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Jean-Pierre Boon, Alain Noullez and Corinne Mommen

ABSTRACT

A piece of music is generally perceived as the time evolution of acoustical signals, but, because of its complexity, the intrinsic dynamics of the music cannot be easily identified or characterized. A musical sequence can also be considered as the time series resulting from a dynamical phenomenon. We show that the theory of dynamic systems provides interesting tools for the identification of complex dynamics in music.

About fifteen years ago, Voss and Clarke (1975, 1978) found that loudness variations and pitch fluctuations in music exhibit 1/f power spectra in the low frequency range ($f \leq 10$ Hz) independently of the kind of music considered. This appeared as a surprising and interesting observation not only *per se*, but also in view of the fact that music, the manifestation of an intellectual process, would follow a statistical law encountered in many natural phenomena (Mandelbrot & Voss, 1983). Recently, Klimontovich and Boon (1987) presented a theoretical analysis showing that any music should indeed behave as "1/f noise" when considered over long times (of the order of the duration of an entire piece, such as a symphony or a concerto). So there exists a long time scale over which all musics, as dynamical phenomena, have a common feature characterized by 1/f spectral behavior. On the other hand, the harmonic and counterpoint time scale is set by the dynamics which governs correlations over a few bars or a few tens of successive sounds. On this *short time scale*, notes (or sounds) are strongly correlated: in this sense, short time sequences are highly predictible. And - except for purely random compositions based on a white noise scheme - this must be true for any short musical sequence.

So it is over an intermediate time scale (intermediate between long and short times) that one should expect to identify the specific dynamics of a musical sequence, and thereby to characterize dynamically a musical style, a composer, a work. However, such dynamics is not easily accessible, as it integrates all the complexity folded into the musical development. By considering the musical sequence as a time series (i.e. as the variations of a quantity as a function of time), the characterization of musical dynamics can be interpreted as a problem in dynamical systems theory (Schuster, 1984), in the way the theory has been used to identify chaotic behavior complex systems (e.g. turbulence) (Bergé, Pomeau & Vidal, 1986). Time irreversibility is intrinsic to the musical language; so starting from the idea that we receive musical messages as the time evolution of acoustical signals, one can associate the concept of time series to the

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the development of the musical sequence and therefore analyse musical dynamics from a spatial representation of the time series.

MUSIC AS A DYNAMICAL SYSTEM

In order to materialize the idea, consider simple examples such as an ascending and descending major scale, a children's song, or a single-part piece like one of J. S. Bach's cello suites or Debussy's Syrinx. The time series is given by the instantaneous pitch as a function of time, say X(t). Now, considering the harmonic and counterpoint structure of even a single-part piece like a Bach's suite, it is quite clear that the time series shows only one of the multiple variables in the composition: obviously there are unknown variables. This is quite similar to the situation encountered in the experimental study of dynamical systems, whether in the laboratory (Bergé et al., 1986) or for natural phenomena (Nicolis & Nicolis, 1984): often one has access to only one of the physical variables defining the state of the system. Nevertheless the knowledge of one significant quantity should be sufficient since its time variations are the manifestation of the underlying full dynamics.

The state of a dynamical system is defined at a given time t by the value of the quantities which are necessary and sufficient to describe the system and which obey the equations governing its evolution. For instance, a three-variable model reads

$$\begin{aligned} \dot{X} &= F\,(X, Y, Z; r) \\ \dot{Y} &= G\,(X, Y, Z; r) \\ \dot{Z} &= H\,(X, Y, Z; r) \end{aligned} \tag{1}$$

where the dot indicates the time derivative and r is a control parameter whose value measures the amplitude of the constraint imposed to the system (for hydrodynamic flows, r is the Reynolds number, whose value is proportional to the fluid velocity). F, G, and H are generally non-linear functions and, as a result, when r exceeds some threshold value, the model sytem (1) can exhibit complex dynamical behavior. Such a behavior is well depicted by a phase-space representation: the evolution of the system is shown as a trajectory in the space sustained by the X, Y, Z coordinate axes, and the trajectory is the locus of all points $\underline{X}(t) = (X(t), Y(t), Z(t))$ starting from an initial condition $(\underline{X}(t_o) = (X(t_o), Y(t_o), Z(t_o))$. When only representative quantity of the system is accessible, the (X, Y, Z) representation is not possible. One then invokes a theorem established by Ruelle and Takens (Ruelle 1981; Takens, 1981) who noted that since the system (X, Y, Z) can be rewritten as $(X, \dot{X}, \ddot{X})$, it can be replaced by a system $(X(t), X(t + n\Delta t), X(t + m\Delta t))$ where Δt is a time delay, and n and m are in principle arbitrary numbers. The important feature is that, although the phase portraits in spaces (X, Y, Z), $(X, \dot{X}, \ddot{X})$, $(X(t), X(t + n\Delta t), X(t + m\Delta t))$ are nonidentical, they are topologically equivalent, i.e. the structural and dynamical properties of the trajectories are conserved. Thus the dimensionality and the information entropy (Schuster, 1984) can be obtained from the phase portrait reconstructed from the time series of the successive values of one single quantity.

For the analysis of musical dynamics, we proceed as follows. The piece is played on a musical keyboard interfaced to a micro-computer; the key depressions corresponding to the pitch are converted into digital data which are stored in the computer memory: the musical sequence is thus transfered on disk as a time series, X(t). We programmed the computer to treat the series so as to obtain its phase portrait, its correlation function C(t), and the corresponding power spectrum S(f). For a single part piece, the portrait is constructed by the time delay method: $X(t), X(t+n\Delta t), X(t+m\Delta t)$; a multi-part piece (e.g a fugue) is treated by constructing the phase-space spanned by the set of variables X(t), Y(t), Z(t),... obtained from the time series of each part.

PHASE PORTRAITS IN MUSIC

A simple example is provided by a major scale, going up and down repeatedly over three octaves. The time series (in fact a graphical representation of the score) shows a triangular wave shape with a superimposed staircase fine structure arising from the discrete nature of the variable (pitch varies by integer numbers of half-tones). The correlation function of a periodic signal is obviously an oscillating function and its power spectrum consists of a series of peaks corresponding to the fundamental frequency (reciprocal of the period) and its harmonics. The phase portrait is framed in the space sustained by the quantity X(t) and its delayed values, $X(t+n\Delta t)$ and $X(t+m\Delta t)$, each one extending over the range of the possible variations of X, i.e. over three octaves; thus the phase-space is a cube with three octaves edge length. The phase portrait of the ascending-descending scale is a cycle with dimensionality one (Fig. 1a). This is the basic structure that should be common to musical pieces with repetitive character.

Opposite to the above example of a sequence without unexpected elements, is the case of random music. By generating random changes over three octaves, we programmed the computer to "compose" a random musical sequence. The result is a "white noise music score" whose phase portrait fills homogeneously the entire phase-space (Fig. 1b) and so has dimensionality three (in 3-D phase space). It is noteworthy to mention that, when considering the case of random music, we also "improvised" on the keyboard what we felt should be a random sequence, whose phase portrait was then constructed. The result is striking: the portrait does not feature at all the homogeneous character of "white noise music" (compare Figs. 1b and 1c). The structure of a spontaneous improvisation intended as random clearly reflects the unconscientious manifestation of cultural background. Yet experience showed that the piece was judged by listeners as "random music"!

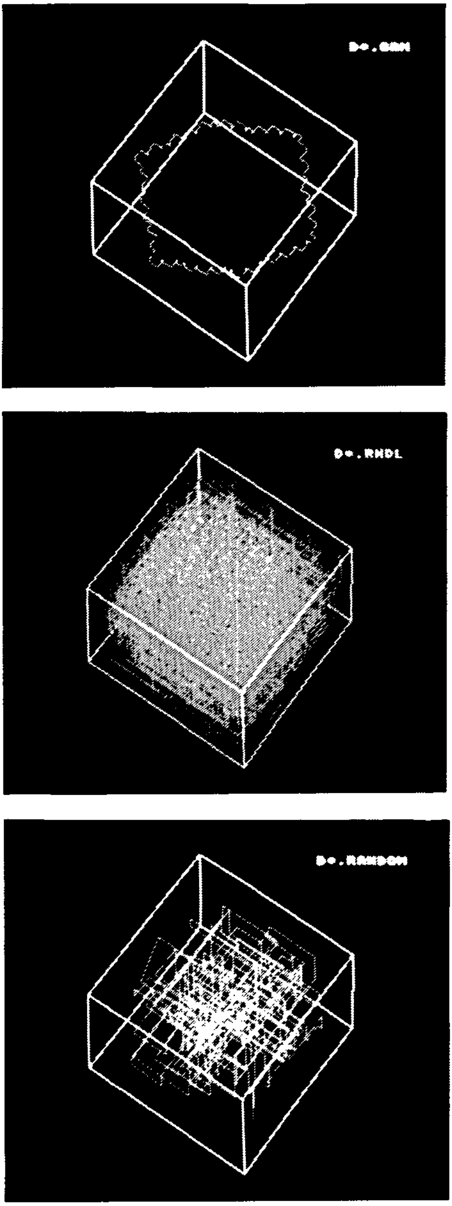

Figure. 1 3-D phase portraits of ascending and descending major scale (a), white noise music (b), and pseudo-random music (c).

Consider now a piece by J. S. Bach: the prelude of the second suite for cello. The piece is played on the keyboard and "recorded" by the computer as a time series (Fig. 2a) wherefrom the correlation function and power spectrum are constructed (Figs. 2b and 2c). Typical features such as peaks (in S(f)) and bumps (in C(t)) indicate correlations and recorrelations in the dynamical structure. Such a structure also emerges from the phase portrait (Fig. 2d): the trajectory develops into a limited portion of phase space, while "exploring" this region densely. The complexity of the trajectory is an indication of unpredictability in the musical content. When a piece by Mozart was analyzed with this procedure, we obtained an apparently less intricate exploration of phase space; we therefore would expect a lower dimensionality and a lower entropy. Should this observation be interpreted as an indication that the "topological signatures" of Bach and Mozart are significantly different?

So far we have examined single-part sequences; what about pieces with several parts? For obvious reasons of graphical representation, we shall

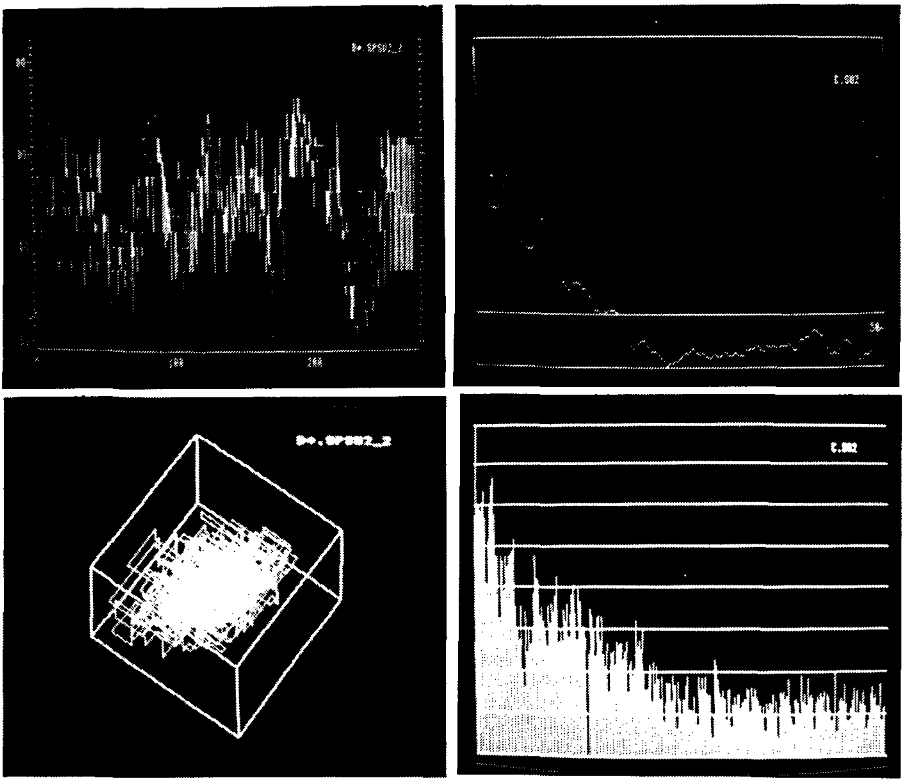

Figure. 2 J. S. Bach's second cello suite (prelude): time series X(t) (unit on vertical scale = half-tone, unit on time scale = 1 sec.) (a); normalized autocorrelation function C(t) (time scale unit = 1 sec.) (b); power spectrum S(f) (frequency range 0-5 Hz) (c); phase portrait ($X(t), X(t+n\,\Delta t), X(t+m\,\Delta t)$; $n=2, m=3.9$) (d).

consider three-part pieces, like fugues. Each part yields a time series, X(t), Y(t), Z(t). So a three-part piece can be considered as a three-variable system, and its dynamical behavior can be depicted from the original phase portrait (without the time delay method). As a simple illustration, we can construct a "three scale canon", by superimposing to the ascending and descending scale (X), a second (Y), then a third scale (Z). The resulting phase-space trajectory first moves along the X-axis, then, when the second part enters, describes a cycle in the (X, Y) plane, and, when the third part combines to the first two, the cycle develops in 3-D space.

We analyzed in this way three-part pieces from several composers and various styles; as typical examples we present works by Bach, Mozart, and Schumann. The Ricercar of the Musical Offering yields a phase portrait (Fig. 3a) whose topology appears to be typical of the composer in that it bears structural similarity to the portrait of the cello suite (Fig. 2d). The structure is

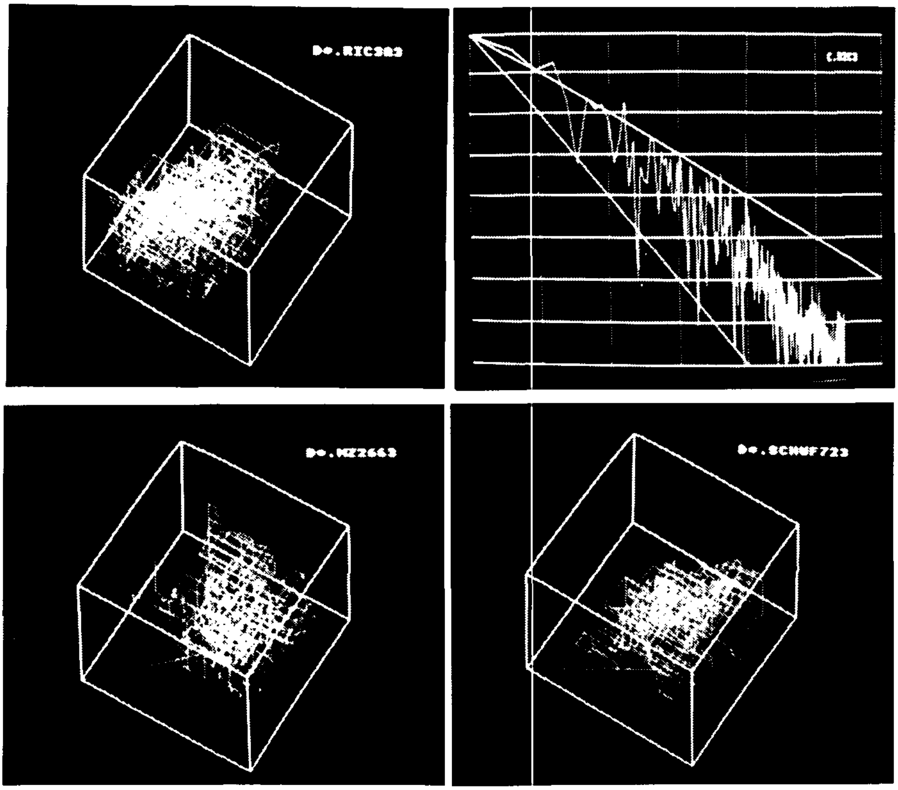

Figure. 3 (X, Y, Z) phase portraits of J. S. Bach's Musical Offering (Ricercar) (a), Mozart's string trio KV266 (adagio) (b), and Schumann's second fugue for piano (c). Power spectrum of first part of J. S. Bach's Ricercar (log-log plot of S(f), frequency range 0-5Hz); the straight lines have slope one ("1/f noise") and two ("red noise") respectively.

also topologically conserved in the portrait reconstructed from one of the parts alone (using the time delay method) as implied by the Ruelle-Takens theorem. The same observation followed from the analysis of Mozart's string trio KV266. On the other hand Bach's and Mozart's topological features seem to be well differentiated from their respective phase portraits (see Figs. 3a and 3b). As an example of a romantic music piece, we show the portrait constructed from Schumann's second fugue for piano (Fig. 3c): the trajectory develops a structure that distinguishes from Bach's and Mozart's while incorporating elements from both.

We also show (Fig. 3d) a typical log-log plot of the power spectrum of the time series (here the first part of Bach's Ricercar). The major trends in the spectrum can be identified as characterizing three spectral regions. One observes, in the midregion - 0.3 Hz to 3 Hz (the dynamic range considered in this study) - a decay with average slope between one and two; in the low frequency range, a smooth transition towards the 1/f line; and white noise type fluctuations in the high frequency region. These high frequency fluctuations cannot be characteristic of the dynamics of the musical sequence itself, but should be interpreted as tempo fluctuations.

DIMENSIONALITY AND ENTROPY

Dynamical systems theory so appears to provide means for the representation of dynamical complexity in music. Beyond the phase portrait representation the question arises as to how the information contained in this representation could be quantified.

We first reconsider the abstract space formed by the phase space spanned by the variables of the system, i.e. the three parts of the pieces considered here. Then the phase space is a three-dimensional space where a point represents the state of the system at a given time. The evolution of the system is given by a succession of points, which describe a trajectory in this space: the phase trajectory, which is representative of the dynamics of the system (that is of the development of the music in the course of time). Thus periodic dynamics (e.g. a piece of music with strong periodicity) produces a phase portrait which is a closed loop (e.g. the repeated ascending/descending scale shown in Fig. 1a). Such a loop is a line and has dimension one. On the other hand for a random process, all states are equally possible: any point in phase space can be visited. As a result, the phase trajectory representative of a random process develops so as to "fill" the whole accessible space. The phase-space considered here being three-dimensional, the mathematical object constructed by such a trajectory has dimension three (See Fig. 2b).

Such examples are, of course, limiting cases. A cycle with dimension one is a typical example of a system with simple deterministic dynamics (e.g., the rule for constructing a scale); any state is unambiguously fixed and therefore fully predictible. In contradistinction, a random sequence results from a process where the behavior of the system is unpredictible: the phase trajectory visits all available space and so defines an object which has the dimension of the embedding space (here three).

Where, with respect to these schematic cases, do musical sequences stand? What is the dimensionality of their phase trajectories? On the basis of the techniques used in dynamical systems theory, we have developped a method for the quantitative analysis of the characteristics of the trajectories obtained from the time series of musical sequences. The first quantity considered is the measure of the Hausdorff dimension, which generalizes the Euclidian dimension to non-integer or *fractal* dimensions. For all musical pieces which we analyzed, we found dimensionality values around two (between 1.75 and 2.25). We are so led to the preliminary global conclusion that there exists some structural universality in musical dynamics (in the same sense as the spectral analysis yields a global 1/f type property): structurally and dynamically, musics are characterized by some dynamics which is intermediate between deterministic (predictable) dynamics and random (unpredictable) dynamics. (In physical sciences, the term "deterministic chaos" is now commonly used to denote the behavior of systems which, although governed by deterministic rules, exhibit unpredictable chaotic behavior).

The second important quantity, which is complementary to the dimensionality, for quantifying the topological complexity of phase trajectories, is the *information entropy*. This entropy is a measure of the rate of unpredictability in a sequence of events or states. Let us reconsider the example of the ascending/descending scale: the sequence of notes is governed by a simple rule which determines the value of a note from the value of the previous one, with a minor uncertainty related to the fact that the scale can be ascending or descending. Thus there is essentially no unexpected event in such a sequence: any new event or new state (a note) brings no additional information. Then the entropy has the value zero (or close to zero). The opposite situation is when all previous knowledge provides no way to predict what is coming next, like in a random sequence, where the information is renewed with each new event. Then the entropy has maximum value.

The unexpected elements in a piece of music can be found in the deviations from established rules and the violation or even the mere rejection of such rules. In the context of classical forms, these deviations are mostly related to the liberty taken with respect to tonality. Thus when Leibowitz (1951) considers *the complexity of musical language,* he argues that *Bach's and Haendel's complex polyphonic style is commonly opposed to what has been called the homophony of Haydn and Mozart (...). According to which criteria does one evaluate simplicity and complexity? Only one: the counterpoint (...). However, continues Leibowitz, the counterpoint is hardly the only constituting element in music, and, even more, it should be obvious that music can be simple or complex independently of any notion of counterpoint.* He then considers the problem of harmony and so observes that the composer's *audacity as well as harmonic complexity may and must be* *evaluated according to further criteria.* Those then invoked *concern the principles of tonality expansion* and here - as argued on the basis of specific examples - *Haydn's and Mozart's works appear more audacious than those of their precursors.* Obviously the argument is of considerable importance as it leads the author to the concept of *increasing complexity that should determine the overall evolution of musical tradition.*

By seeking to introduce in our analysis a measure for quantifying the information entropy in a piece of music, we defined a new quantity: the *parametric entropy,* which yields a quantitative evaluation of deviations from tonality. When we arrived at this idea, we realized that our approach converges towards Leibowitz' analysis, but in addition provides a quantitative estimate of the measure of complexity. It then remains to be seen to what extent the analysis supports the hypothesis of *increasing complexity.* In practice, the new entropy that we introduce provides a statistical evaluation of the range of the interval between successive notes, including a weighting factor that takes into account whether or not the interval belongs to the tonality. Mathematical details are unimportant here and so are omitted (they can be found elsewhere; Mommen, 1988). The fact that a given interval belongs to the tonality is expressed by a parameter (hence the expression: parametric entropy). The operational result is that a large value of the entropy is indicative of frequent excursions away from the tonality, with transitions over intervals uniformly distributed over a large number of notes. On the contrary, the entropy will be low when a note determines almost unambiguously the next one, in particular when the next note remains in the range of the tonality. Table 1 shows the results so obtained for pieces by Bach, Mozart and Schumann; the data are normalized with respect to the maximum entropy (corresponding to the random sequence) and are presented in increasing order of the entropy values.

Table 1. Normalized Parametric Entropy

Ascending/descending scale (3 octaves)	0.159	-
Mozart: String trio KV266 (BbM)	0.233	BbM
Bach: Well tempered clavier, fugue Nr 1 (CM)	0.240	GM
Schumann: Fugue Nr 2 for clavier (Dm)	0.302	FM
Bach: Cello suite Nr 2 (Em)	0.311	FM
Mozart: String trio KV 563 (EbM)	0.353	EbM
Bach: Ricercar from Musical Offering (Cm)	0.418	EbM
Random music	1.000	-

These results show how pieces of music can be characterized in a quantitative way according to the criterion of deviations from tonality. The computations were performed not only with the clef (shown in parenthesis) as the reference tonality but also with respect to other tonalities. It would be expected that, as a general rule, the minimum entropy should be obtained for the tonality corresponding to the clef. We found that this is not necessarily the case. The values given in Table 1 are the minimum entropies obtained and, as shown in the last column, they may as well correspond to a neighboring tonality or to the corresponding major tonality.

So a quantitatively measurable entropy can be defined as a significant evaluation of the dynamical complexity of a musical sequence. The results obtained show that this complexity - in contrast to Leibowitz' hypothesis - appears to be characteristic of the composition rather than of the composer. Whether this observation can be interpreted in more general terms remains to be confirmed in the light of a systematic investigation of a large number of pieces by a large number of composers.

CONCLUDING REMARKS AND PERSPECTIVES

We have presented a method where the tools of the theory of dynamical systems have been shown to be applicable to a new approach in music analysis. It has been suggested that this method could offer interesting perspectives for the identification of coherence and structure in the analysis of new music (C. Deliège, private communication). Whether this potentially can materialize remains to be shown. In particular no conclusion should be drawn before a systematic analysis be performed covering works of different styles and periods extending over whole music history. Extension of the method could also be considered for the analysis of other types of acoustical expression like e.g. bird songs or human languages.

As of interest to the composer, we note that, by following a reciprocal procedure, the method could be exploited as a technical tool in music composition. Starting from a thre-dimensional figure in the phase-space representation, one can generate time series that can be "recoded" as a score or played on a synthetizer interfaced to the computer. When the time delays $n\Delta t$ and $m\Delta t$ are set, there is a unique path from the time series to the phase portrait. However, when operating backwards, one must take into account that the discrete value of the pitch variations implies possible intersections in the trajectory: a point in phase space can be visited more than once. As a result one single trajectory can be unfolded in various ways to generate many different time series. One could so "compose multiple variations" from one single figure in phase space.

With the definition of the parametric entropy, we have deviated from the phase-space representation to return to the original time series (i.e. the time evolution of the piece of music as given by the score). As a result the entropy so measured "quantifies" the harmonic and counterpoint aspects of musical dynamics, but ignores the rhythmic characteristics. Indeed the "time signature" so obtained takes into account the pitch correlations in the time series, independently of the duration of the notes.

To quantify the entropy content of a composition, we have been led to introduce a non-physical concept: tonality, which, as a matter of fact, is not a property intrinsic to music either. Tonality is a reference frame which has been constructed progressively as a particular feature of western culture. So parametric entropy should not be viewed as an absolute characterization, but rather as relative to the specific context of tonality. This is of course the frame within which occidental musical sensitivity has developped. It thus appears as quite logical that our analysis should account for this given framework. On the other hand the validity of the approach is by no means subject to this limitation; indeed adapting the parametrization to fit the requirements (or the lack of requirements) imposed by different frameworks offers the possibility to develop similar procedures for the investigation of other musical languages. Interesting perspectives should arise in the sense that such an approach could possibly shed some light on the specificities and generalities of various forms of musical expression - at least from the viewpoint of a quantitative analysis, where much remains to be discovered.

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REFERENCES

Bergé, P., Pomeau, Y. & Vidal, C. (1986). *Order within Chaos,* New York; Wiley.
Klimontovich, Y. L. & Boon, J. P. (1987). *Eur. Phys. Lett.* 3, 395-399.
Leibowitz R., (1951). *L'évolution de la musique de Bach à Schönberg*, Paris: Correa, chapt. II.
Mandelbrot, B. B. & Voss, R. F. (1983). In *Noise in Physical Systems and 1/f Noise* (eds Savelli, M., Lecoy G. & Nougier, J. P.) Amsterdam: Elsevier Science Publishers. 31-39.
Mommen, C. (1988). *Systèmes complexes et structure musicale,* Mémoire de mathématiques de l'Université de Bruxelles, Bruxelles.
Nicolis, C. & Nicolis, (1984). *Nature* 311, 529-532.
Ruelle, D. (1981). In *Nonlinear Phenomena in Chemical Dynamics* (Eds Pacault, A. & Vidal, C.) Berlin; Springer.
Schuster, H. G. (1984). *Deterministic Chaos.* Weinheim: Physik-Verlag.
Takens, F. (1981). *Lecture Notes in Mathematics* 898, 366-376.
Voss, R. F. & Clarke, J. (1975). *Nature* 258, 317-318.
Voss, R. F. & Clarke, J. (1978). *J. Acc. Soc. Am.* 63, 258-262.

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