Reliable communication and control for the Smart Grid

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Lab: LCA2



February 14, 2012

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The world today

- Energy producers are moving from traditional sources
- Renewables have high variability

Managing the difference

- ► Storage at source
- ► Fossil fuel backups
- Demand response
- ▶ ...

Smart grids enable Demand response



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The Demand-Response problem

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Demand-Response is an mechanism to manage customer consumption of electricity (demand) in response to supply conditions.

Current approaches:

- 1. Dynamic Pricing
 - ▶ Interesting but has problems^a
- 2. Delayed supply
 - Promising but currently ad-hoc; deployed by Votalis, Peaksaver

Le Boudec et. al. formalize 2 via service curve contracts.

^aMay expose consumers to price volatility

Outline

Reliable communication and control

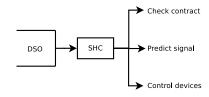
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Problem

Definition

Objectives



Intermediate goals

- ► A Communication protocol.
- ► Smart Home Controller (SHC) architecture

Design constraints

- Reliability and robustness.
- Computational feasibility

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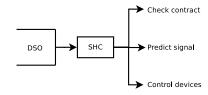
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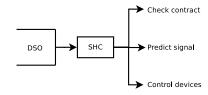
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Tools of trade

- ► Machine running Linux
- Capable of communication
- UI/Algo module can be distributed



Fig: ZPlug from Cleode



- ZigBee SE profile compliant
- ► Passive monitoring devices
- Can turn devices ON/OFF

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Contract

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Outline

Verifying the contract

Service curve $\beta(\cdot)$

- Borrowed from Network Calculus
- ▶ Minimum energy in time $t = \beta(t)$.
- Usually made by repeating the derivative with period t'
- Are deterministic.
- Slight abuse of notation on slides.

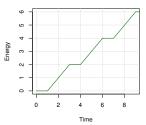


Fig: Serv. cur. (period = 3)

What makes it feasible

Keeping just one period t' of $u(\cdot)$ is sufficient to verify the service curve constraint, for binary service curves and bounded $u(\cdot)$.

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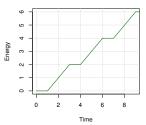


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Service curves constraint

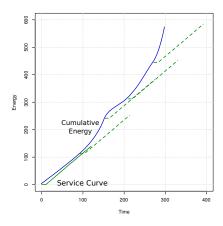


Fig: Control signal constrained by the service curve

$$\forall t_1, t_2. \int_{t_1}^{t_2} u(\tau) d\tau \geq \beta(t_2 - t_1) \implies U(t_2) - U(t_1) \geq \beta(t_2 - t_1)$$

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Service curve violation

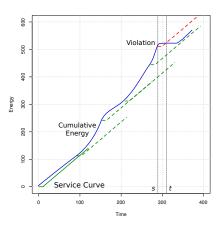


Fig: Service curve violation

$$\exists t, s. \ U(t) - U(s) \leq \beta(t-s)$$

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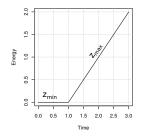
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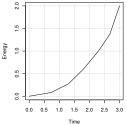
s, t are not unique

Generalized the result

Theorem: Keeping just one period of $u(\cdot)$ is sufficient to verify service curve constraint.



Binary serv. cur.



Star shaped serv. cur.

Constr. dropped

- $u(\cdot)$ was bdd above by z_{max}
- $\triangleright \beta(\cdot)$ was a binary serv. cur.

Shortcomings

Computationally more complex Reliable communication and

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Predicting guaranteed signal

Minimum guaranteed signal

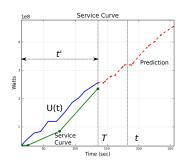


Fig: Minimum signal guaranteed

$$\hat{U}(t) \geq \sup_{s \in [0,T]} \left\{ U(s) + \beta(t-s) \right\}$$

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Minimum guaranteed signal

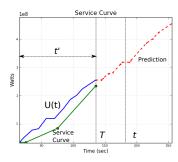


Fig: Minimum signal guaranteed

$$\hat{U}(t) \geq \sup_{s \in [t-t',T]} \left\{ U(s) + \beta(t-s) \right\}$$

- Naive algorithm (discrete Max-convolution)^b $\approx O(S^2)$
- ▶ Quota allocation policy algorithm (Binary service curve): $\approx O(S)$

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Prediction

Issues with naive algorithm

- Worked in discrete time, limited accuracy
- ► Was computationally expensive (5 sec. for 24 hours ≈ 300 million ops./calc.)
- Efficient version was limited to binary service curves

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New algorithm

- ▶ Works in continuous time, has more accuracy.
- ► Has complexity^c O(N · K)
 - More efficient than naive version even if control signal changes each sampling time
 - Is equal to complexity of efficient algorithm for binary curves
- Non-trivial, but proved correct.
- Complexity depends on input

Design recommendation

► Control *N* and *K* while forming the contract

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 $^{{}^{\}mathrm{c}}K=\#$ changes in $u(\cdot)$ in the last period, N= segments in Service Curve

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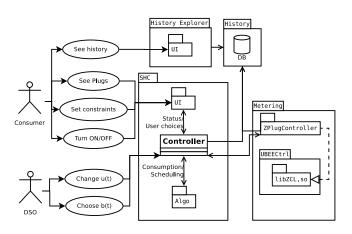


Fig: The Architecture

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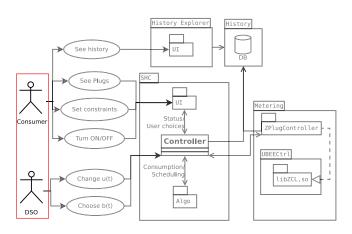


Fig: Actors

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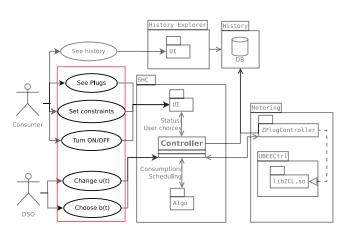


Fig: Actions

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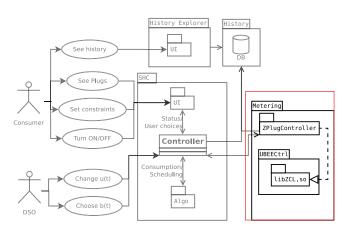


Fig: ZPlugs

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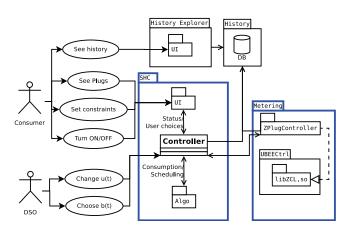


Fig: SHC architecture

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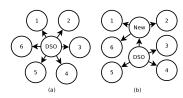


Fig: Messaging layer can manage overloading and grouping

- Communication happens via discrete messages
 - no continuous control
 - ► Data representation scheme is designed
- ▶ Pub-Sub model is employed instead of Req-Rep.

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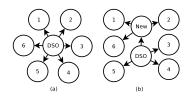


Fig: Messaging layer can manage overloading and grouping

Pub-Sub v/s Req-Rep

- Scalability
- ► Separation of concerns
 - Grouping subscribers

- Logging & Diagnostics
- Simplicity

Problem

- Newly started SHC
- Periodic broadcast

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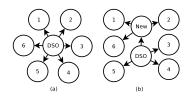


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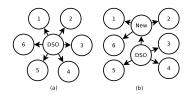


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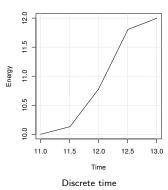
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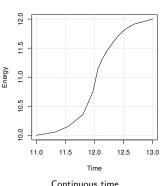
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Discrete v/s continuous time





Discrete

- Ease of presentation
- Sampling period trade-off:
 - Accuracy
 - Scalability

Continuous

- Better accuracy
- One less parameter
- Makes prediction trickier

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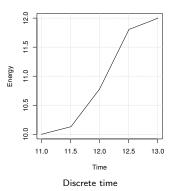
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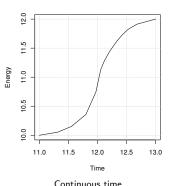
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Discrete v/s continuous time





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Data representation

Control signals

- ▶ Discrete signals ⇒ inform only of changes
- Cumulative energy is piecewise linear

Data representation

Control signals

- ▶ Discrete signals ⇒ inform only of changes
- ▶ Cumulative energy is piecewise linear

Service Curves

- ► As a piecewise linear function
- Can approximate any function
- Usually just binary

Piecewise linear continuous functions

Store end points of all segments.

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Reliability

Primary problems

- ► Crashes: Application or a dependency might crash
- ▶ Live-locks: Operations might start taking too long or freeze
- ▶ Byzantine faults: Nonsensical output (hopefully)

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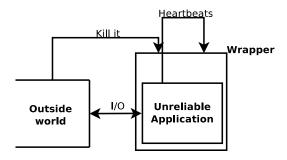


Fig: Wrapper design

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Hosted at: https://github.com/musically-ut/pyoolproof

Reliability

Primary problems

- Crashes: Check for error codes, restart application, persist data
- ▶ Live-locks: Listen to heartbeats, kill and restart if too slow
- Byzantine faults: Kill on request (STONITH)

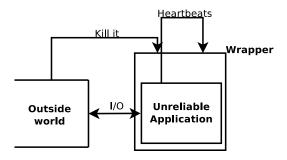


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Implementation

Language Lines of code Comments Component Python 4,300 1,300 UI, Test harness C++220 ZPlug wrapper 800 Ocaml SHC 2,000 460 Total 7,100 2,080

Tbl: Details about the code in the implementation

- ▶ Battery of unit tests
- ► Good documentation and examples

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Skills acquired

Demand-response using Service Curves

- Network Calculus fundamentals
- Demand response

Robust software design

- ▶ Ocaml: Functional + OO paradigm
- Modular design: Separating points of failures

Tools

- ZeroMQ: TCP sockets on steroids
- **SQLite:** Embedded reliable persistence.
- Also learned about ZigBee and ZPlugs.

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Problem

Contract

Service Curve Generalization

Prediction

New algorithn

mplementation

Architecture
Communication
PUB-SUB model
Continuous time
Data representatio
Reliable wrapper

emo

essment

Learning experience

Assessificite

Assessment

Reliable communication and control

Utkarsh Upadhyay

Assessment

State of testbed

- Currently fully functional
- Easy to extend

Self assessment

- Stress testing
- Deployment
- ▶ Theoretical results were time consuming

Reliable communication and control

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Future work

Future work

Reliable communication and control

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Future work

Testbed

Systematic stress testing.

Formal verification.

Prediction and Optimization

Making the algorithm incremental

Design

Security

Reliable communication and control

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Future work

Thank you

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Appendix

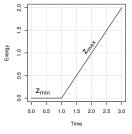
Prediction algorithm Theorems

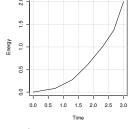
${\sf Appendix}$

10: **end for** 11: **return** $\hat{V}_K(\cdot)$

```
Input: \beta(\cdot) = [(s_1, \beta(s_1)), \dots, (s_N, \beta(s_N))],
U(\cdot) = [(t_1, U(t_1)), \dots, (t_K, U(t_K))]
Output: predicted segments/function

1: \hat{V}_1 = [(t_K + s_i, U(t_K) + \beta(s_i)) \text{ for } s_i \text{ in } \mathbf{T}_{\beta}]
2: scTip = (t_K, \beta(s_N) + U(t_K))
3: for \ j = K - 1 \text{ to } 1 \text{ do}
4: t_j = \mathbf{T}_U[j]
5: C = [(t_j + s_i, U(t) + \beta(s_i)) \text{ for } s_i \text{ in } \mathbf{T}_{\beta}]
6: C.add(scTip)
7: \hat{V}_{(K-j+1)} = \max Cover(\hat{V}_{(K-j)}, C)
8: \hat{V}_{(K-j+1)}.limitTime(t_K, t_K + s_N)
9: scTip = (t_j + s_N, U(t_j) + \beta(s_N))
```





Binary serv. cur.

Star shaped serv. cur.

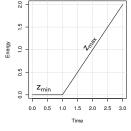
u(t) is control signal defined up to some time horizon T.

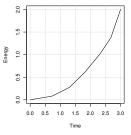
Theorem: If $u(t) \le z_{max}$ for all $t \le T$, then these are equivalent:

- 1. $\int_{s}^{t} u(\tau) d\tau \ge \beta(t-s)$ for all $s < t \le T$,
- 2. $\forall \tau \leq T$. $u(\tau) \geq z_{min}$ and $\int_t^{t+t'} u(\tau) d\tau \geq \beta(t')$, $\forall t$ such that $t+t' \leq T$.

where $\beta(\cdot)$ is a binary service curve.







Binary serv. cur.

Star shaped serv. cur.

u(t) is control signal defined up to some time horizon T. **Theorem:** If control signal $u(\cdot)$ violates service curve $\beta(\cdot)$ for the first time at time T, then

$$\exists t \in [T - t', T). \ U(t) + \beta(T - t) > U(T)$$

Theorems

Previous result: If $u(t) < z_{max}$ for all t < T, then these are equivalent:

- 1. $\int_{s}^{t} u(\tau) d\tau \geq \beta(t-s)$ for all $s < t \leq T$,
- 2. $\forall \tau \leq T$. $u(\tau) \geq z_{min}$ and $\int_{t}^{t+t'} u(\tau) d\tau \geq \beta(t')$, $\forall t$ such that t+t' < T

where $\beta(\cdot)$ is a binary service curve.

New result: If control signal $u(\cdot)$ violates service curve $\beta(\cdot)$ for the first time at time T. then:

$$\exists t \in [T-t',T).\ U(t)+\beta(T-t)>U(T)$$

Constr. dropped

- $\triangleright u(\cdot)$ was bdd above by z_{max}
- $\triangleright \beta(\cdot)$ was binary serv. cur.

Restrictions

Max-plus convolution