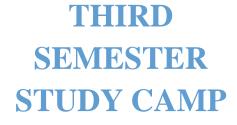
MAJLIS ARTS AND SCIENCE COLLEGE PURAMANNUR



Probability Distributions and Sampling Theory

MODULE-1(continued..)

MAJLIS ARTS AND SCIENCE COLLEGE

Affiliated to the University of Calicut,
Approved by the Government of Kerala,



11. Mean and variance of a binomial distribution are $\frac{8}{3}$ and $\frac{16}{9}$. Find (i) P(x=1) and (ii) P(x=1) and The Binomial distribution $f(x) = n(x) p^{x} e^{n-x}, x = 0,1,2,...,n$ p+2=1Civen-hard

Mean = 8/3 and Variance = 16/9

in
$$p = 86$$
 for $pp = 16/q$

$$\frac{np2}{np} = 2 \Rightarrow \frac{16/q}{q} \times \frac{8}{8}$$

$$= \frac{2}{3}$$

$$p = 1 - 2 = 1 - 26 = \frac{1}{3}$$

$$= \frac{8}{3} \times \frac{3}{1}$$

$$= \frac{8}{3} \times \frac{3}{1} \times \frac{3}{1}$$

$$= \frac{8}{3} \times \frac{3}{1} \times \frac{$$

$$= 0.039 + 0.156$$
 $= 6.195$

Essay

Derive the my fof a normal distribution with parameters pe and or

Moment generating function

$$M_x(t) = E[e^{tx}]$$

$$= \int_{-\infty}^{\infty} e^{tx} f(x) dx$$

$$= \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sigma \sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{t(\mu+\sigma z)}e^{-z^2/2}.\sigma dz$$

$$= \frac{e^{\mu t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t\sigma z - z^2/2} dz$$

Put
$$\frac{x-\mu}{\sigma} = z$$

when $x = \pm \infty$

$$= \frac{e^{\mu t}}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-1/2(z^2 - 2t\sigma z + t^2\sigma^2) + 1/2t^2\sigma^2} dz_{dX = \sigma dz}$$

$$dz_{dX} = \sigma dz$$

$$= \frac{e^{\mu t + \frac{1}{2}t^2\sigma^2}}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-1/2(z-t\sigma)^2} dz$$

$$z = \pm \infty$$

$$= \frac{e^{\mu t + \frac{1}{2}t^2\sigma^2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-u^2/2} du$$

Put
$$z - t\sigma = u$$

 $dz = du$
when $z = 0$,
 $u = \pm \infty$

$$= \frac{e^{\mu t + \frac{1}{2}t^2\sigma^2}}{\sqrt{2\pi}} 2 \int_{0}^{\infty} e^{-u^2/2} du$$

$$= \frac{e^{\mu t + \frac{1}{2}t^{2}\sigma^{2}}}{\sqrt{2\pi}} 2\int_{0}^{\infty} e^{-v} \frac{dv}{\sqrt{2v}}$$

$$= \frac{e^{\mu t + \frac{1}{2}t^{2}\sigma^{2}}}{\sqrt{\pi}} \int_{0}^{\infty} v^{\frac{1}{2}-1} e^{-v} dv$$

$$= \frac{e^{\mu t + \frac{1}{2}t^{2}\sigma^{2}}}{\sqrt{\pi}} \times \frac{1}{1/2}$$

$$= \frac{e^{\mu t + \frac{1}{2}t^{2}\sigma^{2}}}{\sqrt{\pi}} \times \sqrt{\pi}$$
Thus, $M_{x}(t) = e^{\mu t + \frac{1}{2}t^{2}\sigma^{2}}$

Put
$$\frac{u^2}{2} = v$$

 $u^2 = v$
2udu = 2dv
 $\frac{dv}{u} = \frac{dv}{\sqrt{2v}}$
when $u = 0$, $v = 0$
 $u = -\infty$, $v = \infty$

13. Obtain the mgf of X following binomial cliffibed.

From with parameters n and p. Hence state and prove the additive property of binomial dutribution.

If x and Y are independent binomial random variables with parameters (6,0.5) and (4,0.5) respectively, calculate P(x+4=3)

od.

X -> B (6,0.5)

y -> B (4,0.5)

By additive property x+Y > B (6+4,0.5)

Let
$$z = x+y$$

 $z \to B(10,0.5)$

Then

$$p(z \ge 3) = 1 - p(z \le 3)$$

= $1 - \left[p(z = 0) + p(z = 1) + p(z = 0)\right]$

$$f(z) = nC_z p^z 2^{n-z}$$
, $z = 0,1/2,-10$
 $n = 10 p = 0.5$

$$(\varepsilon < v < 0) = 0.00097$$

$$P(z=1) = 10C, (4)(4)^{9}$$

$$P(z=3) = 1 - P(z=3)$$

$$= 1 - \left[0.00097 + 0.0097 + 0.044\right]$$

$$= 1 - 0.0546$$

$$= 0.9454$$