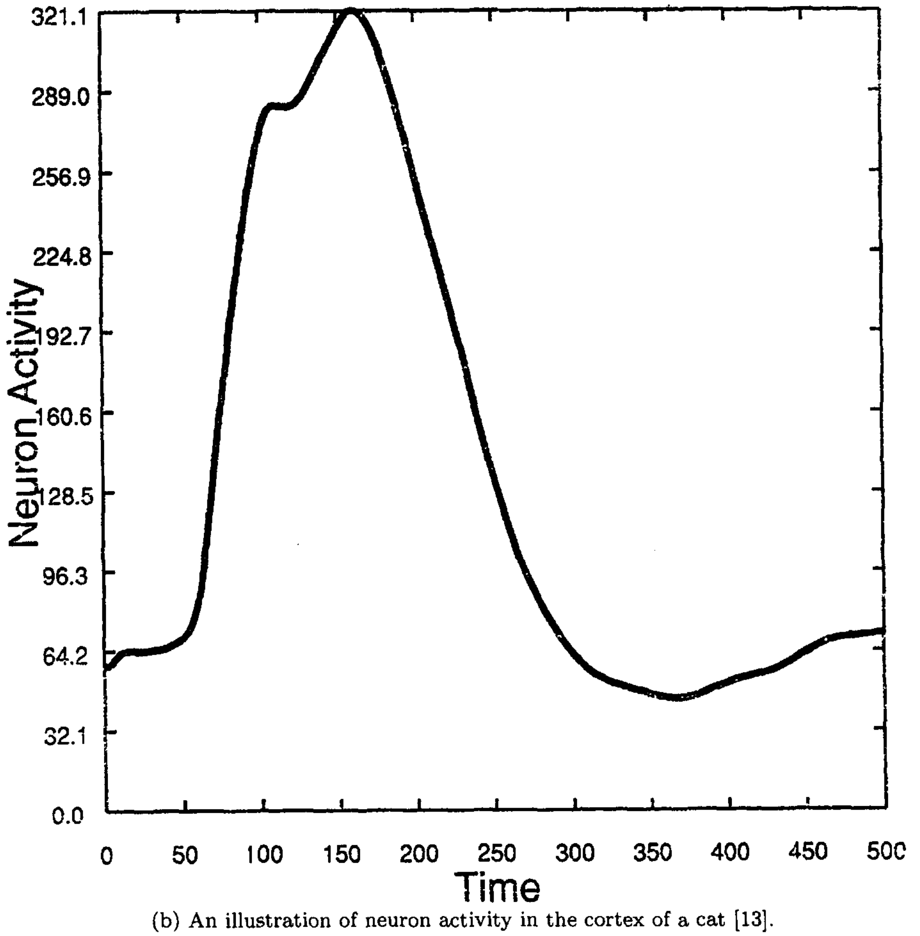
Figure 14a. Assume that we have drawn a crosshair through the point (x, y). This crosshair divides the shaded rectangle into four smaller boxes which we will label R_1, R_2, R_3, and R_4 as illustrated in Figure 14b. Let A_1, A_2, A_3, and A_4 be the corresponding area associated with each rectangle. It is straightforward to demonstrate that these areas are as given in Table 3, where x and y are the coordinates of the model dynamics point at time t.

Table 3. Areas for each area element A_i in Figure 14b.

Area Element	Area
A_1	$(x-G)(y-E)$
A_2	$(y-G)(H-x)$
A_3	$(F-y)(H-x)$
A_4	$(x-G)(F-y)$



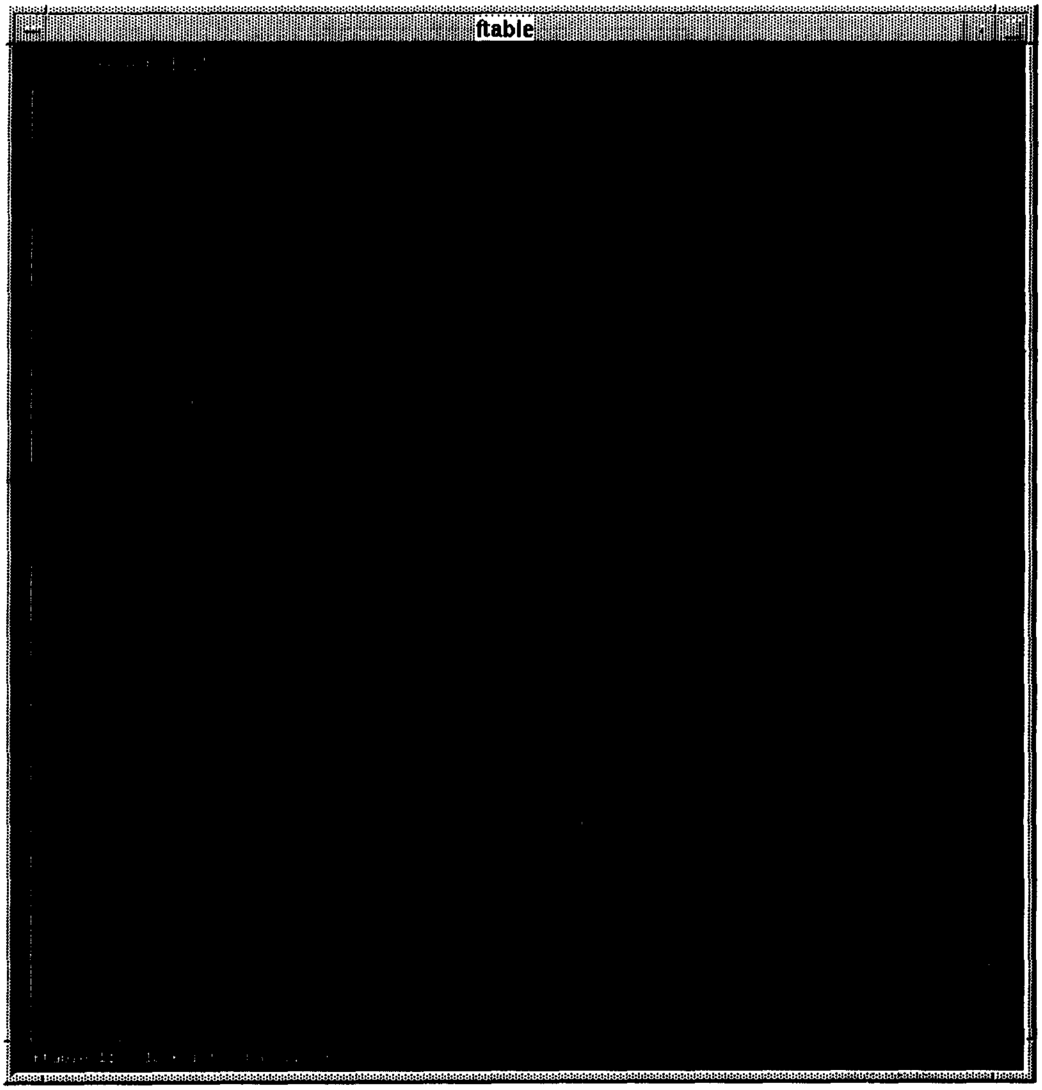
(a) An illustration of the basic ADSR components of a digital signal and how the area region transform specified in Table 4 can be used to modify the construction of the musical signal transformation for the dynamics of the equation under study.



(b) An illustration of neuron activity in the cortex of a cat [13].

Figure 15. (continued on next page)

Control Envelope for Neuron Simulation



(c) An illustration of how the neuron activity illustrated in Figure 15b is used to create a CSound ftable control envelope for simulating the sound of neurons firing.

Figure 15. (continued)

Table 4. Attribute factors for each area element A_i.

Factor	Attribute Value
F_1	$\frac{(x-G)(y-E)}{(H-G)(F-E)}$
F_2	$\frac{(y-G)(H-x)}{(H-G)(F-E)}$
F_3	$\frac{(F-y)(H-x)}{(H-G)(F-E)}$
F_4	$\frac{(x-G)(F-y)}{(H-G)(F-E)}$

If we now divide each subarea A_i, $i = 1,2,3,4$, by the overall total area of the rectangle $A = (H-G)(F-E)$, it is easy to show that the factors F_i $i = 1,2,3,4 \in [0,1]$ for all (x,y) in the shaded rectangle. This is a simple, yet powerful way to associate an attribute quadruple with each point (x,y) of the dynamics. That is, we define a quadruple $A = (A_1, A_2, A_3, A_4)$

that relates the value of the i^{th} factor F_i with a particular attribute A_i. For example, we might scale F_1 so that it takes on the integer values in the range $[0, 127]$. Thus, attribute A_1 might be assigned the `midi-note-number`. It is straightforward to see how such assignments might be made for `volume`, `timbre(voice)`, and other musical attributes.

Clearly, there are all sorts of possible musical relationship assignments that one could make. For example, the attributes might be associated with the ADSR components of a digital signal (see Figure 15) where each factor F_i is associated with some scaled duration of the signal component as measured in milliseconds. We can enhance the sonic cues by changes in the instrumentational component as the time series evolves in space. For example, downstage right might be clarinet, upstage right might be trumpet and, as the sound evolves in space, the spatial cue changes and is enhanced by an orchestral cue change.

Pinkston (personal discussion) points out that assignments that associate the factors with musical attributes such as `volume`, `midi-note-number`, etc., are not likely to yield musically interesting (artistic) rendering of the dynamical behavior of a system under examination. That isn't to say that these assignments would not offer a scientific insight. Pinkston suggests that the assignments look more to esoteric qualities of the music—qualities such as `timbre(voice)`, and other nonbasic musical qualities. For example, one could have a number of digitized sound samples that are associated with various midi channel numbers. These sound samples could be associated with one of the factors F_i, while other factors give the samples their necessary color, volume, duration.

REFERENCES

1. P. Beyls, The musical universe of cellular automata, In *Proc. 1989 Int. Computer Music Conference*, Computer Music Association, San Francisco, CA, (1989).
2. B. Buxton, Using our ears: An introduction to the use of nonspeech audio cues, *SPIE: Extracting Meaning from Complex Data: Processing, Display, Interaction* **1259** (1990).
3. B. Evans, Correlating sonic and graphic materials in scientific visualization, *SPIE: Extracting Meaning from Complex Data: Processing, Display, Interaction* **1259** (1990).
4. S.P. Frysinger, Applied research in auditory data representation, *SPIE: Extracting Meaning from Complex Data: Processing, Display, Interaction* **1259** (1990).
5. W.W. Gaver, The SonicFinder: An interface that uses auditory icons, *Human-Computer Interaction* **4** (1), 67–94 (1989).
6. W.W. Gaver, Auditory icons: Using sound in computer interfaces, *Human-Computer Interaction* **2**, 167–177 (1986).
7. G. Kramer and S. Ellison, Audification: The use of sound to display multivariate data, (preprint) (1990).
8. D. Lunney and R.C. Morrison, Auditory presentation of experimental data, *SPIE: Extracting Meaning from Complex Data: Processing, Display, Interaction* **1259** (1990).
9. D.L. Mansur, M.M. Blattner and K.I. Jo, Sound graphs: A numerical data analysis method for the blind, *J. Medical Systems* **9** (3), 163–174 (1985).
10. J. Pressing, Nonlinear maps as generators of musical design, *Computer Music Journal* **12** (2), 35–46 (1988).
11. D.A. Rabenhorst, E.J. Farrell, D.H. Jameson, T.D. Linton and J.A. Mandelman, Complementary visualization and sonification of multidimensional data, *SPIE: Extracting Meaning from Complex Data: Processing, Display, Interaction* **1259** (1990).
12. E.S. Yeung, Pattern recognition by audio representation, *Anal. Chem.* **52** (7), 1120–1123 (1980).
13. M. Witten and R.E. Wyatt, Increasing our understanding of biological models through visual and sonic representations: A cortical case study, *International Journal of Supercomputer Applications* **6** (3), 257–280 (1992).
14. C. Dodge, "Earth's Magnetic Field", Nonesuch Records, New York, (1970).
15. P. Lansky, "Quakerbridge", GrimTim Music, Princeton Junction, NJ, (1989).
16. G. Schöner, W.Y. Yiang and J.A.S. Kelso, A synergetic theory of quadrupedal gaits and gait transitions, *J. Theor. Biol.* **142**, 359–391 (1990).
17. D.A. Wood, Modelling hexapod gaits by coupled nonlinear oscillators, 3rd Year Applied Mathematics Project Report, University of Warwick, (1991).
18. M. Witten, "Bugged 1.0", Sounds of Science, Austin, TX, (1990).
19. H. Degn, A.V. Holden and L.F. Olsen, *Chaos in Biological Systems*, Plenum Press, New York, (1987).
20. G.P. Scott, Editor, *Time, Rhythms, and Chaos in the New Dialogue with Nature*, Iowa State University Press, Ames, IA, (1991).

21. J. Sundberg, Computer synthesis of music performance, In *Generative Processes in Music*, (Edited by J.A. Sloboda), Clarendon Press, Oxford, (1988).
22. C. Scaletti and A.B. Craig, Using sound to extract meaning from complex data, *SPIE/SPSE Conference Proceedings 1991* (preprint).
23. J.-P. Collet and P. Eckmann, *Iterated Maps on the Interval as Dynamical Systems*, Birkhäuser, Boston, (1980).
24. P.J. Holmes, *New Approaches to Nonlinear Problems in Dynamics*, SIAM Press, Philadelphia, (1980).
25. T.S. Parker and L.O. Chua, *Practical Numerical Algorithms for Chaotic Systems*, Springer-Verlag, New York, (1989).
26. H.O. Peitgen and P.H. Richter, *The Science of Fractal Images*, Springer-Verlag, New York, (1986).
27. H.O. Peitgen and D. Saupe, Editors, *The Science of Fractal Images*, Springer-Verlag, New York, (1988).
28. H.L. Swinney, Observation of order and chaos in nonlinear systems, *Physica D* **7**, 3–15 (1983).
29. S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, Springer-Verlag, New York, (1990).
30. M. Witten, Fitness and survival in logistic models, *Journal of Theoretical Biology* **74**, 23–32 (1978).
31. M. Witten, Some generalized conjugacy theorems and the concepts of fitness and survival in logistic models, *Bulletin of Mathematical Biology* **42**, 507–528 (1980).
32. D.S. Dendrinos and M. Sonis, *Chaos and Socio-Spatial Dynamics*, Springer-Verlag, New York, (1990).
33. E. Di Cera, P.E. Phillipson and J. Wyman, Limit-cycle oscillations and chaos in reaction networks subject to conservation of mass, *Proc. Nat. Acad. Sci. USA* **86**, 142–146 (1989).
34. W.M. Schaffer, Order and chaos in ecological systems, *Ecology* **66** (1), 93–106 (1985).
35. C. Grebogi, E. Ott and J.A. Yorke, Chaos, strange attractors, and fractal basin boundaries in nonlinear dynamics, *Science* **238**, 632–638 (1987).
36. J. Guckenheimer, G.F. Oster and A. Ipaktchi, The dynamics of density-dependent models, *Journal of Mathematical Biology* **4**, 101–147 (1977).
37. M. Henon, A two-dimensional mapping with a strange attractor, *Communications in Mathematical Physics* **50**, 69–78 (1976).
38. E.N. Lorenz, Deterministic nonperiodic flow, *Journal of Atmospheric Science* **357**, 282–291 (1963).
39. E.N. Lorenz, Noisy periodicity and reverse bifurcation, *Annals of the New York Academy of Sciences* (1980).
40. D. Ruelle, Strange attractors, *The Mathematical Intelligencer* **2**, 126–137 (1980).
41. T. Matsumoto, L.O. Chua and M. Komuro, Birth and death of the double scroll, *Physica D* **24**, 97–124 (1987).
42. R.M. May, Biological populations with nonoverlapping generations: Stable points, stable cycles, and chaos, *Science* **186**, 645–647 (1974).
43. R.M. May, Simple mathematical models with very complicated dynamics, *Nature* **261**, 459–467 (1976).
44. T. Rogers, Chaos in systems in population biology, *Progress in Theoretical Biology* **6**, 91–146 (1981).
45. M. Witten, Biological extinction and iterates of functions with finite domain, Ph.D. Thesis, SUNY at Buffalo, Buffalo, NY, (1977).
46. S.R. Channon and J.L. Lebowitz, Numerical experiments in stochasticity and homoclinic oscillation, *Annals of the New York Academy of Sciences*, 108–118 (1980).
47. J.M.T. Thompson and R.J. Thompson, Numerical experiments with a strange attractor, *The Institute of Mathematics and Its Applications* **16**, 150–154 (1980).
48. P. Choissat and M. Golubitsky, Iterates of maps with symmetry, *SIAM J. Math. Anal.* **19**, 1259–1270 (1988).
49. P. Choissat and M. Golubitsky, Symmetry-increasing bifurcation of chaotic attractors, *Physica D* **32**, 423–436 (1988).
50. M.J. Field and M. Golubitsky, Symmetric chaos, *Computers in Physics* (Sept/Oct), 470–479 (1990).
51. D.R. Begault and E.M. Wenzel, Techniques and applications for binaural sound manipulation in human-machine interfaces, NASA Technical Memorandum 102826, (August 1990).
52. D.R. Begault and E.M. Wenzel, Technical aspects of a demonstration tape for three-dimensional sound displays, NASA Technical Memorandum 102826, (October 1990).
53. G.S. Kendall, W.L. Martens and S.L. Decker, Spatial reverberation: Discussion and demonstration, In *Current Directions in Computer Music Research*, (Edited by M.V. Mathews and J.R. Pierce), MIT Press, Cambridge, MA, (1989).
54. F.R. Moore, Spatialization of sounds over loudspeakers, In *Current Directions in Computer Music Research*, (Edited by M.V. Mathews and J.R. Pierce), MIT Press, Cambridge, MA, (1989).
55. E.M. Wenzel, S.H. Foster, F.L. Wightman and D.J. Kistler, Realtime digital synthesis of localized auditory cues over headphones, *Proc. 1989 IEEE ASSP Workshop on Applications of Signal Processing to Audio and Acoustics.*
56. E.M. Wenzel, P.K. Stone, S.S. Fishe and S.H. Foster, A system for three-dimensional acoustic visualization in a virtual environment workstation, In *Visualization '90 Conference Proceedings*, San Francisco, CA, (1990).
57. J.B. Delos, Catastrophes and stable caustics in bound Hamiltonian systems, *J. Chem. Physics* **86**, 425–439 (1987).
58. O. Gurel and O.E. Roessler, Editors, *Bifurcation Theory and Applications in Scientific Disciplines*, New York Academy of Sciences, New York, (1979).
59. F. Kaiser, Limit cycle model for brain waves, *Biol. Cyber.* **27**, 153–163 (1977).

60. F. Kaiser, Biophysical models related to Fröhlich excitations, *Nanobiology* **1**, 149–161 (1992).
61. I. Peterson, Basins of froth, *Science News* **142** (20), 329–330 (1992).
62. C. Scholz, HMSL software language, *Music Technology* (September), 82–83 (1988).