Kalman Filter Theory and Applications
Heuristic Overview

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July 24, 2020

- Understanding the Equations: Heuristic Introduction

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- Equation Drilldown: Taking the Equations Apart

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- State Space Concepts

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- Recursive Predictor-Corrector Algorithms

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- **Running Averages (Part A)**

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- Prediction (Extrapolation)

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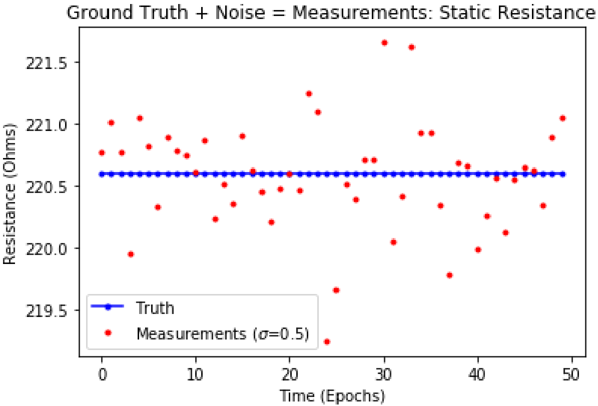

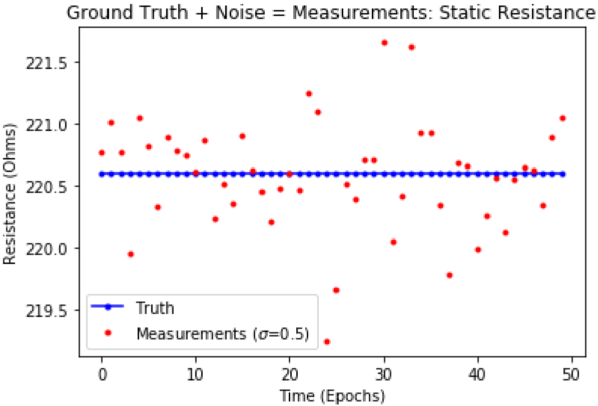
Ground Truth + Noise = Measurements: Static Resistance
Resistance (Ohms)
221.5
221.0
220.5
220.0
219.5
Time (Epochs)
0
10
20
30
40
50
Truth
Measurements (σ=0.5)

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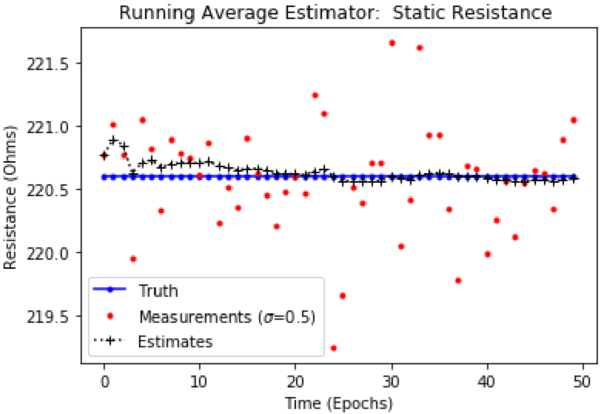

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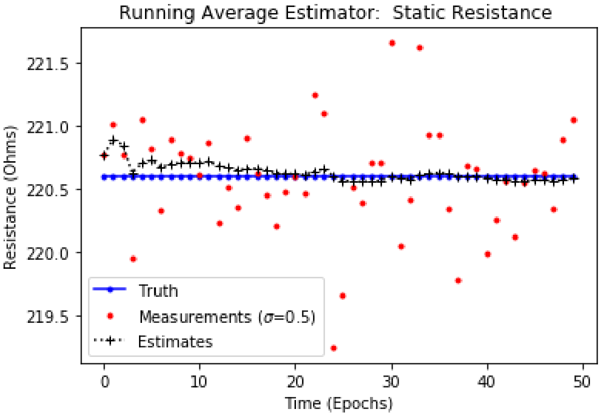

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  - When non-constant state dynamics come into play, the predicted measurement will certainly differ from the previous updated estimate, but we expect this predicted measurement to be fairly close to the actual measurement (if our dynamic model is any good)

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- A: We have assumed that the state in this case is not changing with time. Therefore, in the absence of new information the previous estimate is the best answer thus far!
    - When non-constant state dynamics come into play, the predicted measurement will certainly differ from the previous updated estimate, but we expect this predicted measurement to be fairly close to the actual measurement
(if our dynamic model is any good)

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