Kalman Filter Theory and Applications
Heuristic Overview

Michael L. Carroll

July 27, 2020

©2020 by Michael L. Carroll

Part I
The Basic Kalman Equations
Topics

- Understanding the Equations: Heuristic Introduction

- Understanding the Equations: Heuristic Introduction
- Equation Drilldown: Taking the Equations Apart

- Understanding the Equations: Heuristic Introduction
- Equation Drilldown: Taking the Equations Apart
- State Space Concepts

- Understanding the Equations: Heuristic Introduction
- Equation Drilldown: Taking the Equations Apart
- State Space Concepts

- Recursive Predictor-Corrector Algorithms

- Recursive Predictor-Corrector Algorithms
- **Running Averages (Part B)**

- Recursive Predictor-Corrector Algorithms
- **Running Averages (Part B)**
- Prediction (Extrapolation)

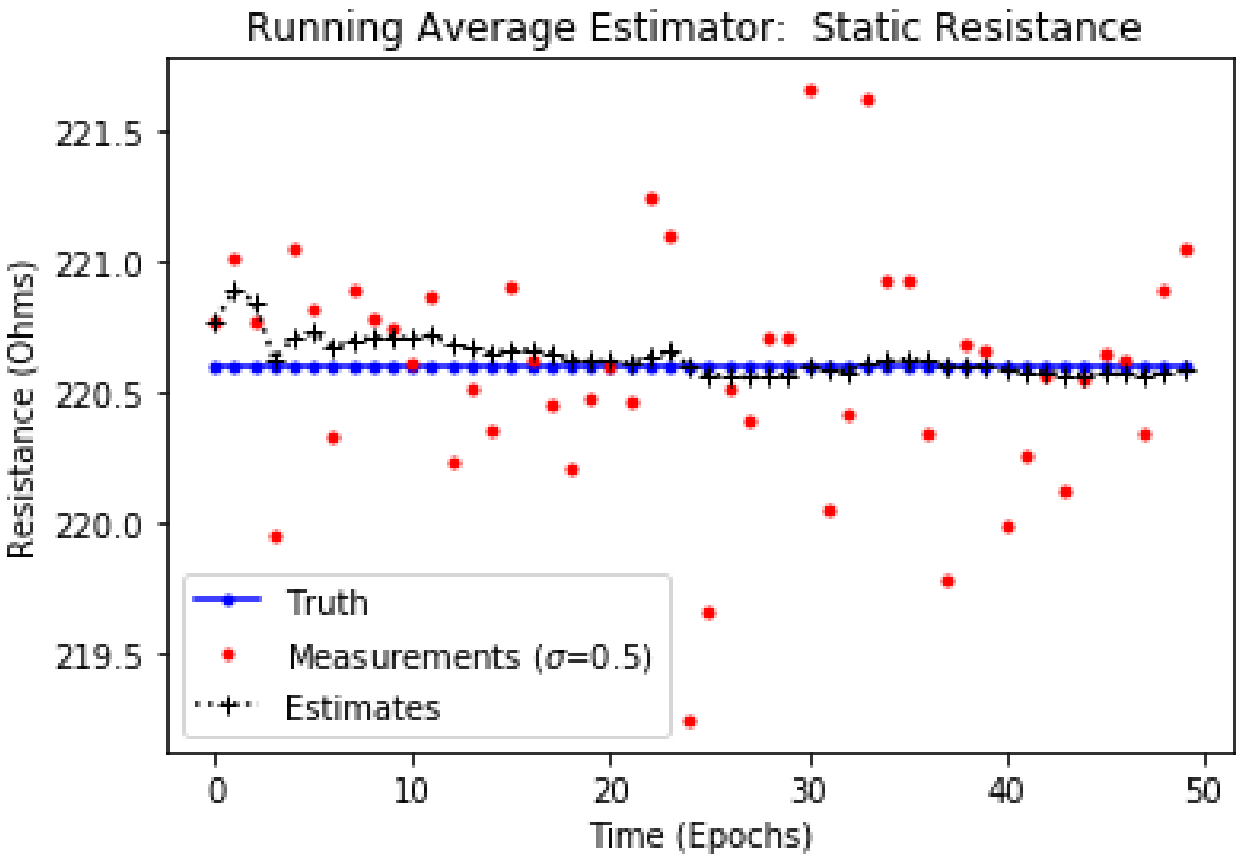
- Recursive Predictor-Corrector Algorithms
- **Running Averages (Part B)**
- Prediction (Extrapolation)
- Correction (Measurement Update)

- Recursive Predictor-Corrector Algorithms
- **Running Averages (Part B)**
- Prediction (Extrapolation)
- Correction (Measurement Update)
- Gain Computation

- Recursive Predictor-Corrector Algorithms
- **Running Averages (Part B)**
- Prediction (Extrapolation)
- Correction (Measurement Update)
- Gain Computation

Recursive Running Average Python Scripts: dyn_simple_moving_avg_demo.py

Recursive Running Average Python Scripts: dyn_simple_moving_avg_demo.py



Running Average
Shortcomings

- Uses (implicitly) all data points, even those from the "remote" past

Running Average
Shortcomings

- Uses (implicitly) all data points, even those from the "remote" past
- Gain gets smaller and smaller ... running average filter goes to sleep, becomes insensitive to new information

Running Average
Shortcomings

- Uses (implicitly) all data points, even those from the "remote" past
- Gain gets smaller and smaller ... running average filter goes to sleep, becomes insensitive to new information
- Can improve things with a simple moving average (sliding window)

- Uses (implicitly) all data points, even those from the "remote" past
- Gain gets smaller and smaller ... running average filter goes to sleep, becomes insensitive to new information
- Can improve things with a simple moving average (sliding window)

Simple Moving Average
Addressing the Shotcomings

- Simple Moving Average of Window Size M only averages the M most recent measurements

Simple Moving Average
Addressing the Shotcomings

- Simple Moving Average of Window Size M only averages the M most recent measurements
- Has a fixed gain of $1/M$ which never goes to zero

- Simple Moving Average of Window Size M only averages the M most recent measurements
- Has a fixed gain of $1/M$ which never goes to zero
- Assume the number of available data points is $N > M$

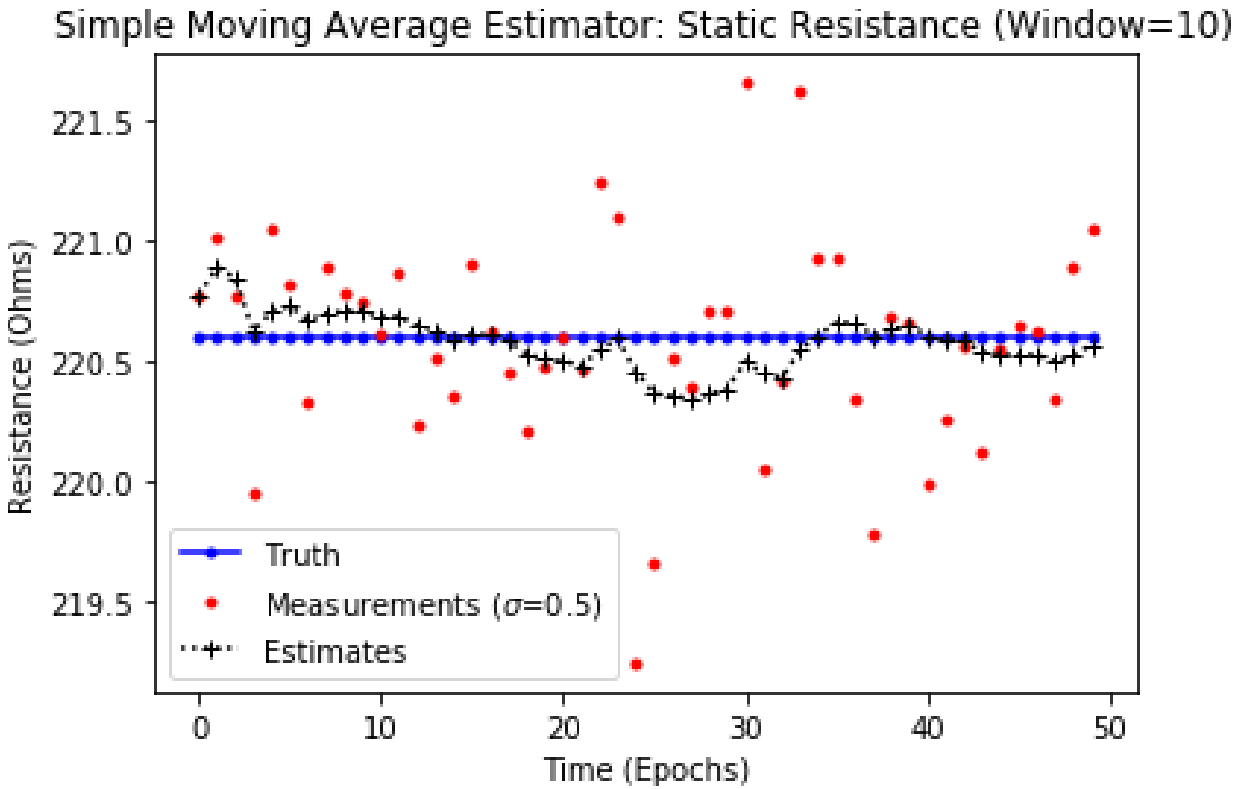
- Simple Moving Average of Window Size M only averages the M most recent measurements
- Has a fixed gain of $1/M$ which never goes to zero
- Assume the number of available data points is $N > M$
- Simple Moving Average of window size M is $s(k, M) = \frac{1}{M} \sum_{j=k-M+1}^{k} z(j)$, valid for $k \geq M - 1$ if indexing starts at 0 (we can set the SMA equal to the running average for $k < M - 1$)

- Simple Moving Average of Window Size M only averages the M most recent measurements
- Has a fixed gain of $1/M$ which never goes to zero
- Assume the number of available data points is $N > M$
- Simple Moving Average of window size M is $s(k, M) = \frac{1}{M} \sum_{j=k-M+1}^{k} z(j)$, valid for $k \geq M - 1$ if indexing starts at 0 (we can set the SMA equal to the running average for $k < M - 1$)
- Recursive SMA formula: $s(k, M) := s(k-1, M) + \frac{1}{M} [z(k) - z(k - M + 1)]$

- Simple Moving Average of Window Size M only averages the M most recent measurements
- Has a fixed gain of $1/M$ which never goes to zero
- Assume the number of available data points is $N > M$
- Simple Moving Average of window size M is $s(k, M) = \frac{1}{M} \sum_{j=k-M+1}^{k} z(j)$, valid for $k \geq M - 1$ if indexing starts at 0 (we can set the SMA equal to the running average for $k < M - 1$)
- Recursive SMA formula: $s(k, M) := s(k-1, M) + \frac{1}{M} [z(k) - z(k - M + 1)]$

Simple Moving Average Python Scripts: simple_moving_avg_demo.py of window size 10

Simple Moving Average Python Scripts: simple_moving_avg_demo.py of window size 10



Running Average
Addressing the Shotcomings

- Note that the recursive SMA seems to have shortcomings of its own

- Note that the recursive SMA seems to have shortcomings of its own
- First, it seems overly conservative about updating the previous SMA value

- Note that the recursive SMA seems to have shortcomings of its own
- First, it seems overly conservative about updating the previous SMA value
  - However, this is because unchanging process dynamics means that measurements do not change much

- Note that the recursive SMA seems to have shortcomings of its own
- First, it seems overly conservative about updating the previous SMA value
  - However, this is because unchanging process dynamics means that measurements do not change much
  - Performs worse than running average on constant dynamics, but better with changing dynamics (See next slide)

- Note that the recursive SMA seems to have shortcomings of its own
- First, it seems overly conservative about updating the previous SMA value
  - However, this is because unchanging process dynamics means that measurements do not change much
  - Performs worse than running average on constant dynamics, but better with changing dynamics (See next slide)
- A more aggresive formulation of the recursive SMA can be defined by using a weighting scheme whereby the oldest measurement is weighted less than the most recent

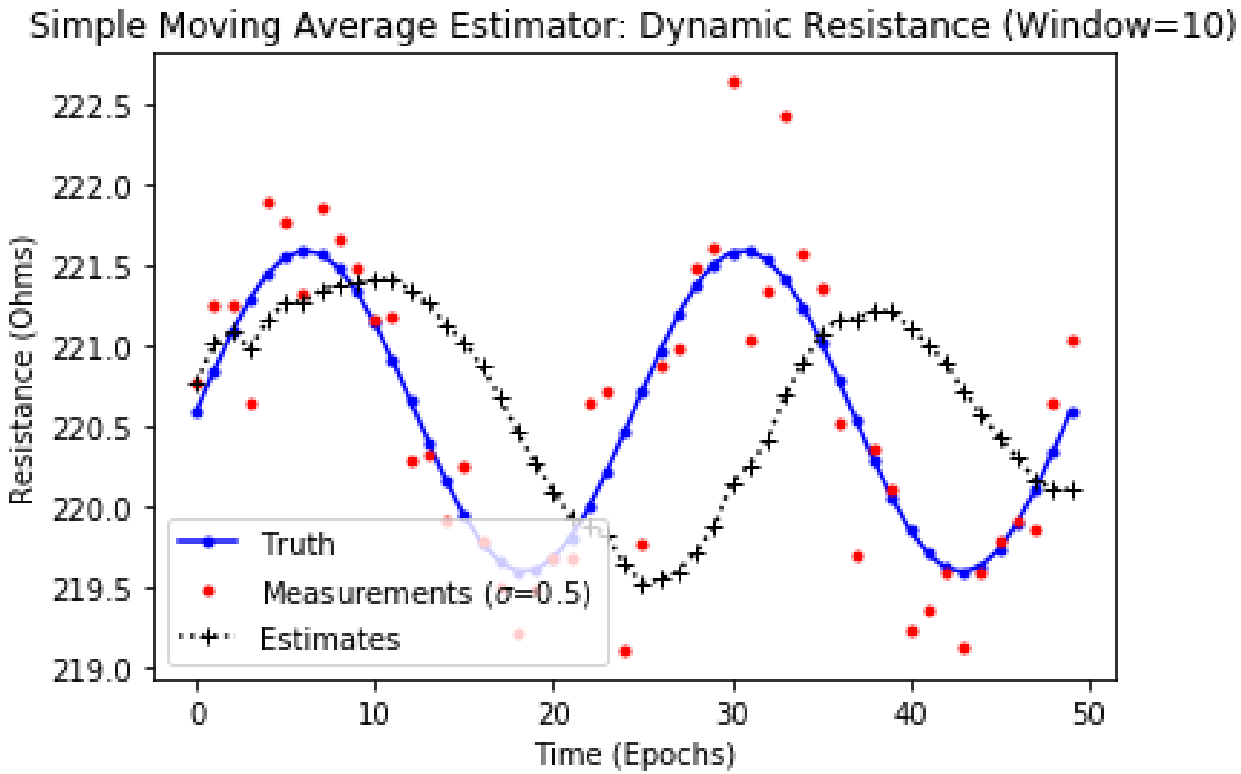
- Note that the recursive SMA seems to have shortcomings of its own
- First, it seems overly conservative about updating the previous SMA value
  - However, this is because unchanging process dynamics means that measurements do not change much
  - Performs worse than running average on constant dynamics, but better with changing dynamics (See next slide)
- A more aggresive formulation of the recursive SMA can be defined by using a weighting scheme whereby the oldest measurement is weighted less than the most recent
- Neither of these averaging filters explicitly takes into account the degree of uncertainty in the process dynamics

- Note that the recursive SMA seems to have shortcomings of its own
- First, it seems overly conservative about updating the previous SMA value
  - However, this is because unchanging process dynamics means that measurements do not change much
  - Performs worse than running average on constant dynamics, but better with changing dynamics (See next slide)
- A more aggresive formulation of the recursive SMA can be defined by using a weighting scheme whereby the oldest measurement is weighted less than the most recent
- Neither of these averaging filters explicitly takes into account the degree of uncertainty in the process dynamics

---

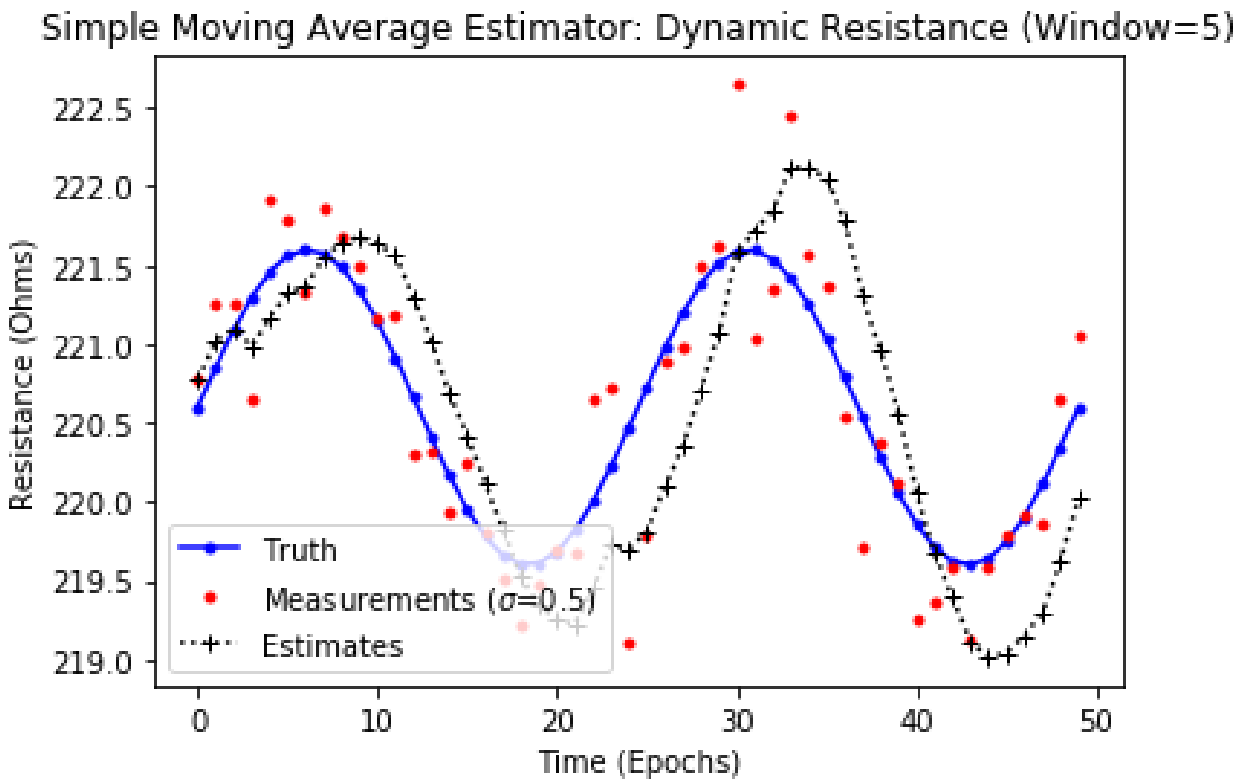
Simple Moving Average Python Scripts: dyn_simple_moving_avg_demo.py of window size 10

Simple Moving Average Python Scripts: dyn_simple_moving_avg_demo.py of window size 10



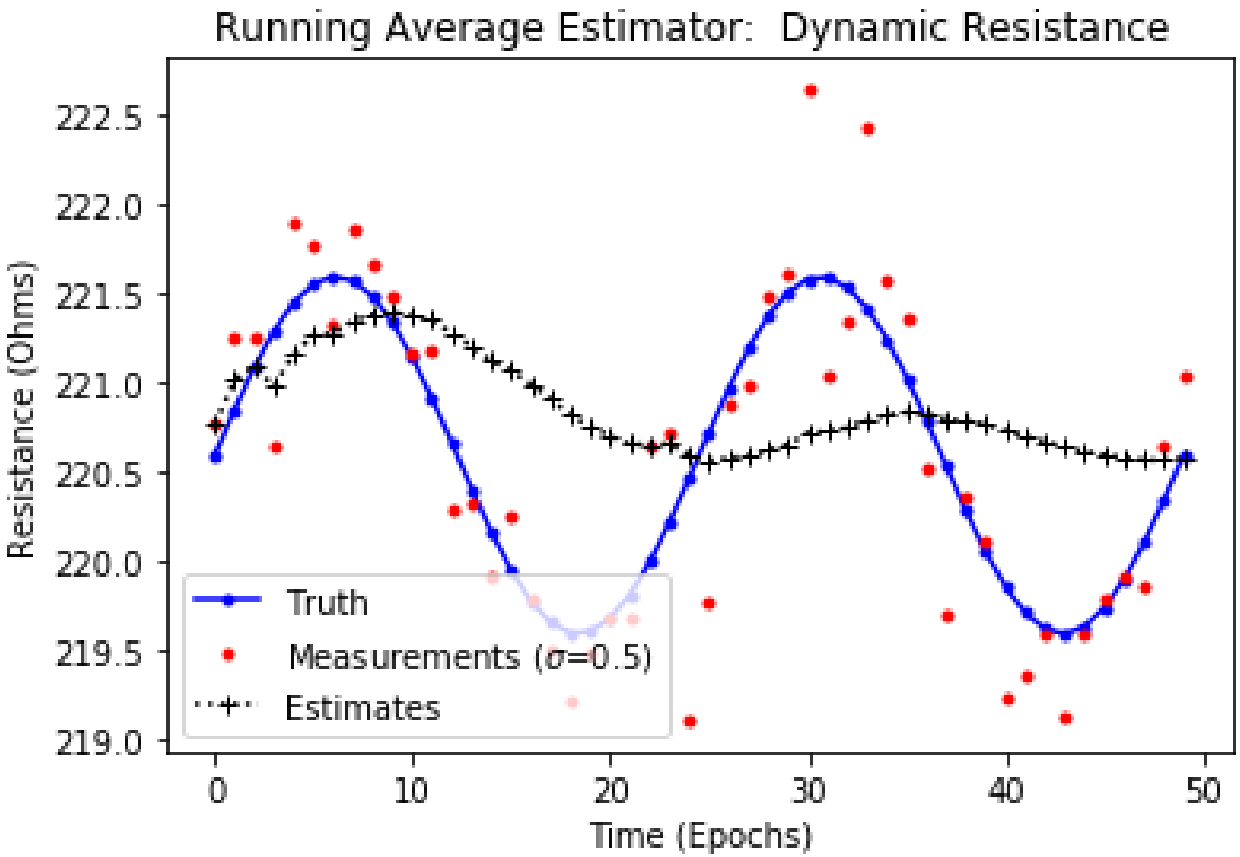
Simple Moving Average Python Scripts: dyn_simple_moving_avg_demo.py of window size 5

Simple Moving Average Python Scripts: dyn_simple_moving_avg_demo.py of window size 5



Simple Moving Average Python Scripts: dyn_simple_moving_avg_demo.py of window size 5

Simple Moving Average Python Scripts: dyn_simple_moving_avg_demo.py of window size 5



Running Average Summary
Vector Check

Running Average Summary
Vector Check

- Where are we?

Running Average Summary
Vector Check

- Where are we?
  - Looked at some Python implementations of the running average and simple moving average

- Where are we?
  - Looked at some Python implementations of the running average and simple moving average
  - Ran some simulations of both constant and dynamic scalar processes and examine how the two types of averages performed

- Where are we?
  - Looked at some Python implementations of the running average and simple moving average
  - Ran some simulations of both constant and dynamic scalar processes and examine how the two types of averages performed
- What's next?

- Where are we?
  - Looked at some Python implementations of the running average and simple moving average
  - Ran some simulations of both constant and dynamic scalar processes and examine how the two types of averages performed
- What's next?
  - In the next video we will look at the prediction part of the algorithm

- Where are we?
  - Looked at some Python implementations of the running average and simple moving average
  - Ran some simulations of both constant and dynamic scalar processes and examine how the two types of averages performed
- What's next?
  - In the next video we will look at the prediction part of the algorithm
  - Especially in the case where the state of the system (process) is dynamically changing

- Where are we?
  - Looked at some Python implementations of the running average and simple moving average
  - Ran some simulations of both constant and dynamic scalar processes and examine how the two types of averages performed
- What's next?
  - In the next video we will look at the prediction part of the algorithm
  - Especially in the case where the state of the system (process) is dynamically changing

---