JaQo - Partitioned Quantum Cellular Automata based Music Generation

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ABSTRACT

The objective of this research is to combine musical theory, cellular automata and quantum circuits in order to create a unique tool for original music composition. While research in this direction is not unprecedented, this study places a strong emphasis on musical aspects to enhance the quality and appeal of the resulting scores. Also, it explores increasingly complex and intriguing quantum circuit designs, which will hopefully lead to more intersting and fun results.

The composing algorithm, heavily based on an already existing research which will be discussed later in this document, is able to choose the fundamental node, the triads and the instrumentation that will play the score, which will be split in three parts. The composing process is of course totally automatic, without any human intervention.

A key feature of our system is a user-friendly graphical interface that allows for the customization of various properties in the generated songs, such as tempo, time signature, octave and preferred instruments, making it accessible to both musicians and quantum computing enthusiasts. A survey has been conducted in order to gather opinions about generated scores, which has provided valuable insights into the perceived quality of the compositions produced by our tool.

After a brief introduction to Quantum Computing and a quick overview on the existing literature, with a particolar focus on the ones that inspired JaQo the most, this report will cover the development and implementation of the algorithm.

1. Introduction

In recent months, we have witnessed a seemingly increasing trend in Generative Art created by Artificial Intelligence systems. These AI models typically rely on existing art and music pieces to generate new ones, an approach that offers several advantages. Primarily, it tends to ensure pleasing results from a creative standpoint, as the generation is based on human-created works. Additionally, this method facilitates easier evaluation and measurement of AI model performance, since it will be trivial to compare generated scores and pieces of art with original ones.

This increasing trend in AI Generative Art, however, has been facing significant backlash from the art and music communities (Vyas, 2022). Some professionals fear that AI might threaten their livelihoods, while others argue that it's unfair for AI "artists" to achieve decent results simply by writing unambiguous prompts. This controversy has sparked debates about the nature of creativity, originality, and the role of technology in art.

Acknowledging these dynamics, our research aims to take a step back and explore a different path, while still keeping an eye on the future of technology. We decided to develop a system that supports the creation of entirely original music, leveraging quantum computing technologies which are undoubtedly at the forefront of computational advancement (Rietsche, Dremel, Bosch, Steinacker, Meckel, and Leimeister, 2022) (Miranda, 2022). This approach allows us to address some of the concerns raised by the

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artistic community. By creating music from first scratch rather than learning from already existing compositions, our system avoids the ethical issues associated with training on copyrighted works. Additionally, it explores new frontiers in computational creativity, potentially leading to relevant artistic results.

It's important to clarify that algorithmic music is not a weird sci-fi trend, but it actually has some potential. For example, something that should be mentioned before proceeding any further is the concept of stochastic music, which refers to composition of musical pieces by the use of the laws of probability. Relying on randomness in order to create art is not something too crazy: in visual art, for example, one could think to convert a shortest route problem into a work of art (Constant, 2015). Speaking of music, even the notorious Viennese musician Wolfgang Amadeus Mozart himself, among the others, is said to have overcame the writer's block thanks to a dice-based composition system (Wayne, 2023). In more recent times, speaking of stochastic music can not lead to speak about Iannis Xenakis, an engineer and artist which is often referred to as an "architect of music" (Capanna, 2001). Xenakis explored a wide number of probabilistic models which could lead to compose interesting musical composition, such as his songs Analogique A and Analogique B, composed leveraging Markovian chains (Ames, 1989), or his orchestra piece *Horos*, which is totally based on cellular automata (Solomos, 2005).

In the Background and Related Works chapter, we will briefly discuss some principles of quantum computing and cellular automata, with a keen eye to explain how these concepts have been (and will surely be) used in automatic music generation. We will then focus on the developing of JaQo (Just Another Quantum Orchestra), our tool which generates music pieces based on cellular automata and quantum circuits, providing a wide number of musical tweaks to enhance the artistic value of generated music and a user interface to customize some musical aspect of the generated scores. A survey has then been conducted, trying to gather opinions about how such scores have been perceived by the public, eventually reaching a conclusion and discussing some further works that could bring on scientific research in this direction. In the Appendix, the reader in need will find some concepts of music theory and the scores of generated music pieces.

2. Background and Related Works

We will now dive in some key concepts that will surely help to a better understanding on how this all works, leading to meaningful insights for the reader.

2.1. Quantum Computing

Quantum computing represents a rapidly evolving field of technology that harnesses quantum-mechanical phenomena, such as superposition and entanglement, to perform computational operations. This paradigm shift in information processing offers the potential for solving complex problems that are intractable for classical computers (Ray, 2011).

While classical computation relies on the *bit* as its fundamental unit of information, quantum computing introduces a more versatile and powerful unit: the *qubit*. Qubits operate at the subatomic level, thereby becoming subject to the laws of quantum physics (Miranda and Weaver, 2022). This quantum nature allows qubits to exist in multiple states simultaneously, dramatically expanding the computational space available for problem-solving.

A useful visualization of a qubit's state is the *Bloch sphere* (Glendinning, 2005), a three-dimensional representation that encapsulates the quantum state's complexity. The Bloch sphere has two opposite poles, labeled $|0\rangle$ and $|1\rangle$, analogous to the classical bit states. However, unlike classical bits, a qubit's state is represented by a vector - the *state vector* - originating from the sphere's center and pointing to any point on its surface.

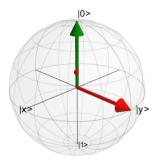


Figure 1: Visual representation of a Bloch sphere.

Mathematically, the Bloch sphere is described using spherical coordinates, where θ (theta) represents the latitude and ϕ (phi) represents the longitude. These angles precisely define the qubit's state on the sphere's surface, allowing for a continuum of possible states between $|0\rangle$ and $|1\rangle$.

The concept of *reading* a bit's value in classical computing evolves into *measuring* in quantum computing. When a qubit is measured, its state vector collapses to one of the poles, yielding either 0 or 1 as the result. This measurement process is fundamental to quantum computing and has profound implications for algorithm design and implementation.

Measurement plays a crucial role in collapsing a superposition state - a state in which a qubit exists in multiple states simultaneously - into a definite one (Hughes, Isaacson, Perry, Sun, and Turner, 2021). Superposition is a key feature that enables quantum computers to process vast amounts of information in parallel, potentially leading to exponential speedups for certain classes of problems.

Another critical quantum phenomenon is entanglement, a state in which two or more qubits become inextricably linked. Entangled qubits cannot be described independently, even when separated by large distances. This property is central to many quantum algorithms and is a key resource in quantum information processing.

The unique properties of qubits - superposition, entanglement, and the continuous state space represented by the Bloch sphere - form the foundation of quantum computing's power. These characteristics enable quantum computers to approach certain problems in fundamentally different ways than classical computers, potentially revolutionizing fields such as cryptography, drug discovery, financial modeling, and artificial intelligence (Shafique, Munir, and Latif, 2024).

Quantum gates are the fundamental building blocks of quantum circuits, analogous to logic gates in classical circuits (Varga, Aragonés-Soria, and Oriol, 2024). However, unlike classical gates, quantum gates are always reversible and operate on quantum states. We now propose a brief definition for some gates which have been used in developing JaQo.

- Hadamard Gate (H): The Hadamard gate is one of the most widely used quantum gates. It creates an equal superposition between the |0⟩ and |1⟩ states. Mathematically, it transforms the state |0⟩ into (|0⟩ + |1⟩)/√2 and the state |1⟩ into (|0⟩ |1⟩)/√2. It is often used to initialize qubits in a uniform superposition.
- CNOT Gate (Controlled-NOT): The CNOT gate is a
 two-qubit gate that flips the state of the target qubit
 if and only if the control qubit is in the |1⟩ state. It
 is crucial for creating entanglement between qubits
 and implementing conditional operations in quantum
 algorithms.
- Z Gate: The Z gate applies a phase shift of π radians to the |1⟩ state, leaving the |0⟩ state unchanged. In terms

of rotation on the Bloch sphere, the Z gate rotates the state around the z-axis by π radians.

- SWAP Gate: As the name suggests, the SWAP gate exchanges the states of two qubits. It is useful in situations where it's necessary to rearrange the order of qubits in a quantum register.
- T Gate: The T gate is a phase rotation gate that rotates the quantum state by $\pi/4$ radians around the z-axis of the Bloch sphere. It is particularly important in fault-tolerant quantum computation and implementing universal quantum algorithms.
- **Ry Gate**: The Ry(θ) gate performs a rotation of θ radians around the y-axis of the Bloch sphere. It is parameterized by the angle θ , allowing arbitrary rotations in the xz-plane of the Bloch sphere.

2.1.1. Quantum Computing for Music

Quantum Computing has been experimented for a various number of task, but we are now interested to see how it impacted the research on **algorithmic computer music**; conducting research on music can not only provide useful insights about quantum computing itself, but also encourages researches to have fun and put creativity in their projects.

One interesting example is the work conducted in 2021 by Dr. Allen and Dr. Bulmer from the Quantum Engineering Technology Labs at the University of Bristol (UK) and the music artist Simon Small. In their paper "Making Music Using Two Quantum Algorithms" (Allen, Bulmer, and Small, 2022) explored how *quantum walks* and *Grover's Algorithm*, which the reader can learn about in the aforementioned research. The output has then been played, leading to interesting results from a creative point of view, and can be listened on the *Bohm Electronic Collective's Bandcamp*.

Another interesting work has been conducted to try and fuse a Quantum Natural Language Processing pipeline to the world of algorithmic music, leading to the creation of the *Quanthoven* project (Miranda, Yeung, Pearson, Meichanetzidis, and Coecke, 2021). Typical QNLP task is structured on a known grammar, which helps the machine to "understand" a given sentence and potentially predict how said sentence could be expanded avoiding to violate any grammar rule. Miranda et al. have then created a set of grammatical rules for music composition, starting from a set of music snippets extracted from already existing scores. The results, which the reader can listen on Quanthoven's GitHub repository, have been played and recorded by the Lithuanian pianist Lauryna Sableveciute.

Miranda's name is certainly not new in the algorithmic music field of research, being a pioneer in this field, which works have inspired a wide number of other researches, even the one which we are proposing now! One of the works he's most recognized for is *Q1Synth* (Miranda, Thomas, and Itaboraí, 2023). Q1Synth is presented as a Quantum Computer Musical Instrument, which produces sounds from the measuring of the quantum state vectors of a qubit,

using the principles of frequency modulation and substractive/granular synthesis. The instrument can be controlled with a mouse (please try the demo here) and - even more significantly - with a VR glove, and is planned to be expanded in a whole quantum orchestra, named *Spinnings*. Miranda proposed some live performances presenting Q1Synth and Spinnings, gaining obtaining a certain visibility and interest towards the subject by not only the scientific community, but even the musicians' one. In the 2022 performance at the Goethe-Institute of London, the famous musician and activist *Brian Eno* - producer of *David Bowie*, *Slowdive* and *Talking Heads*, among the others - was in the audience (Krishnan, 2024), and said to be "[...] fascinated to know how this works [...].

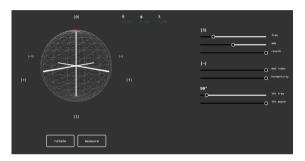


Figure 2: Q1Synth by Prof. Edoardo Miranda et al.

2.2. Cellular Automata

Before, we already mentioned how cellular automata have been using by Xenakis, among the others, to music production. We propose a more detailed definition of the concept of cellular automata.

Cellular Automata (CA) are mathematical models used to simulate complex systems through simple, local interactions. These models consist of a grid of cells, each in one of a finite number of states, that evolve over discrete time steps according to a set of rules based on the states of neighboring cells. They have proved to be versatile and useful in various fields like biology, chemistry and so on.

They rely on the following key concepts:

- Grid: A regular lattice of cells, which can be in any number of dimensions, though one- and twodimensional grids are most common.
- **States:** Each cell can be in one of a finite number of states, typically represented by integers or colors.
- Neighborhood: A defined set of cells that are considered adjacent to each cell.
- Rules: A set of transition rules that determine how the state of a cell changes based on its current state and the states of its neighbors.
- **Time:** The system evolves in discrete time steps, with all cells updating simultaneously.

One of the most famous examples of a Cellular Automaton is Conway's Game of Life, developed by British mathematician John Conway in 1970 (Gardner, 1970). It is a two-dimensional CA with binary cell states (alive or dead) and a set of rules that mimic the birth, survival, and death of a population. The Game of Life is played on an infinite grid of square cells, each in one of two possible states: alive (usually represented as black) or dead (usually white). The game evolves in discrete time steps, where the state of each cell in the next generation is determined by its current state and the states of its eight neighbors (horizontal, vertical, and diagonal). The rules of the Game of Life are as follows:

- Any live cell with fewer than two live neighbors dies, as if by underpopulation.
- 2. Any live cell with **two or three live neighbors lives** on to the next generation.
- 3. Any live cell with more than three live neighbors dies, as if by overpopulation.
- 4. Any dead cell with exactly three live neighbors becomes a live cell, as if by reproduction.

For now, we introduce the concept of **Partitioned Quantum Cellular Automata**, which consists in an approach to implement CA on quantum computers (Miranda and Shaji, 2023). PQCA are preferred to standard QCA due to computational issues which can slow down processing: in fact, PQCA limits calculations to a selected partition of cells, which become tessellating supercells, which will be used together with update frames in order to build a global update circuit. It's trivial to think that if we map notes and durations to each state of the PQCA, we can produce music.

An example of using CA and PQCA for music production will be discussed later, since these concepts represent the backbone of the research we are proposing.

3. Methodology

In this section, we present the JaQo pipeline, describing how it works, how it has been developed and the reasons behind a number of choices that have been made.

Please keep in mind that the bearing structure of the project has been developed starting from the PQCA Tutorial repository associated to the 'Evolving Cellular Automata Music: From Sound Synthesis to Composition' (Miranda, 2003) paper we already discussed before, published by Prof. E. Miranda from the Interdisciplinary Centre for Computer Music Research (ICCMR) at University of Plymouth (UK). For a better understanding of the whole picture, the reader is invited to carefully read this meaningful work. To be thorough, the said base algorithm will briefly be described first, leading then to our tweaks, to the interface development and, eventually, to the survey which has been conducted.

Another important thing to clarify is that the project involves a **simulation** of a quantum circuit, for budgeting and time reasons. Results will surely be different if run on an actual quantum computer.

3.1. Introduction to CAMUS Algorithm

The original algorithm is based on a system called **CA-MUS** (Cellular Automata MUSic), which utilizes the *Game of Life* ruleset as discussed in the *Background and Related Works* chapter. At its core, CAMUS operates on a 9×4 matrix, where each cell's state (alive or dead) determines various musical parameters:

- The **first** column dictates the fundamental pitch of the triad sequence.
- Columns 2 5 describe a sequence of triads positioned above the fundamental by a specified number of semitones.
- Columns 6 9 define the instrumentation that will play these notes.

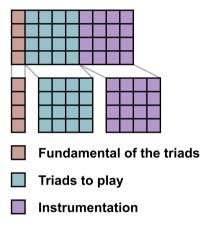


Figure 3: Visual Representation of the CAMUS matrix

The fundamental pitch column provides 16 possible combinations (2^4), corresponding to the binary states of its four cells. These combinations are translated into semitone intervals, with the default sequence [0,0,0,0] representing the base note, C4 (middle C) by default.

In the CAMUS protocol, each alive cell in the triaddefining columns (2-5) is interpreted as a component of a musical triad. The *x* coordinate of the cell determines the interval between the fundamental and the first note of the triad, while the *y* coordinate specifies the interval between the first and second notes. This approach allows for a wide variety of chord structures to emerge from the cellular automaton's state

The instrumentation selection process (columns 6-9) is based on the neighborhood of alive cells. By examining the states of adjacent cells, the algorithm determines which available instruments will play the generated notes.

Additionally, the CAMUS system employs a similar neighborhood-based approach to determine the start and end points of musical bars. This feature adds rhythmic complexity by varying the timing and duration of musical phrases, preventing a monotonous and predictable beat structure.

While the original CAMUS algorithm has produced intriguing results, extended listening reveals certain limitations. The compositions tend to become somewhat predictable and potentially monotonous over longer durations. This observation served as the primary motivation for our research team to explore expansions and modifications to the existing framework.

3.2. Proposed Enhancements and Tweaks

As we said before, our goal is to build upon the innovative foundation laid by CAMUS, addressing its limitations while preserving its core strengths. By introducing new rules, new musical tweaks and expanding probabilistic elements leveraging the quantum principles, we aim to enhance the algorithm's capacity for generating diverse, engaging, and sustained musical compositions. We now describe in detail our proposed enhancements and their impact on the musical output.

First, we sought to maximize the potential of the simulated quantum circuit involved in generating the initial bit grid for the CAMUS algorithm. To encourage variety in composed scores, we expanded the random component by incorporating a wide array of quantum gates, including the Ry, Z, CNOT, and Hadamard gates discussed in the *Background and Related Works* chapter.

A significant modification that yielded interesting results was the decision to double the generated bit grids, appending them together to produce longer and more varied compositions.

We introduced several functions to refine the musical output:

- snap_to_scale: This function helps avoid undesirable off-scale notes.
- harmonic_attraction: Works in conjunction with the snap to scale function to tie generated pitches to a certain reference pitch.
- ensure_minimal_duration: Ensures notes have a minimal duration, enhancing musical coherence.

A time quantization function (Cemgil, Kappen, and Desain, 2000) was initially considered but later discarded as scores involving musical time quantization received poorer subjective evaluations during analysis.

Adjustments were made to guarantee variety in chosen intervals, allowing for the inclusion of major intervals in the score. The balancing of intervals and pitches has apparently ensured greater equilibrium in the score, leading to less monotonous results.

Lastly, we developed an interface to allow end-users to experiment with the system. Currently, users can modify the following parameters:

- **Tempo**: Set the beats per minute (BPM) of the score.
- Time Signature: Choose both the numerator and denominator.

- Octave: Select the pitch range of the composition.
- **Instrumentation**: Define the pool of possible instruments (max. 3) from which the algorithm can choose.

Initially, we considered allowing users to choose the scale and mode of the composition. However, this option was ultimately discarded as it deviated too far from the original CAMUS algorithm's principles.



Figure 4: JaQo's simple interface

Whem the user selects "Generate", the system will create the .midi file and display a graphic representation of the generated quantum circuit.

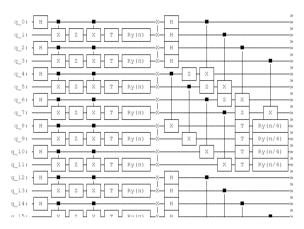


Figure 5: Example of a generated quantum circuit

The score can be opened in *ad hoc* software like MuseScore, where it can be listened and worked on.



Figure 6: Score generated from the quantum circuit in *Figure* 5

4. Experimental Analysis

Objectively evaluating a music generator presents significant challenges. While it is possible to assess whether a score adheres to Western music standards, compliance with these standards does not guarantee aesthetic appeal or musical quality. Art, in all its forms, is *inherently subjective*. Individual preferences can vary widely, even for the most unconventional pieces of art or music. As noted by Loughran and O'Neill (2017), the subjective nature of music makes evaluation difficult, even for human-composed pieces.

4.1. Survey

To capture a broader range of perspectives and to quantify subjective responses, we implemented a survey-based approach to gather opinions on the generated music. The survey has been advertised via *word-of-mouth* technique and sharing by social media, i.e. *Instagram stories*. Communicating via social-media led to a low average range (18-30), and could be something relevant while analyzing the results.

The sample is composed of 89 elements, and we have asked them:

 their estimated level of knowledge of Western music theory;

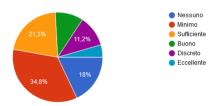


Figure 7: Pie-chart representing the level of Western theory knowledge

• their favourite **music genres** (max. 2);

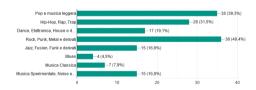


Figure 8: Histogram representing the favourite genres distribution

• the musical instrument they play, if any.

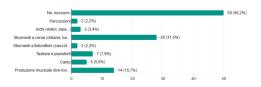


Figure 9: Histogram representing the instruments distribution

At the start of the survey, it has been made clear that the proposed songs are not AI generated and have not been post-produced at all. After listening each track, the user had to rate them in a 1-5 scale. The final set of questions has been inspired by those proposed in projects like *EvoComposer* (De Prisco, Zaccagnino, and Zaccagnino, 2019) and *Pulsate* (?), and follows the Consensual Assessment Technique, first proposed by Amabile (1982).

In the end, the users have been asked if they agreed, partially agreed, partially disagreed or totally disagreed with the following sentences:

- Proposed tracks may provide interesting creative and original ideas to a composer.
- Proposed tracks are musically accurate, coherent and structurally organized.

4.2. Results Analysis

First of all, let's give a sneak peek to the rating given to the tracks by the interviewed sample.

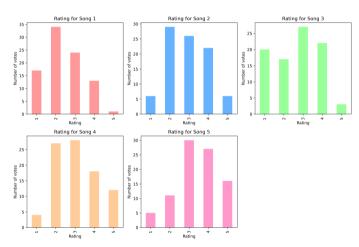
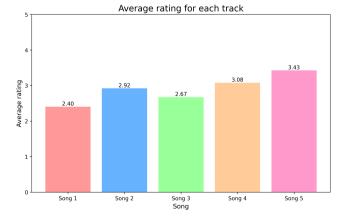


Figure 10: Rating for each proposed track



Apparently, the fifth song seems the most appreciated, while the first one has not received much love: this was expected since the first track presents some off-beat notes

which aren't typical of Western music, while the fifth is more classical and listening-friendly. However, the average rating of the tracks is not that high; this was expected since the MuseScore sound output is a bit weak and not that exciting to the listener. Related to this, a lot of comments said that the proposed songs reminded them of videogame music, in particular the first one has been said to be similar to horror games such as *Silent Hill* music, by the Japanese composer Akira Yamaoka, and the second one has been said to seem like *The Legend of Zelda* music, by Koji Kondo. The fifth one, however, has been said to sound more natural, clear and familiar to the users.

Let's see how people rated to the tracks basing on their musical knowledge level.

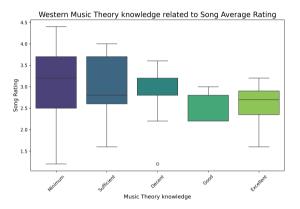


Figure 11: Box-plot correlating Music Theory knowledge and average rating of the songs

With that said, it has been made clear in the survey is that the focus of the study is not on the sound appeal of the tracks, but on their creative potential and correctness. Most the sample partially agrees on the fact that the given output tracks can have some interesting musical creative ideas in them.

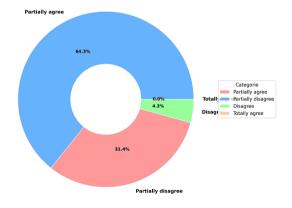


Figure 12: Pie-chart about Creativity Question

Speaking of correctness, however, most people don't think the algorithm produced decent music outputs from a technical standpoint. The reason could this may be found in the off-beat notes we discussed before, or even in the absence of post-production in the tracks.

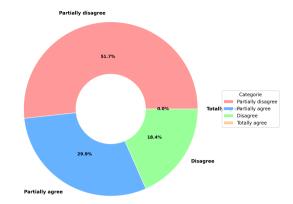


Figure 13: Pie-chart about Correctness and Coherence Question

5. Conclusions and Future Work

In this work, we have reworked and implemented an already existing Music Generator based on Partitioned Quantum Cellular Automata, maximizing the stochastic components and lending concepts from the world of Quantum Computing. We have then performed a qualitative evaluation to measure the creativity and the correctness of the composed tracks, finding out that the majority of the sample agrees that, despite the algorithm could surely provide interesting ideas to a musician, at the time being the outputs are not pleasant to listen to at all.

Future work might try to run the quantum circuit generation on a quantum processor, which has not been tried for budgeting and time reasons. Some could even think to reengineer the original algorithm, to create and deliver more appealing compositions with even more interesting musical structures and patterns, trying to focus more on the rhythmic aspect.

Wrapping up, we think that this report can provide a good overview on the state of quantum computing algorithms performance in the task of music generation.

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CRediT authorship contribution statement

Coccorullo David: Conceptualization of this study, Methodology, Software.

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