Kalman Filter Theory and Applications
Heuristic Overview

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Part I

The Basic Kalman Equations

Topics

- Understanding the Equations: Heuristic Introduction

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- Equation Drilldown: Taking the Equations Apart

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  - Note that the measurement model is a direct algebraic equation, not a differential equation nor a difference equation

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