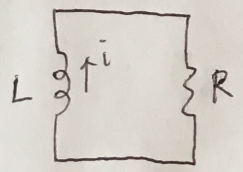
First-order circuits

- RL circuit
- RC circuit

Natural Response : No external sources

* RL circuit



$t = 0^+$

KVL: $L\frac{di}{dt} + Ri = 0$

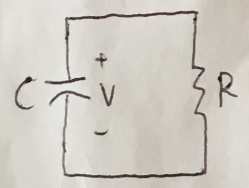
$i(t) = i(0)e^{-\frac{R}{L}t}$

$V(t) = Ri(0)e^{-\frac{R}{L}t}$

$i_L(0^-) = i_L(0^+) = i_L(0)$

(Inductors don't allow instantaneous changes in current)

* RC circuit



$t = 0^+$

KCL: $C\frac{dv}{dt} + \frac{V}{R} = 0$

$V(t) = V(0)e^{-\frac{t}{RC}}$

$i(t) = \frac{V(0)}{R}e^{-\frac{t}{RC}}$

$V_C(0^-) = V_C(0^+) = V_C(0)$

* Energy

- power dissipated in R

$$p = iV = i^2R = \frac{V^2}{R} = [i(0)]^2 R e^{-2\frac{R}{L}t} \quad (RL)$$

$$\frac{[V(0)]^2}{R} e^{-2\frac{t}{RC}} \quad (RC)$$

- energy dissipated by R or delivered in L/C

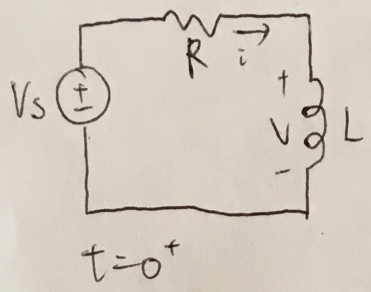
$$\Delta W_R = \int_0^t p\,dt$$

$$\Delta W_L = -\Delta W_R \quad (RL)$$

$$\Delta W_C = -\Delta W_R \quad (RC)$$

Step Response

* RL circuit

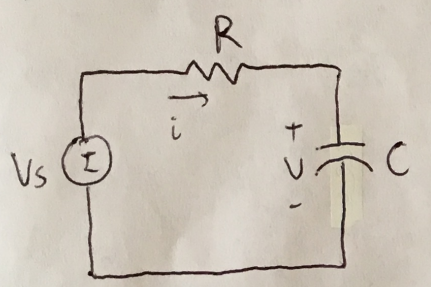


KVL: $-V_s + L\frac{di}{dt} + iR = 0$

$$i(t) = \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$v(t) = L\frac{di}{dt} = V_s e^{-\frac{R}{L}t}$$

* RC circuit

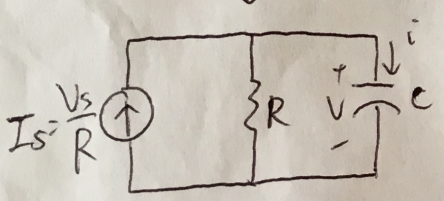


KCL: $-I_s + \frac{V}{R} + C\frac{dV}{dt} = 0$

$$V(t) = I_s R\left(1 - e^{-\frac{t}{RC}}\right)$$

$$i(t) = C\frac{dV}{dt} = I_s e^{-\frac{t}{RC}}$$

$\Downarrow$



Time constant

RL: $\tau = \frac{L}{R}$, RC: $\tau = RC$

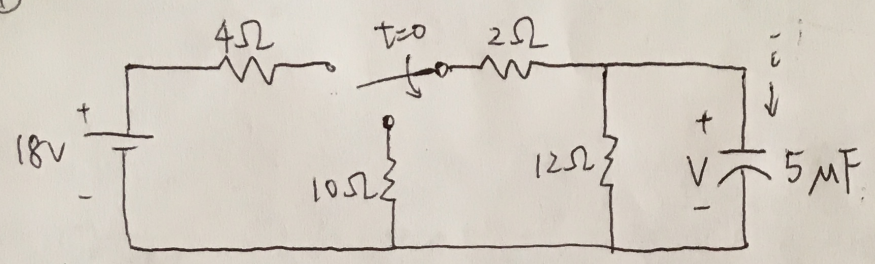
General Form

$$x(t) = \underbrace{x_\infty}_{\text{steady state}} + \left[x(t_0) - x_\infty\right] e^{-\frac{t-t_0}{\tau}}$$

exponential decay

difference between initial value and steady state value

①



The switch has been in the upper position before $t=0$.

Find $V(t)$ and $i(t)$ for $t>0$

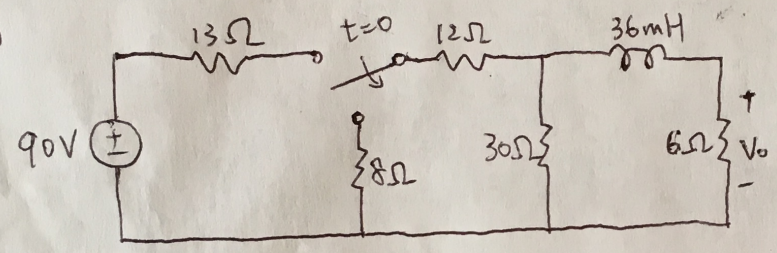
$$V(0) = 18 \times \frac{12}{4+2+12} = 12\ V$$

$$R_{eq} = (10+2) \parallel 12 = 6\Omega$$

$$\tau = R_{eq} \cdot C = 30\mu s$$

$$V(t) = 12 e^{\frac{-t}{30\mu}}\ V, \quad i(t) = -\frac{V(t)}{R_{eq}} = -2e^{\frac{-t}{30\mu}}\ A$$

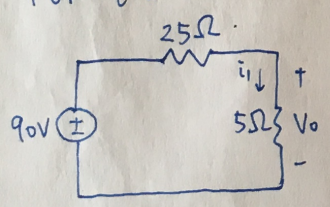
②



(a) Find $V_o(t)$ for $t>0$

(b) How much energy is dissipated in the 6Ω resistor for $t>0$

(a) For $t<0$

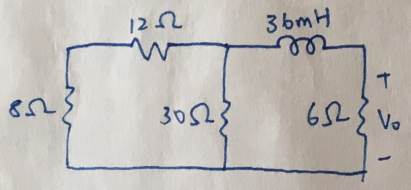


$$i_1(0^+) = i_1(0^-) = i_1(0)$$

$$V_o(0^+) = 5\, i_1(0^+) = 5\, i_1(0^-) = V_o(0^-) = V_o(0)$$

$$= 90 \cdot \frac{5}{25+5} = 15\ V$$

For $t>0$



$$V_o(\infty) = 0$$

$$R_{eq} = [(8+12) \parallel 30] + 6 = 18\ \Omega$$

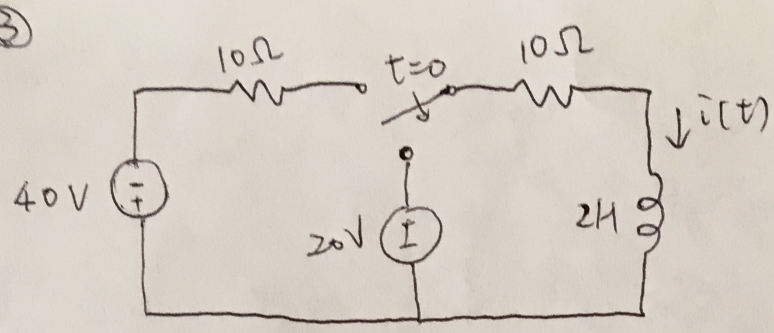
$$\tau = \frac{L}{R_{eq}} = \frac{36m}{18} = 0.002$$

$$V_o(t) = 15 e^{-\frac{t}{0.002}}\ V \ \star$$

(b) $$P = \frac{V^2}{R} = \frac{75}{2} e^{-\frac{t}{10^{-3}}}$$

$$W = \int_0^\infty \frac{75}{2} e^{-\frac{t}{10^{-3}}} dt = \frac{75}{2}(-10^{-3}) e^{-\frac{t}{10^{-3}}} \Big|_{t=0}^{\infty} = 37.5\ mJ \ \star$$

③



The switch has been at the upper position for a long time before $t=0$, at which it goes to the lower position. The switch return back to the upper position at $t=1s$. Find $i(t)$ for $t>0$

(i) For $0<t<1$

$$i(0^+) = i(0^-) = -40/20 = -2\ A$$

$$i_\infty = \frac{20}{10} = 2\ A$$

$$\tau_1 = \frac{2}{10} = 0.2$$

$$i(t) = 2 - 4e^{-5t}$$

(ii) $t \geq 1$

Let $t_0 = 1s$

$$i(t_0^+) = i(t_0^-) = 2 - 4e^{-5\cdot 1} = 1.973\ A$$

$$i_\infty = -\frac{40}{10+10} = -2\ A$$

$$\tau_2 = \frac{2}{20} = 0.1$$

$$i(t) = -2 + 3.973e^{-10(t-1)}$$

$$\therefore\ i(t) = \begin{cases} 2-4e^{-5t}, & 0<t<1 \\ -2+3.973e^{-10(t-1)}, & t \geq 1 \end{cases}$$