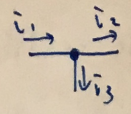
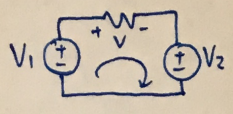
Basic Tools

1. KCL : $\sum_{node} i_j = 0$



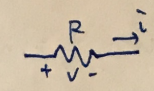
$-i_1 + i_2 + i_3 = 0$

2. KVL : $\sum_{loop} V_k = 0$



$-V_1 + V + V_2 = 0$

3. Ohm's Law : For a "resistor"



$V = iR$

Node Voltage Method

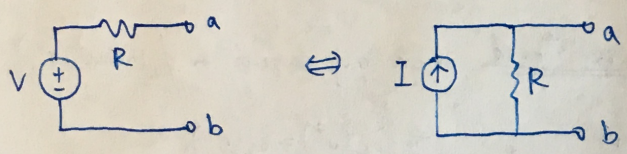
1. Find reference node
2. Mark all the voltages at essential nodes
3. Apply KCL at each node (except for the reference node)

* Supernode : a voltage source between 2 essential nodes (apply KCL to the sup rnode)

Mesh Current Method

1. Identify all the meshes
2. Identify supermesh (2 meshes share a current Source)
3. Apply KVL to each mesh

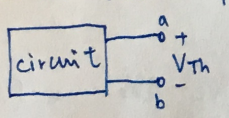
Source Transformation



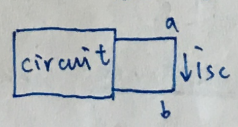
$V = IR$

Thevenin and Norton Equivalent

1. Determine V_{Th} – open circuit



2. Determine i_{sc} – short circuit

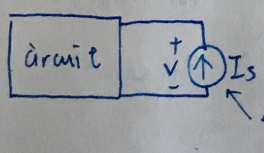


3. Determine R_{Th}

① $R_{Th} = \frac{V_{Th}}{i_{sc}}$

② Turn off all the "independent" sources (voltage source ⇒ short, current source ⇒ open) and then find the equivalent resistance.

③ For circuit containing dependent sources, you may obtain R_{Th} from ①
Or, turn off all the independent sources and apply a test source at the terminals.

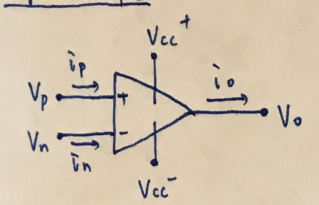


find $V \Rightarrow R_{Th} = \frac{V}{I_s}$

Superposition (for resistive circuits)

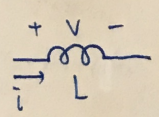
1. Have one source switched on at a time 2. Add the results

Op-Amps



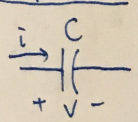
1. virtual short : $V_p = V_n$
2. ∞ input resistance : $i_p = i_n = 0$
3. Working region : $V_{cc}^- \le V_o \le V_{cc}^+$

Inductor



* $V = L\frac{di}{dt}$
* continuous current ($i(t^+) = i(t^-)$)
* $i(t) = \frac{1}{L}\int_{t_0}^{t} V\,d\tau + i(t_0)$
* energy stored : $\frac{1}{2}Li^2$

Capacitor



* $i = C\frac{dV}{dt}$
* continuous voltage ($V(t^+) = V(t^-)$)
* $V(t) = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + V(t_0)$
* energy stored : $\frac{1}{2}CV^2$

First-Order circuits (RC, RL)

1. $\tau = RC$ or $\tau = \frac{L}{R}$ (R : Find <u>Thevenin resistance</u> seen by capacitor / inductor)
2. $x(t) = x_\infty + (x(0) - x_\infty)e^{-\frac{t}{\tau}}$
   - natural response : $x(t) = x(0^+)e^{-\frac{t}{\tau}}$, $t > 0$
   - step response : $x(t) = x_\infty - x_\infty e^{-\frac{t}{\tau}}$, $t > 0$

* $i_L(0^+) = i_L(0^-)$
  $V_C(0^+) = V_C(0^-)$

Second-Order Circuits (RLC)

1. Determine <u>Thevenin resistance</u> seen by
2. Find parameters α and ω_0

   $\alpha = \frac{1}{2RC}$ (parallel) or $\frac{R}{2L}$ (serial), $\omega_0 = \frac{1}{\sqrt{LC}}$
3. Determine the system

   $\alpha^2 - \omega_0^2 > 0$: overdamped $\Rightarrow s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

   $\alpha^2 - \omega_0^2 = 0$: critically damped $\Rightarrow s = -\alpha$

   $\alpha^2 - \omega_0^2 < 0$: underdamped
4. Have the equation

   overdamped : $x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + x(\infty)$

   critically damped : $x(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t} + x(\infty)$

   underdamped : $x(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t) + x(\infty)$

   $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$
5. Solve the coefficients

   <i> initial value

   $x(0) = A_1 + A_2 + x(\infty)$

   $x(0) = D_2 + x(\infty)$

   $x(0) = B_1 + x(\infty)$

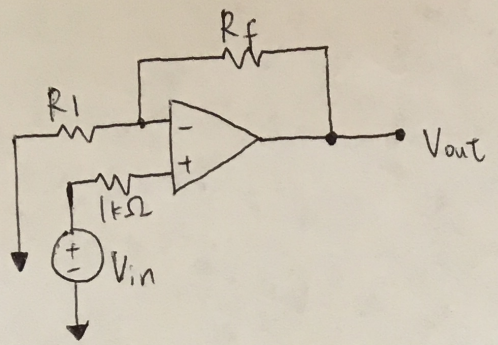
   <ii> first-order derivative

   - find the first order derivatives of $x(t)$
   - get $\frac{dx}{dt}(0^+)$

   for inductor, $\frac{di_L}{dt}(0^+) = \frac{V_L(0^+)}{L}$

   for capacitor, $\frac{dV_C}{dt}(0^+) = \frac{i_C(0^+)}{C}$

①



$$\frac{V_{in}}{R_1} + \frac{V_{in} - V_{out}}{R_f} = 0$$

$$\frac{0.1\cos(2000t)}{R_1} + \frac{-102.5\cos(2000t)}{R_f} = 0$$

$$\Rightarrow R_f = 1025 R_1$$

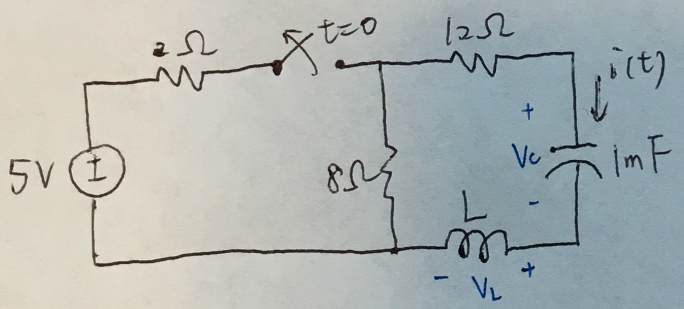
choose $R_1 = 100\Omega$, $R_f = 102.5\,k\Omega$

$V_{in}(t) = 0.1\cos(2000t)$ V

Pick R_1, R_f to have

$V_{out}(t) = 102.6\cos(2000t)$ V

②



The circuit has been closed for a long time before $t=0$.

(a) Find L such that the circuit is critically damped.

(b) Find $i(t)$ when it is critically damped.

(a) RLC in series

$$\alpha = \frac{R}{2L} = \frac{20}{2L}, \quad \omega_0^2 = \frac{1}{LC} = \frac{1000}{L}$$

critically damped

$$\Rightarrow \left(\frac{20}{2L}\right)^2 = \frac{1000}{L} \Rightarrow L = 0.1\ H$$

(b)

$$\alpha = \frac{R}{2L} = 100$$

$$i(t) = D_1 t e^{-100t} + D_2 e^{-100t}$$

$$i(0) = 0 = D_2$$

$$\frac{di}{dt}(0^+) = \frac{1}{L} V_L(0^+)$$

$$= \frac{1}{L}\left[-V_c(0^+)\right] \quad \text{(no current, KVL)}$$

$$= \frac{1}{0.1}\cdot\left(-5\cdot\frac{8}{2+8}\right)$$

$$= -40 = D_1 - 100 D_2$$

$$\Rightarrow D_1 = -40$$

$$\therefore i(t) = -40 t e^{-100t} \text{ A}$$