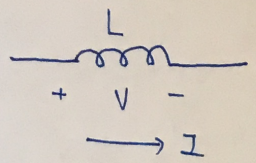
Inductor



$$V = L\frac{dI}{dt}$$

* $I = \text{const} \Rightarrow V = 0 \Rightarrow$ short circuit

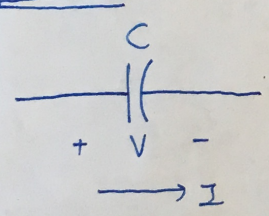
* $I(t) = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + I(t_0)$

* power: $p = VI = L\frac{dI}{dt}\cdot I = V\cdot\frac{1}{L}\left[\int_{t_0}^{t} v\,d\tau + I(t_0)\right]$

* energy: $p = \frac{dw}{dt} = L\frac{dI}{dt}\cdot I \Rightarrow dw = LI\,dI$

$$\Rightarrow w = \frac{1}{2}LI^2$$

Capacitor



$$I = C\frac{dV}{dt}$$

* $V = \text{const} \Rightarrow I = 0 \Rightarrow$ open circuit

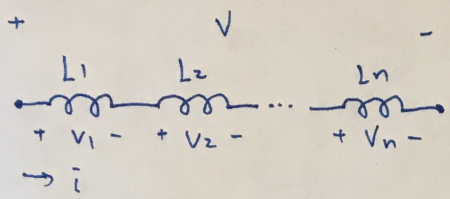
* $V(t) = \frac{1}{C}\int_{t_0}^{t} I\,d\tau + V(t_0)$

* power: $p = VI = V\cdot C\frac{dV}{dt} = \left[\frac{1}{C}\int_{t_0}^{t} I\,d\tau + V(t_0)\right]\cdot I$

* energy: $p = \frac{dw}{dt} = VC\frac{dV}{dt} \Rightarrow dw = VC\,dV$

$$\Rightarrow w = \frac{1}{2}CV^2$$

Inductors in series

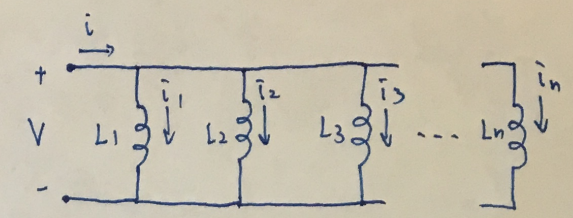


$$V = V_1 + V_2 + \cdots + V_n$$

$$= (L_1 + L_2 + \cdots + L_n)\frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + \cdots + L_n$$

Inductors in Parallel



$$i = i_1 + i_2 + \cdots + i_n$$

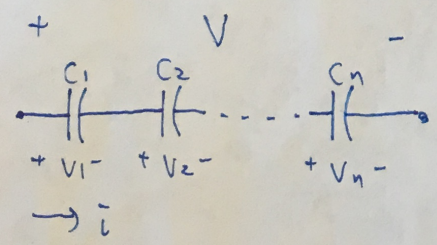
$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}\right)\int_{t_0}^{t} v\,d\tau$$

$$+ i_1(t_0) + i_2(t_0) + \cdots + i_n(t_0)$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n},$$

$$i(t_0) = i_1(t_0) + i_2(t_0) + \cdots + i_n(t_0)$$

Capacitors in Series



$$V = V_1 + V_2 + \cdots + V_n$$

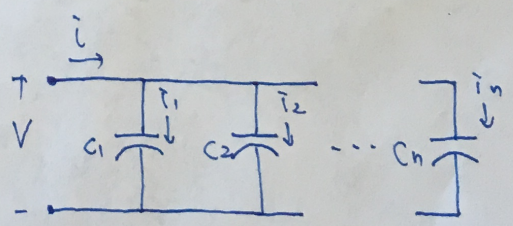
$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}\right)\int_{t_0}^{t} i\,d\tau$$

$$+ V_1(t_0) + V_2(t_0) + \cdots + V_n(t_0)$$

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_n}$$

$$V(t_0) = \sum_{i=1}^{n} V_i(t_0)$$

Capacitors in Parallel

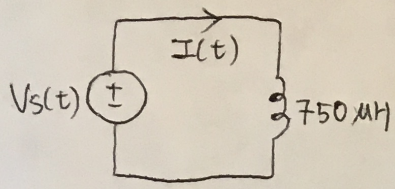


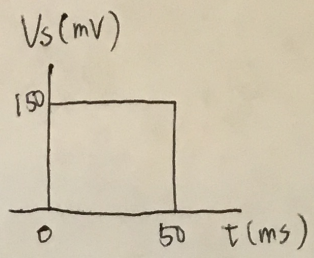
$$i = i_1 + i_2 + \cdots + i_n$$

$$= (C_1 + C_2 + \cdots + C_n)\frac{dV}{dt}$$

$$C_{eq} = \sum_{i=1}^{n} C_i$$

①





$I(t)=0$ for $t \le 0$

(a) Find $I(t)$ for $t \geq 0$

(b) Sketch I versus t, $t \geq 0$

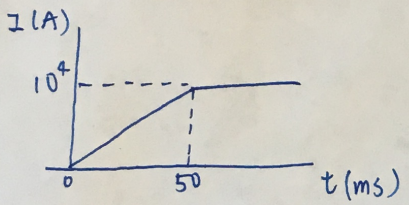
(a) $I(0) = 0$

$$I(t) = \frac{1}{L}\int_0^t V\,d\tau + I(0) = \frac{1}{750\mu}\cdot 150m\cdot t = 200t$$

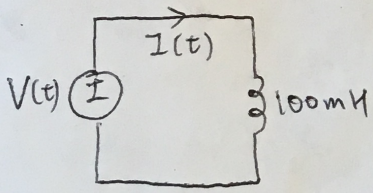
$$I(50) = \frac{1}{750\mu}\int_0^{50} 150m\,d\tau + 0 = 10^4 \text{ mA}$$

$$\Rightarrow I(t) = \begin{cases} 200t, & 0<t<50 \\ 10^4, & t \geq 50 \end{cases}$$

(b)



②



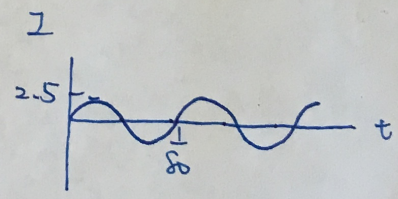
$V(t) = 20\cos 80t$, $I(0) = 0$ A

Find I, P, and W for $t \geq 0$

$$I(t) = \frac{1}{0.1}\int_0^t 20\cos 80\tau\,d\tau + 0$$

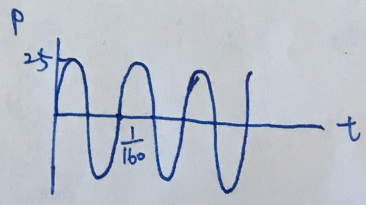
$$= \frac{1}{0.1}\cdot 20\cdot\frac{1}{80}\cdot\sin 80t$$

$$= 2.5\sin 80t \text{ A}$$



$$p(t) = VI = 20\cos 80t\cdot 2.5\sin 80t$$

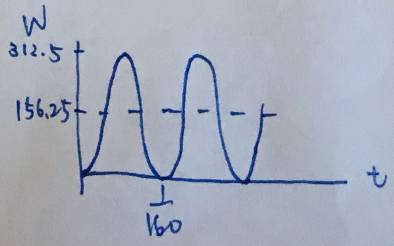
$$= 25\sin 160t \text{ W}$$



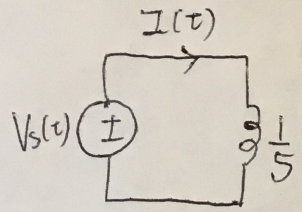
$$W(t) = \frac{1}{2}LI^2$$

$$= 312.5\sin^2 80t$$

$$= -156.25\cos 160t + 156.25 \text{ mJ}$$



③



$$V_s(t) = \begin{cases} 0 & , t<0 \\ 20te^{-10t} & , t>0 \end{cases}$$

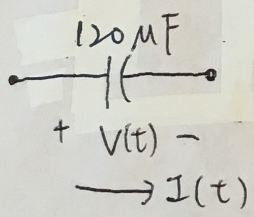
Find $I(t)$

$$I(t) = 5\int_0^t 20\tau e^{-10\tau}\,d\tau + I(0)$$

$$= 5\cdot\left(-\frac{1}{5}\right)\left(e^{-10\tau}(10\tau+1)\right)\Big|_0^t + I(0)$$

$$= 1 - e^{-10t}(10t+1) + I(0)$$

④



$$V(t) = \begin{cases} 60\ V & , t\le 0 \\ 30 + 5e^{-500t}(6\cos 2000t + \sin 2000t)\ V & , t>0 \end{cases}$$

Find $I(t)$

$$I(t) = C\frac{dV}{dt} = \begin{cases} 0 & , t<0 \\ \left[-2500e^{-500t}(6\cos 2000t + \sin 2000t) + 5e^{-500t}(-12000\sin 2000t + 2000\cos 2000t)\right]\cdot 120\mu & , t>0 \end{cases}$$

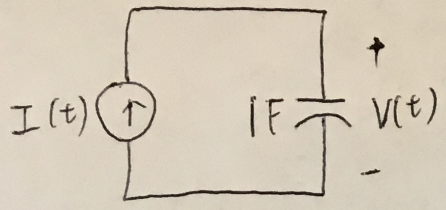
$$= \begin{cases} 0 & , t<0 \\ -0.3e^{-500t}(2\cos 2000t + 25\sin 2000t) & , t>0 \end{cases}$$

How much energy is stored in the capacitor at $t=\infty$?

$V(\infty) = 30$

$W = \frac{1}{2}CV^2 = 54\ mJ$

⑤



$$I(t) = \begin{cases} 0, & t<0 \\ 10, & 0<t<10 \\ 0, & t>10 \end{cases} \quad \left(I(t) = 10(u(t)-u(t-10)) \right)$$

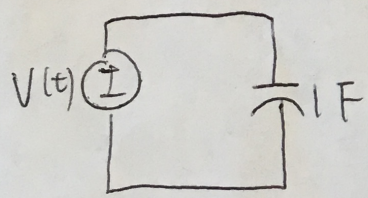
Find $V(t)$ and energy stored in the capacitor at $t=\infty$

$$V(t) = \frac{1}{C}\int_0^t I(\tau)\,d\tau = \begin{cases} 0, & t<0 \\ 10t, & 0<t<10 \\ 100, & t>10 \end{cases}$$

$V(\infty) = 100$

$W = \frac{1}{2}CV^2 = 5000\ J$

⑥



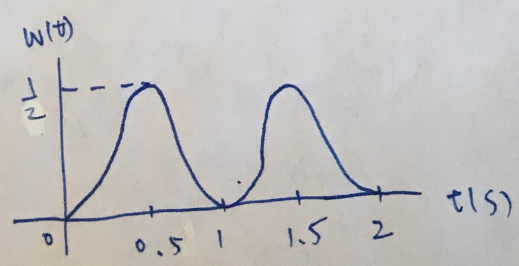
Find $P(t)$, $W(t)$

When is energy being stored? released?

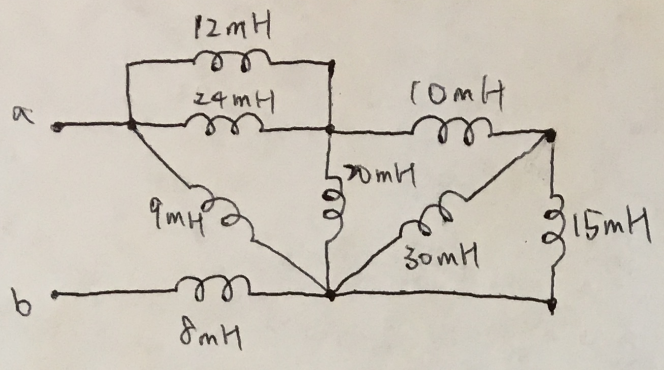
$V(t) = \sin(\pi t)$

$$P(t) = V(t)\cdot C\frac{dV}{dt} = 1\cdot \sin(\pi t)\cdot \pi\cos(\pi t) = \frac{\pi}{2}\sin(2\pi t)$$

$$W(t) = \int_0^t P(\tau)\,d\tau = \frac{1}{2}CV^2 = \frac{1}{2}\left[\sin(\pi t)\right]^2 = \frac{1-\cos(2\pi t)}{4}$$



⑦ Find the equivalent inductance with respect to a, b



$15 \parallel 30 = 10$

$10 + 10 = 20$

$20 \parallel 20 = 10$

$12 \parallel 24 = 8$ $\Big\rangle$ $10 + 8 = 18$

$18 \parallel 9 = 6$

$6 + 8 = 14$ mH