Kalman Filter Theory and Applications Equation Drilldown

https://github.com/musicarroll/kalman_course

Michael L. Carroll

June 22, 2023

- Understanding the Equations: Heuristic Introduction

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- **Equation Drilldown: Taking the Equations Apart**

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- State Space Concepts

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- Drilldown on State Dynamics and Covariance Extrapolation Equations

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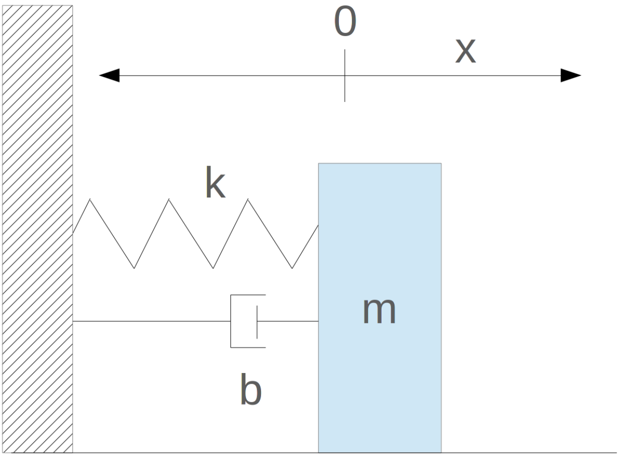
Damped Harmonic Oscillator

Spring, Mass, Damper System



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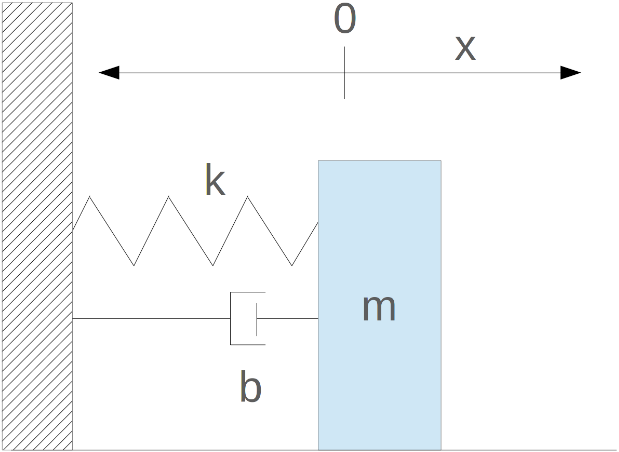

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MATLAB/Python Run

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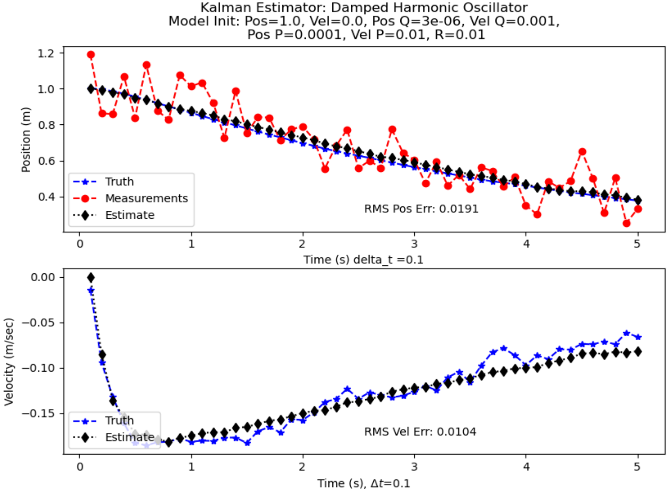

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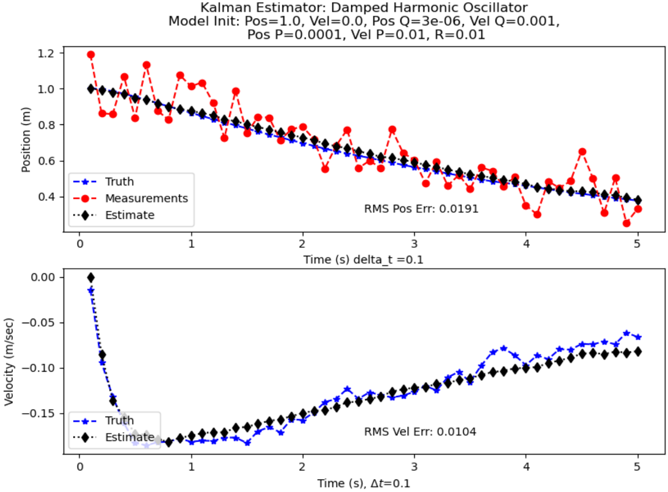
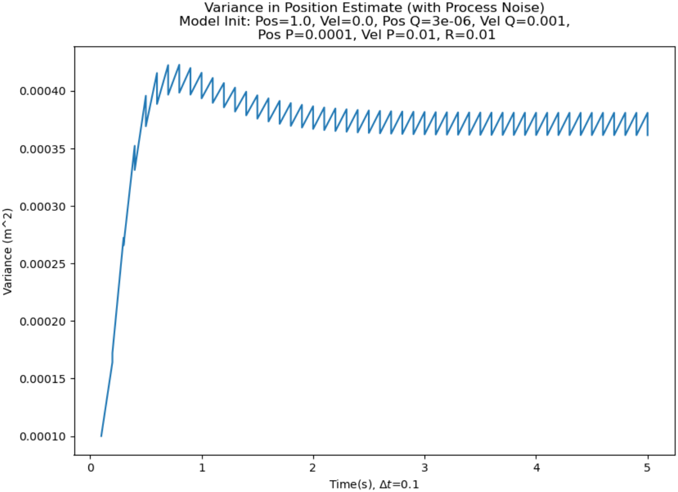


Damped Harmonic Oscillator Estimator

Sawtooth Plots



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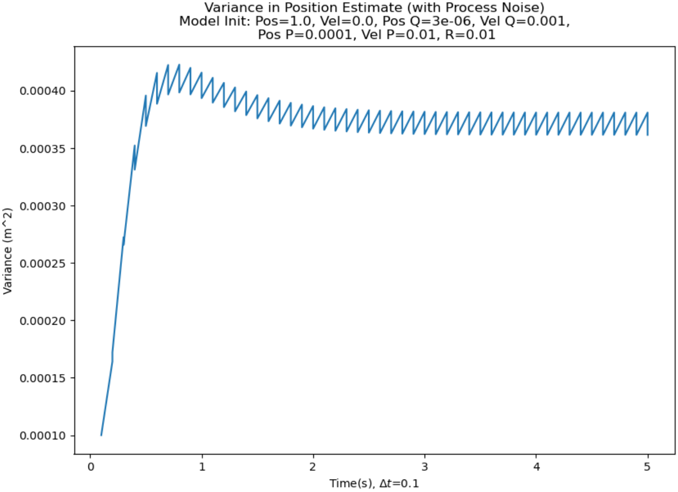

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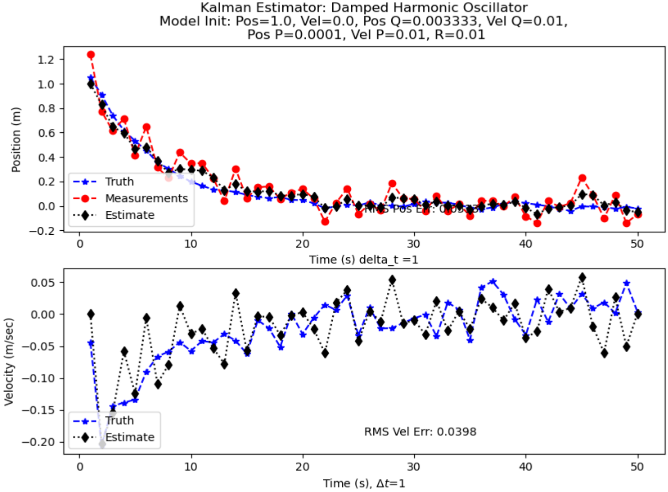

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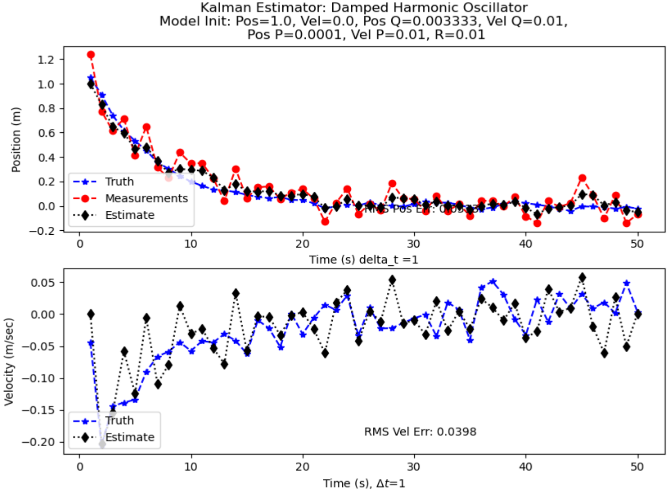

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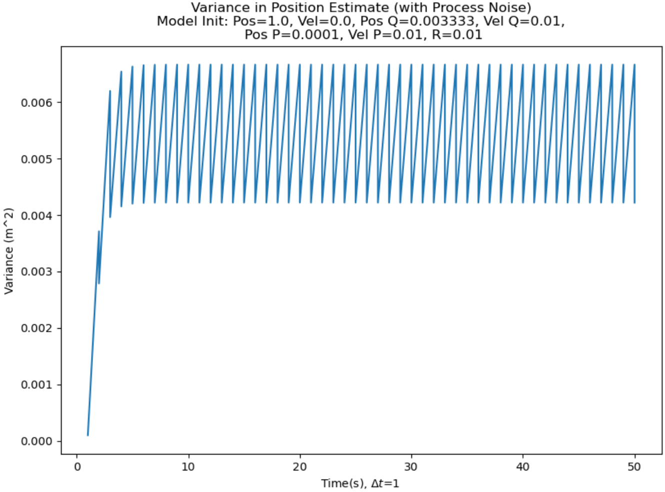

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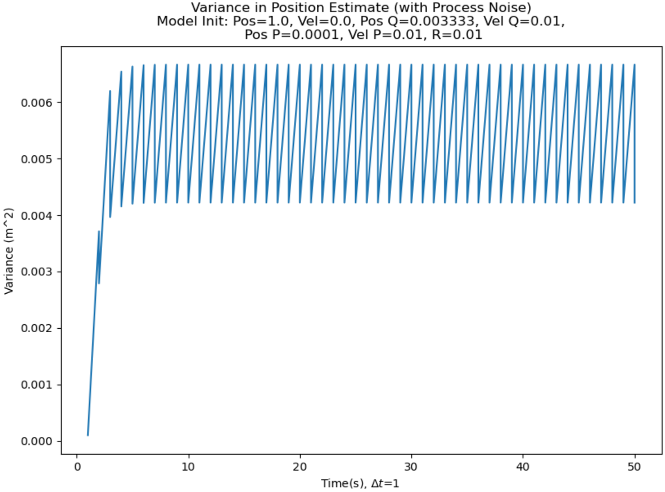

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