# MAJLIS ARTS AND SCIENCE COLLEGE PURAMANNUR

## THIRD SEMESTER STUDY CAMP

**Probability Distributions and Sampling Theory**

**MODULE-1(continued..)**

**MAJLIS ARTS AND SCIENCE COLLEGE**

Affiliated to the University of Calicut,

Approved by the Government of Kerala,

11. Mean and variance of a binomial distribution are $\frac{8}{3}$ and $\frac{16}{9}$. Find (i) $P(X=1)$ and (ii) $P(X \leq 1)$

A The Binomial distribution

$$f(x) = nC_x\, p^x q^{n-x}, \quad x = 0, 1, 2, \dots, n$$

$$p+q=1$$

Given that

Mean $= 8/3$ and Variance $= 16/9$

ie, $np = 8/3$ & $npq = 16/9$

$$\therefore \frac{npq}{np} = q \Rightarrow \frac{16}{9} \times \frac{3}{8}$$

$$= \underline{\underline{2/3}}$$

$$\Rightarrow P = 1 - q = 1 - 2/3 = \underline{\underline{1/3}}$$

$$np = 8/3 \Rightarrow n = 8/3 / p$$

$$= 8/3 \times 3/1$$

$$= 8$$

~~We~~ get

So we have $n = 8$, $p = 1/3$ & $q = 2/3$

(i) $P(X=1) = nC_1 p^1 q^{n-1}$

$$= 8C_1 (1/3)^1 (2/3)^7$$

$$= \underline{\underline{0.156}}$$

(ii) $P(X \leq 1) = P(X=0) + P(X=1)$

$$= nC_0 p^0 q^{n-0} + nC_1 p^1 q^{n-1}$$

$$= 8C_0 (1/3)^0 (2/3)^8 + 8C_1 (1/3)^1 (2/3)^7$$

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$$= 0.039 + 0.156$$

$$= \underline{\underline{0.195}}$$

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# Essay

12. Derive the m.g.f of a normal distribution with parameters $\mu$ and $\sigma^2$

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# Moment generating function

$$M_x(t) = E[e^{tx}]$$

$$= \int_{-\infty}^{\infty} e^{tx} f(x)\,dx$$

$$= \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t(\mu+\sigma z)} . e^{-z^2/2} . \sigma dz$$

$$= \frac{e^{\mu t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t\sigma z - z^2/2} dz$$

Put $\frac{x-\mu}{\sigma} = z$

$x - \mu = \sigma z$

$$= \frac{e^{\mu t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-1/2(z^2 - 2t\sigma z + t^2\sigma^2) + 1/2\, t^2\sigma^2} dz$$

$dx = \sigma dz$

when $x = \pm\infty$

$z = \pm\infty$

$$= \frac{e^{\mu t + \frac{1}{2}t^2\sigma^2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-1/2(z-t\sigma)^2} dz$$

Put $z - t\sigma = u$

$dz = du$

when $z = 0,$

$u = \pm\infty$

$$= \frac{e^{\mu t + \frac{1}{2}t^2\sigma^2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-u^2/2} du$$

$$= \frac{e^{\mu t + \frac{1}{2}t^2\sigma^2}}{\sqrt{2\pi}} 2\int_{0}^{\infty} e^{-u^2/2} du$$

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$$= \frac{e^{\mu t+\frac{1}{2}t^2\sigma^2}}{\sqrt{2\pi}} 2\int_0^\infty e^{-v} \frac{dv}{\sqrt{2v}}$$

Put $\frac{u^2}{2} = v$

$u^2 = v$

$2udu = 2dv$

$\frac{dv}{u} = \frac{dv}{\sqrt{2v}}$

when $u = 0, v = 0$

$u = -\infty$ , $v = \infty$

$$= \frac{e^{\mu t+\frac{1}{2}t^2\sigma^2}}{\sqrt{\pi}} \int_0^\infty v^{\frac{1}{2}-1} e^{-v} dv$$

$$= \frac{e^{\mu t+\frac{1}{2}t^2\sigma^2}}{\sqrt{\pi}} \times \frac{\overline{|1/2}}{1^{1/2}}$$

$$= \frac{e^{\mu t+\frac{1}{2}t^2\sigma^2}}{\sqrt{\pi}} \times \sqrt{\pi}$$

Thus, $M_x(t) = e^{\mu t+\frac{1}{2}t^2\sigma^2}$

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13. Obtain the mgf of x following binomial distribution with parameters n and p. Hence state and prove the additive property of binomial distribution.

If x and y are independent binomial random variables with parameters (6, 0.5) and (4, 0.5) respectively, calculate $P(x+y \geq 3)$

a.

$$X \rightarrow B(6, 0.5)$$

$$Y \rightarrow B(4, 0.5)$$

By additive property $X+Y \rightarrow B(6+4, 0.5)$

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Let $z = x+y$

$\therefore z \to B(10, 0.5)$

Then

$$P(z \geq 3) = 1 - P(z<3)$$
$$= 1 - [P(z=0)+P(z=1)+P(z=2)]$$

$$f(z) = nC_z \, p^z q^{n-z} \quad , \quad z = 0,1,2,\dots 10$$

$n = 10 \quad p = 0.5$

$$\therefore P(z=0) = 10C_0 (1/2)^0 (1/2)^{10}$$
$$= 0.00097$$

$$P(z=1) = 10C_1 (1/2)^1 (1/2)^9$$
$$= 0.0097$$

$$P(z=2) = 10C_2 (1/2)^2 (1/2)^8$$

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$$= \underline{\underline{0.044}}$$

$$\therefore P(Z \geq 3) = 1 - P(Z < 3)$$

$$= 1 - [0.00097 + 0.0097 + 0.044]$$

$$= 1 - 0.0546$$

$$= \underline{\underline{0.9454}}$$

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