Kalman Filter Theory and Applications
Equation Drilldown

https://github.com/musicarroll/kalman_course

Michael L. Carroll

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- Understanding the Equations: Heuristic Introduction

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- **Equation Drilldown: Taking the Equations Apart**

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