GENERAL ASSEMBLY

LINEAR ALGEBRA REVIEW (OVERVIEW)

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AGENDA

- Notation
- Linear Operations
- Advanced Topics

WHAT IS LINEAR ALGEBRA?

- Linear algebra is the math of vectors and matrices.
- It is distills math problems down to their fundamental principles, which enables us to solve complex problems in a simpler way

WHAT IS LINEAR ALGEBRA?

- Solve this:
- $4x_1 – 5x_2 = -13$
- $-2x_1 + 3x_2 = 9$

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- Solve this:
- $4x_1 - 5x_2 = -13$
- $-2x_1 + 3x_2 = 9$

- $X_2 = 5$
- $X_1 = 3$

WHAT IS LINEAR ALGEBRA?

- Matrix algebra:

$$A = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad b = \begin{bmatrix} -13 \\ 9 \end{bmatrix}$$

BASIC NOTATION - MATRIX

- A matrix with real-valued entries, m rows, and n columns

$$A \in \mathbb{R}^{m \times n}$$

$$\begin{array}{cc} n & \\ \begin{bmatrix} \quad & \quad \end{bmatrix} & m \end{array}$$

- Denote a single entry in this matrix

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{bmatrix} \qquad A_{ij}$$

Linear_Algebra_Review.pdf

BASIC NOTATION - VECTOR

- A (column) vector with n real-valued entries

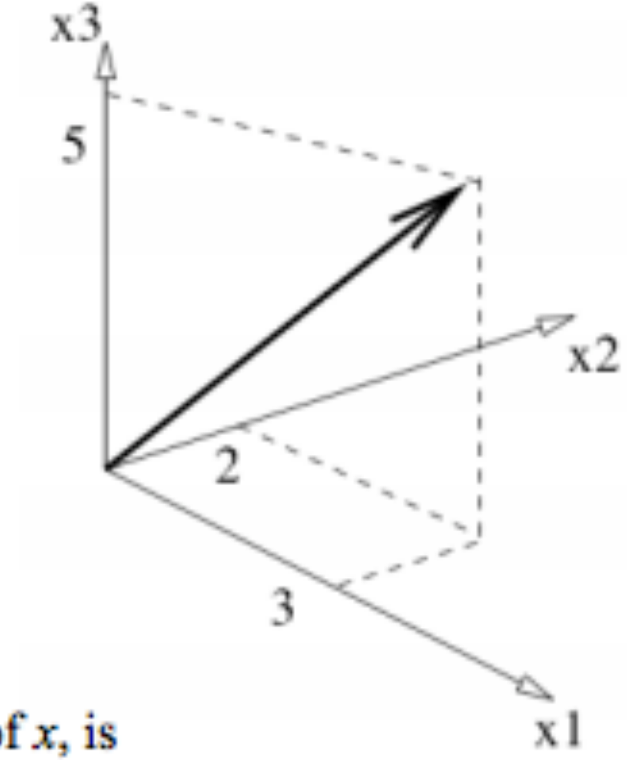
$x \in \mathbb{R}^n$ $\in$

- Denote a single entry in this vector:

x_i denotes the i^{th} entry

VECTOR: SPATIAL REPRESENTATION

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{pmatrix} \quad \text{e.g.} \quad \boldsymbol{x} = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$$



The **length** of x, a.k.a. the **norm** or **2-norm** of x, is

$$\|x\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

e.g.,

$$\|x\| = \sqrt{3^2 + 2^2 + 5^2} = \sqrt{38}$$

BASIC NOTATION - TRANSPOSE

- The transpose operator A^T switches rows and columns of a matrix

$$A_{ij} = (A^T)_{ji}$$

$$A = \overset{n}{\begin{bmatrix} & \\ & \end{bmatrix}} m \qquad A^T = \overset{m}{\begin{bmatrix} & \\ & \end{bmatrix}} n$$

- Denote a single entry in this vector:

$$x = \underset{1}{\begin{bmatrix} \\ \\ \end{bmatrix}} n \qquad x^T = \underset{n}{\begin{bmatrix} & & \end{bmatrix}} 1$$

BASIC NOTATION – ELEMENTS OF A MATRIX

- We can write a matrix in terms of its columns

$$A = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix}$$

- Denote a single entry in this vector:

$a_1 \in \mathbb{R}^m$ $\quad (a_i)_j = A_{ji}$

LINEAR OPERATIONS - ADDITION

- Vectors of equal dimensions may be added together:

$$u + v = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \end{bmatrix}$$

$A = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix}$

Linear_Algebra_Review.pdf

- ~~Denote a single entry in this vector:~~

Represent a matrix by its rows:

$$A = \begin{bmatrix} - & a^T & - \\ - & a^T & - \\ - & a^T & - \end{bmatrix} \qquad a_i \in \mathbb{R}^n$$

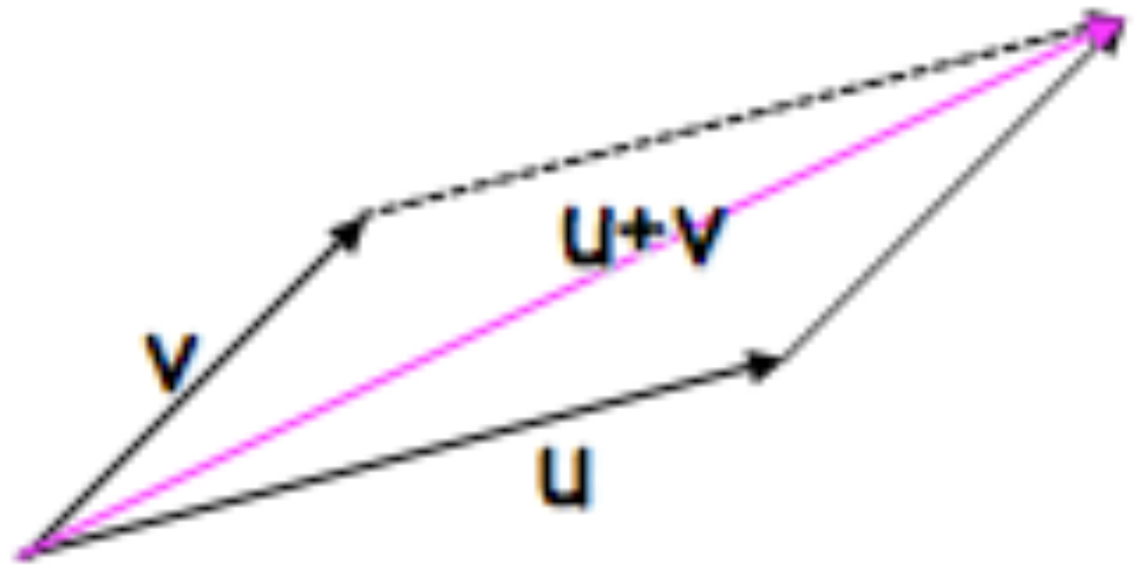
LINEAR OPERATIONS - ADDITION

- Vectors of equal dimensions may be added together:

$$\begin{bmatrix} 4 \\ 8 \end{bmatrix} + \begin{bmatrix} 12 \\ 16 \end{bmatrix} = \begin{bmatrix} 16 \\ 24 \end{bmatrix}$$

LINEAR OPERATIONS - ADDITION

- Vectors of equal dimensions may be added together:



LINEAR OPERATIONS - ADDITION

- Matrices of compatible dimensions may be added together:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$$

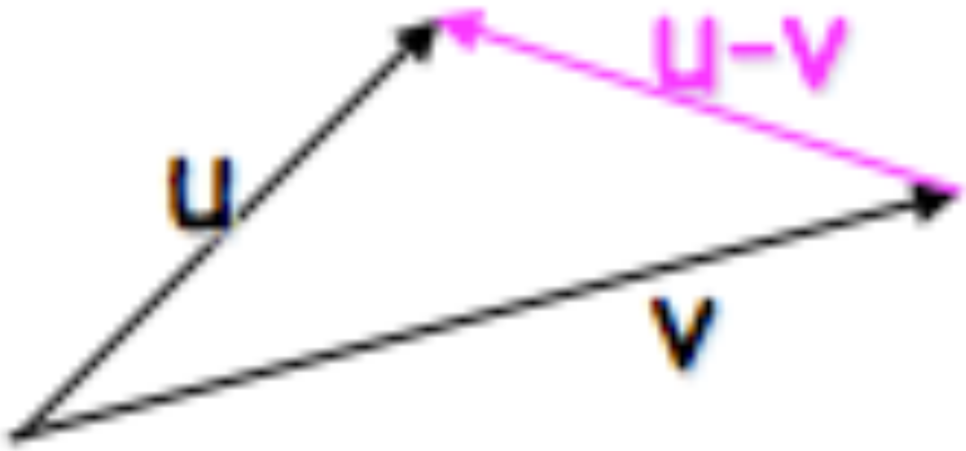
LINEAR OPERATIONS - SUBTRACTION

- Vectors of equal dimensions may be subtracted:

$$u - v = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} - \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} u_1 - v_1 \\ u_2 - v_2 \end{bmatrix}$$

LINEAR OPERATIONS - SUBTRACTION

- Vectors of equal dimensions may be subtracted:



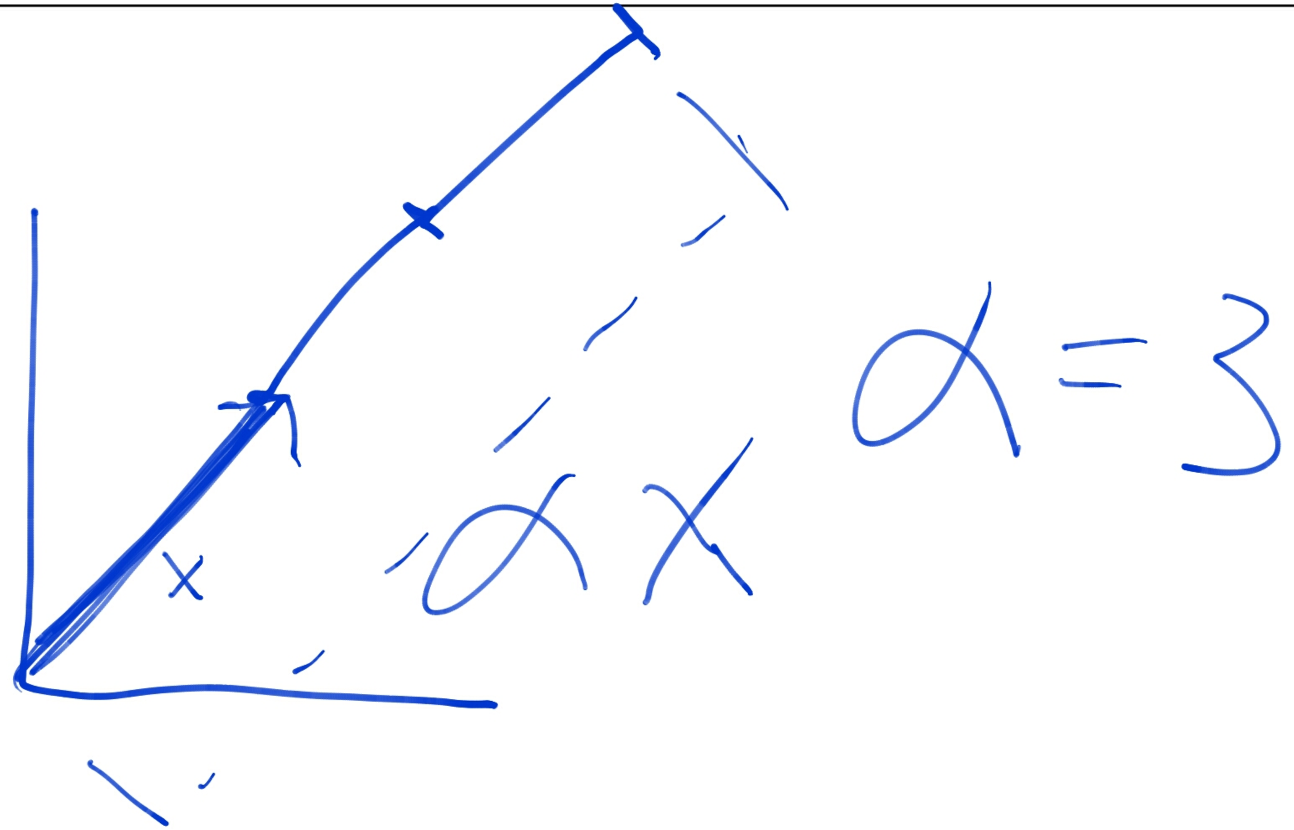
LINEAR OPERATIONS - SCALING

- Scaling a vector: shrinking/growing a vector by some value

$$\boldsymbol{z} = \alpha \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3\alpha \\ 5\alpha \\ 2\alpha \end{pmatrix}$$

LINEAR OPERATIONS - SCALING

- Graphically:



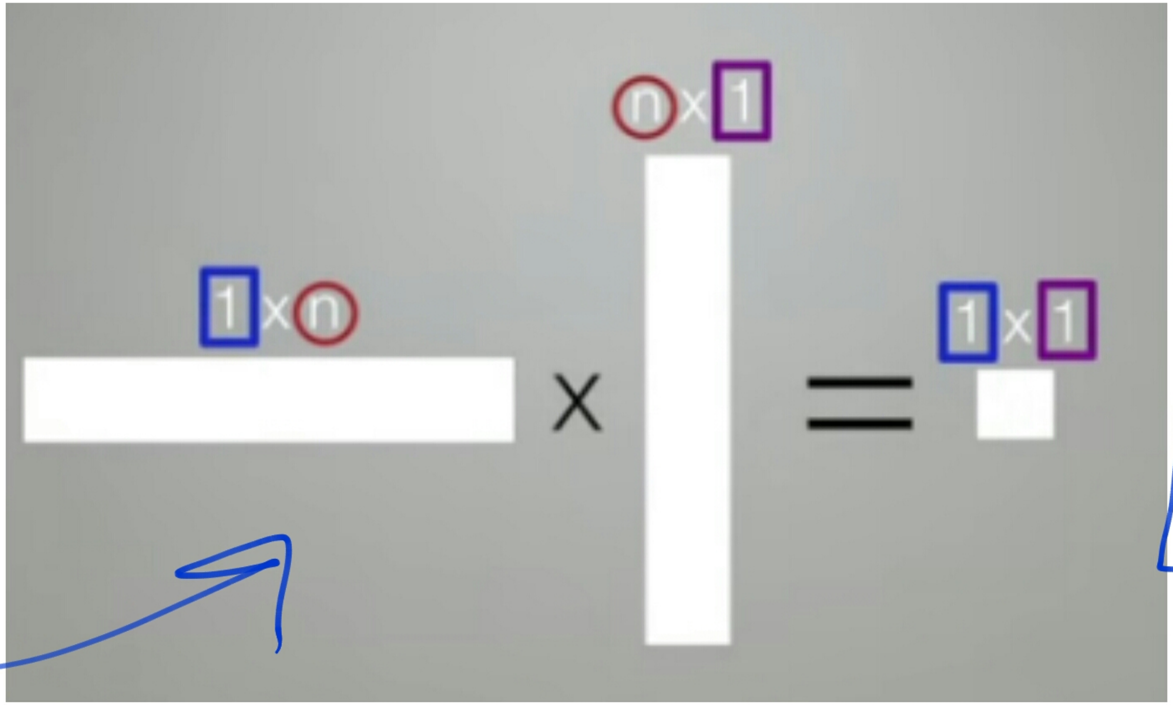
LINEAR OPERATIONS – DOT PRODUCT/INNER PRODUCT/SCALAR

- Multiply vectors of compatible dimensions to produce a scalar

1xn 1xn

=

1xn nx1



1 x n A

1 x n

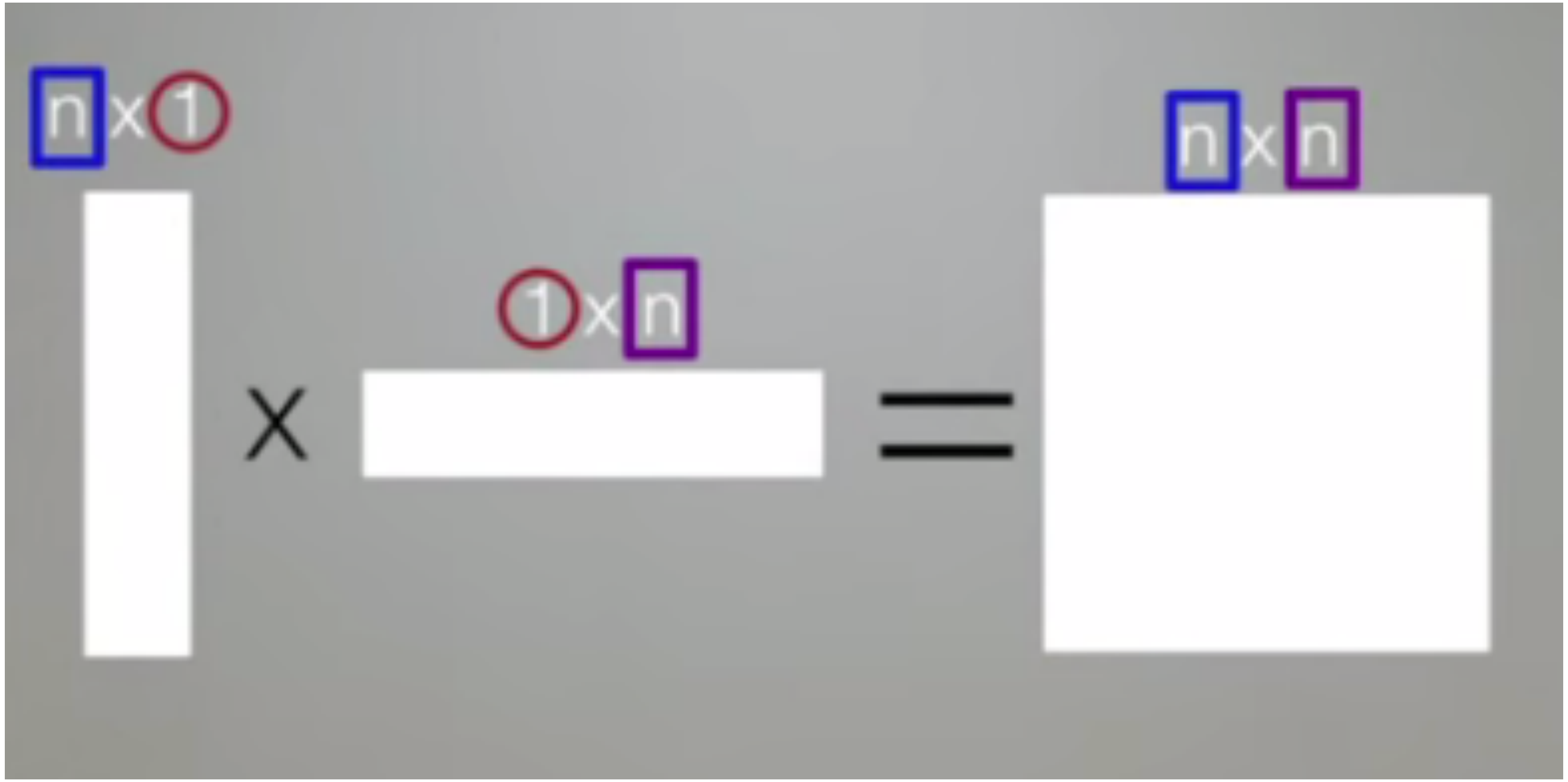
B

- If the dot product of two vectors is zero, they are orthagonal

$A \cdot B \approx A \cdot B^T$

LINEAR OPERATIONS – DOT PRODUCT/INNER PRODUCT/SCALAR

- The inverse (to produce a scalar) does not make sense



LINEAR OPERATIONS – MATRIX MULTIPLICATION

- We can multiple matrixes if the dimensions are compatible

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \quad \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$3 \times 2 \longleftrightarrow 2 \times 2 \quad \checkmark$$

LINEAR OPERATIONS – MATRIX MULTIPLICATION

- We can multiple matrixes if the dimensions are compatible

$$\begin{bmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} & a_{11} \cdot b_{21} + a_{12} \cdot b_{22} \\ a_{21} \cdot b_{21} + a_{21} \cdot b_{22} & a_{21} \cdot b_{21} + a_{22} \cdot b_{22} \\ a_{31} \cdot b_{11} + a_{32} \cdot b_{21} & a_{31} \cdot b_{21} + a_{32} \cdot b_{22} \end{bmatrix}$$

LINEAR OPERATIONS – MATRIX MULTIPLICATION

- We can multiple matrixes if the dimensions are compatible

$$\overset{\boxed{3}\times 2}{\begin{pmatrix}1&2\\3&4\\5&6\end{pmatrix}}\overset{2\times\boxed{4}}{\begin{pmatrix}7&8&9&10\\11&12&13&14\end{pmatrix}} = \overset{\boxed{3}\times\boxed{4}}{\begin{pmatrix}29&32&35&38\\65&72&79&86\\101&112&123&134\end{pmatrix}}$$

LINEAR OPERATIONS – MATRIX MULTIPLICATION

- Matrix multiplication is not commutative, but it is distributive and associative

- $AB \neq AB$
- $A(B+C) = A(B) + A(C)$

LINEAR OPERATIONS – IDENTITY MATRIX

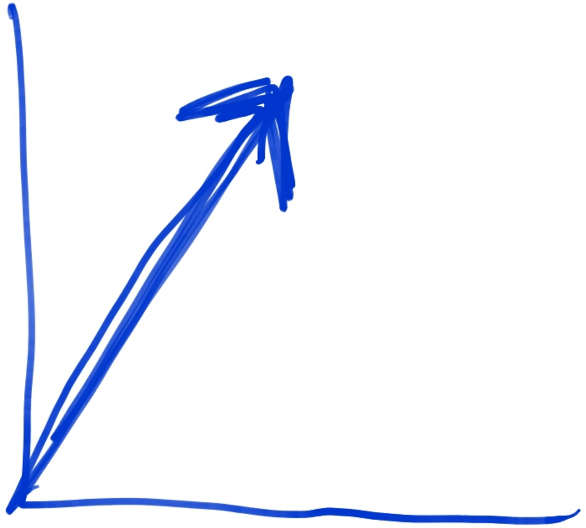
- Any matrix times an identity matrix equals the matrix itself

$$I \in \mathbb{R}^{n\times n} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

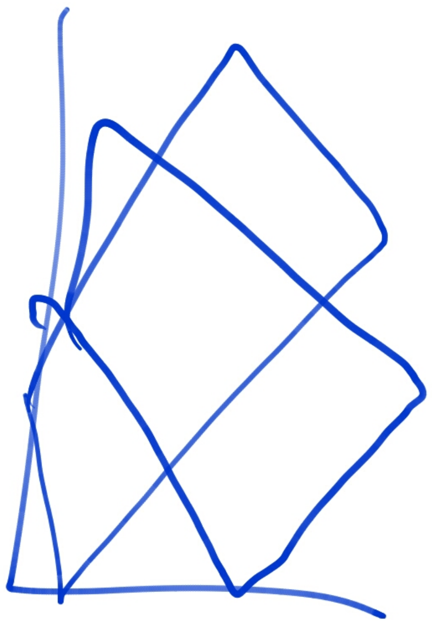
- AI = A

GEOMETRIC INTERPRETATION

$$\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{n} \end{pmatrix}$$



$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{\dots} & \dots \end{pmatrix}$$



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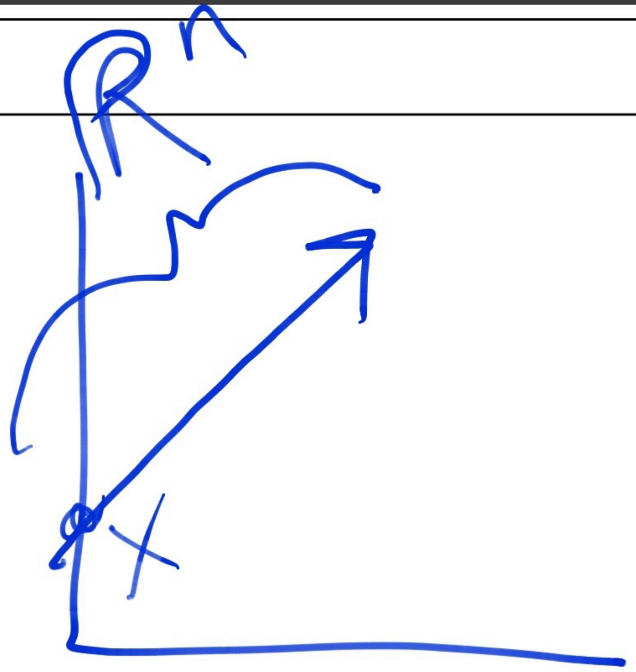


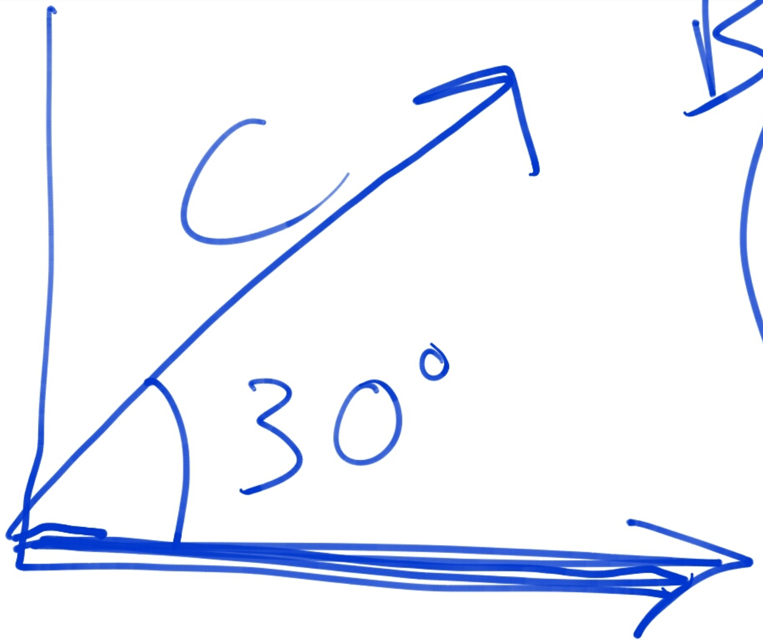
$$\sqrt{x \cdot x}$$

$$\sqrt{x_1^2 + x_2^2}$$

$$A \cdot B = C$$

ANGLE

$$A \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



$$B \begin{pmatrix} .8660 & -.5 \\ .5 & .8660 \end{pmatrix}$$

$$\begin{pmatrix} .8660 \\ .5 \end{pmatrix}$$

$$\begin{bmatrix} .8660 & -.5 \\ .5 & .8660 \end{bmatrix} \begin{bmatrix} .8660 & -.5 \\ .5 & .8660 \end{bmatrix} = 0$$

SCALING, ROTATION, REFLECTION, PROJECTION

$$\lambda \begin{pmatrix} a_{11} \\ a_n \end{pmatrix} \qquad \cos\begin{pmatrix} a_{11} \\ a_n \end{pmatrix}$$