Tesseract Trainer

Introduction

A tesseract is one name for the four-dimensional equivalent of the cube. It is the four-dimensional hypercube, in a progression that goes from the line in 1D, square in 2D, cube in 3D, tesseract in 4D.

For this demo you need to forget about *time being the fourth dimension*. This is a different concept to the four spatial dimensions we're using here. You also need to forget some of what you know about spinning objects in 3D. 4D rotations are different, in that there's no single axis of rotation. Instead there are one or two planes of rotation at right angles to each other, rotating at different speeds. The important thing is not to look for the axis of rotation you'd find in 3D.
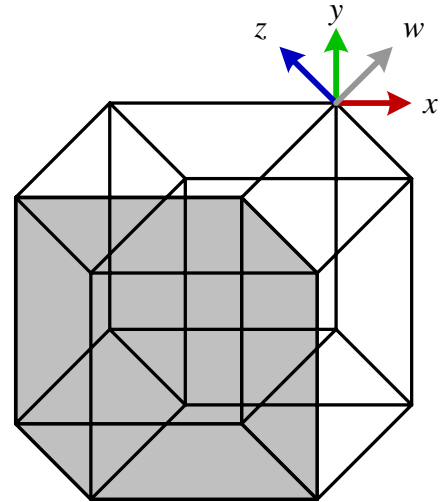
Finally, remember that this demo throws one of the dimensions away when it draws the tesseract on the screen, which can cause some strange effects. The one to watch for is the front of an object becoming the back and vice versa, as if it's been turned inside out. The rest are described a bit later on.

What am I looking at?

The tesseract has eight faces and sixteen corners. On the display, each face has been shrunken slightly so it edges and facets can be seen and colour coded. Four faces have been coloured and filled with dots so that their motion can be seen. Each of the coloured faces has an opposite number displayed in grey.

What am I trying to see?

- The tesseract is a rigid shape, so it should appear to rotate without changing its shape and size.
- The rotation planes should look like they're embedded in the tesseract and rotating with it.
- The shape has an interior extending in four dimensions, just like a 3D cube has an interior extending in three dimensions.
- Each of the eight faces extends in three dimensions and has zero thickness in the other.
- The coloured dots are all inside the face of the same colour. They are neither inside nor outside of the tesseract, but on the boundary between the two.
- Each of the eight faces is the same type of face as the others. There are no special or different faces, just like a 3D cube.
- Each of the four dimensions is the same as the others, but there is a special direction. It's the direction that you're looking in, because that's the direction that is used for shrinking due to perspective.

Stereoscopic view

The stereoscopic view works like the 'magic' 3D pictures (stereograms). Pressing the **S** key once switches to stereoscopic display. In stereoscopic mode the display shows two spinning tesseracts instead of one. The idea is to make the left eye look at the right image, the right eye look at the left image, and for the brain to think that both eyes are looking at the same thing. It's easier to say than do so here's a recipe that works for me.
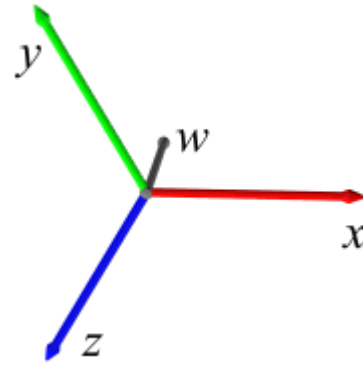
- Sit about 6ft (2m) away from the screen. Any closer and it might be uncomfortable to cross your eyes that much.
- Hold out both hands in front of you, flat with palms towards you, about a foot (30cm) away. Move your right hand so that it stops the right eye seeing the right image but allows it to see the left image. Likewise more your left hand so that it stops your left eye seeing the left image but allows it to see the right image. Closing one eye at a time will help to line this up. You should end up with a gap of about 2 inches (5cm) between your hands.
- Now look at your hands. You should see a fuzzy shape between them. Try to focus on that shape. This may take a while. Darkening the room, or moving your handle closer to or further from your face may help.
- If and when the image comes into focus, you should see the tesseract appearing closer to you than the screen is, and with 3D depth.

Things to try

- Look at the two rotation planes. Visualise that they intersect at a single point – the centre of rotation. The planes don't touch each other anywhere else.
- Try to see all eight faces of the tesseract at once. Start with one and work up.
- Watch a single face and try to work out why it's moving how it is from the rotation planes.
- Switch the rotation planes off, select a new rotation, and try to guess where the planes are before switching them on again.
- Try to visualise the tesseract in your head without using this program.
- Incidentally the music is in seven-time. It should sound weird to start with, but more natural as you keep listening.

Controls

These keys control the application. Pressing **K** will bring up a reminder list.
- **+ and -:** Change display resolution.
- **1:** Cycle through the number of faces that are drawn, 1 to 8.
- **2:** Toggle edge drawing on and off.
- **3:** Toggle face textures on and off.
- **4:** Toggle face dots on and off.
- **5:** Toggle rotation planes on and off.
- **6:** Toggle axis planes on and off.
- **M:** Toggle music on and off.
- **S:** Toggle stereoscopic display.
- **Esc:** Exit.

More features of displaying 4D shapes on a flat screen

As promised in the intro, more about the problems introduced when 4D is displayed on a 2D screen, and one dimension is discarded. The other three are kept as x (left and right), y (up and down), and w (into and out of the screen). Since another dimension, z, has gone missing, some strange things can appear to happen:

- The front of the object becomes the back and *vice versa*. This is because of a rotation plane in only z and w.
- Planes vanish to a point. This happens when the plane is lying in z and w. It's similar to what happens when you look at a line end-on in 3D.
- Faces of the tesseract can squash down into a line as the tesseract rotates. Although the face is thin (it has zero width in one of the dimensions) it should squash down to a plane, not a line, but the missing dimension makes it appear as line.

Registration

Tesseract Trainer is *very* cheap shareware. Please register at mushware.com if you like it.

More information

At mushware.com there are a number of technical documents in the download section. The projection and rendering paper has a lot of detail and diagrams about 4D rendering and viewing. Apart from that, try a web search for 'tesseract'.

Version History

1. 2003-03-13: Created with filename tesseracttrainer1-2005-03-13.doc
2. 2005-05-20: Release for Tesseract Trainer version 0.1.0.