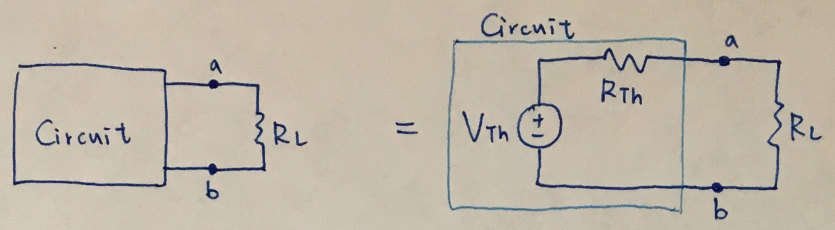
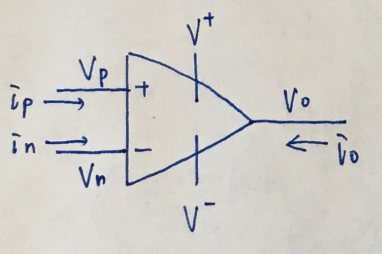
- Maximum Power Transfer



* Max Power when $R_L = R_{Th}$

- Superposition
  - Turn on one "independent" source at a time
  - Turn off
    - ↑ : open
    - ± : short

- Operational Amplifier



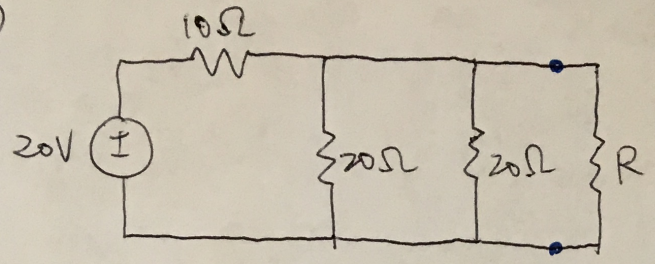
- Characteristic
  1. When $V^- \le V_o \le V^+$, $V_o = A(V_p - V_n)$, where A is the "Gain" of the op-amp
  2. V_o saturates at V^+ and V^-

☆ Two important conditions

1. Virtual short : $V_p = V_n$
2. High input resistance : $i_p = i_n = 0$

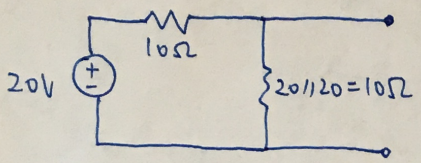
☆ Goal : use the two conditions to find out the relationship between output (usually V_o) and input (usually $V_n - V_p$)

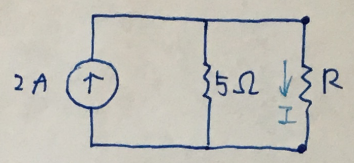
1

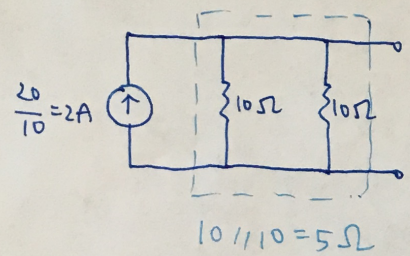


Find R for maximum power transfer and the power, P_l dissipated by load R.

$R = R_{Th}$







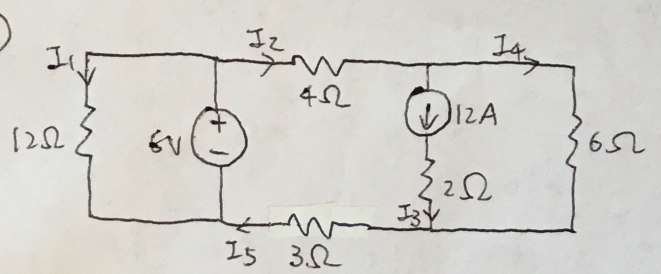
$R_{Th} = 5\Omega$

$R = R_{Th} = 5\Omega$ ※

$I = 2 \cdot \dfrac{5\Omega}{5\Omega + 5\Omega} = 1A$

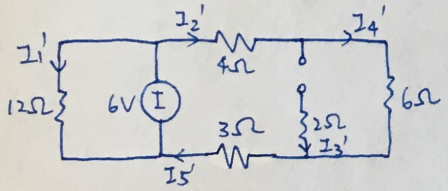
$P_l = I^2 R = 5W$ ※

2



Using superposition find the current through all resistors.

<i> Turn off the current source

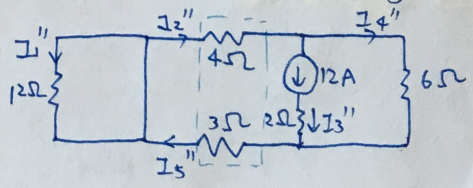


$I_3' = 0$

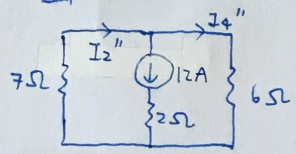
$I_1' = \dfrac{6}{12} = \dfrac{1}{2}$ A

$I_2' = I_4' = I_5' = \dfrac{6}{4+6+3} = \dfrac{6}{13}$ A

<ii> Turn off the voltage source



$I_1'' = 0$ (it is short), $I_3'' = 12A$



(current division)

$I_2'' = I_5'' = \dfrac{6}{7+6} \cdot 12 = \dfrac{72}{13}$

$I_4'' = \dfrac{-7}{7+6} \cdot 12 = \dfrac{-84}{13}$

<iii>

$I_1 = \frac{1}{2} + 0 = \frac{1}{2}$ A

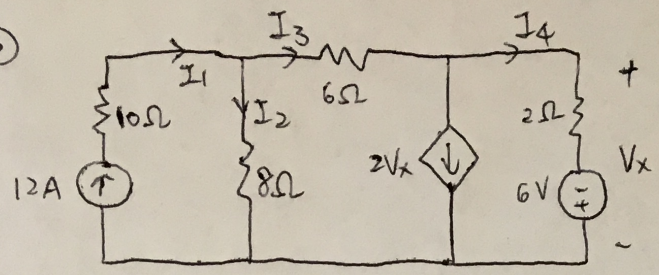
$I_2 = \frac{6}{13} + \frac{72}{13} = 6A$

$I_3 = 0 + 12 = 12A$

$I_4 = \frac{6}{13} + \frac{-84}{13} = -6A$

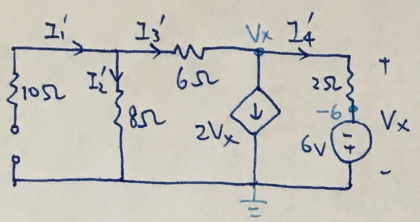
$I_5 = \frac{6}{13} + \frac{72}{13} = 6A$

③



Find I_1, I_2, I_3, I_4 using superposition

<i> Turn off the current source



$I_1' = 0$, $I_2' = -I_3'$

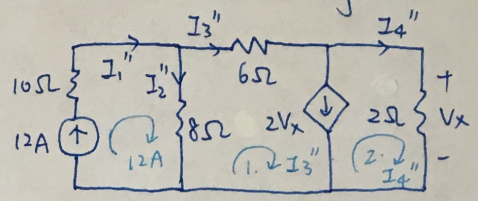
KCL: $\frac{V_x}{6+8} + 2V_x + \frac{V_x-(-6)}{2} = 0$

$\Rightarrow V_x = -\frac{7}{6}$

$I_2' = \frac{V_x}{6+8} = -\frac{1}{12} = -I_3'$

$I_4' = \frac{V_x-(-6)}{2} = \frac{29}{12}$

<ii> Turn off the voltage source



$I_1'' = 12\ A$

Mesh 1, 2 (supermesh)

$8(I_3''-12) + 6I_3'' + 2I_4'' = 0$ — ①

Source:

$I_3'' - I_4'' = 2V_x$ — ②

$V_x = 2I_4''$ — ③

①,②,③ $\Rightarrow I_3'' = \frac{20}{3}$, $I_4'' = \frac{4}{3}$

$I_2'' = 12 - I_3'' = \frac{16}{3}$

<iii>

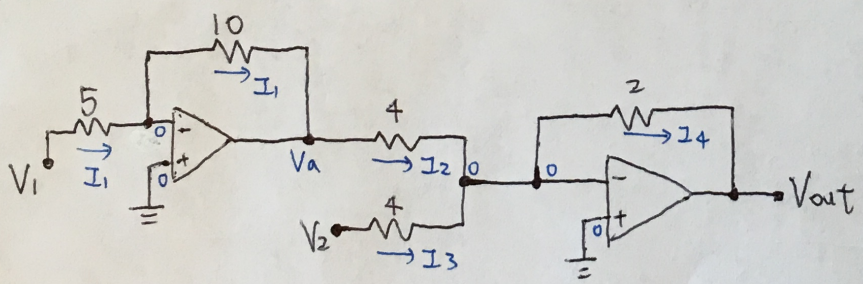
$I_1 = 0 + 12 = 12A$

$I_2 = -\frac{1}{12} + \frac{16}{3} = \frac{21}{4}A$

$I_3 = \frac{1}{12} + \frac{20}{3} = \frac{27}{4}A$

$I_4 = \frac{29}{12} + \frac{4}{3} = \frac{15}{4}A$

④



Find a relationship between V_{out} and V_1, V_2

$I_1 = \frac{V_1 - 0}{5} = \frac{0 - V_a}{10} \Rightarrow 2V_1 = -V_a$

$I_2 = \frac{V_a - 0}{4} = \frac{V_a}{4} = \frac{-2V_1}{4} = \frac{-V_1}{2}$

$I_3 = \frac{V_2 - 0}{4} = \frac{V_2}{4}$

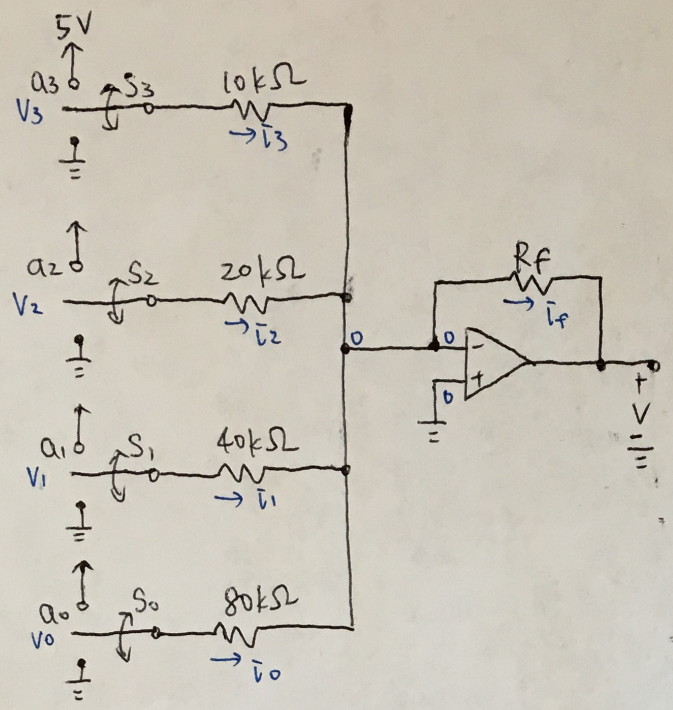
$I_4 = I_2 + I_3 = -\frac{V_1}{2} + \frac{V_2}{4}$

$= \frac{0 - V_{out}}{2} = \frac{-V_{out}}{2}$

$-\frac{V_{out}}{2} = -\frac{V_1}{2} + \frac{V_2}{4}$

$\Rightarrow V_{out} = V_1 - \frac{V_2}{2}$

⑤



a_0, a_1, a_2, a_3 are the inputs of the circuit. $a_i = 0$ when S_i switches to 0V and $a_i = 1$ when S_i switches to 5V. Show

$$V = -\frac{R_f}{16}\left[2^0 a_0 + 2^1 a_1 + 2^2 a_2 + 2^3 a_3\right]$$

$$i_f = i_0 + i_1 + i_2 + i_3 = \frac{0 - V}{R_f}$$

$$= \frac{V_0}{80} + \frac{V_1}{40} + \frac{V_2}{20} + \frac{V_3}{10} = -\frac{V}{R_f}$$

$$-\frac{V}{R_f} = \frac{V_0}{80} + \frac{V_1}{40} + \frac{V_2}{20} + \frac{V_3}{10} = \frac{5a_0}{80} + \frac{5a_1}{40} + \frac{5a_2}{20} + \frac{5a_3}{10}$$

$$\Rightarrow V = -\frac{R_f}{16}\left[2^0 a_0 + 2^1 a_1 + 2^2 a_2 + 2^3 a_3\right] \#$$

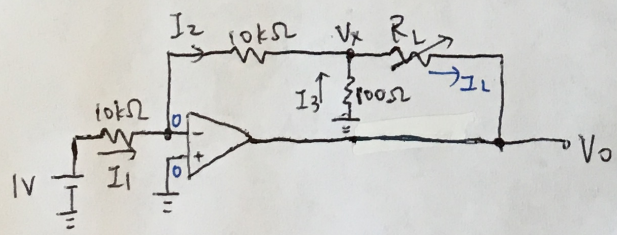
⑤-2 Find the value of R_f so that $-12 \le V \le 0$

$$-\frac{R_f}{16}\left[2^0 \cdot 1 + 2^1 \cdot 1 + 2^2 \cdot 1 + 2^3 \cdot 1\right] \le V \le -\frac{R_f}{16}\left[2^0 \cdot 0 + 2^1 \cdot 0 + 2^2 \cdot 0 + 2^3 \cdot 0\right]$$

$$-\frac{15}{16} R_f \le V \le 0$$

$$-\frac{15}{16} R_f = -12 \Rightarrow R_f = 12.8\ k\Omega \#$$

⑥



(a) Find I_1, I_2, I_3, V_x

(b) If $V_o \geq -13V$, find the maximum allowed value of R

(a) $I_1 = \frac{1-0}{10} = 0.1\ mA$

$I_2 = I_1 = 0.1\ mA$

$= \frac{0 - V_x}{10}$

$\Rightarrow V_x = -1\ V$

$I_3 = \frac{0 - V_x}{0.1k} = 10\ mA$

(b) $I_L = I_2 + I_3 = 10.1\ mA$

$V_o = V_x - I_L R_L$

$= -1 - 10.1 \cdot R_L \geq -13$

$10.1 R_L \le 12$

$R_L \le \frac{12}{10.1} = 1.19\ k\Omega \#$