Kalman Filter Theory and Applications
Heuristic Overview

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- Understanding the Equations: Heuristic Introduction

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- Equation Drilldown: Taking the Equations Apart

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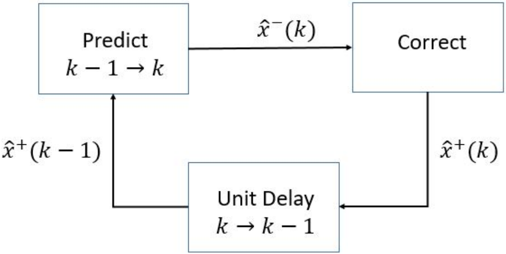
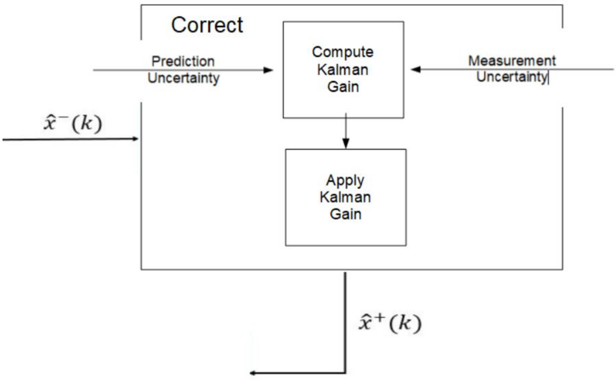


Predict
$k-1 \rightarrow k$
$\hat{x}^-(k)$
Correct
$\hat{x}^+(k-1)$
$\hat{x}^+(k)$
Unit Delay
$k \rightarrow k-1$

Kalman Gain
within Correction (or Update) Block



Gain Summary
Vector Check

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