

suppose we have binomial function $P(m|N, p)$,

[m head in N trials with the chance of head, p

and we pick a prior which is beta distribution $\beta(p|a, b)$,

with bayesian equation :

$$\begin{aligned} P(p|m) &= \frac{\overset{\text{binomial}}{P(m|N, p)} \cdot \overset{\text{beta}}{\beta(p|a, b)}}{\int_0^1 P(m|N, p) \cdot \beta(p|a, b) dp} = \frac{\binom{N}{m} p^m (1-p)^{N-m} \cdot p^{a-1} (1-p)^{b-1} \frac{1}{\beta(a, b)}}{\int_0^1 \binom{N}{m} p^m (1-p)^{N-m} \cdot p^{a-1} (1-p)^{b-1} \frac{1}{\beta(a, b)} dp} \quad \begin{array}{l} \int (\text{joint pdf}) dp \\ \text{it's the normalization factor} \end{array} \\ &= \frac{\cancel{\binom{N}{m}} p^{m+a-1} (1-p)^{N-m+b-1} \cancel{\frac{1}{\beta(a, b)}}}{\int_0^1 \cancel{\binom{N}{m}} p^{m+a-1} (1-p)^{N-m+b-1} \cancel{\frac{1}{\beta(a, b)}} dp} = \frac{p^{m+a-1} (1-p)^{N-m+b-1}}{\beta(m+a, N-m+b)} = \beta(p|m+a, N-m+b) \quad \# \end{aligned}$$