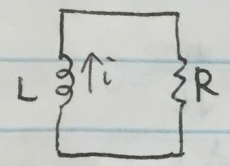
First-order circuits

- RL circuits
- RC circuits

Natural Response

* RL circuit



$t = 0^+$

KVL: $L\frac{di}{dt} + Ri = 0$

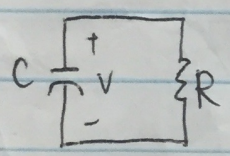
$i(t) = i(0)e^{-\frac{R}{L}t}$

$V(t) = Ri(0)e^{-\frac{R}{L}t}$

$i_L(0^-) = i_L(0^+) = i_L(0)$

(inductors don't allow instantaneous changes in current.)

* RC circuit



$t = 0^+$

KCL: $C\frac{dv}{dt} + \frac{V}{R} = 0$

$V(t) = V(0)e^{-\frac{t}{RC}}$

$i(t) = \frac{V(0)}{R}e^{-\frac{t}{RC}}$

$V_c(0^-) = V_c(0^+) = V_c(0)$

* Energy

- power dissipated in R

$$p = iv = i^2R = \frac{V^2}{R} = [i(0)]^2 R e^{-2\frac{R}{L}t} \quad (RL)$$

$$\frac{[V(0)]^2}{R}e^{-\frac{2t}{RC}} \quad (RC)$$

- energy dissipated by R or delivered in L/C

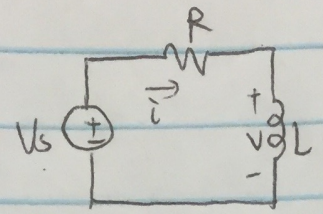
$$\Delta W_R = \int_0^t p\,dt$$

$$\Delta W_L = -\Delta W_R \quad (RL)$$

$$\Delta W_C = -\Delta W_R \quad (RC)$$

Step response

* RL circuit



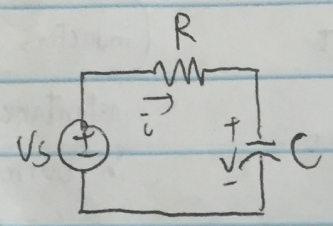
$t = 0^+$

KVL: $-V_s + L\frac{di}{dt} + iR = 0$

$i(t) = \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

$v(t) = L\frac{di}{dt} = V_s e^{-\frac{R}{L}t}$

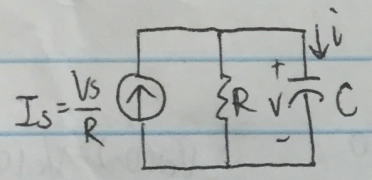
* RC circuit



KCL: $-I_s + \frac{v}{R} + C\frac{dv}{dt} = 0$

$v(t) = I_s R\left(1 - e^{-\frac{t}{RC}}\right)$

$i(t) = C\frac{dv}{dt} = I_s e^{-\frac{t}{RC}}$



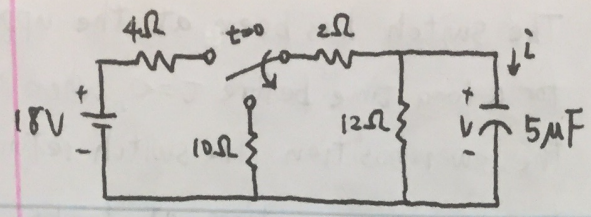
Time Constant

RL: $\tau = \frac{L}{R}$, RC: $\tau = RC$

General Form

$$x(t) = \underbrace{x_\infty}_{\text{steady state}} + \underbrace{\left[x(t_0) - x_\infty\right]}_{\text{difference between initial value and steady state value}} \underbrace{e^{-\frac{t-t_0}{\tau}}}_{\text{exponential decay}}$$

① ② ③



The switch has been in the upper position before $t=0$.

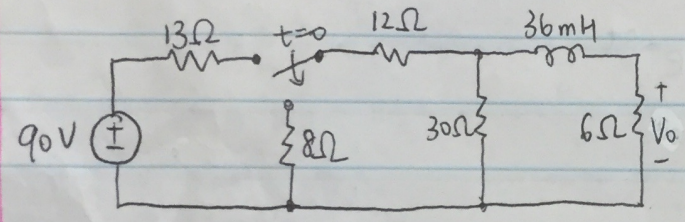
Find $v(t)$ and $i(t)$ for $t>0$.

$$V(0) = 18 \times \frac{12}{4+2+12} = 12\ V$$

$$R_{eq} = (10+2) \parallel 12 = 6\ \Omega$$

$$\tau = 6 \cdot C = 30\ \mu s$$

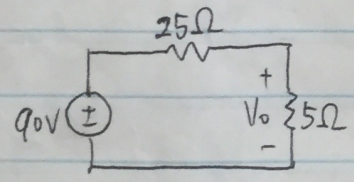
$$v(t) = 12 e^{\frac{-t}{30\mu}}\ V, \quad i(t) = -\frac{V(t)}{R_{eq}} = -2 e^{\frac{-t}{30\mu}}\ A$$



(a) Find $V_o(t)$ for $t>0$

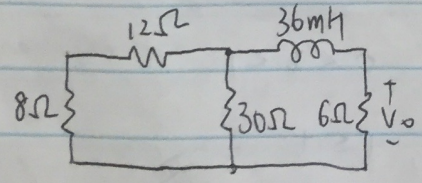
(b) How much energy is dissipated in the 6Ω resistor for $t>0$.

(a) For $t<0$,



$$V_o(0^-) = V_o(0^+) = 90 \cdot \frac{5}{25+5} = 15\ V$$

For $t>0$,



$$V_o(\infty) = 0$$

$$R_{eq} = [(8+12) \parallel 30] + 6 = 18\ \Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{36m}{18} = 2 \cdot 10^{-3}$$

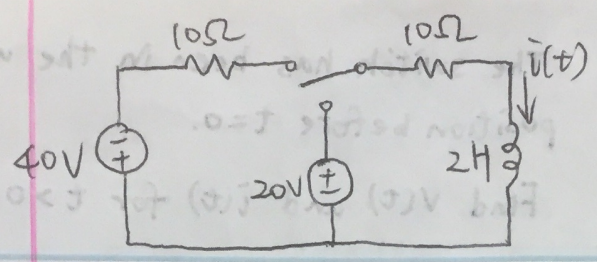
$$V_o(t) = 15 e^{-\frac{t}{2 \cdot 10^{-3}}}\ V \ \#$$

(b) $$p = \frac{V^2}{R} = \frac{75}{2} e^{-\frac{t}{10^{-3}}}$$

$$W = \int_0^\infty \frac{75}{2} e^{-\frac{t}{10^{-3}}} dt = \frac{75}{2} \cdot (-10^{-3}) e^{-\frac{t}{10^{-3}}} \Big|_{t=0}^{\infty}$$

$$= 0 - \frac{75}{2}(-10^{-3}) \cdot 1$$

$$= 37.5\ mJ \ \#$$



The switch has been at the upper position for a long time before $t=0$, when it goes to the lower position. The switch returns to the upper position at $t_0=1$ s.

Find $i(t)$ for $t>0$.

(i) For $0<t<1$

$$i(0^-)=i(0^+)=-\frac{40}{20}=-2\text{ A}$$

$$i_\infty=\frac{20}{10}=2\text{ A}$$

$$\tau_1=\frac{2}{10}=0.2$$

$$i(t)=2-4e^{-5t}$$

(ii) For $t\geq 1$

$$t_0=1\text{ s}$$

$$i(t_0^-)=i(t_0^+)=2-4e^{-5}=1.973\text{ A}$$

$$i_\infty=-\frac{40}{20}=-2\text{ A}$$

$$\tau_2=\frac{2}{20}=0.1$$

$$i(t)=-2+3.973e^{-10t}$$

$$\therefore\ i(t)=\begin{cases}2-4e^{-5t}, & 0<t<1\\ -2+3.973e^{-10t}, & 1\leq t\end{cases}$$

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