Music synthesis based on nonlinear dynamics

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Music Synthesis Based on Nonlinear Dynamics

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Abstract

Music composition from nonlinear dynamics has been a subject of thorough research, producing interesting music tracks. Simple chaotic maps or even more complex iterative schemes have been proposed, taking advantage of the "structured spontaneity" of nonlinear dynamics by directly transforming the mathematical objects to musical entities. In this work we examine the music compositions produced by two nonlinear systems in terms of complexity, expressed through the Shannon Information Entropy of their Pitch Class Profiles. We present a methodology to interpret the information obtained by the iterative equations to several tonal and rhythmic attributes. This methodology is implemented in real-time software. A Graphical User Interface is provided that allows the user to adjust several variables while listening to and observing the derived results.

Introduction

Various methods that simulate natural phenomena have been used for algorithmic music composition. These phenomena are mostly described by simple and deterministic rules, but the resulting output sometimes exhibits great structural complexity. Dynamic systems, either in the form of differential equations or as iterative maps, have been used for sound synthesis and music composition. Especially in the latter case, very simple iterative schemes like the logistic function [7] or more complex ones [2], have produced music with interesting structure. This kind of deterministic complexity can offer music that incorporates an amount of information within the thresholds that a human listener considers satisfactory.

This work presents a methodology to translate information corresponding to a specific iteration of dynamical systems to musical entities that describe the pitch, intensity, duration, onset time and polyphony of note events. This methodology is realized in real-time software by which the sonic output can be controlled in real–time, with parallel graphical representation of the derived orbits. A modification of this application is used to export alternative sets of compositions with two different tonal setups, diatonic and chromatic, and results are reported on the Shannon Information Entropy (SIE) of their Pitch Class Profiles (PCP). Furthermore, the same feature is extracted from string quartets of Beethoven, Haydn and Mozart. The comparison of the SIE of the PCP in all the aforementioned pieces provides insights about the potential "compositional capabilities" of the examined dynamic systems for both tonal setups.

From Dynamic Systems to Music

We have built two applications [3] that allow the user to control several parameters and then listen to the music synthesized by well–known and examined nonlinear systems. These systems are the *Chirikov Standard Map* [1] and the *Hénon Map* [6, 9], the rich dynamical properties of which have been thoroughly examined [6]. These iterative maps are converted to music by assigning each point (or points in the case of polyphony) of the current iteration to notes, with the use of the MIDI protocol.

The Chirikov Standard Map is expressed by the system of equations

$$\begin{aligned} p_{n+1} &= p_n + K\sin(\theta_n) \\ \theta_{n+1} &= \theta_n + p_{n+1} \end{aligned}$$

where $K > 0$ is a constant, for the initial points we have $p_0, \theta_0 \in [0, 2\pi]$ and p_{n+1}, θ_{n+1} are taken modulo 2π. We consider the parameters of the Hénon Map as follows

$$\begin{aligned} x_{n+1} &= \cos(\theta)x_n - \sin(\theta)(y_n + \varepsilon x_n^2) \\ y_{n+1} &= \sin(\theta)x_n + \cos(\theta)(y_n + \varepsilon x_n^2) \end{aligned}$$

where $\theta \in [0, 2\pi]$ and $\varepsilon \in [-2, 2]$ are constants and the initial points, (x_0, y_0), are each in $[-1, 1]$.

Each iteration of these two dimensional mappings provides a point with two coordinates that we denote as x, y. These coordinates are used to describe note events that incorporate several attributes, namely *pitch*, *intensity* (or velocity under the MIDI terminology), *duration*, *inner onset interval* and *polyphony*. The current point of the iteration is used to describe all these music attributes. In the case of a polyphonic note event, which includes multiple points, the current point is considered the last point of the iteration. The interpretation of the x, y coordinates to the aforementioned musical attributes is accomplished separately for each instrument. The complete process can be described as follows:

a) **pitch**: the user provides a tonal range $[t_{\text{min}}, t_{\text{max}}], \in \mathbb{N}$ and the pitch height t of a note event is computed by normalizing the $x \in [x_{\text{min}}, x_{\text{max}}]$ coordinate in the selected range. Hence

$$t = \left[\frac{(x - x_{\text{min}})(t_{\text{max}} - t_{\text{min}})}{x_{\text{max}} - x_{\text{min}}} \right] + t_{\text{min}},$$

where $[x] = \lfloor x + 0.5 \rfloor$ is the nearest integer rounding of a real number x. The latter formula computes the normalized quantities as described below for intensity, duration and polyphony.

b) **intensity**: the user provides an intensity range $[v_{\text{min}}, v_{\text{max}}], \in \mathbb{N}$ and the intensity v of a note event is computed by normalizing the $y \in [y_{\text{min}}, y_{\text{max}}]$ coordinate in the selected range using the formula described for the computation of pitch.

c) **duration**: the user provides a duration constant, α, that determines the overall duration of the note events produced by an instrument, with lower values creating *staccato* and higher values producing *legato* play feelings. We consider d to be the normalized value of $|x - y|$ from within the range of $\left[0, \max\{|x_i - y_j|\}_{i,j \in \{\text{min, max}\}}\right]$ to the range of $[10, 100]$. The duration of the current event in milliseconds is provided as the multiplication of α with d.

d) **inner onset interval**: the user provides four values, $q_r, r \in 1, 2, 3, 4$, that represent the onset duration of a note event that corresponds to quartile of the coordinates of the respective point. I.e., if we suppose that the current note event has x, y coordinates, then the next note event will occur in a time interval of

$$\begin{cases} q_1 & \text{if } x \geqslant 0 \text{ and } y \geqslant 0 \\ q_2 & \text{if } x < 0 \text{ and } y \geqslant 0 \\ q_3 & \text{if } x < 0 \text{ and } y < 0 \\ q_4 & \text{if } x \geqslant 0 \text{ and } y < 0 \end{cases} \quad \text{or} \quad \begin{cases} q_1 & \text{if } x \geqslant \pi \text{ and } y \geqslant \pi \\ q_2 & \text{if } x < \pi \text{ and } y \geqslant \pi \\ q_3 & \text{if } x < \pi \text{ and } y < \pi \\ q_4 & \text{if } x \geqslant \pi \text{ and } y < \pi \end{cases},$$

in the case of the Hénon map or the Standard map respectively, where q_r represents a duration selected by the user among 2^{nds}, 4^{ths}, 8^{ths}, 16^{ths} and 32^{nds}.

e) **polyphony**: the user provides a polyphony index, p, which is the upper limit of simultaneous notes that an instrument is allowed to play. The polyphony of the current note event, p_c, is computed as the normalized value of $|y - x| \in \left[0, \max\{|y_i - x_j|\}_{i,j \in \{\text{min, max}\}}\right]$ to the range of $[1, p]$.

All values that are required along with some additional controls are adjusted by the user through a Graphical User Interface (GUI). These controls include the real–time adjustment of the coefficients and the current iteration point, while graphical monitoring of the derived orbits is also provided. Furthermore, the user can define a global musical scale for the composition, or separate scales for each instrument. The user can assign all the aforementioned properties to different instruments within a list, creating several orchestration combinations.

Results

To examine the complexity potential of the compositions obtained by the proposed approach, as mentioned previously, we computed the Shannon Information Entropy (SIE) [8] of the PCP for 1200 music tracks composed with the Standard and the Hénon maps. The PCP of a piece describes the distribution of pitches throughout the piece and the SIE of this PCP describes the sparseness of this distribution. In particular, these pieces were composed for random initial x, y values and coefficients (K, θ, ε) and were divided into 4 sets with 300 tracks each, composed with different composition scale setups: `SD` (Standard Diatonic) and `HD` (Hénon Diatonic) were composed using major diatonic scales and `SD` (Standard Chromatic) and `HC` (Hénon Chromatic) were composed in the chromatic scale. The tonal center for all these compositions was random.

To compare the SIE values provided by the PCPs for the aforementioned *artificial* compositions with the ones obtained by real music compositions, we also analyzed 150 string quartets, in particular 50 music pieces composed by Beethoven, Haydn and Mozart, denoted as `Bsq`, `Hsq` and `Msq` respectively. Information related to the SIE of the PCP among these string quartets has previously provided promising results for composer identification [4].

Table 1 shows the the mean values (μ) and the standard deviations (σ) of the SIEs from the PCP distribution of all the aforementioned datasets. First, we observe that the string quartets have a greater SIE mean than the artificial pieces. Moreover, the artificial pieces composed in a diatonic scale (`HD` and `SD`) are indicated to be considerably less complex than real–world compositions. The chromatic artificial compositions on the other hand seem to have a less “complex” PCP distribution than the string quartets, but with greater standard deviation. Figure 1 illustrates the distribution of the SIEs among all compositions with box plots. It is indicated that the “compositional” capabilities of the Hénon map in chromatic scale are more flexible than the ones of the Standard map, meaning that a wider range of complexity can be achieved by the corresponding compositions.

	Artificial compositions				String quartets		
	`HD`	`SD`	`HC`	`SC`	`Bsq`	`Hsq`	`Msq`
μ	1.5976	1.3236	2.0620	1.6488	2.2082	2.1355	2.1181
σ	0.2966	0.3844	0.4432	0.5242	0.0933	0.0971	0.1018

Table 1 : *Box plot of the mean values obtained by the PCP distribution among pieces of the `HD`, `SD`, `HC`, `SC`, `Bsq`, `Hsq` and `Msq` data sets.*

Conclusions

In this work we have presented a system that composes music through two well–known nonlinear systems of equations, the Standard (or Chirikov) map and the Hénon map. A methodology for transforming information by these iterative schemes to musical entities has been presented, which is realized in a real-time software application. The utilization of the Shannon Information Entropy (SIE) has allowed a comparative analysis of the compositions of the proposed approach in different modes. Additionally, indications for the complexity of

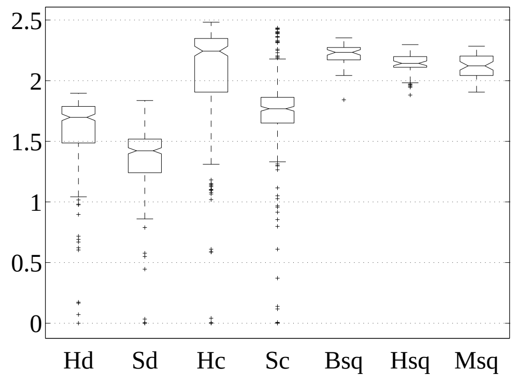

Figure 1 : *Box plots of the PCP distribution SIEs of the examined pieces.*

these compositions compared to real–world pieces were provided by the comparison of the aforementioned SIEs with the respective ones in string quartets of Beethoven, Haydn and Mozart.

These results can be further amplified with the utilization of more sophisticated musical features. For example, the information provided by the PCP is "stationary", in a sense that the transitions between scale degrees is not considered. The transitions can be captured with the utilization of the Markov transition tables [5] (or the N–Gram models). Furthermore, the rhythmic complexity of these compositions should be examined. Finally, we intend to formulate an intelligent system that controls the coefficients of the nonlinear systems in order to produce music that satisfies certain musical criteria. This will allow the exploration of the compositional potentiality of the presented approach more extensively.

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