

Reliable communication and control for the Smart Grid

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Lab: LCA2



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Generalization

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The world today

- ▶ Energy producers are moving from traditional sources
- ▶ Renewables have high variability

Managing the difference

- ▶ Storage at source
- ▶ Fossil fuel backups
- ▶ Demand response
- ▶ ...

Smart grids enable Demand response



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The Demand-Response problem

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Demand-Response is an mechanism to manage customer consumption of electricity (**demand**) in **response** to supply conditions.

Current approaches:

1. Dynamic Pricing

- ▶ Interesting but has problems^a

2. Delayed supply

- ▶ Promising but currently ad-hoc; deployed by Votalis, Peaksaver

Le Boudec *et. al.* formalize 2 via service curve contracts.

^aMay expose consumers to price volatility

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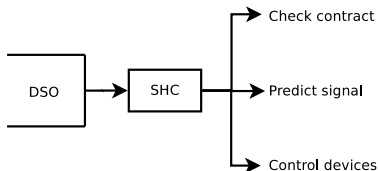
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Definition

Objectives



Intermediate goals

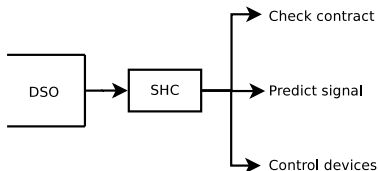
- ▶ A Communication protocol.
- ▶ Smart Home Controller (SHC) architecture.

Design constraints

- ▶ Reliability and robustness.
- ▶ Computational feasibility.

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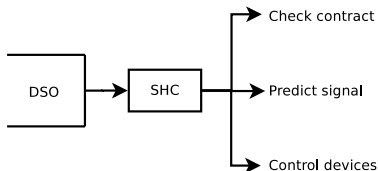
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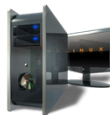
Future work

Tools of trade

- ▶ Machine running Linux
- ▶ Capable of communication
- ▶ UI/Algo module can be distributed



Fig: ZPlug from Cleode



- ▶ ZigBee SE profile compliant
- ▶ Passive monitoring devices
- ▶ Can turn devices ON/OFF

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Service curve $\beta(\cdot)$

- ▶ Borrowed from Network Calculus
- ▶ Minimum energy in time $t = \beta(t)$.
- ▶ Usually made by repeating the *derivative* with period t'
- ▶ Are deterministic.
- ▶ Slight abuse of notation on slides.

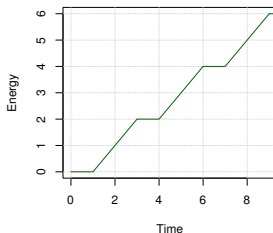


Fig: Serv. cur. (period = 3)

What makes it feasible

Keeping just one period t' of $u(\cdot)$ is sufficient to verify the service curve constraint, for binary service curves and bounded $u(\cdot)$.

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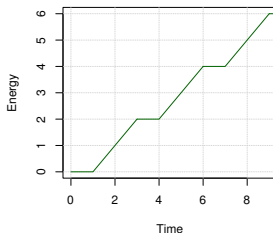


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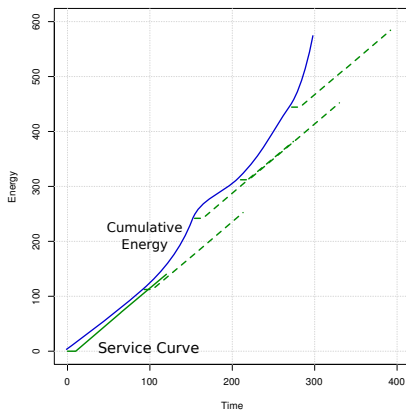


Fig: Control signal constrained by the service curve

$$\forall t_1, t_2. \int_{t_1}^{t_2} u(\tau) d\tau \geq \beta(t_2 - t_1) \implies U(t_2) - U(t_1) \geq \beta(t_2 - t_1)$$

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Service curve violation

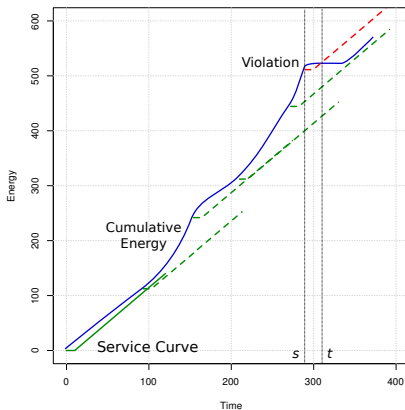


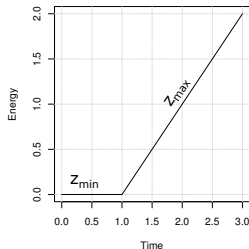
Fig: Service curve violation

$$\exists t, s. U(t) - U(s) \leq \beta(t - s)$$

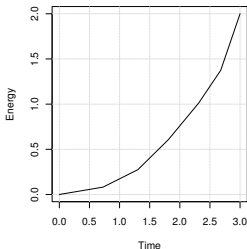
s, t are not unique

Generalized the result

Theorem: Keeping just one period of $u(\cdot)$ is sufficient to verify service curve constraint.



Binary serv. cur.



Star shaped serv. cur.

Constr. dropped

- ▶ $u(\cdot)$ was bdd above by z_{\max}
- ▶ $\beta(\cdot)$ was a binary serv. cur.

Shortcomings

- ▶ Computationally more complex

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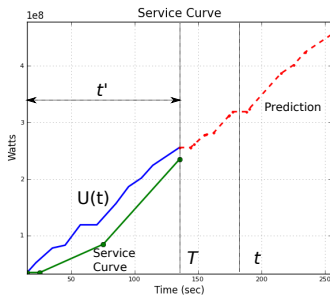


Fig: Minimum signal guaranteed

$$\hat{U}(t) \geq \sup_{s \in [0, T]} \{U(s) + \beta(t - s)\}$$

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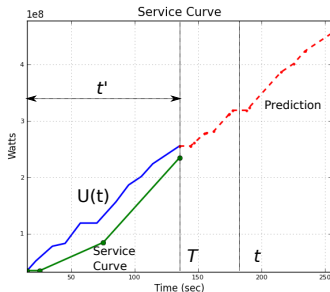


Fig: Minimum signal guaranteed

$$\hat{U}(t) \geq \sup_{s \in [t-t', T]} \{U(s) + \beta(t-s)\}$$

- ▶ Naive algorithm (discrete Max-convolution)^b $\approx O(S^2)$
- ▶ Quota allocation policy algorithm (Binary service curve): $\approx O(S)$

^bComplexity per prediction calculation, S is number of samples in one period

Prediction algorithm

Issues with naive algorithm

- ▶ Worked in discrete time, limited accuracy
- ▶ Was computationally expensive (5 sec. for 24 hours \approx 300 million ops./calc.)
- ▶ Efficient version was limited to binary service curves

New algorithm

- ▶ Works in continuous time, has more accuracy.
- ▶ Has complexity^c $O(N \cdot K)$
 - ▶ More efficient than naive version even if control signal changes each sampling time
 - ▶ Is equal to complexity of efficient algorithm for binary curves
- ▶ Non-trivial, but proved correct.
- ▶ Complexity depends on input

Design recommendation

- ▶ Control N and K while forming the contract.

^c $K = \#$ changes in $u(\cdot)$ in the last period, $N =$ segments in Service Curve

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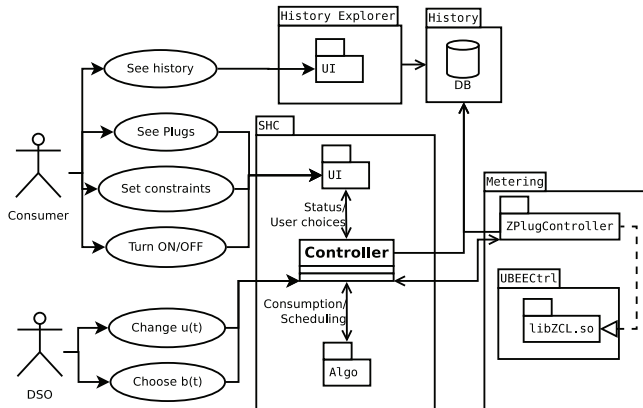


Fig: The Architecture

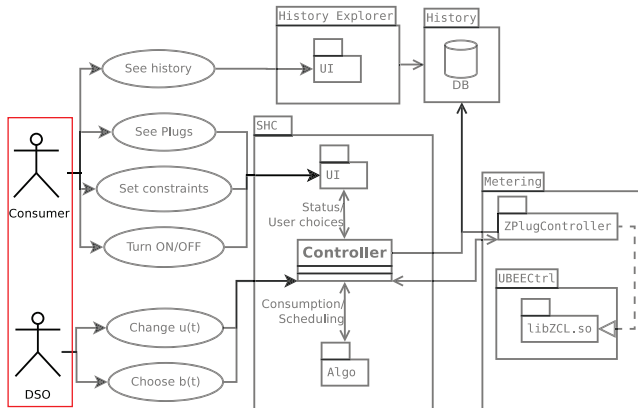


Fig: Actors

Smart Home Controller

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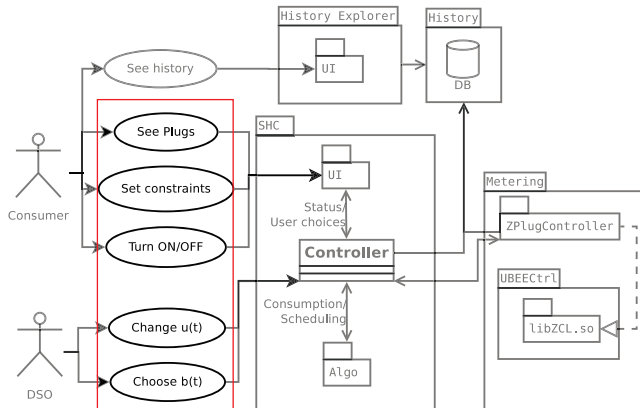


Fig: Actions

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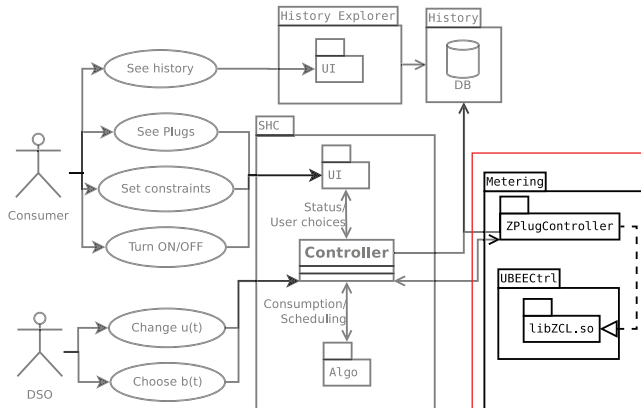


Fig: ZPlugs

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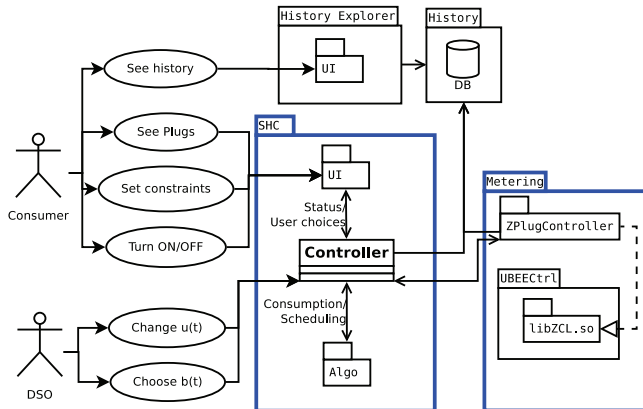


Fig: SHC architecture

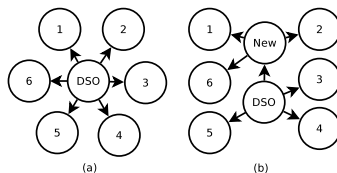


Fig: Messaging layer can manage overloading and grouping

- ▶ Communication happens via discrete messages
 - ▶ *no continuous control*
 - ▶ Data representation scheme is designed
- ▶ PUB-SUB model is employed instead of REQ-REP.

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Communication protocol

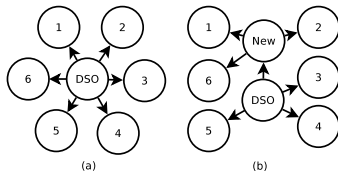


Fig: Messaging layer can manage overloading and grouping

PUB-SUB v/s REQ-REP

- ▶ Scalability
- ▶ Separation of concerns
 - ▶ Grouping subscribers
- ▶ Logging & Diagnostics
- ▶ Simplicity

Problem

- ▶ Newly started SHC
- ▶ Periodic broadcast

Communication protocol

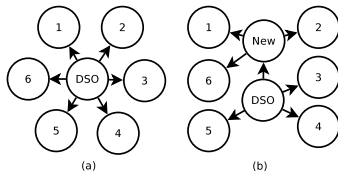


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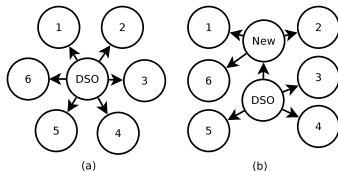


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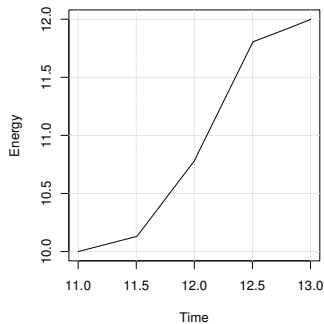
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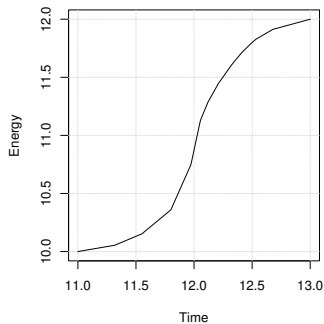
Problem

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Discrete v/s continuous time



Discrete time



Continuous time

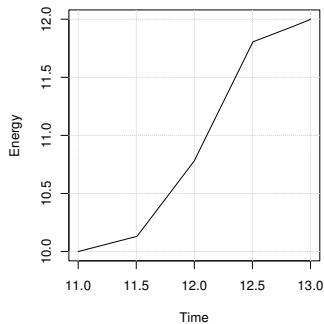
Discrete

- ▶ Ease of presentation
- ▶ Sampling period trade-off:
 - ▶ Accuracy
 - ▶ Scalability

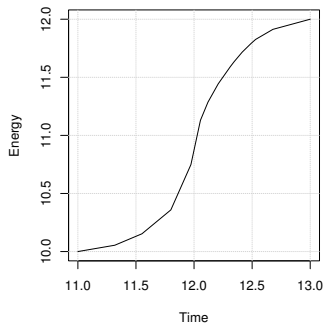
Continuous

- ▶ Better accuracy
- ▶ One less parameter
- ▶ Makes prediction trickier

Discrete v/s continuous time



Discrete time



Continuous time

Discrete

- ▶ Ease of presentation
- ▶ Sampling period trade-off:
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Continuous

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Control signals

- ▶ Discrete signals \implies inform only of changes
- ▶ Cumulative energy is piecewise linear

Service Curves

- ▶ As a piecewise linear function
- ▶ Can approximate any function
- ▶ Usually just binary

Piecewise linear continuous functions

- ▶ Store end points of all segments.

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Primary problems

- ▶ **Crashes**: Application or a dependency might crash
- ▶ **Live-locks**: Operations might start taking too long or freeze
- ▶ **Byzantine faults**: Nonsensical output (hopefully)

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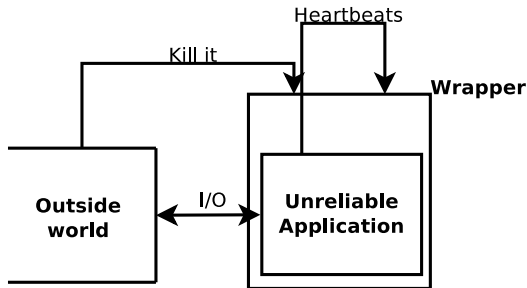


Fig: Wrapper design

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Primary problems

- ▶ **Crashes:** Check for error codes, restart application, persist data
- ▶ **Live-locks:** Listen to heartbeats, kill and restart if too slow
- ▶ **Byzantine faults:** Kill on request (STONITH)

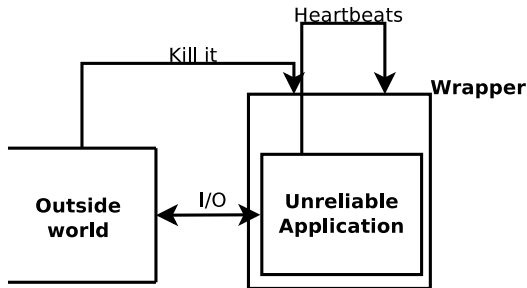


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Language	Lines of code	Comments	Component
Python	4,300	1,300	UI, Test harness
C++	800	220	ZPlug wrapper
Ocaml	2,000	460	SHC
Total	7,100	2,080	–

Tbl: Details about the code in the implementation

- ▶ Battery of unit tests
- ▶ Good documentation and examples

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Demand-response using Service Curves

- ▶ Network Calculus fundamentals
- ▶ Demand response

Robust software design

- ▶ **Ocaml**: Functional + OO paradigm
- ▶ **Modular design**: Separating points of failures

Tools

- ▶ **ZeroMQ**: TCP sockets on steroids
- ▶ **SQLite**: Embedded reliable persistence.
- ▶ Also learned about ZigBee and ZPlugs.

Introduction

Problem

Contract

Service Curves
Generalization

Prediction

New algorithm

Implementation

Architecture
Communication
PUB-SUB model
Continuous time
Data representation
Reliable wrapper
Testbed

Demo

Assessment

Learning experience
Assessment

Future work

State of testbed

- ▶ Currently fully functional
- ▶ Easy to extend

Self assessment

- ▶ Stress testing
- ▶ Deployment
- ▶ Theoretical results were time consuming

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Predicting guaranteed signal

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Reliable
communication and
control

Utkarsh Upadhyay

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- ▶ Systematic stress testing.
- ▶ Formal verification.

Prediction and Optimization

- ▶ Making the algorithm incremental

Design

- ▶ Security

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Thank you

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Reliable
communication and
control

Utkarsh Upadhyay

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Appendix

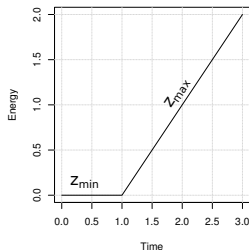
Prediction algorithm
Theorems

Appendix

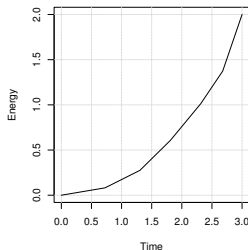
Input: $\beta(\cdot) = [(s_1, \beta(s_1)), \dots, (s_N, \beta(s_N))]$,
 $U(\cdot) = [(t_1, U(t_1)), \dots, (t_K, U(t_K))]$

Output: predicted segments/function

```
1:  $\hat{V}_1 = [(t_K + s_i, U(t_K) + \beta(s_i)) \text{ for } s_i \text{ in } \mathbf{T}_\beta]$ 
2:  $\text{scTip} = (t_K, \beta(s_N) + U(t_K))$ 
3: for  $j = K - 1$  to  $1$  do
4:    $t_j = \mathbf{T}_U[j]$ 
5:    $\mathbf{C} = [(t_j + s_i, U(t_j) + \beta(s_i)) \text{ for } s_i \text{ in } \mathbf{T}_\beta]$ 
6:    $\mathbf{C.add(scTip)}$ 
7:    $\hat{V}_{(K-j+1)} = \text{maxCover}(\hat{V}_{(K-j)}, \mathbf{C})$ 
8:    $\hat{V}_{(K-j+1)}.limitTime(t_K, t_K + s_N)$ 
9:    $\text{scTip} = (t_j + s_N, U(t_j) + \beta(s_N))$ 
10: end for
11: return  $\hat{V}_K(\cdot)$ 
```



Binary serv. cur.



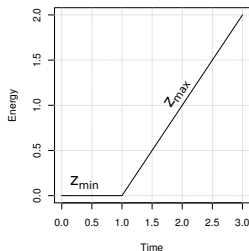
Star shaped serv. cur.

$u(t)$ is control signal defined up to some time horizon T .

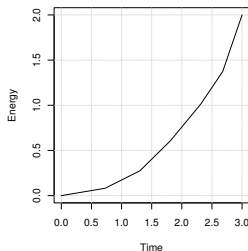
Theorem: If $u(t) \leq z_{\max}$ for all $t \leq T$, then these are equivalent:

1. $\int_s^t u(\tau) d\tau \geq \beta(t-s)$ for all $s < t \leq T$,
2. $\forall \tau \leq T. u(\tau) \geq z_{\min}$ and $\int_t^{t+t'} u(\tau) d\tau \geq \beta(t')$, $\forall t$ such that $t+t' \leq T$.

where $\beta(\cdot)$ is a *binary service curve*.



Binary serv. cur.



Star shaped serv. cur.

$u(t)$ is control signal defined up to some time horizon T .

Theorem: If control signal $u(\cdot)$ violates *service curve* $\beta(\cdot)$ for the first time at time T , then

$$\exists t \in [T - t', T). U(t) + \beta(T - t) > U(T)$$

Previous result: If $u(t) \leq z_{\max}$ for all $t \leq T$, then these are equivalent:

1. $\int_s^t u(\tau) d\tau \geq \beta(t-s)$ for all $s < t \leq T$,
2. $\forall \tau \leq T. u(\tau) \geq z_{\min}$ and $\int_t^{t+t'} u(\tau) d\tau \geq \beta(t')$, $\forall t$ such that $t+t' \leq T$.

where $\beta(\cdot)$ is a **binary service curve**.

New result: If control signal $u(\cdot)$ violates **service curve** $\beta(\cdot)$ for the first time at time T , then:

$$\exists t \in [T - t', T). U(t) + \beta(T - t) > U(T)$$

Constr. dropped

- ▶ $u(\cdot)$ was bdd above by z_{\max}
- ▶ $\beta(\cdot)$ was binary serv. cur.

Restrictions

- ▶ Max-plus convolution