Optimization Project 1 - Linear Programming

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Aim

Research says that marketing budgets now constitute 11 percent of that total company budget. The main question here is how do we utilize this budget in the most efficient manner i.e profit-maximizing manner?

The problem that we're grappling with is how the effectiveness of marketing varies across companies. What might work for one company might not work for the other. To address this problem, we are making use of linear programming to come up with a marketing budget allocation strategy.

Methodology

To come up with an optimum marketing budget allocation strategy, we require information on what the return on investment of each marketing channel would be. This data has been outsourced from two consulting firms, based on which the analysis is conducted.

Coming to the Analysis, our objective is to maximize revenue. This strategy would inform the marketing spend of the following year. The analysis has been done at an annual and a monthly level. The optimization technique used is Linear programming. This analysis has been done using the Gurobi package of Python.

Constraints

1. The total Budget allocated for marketing is $10M
2. The amount invested in print and TV should be no more than the amount spent on Facebook and Email
3. The Total amount used in social media should be twice that of SEO and AdWords
4. For each platform, the amount invested should be no more than $3M

Analysis

Case 1: Marketing Budget Allocation based on ROI data from the first consulting firm

Code

```
# 1. Objective
obj_1 = array(roi_data.iloc[0].values)
n_var = len(roi_data.iloc[0].values)

# 2. Matrix A - Coefficients
A = zeros((3, n_var))
    # The amount invested in print and TV should be no more than the amount spent on Facebook and Email.
A[0,indices_cons_1] = cons_1
    # The total amount used in social media should be at least twice of SEO and AdWords
A[1,indices_cons_2] = cons_2
    # The total investment should not exceed the budget
A[2,:] = [1.]*n_var

# 3. Sense
sense = array([cons_1_sense, cons_2_sense, '<'])

# 4. RHS
rhs = array([0, 0, budget])

# 5. Upper bound - For each platform, the amount invested should be no more than $3M
ubound = array([upper_bound_value]*n_var)


# 6. Solve
objModX_1, objModCon_1, objModel_1 = solve_inequations(A, n_var, sense, rhs, obj_1, ubound=ubound)

print("The Maximum value of the objective function - i.e. First ROI: ", end="")
display(round(objModel_1.objVal, 5))
print("\nThe investments to obtain the objective function: ", end="")
Alloc1= DataFrame(objModX_1.x, index=roi_data.columns, columns=["Allocation"])
display(Alloc1)
```

```
The Maximum value of the objective function - i.e. First ROI:

456000.0
```

Result

The maximum return obtained based on the ROI data from the first consulting firm and the above-mentioned constraints laid out is **$456,000**.

The budget allocation to obtain this return is illustrated below:

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Type										
Variable	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
ROI 1	0.03100	0.04900	0.02400	0.03900	0.01600	0.02400	0.04600	0.02600	0.03300	0.04400
Allocation 1	0.00000	3000000.00000	0.00000	1000000.00000	0.00000	0.00000	3000000.00000	0.00000	0.00000	3000000.00000

These results look promising but it's always wise to not rely on one source of data for critical information especially when there is so much variation across each channel's allocation. We address this issue by re-running the analysis with data from another consulting firm in the next step.

Case 2: Marketing Budget Allocation based on ROI data from the second consulting firm

Code

The code remains the same, only the ROI data has been replaced with that of the second consulting firm.

Result

The maximum return obtained based on the ROI data from the second consulting firm and the above-mentioned constraints laid out is $456,000.

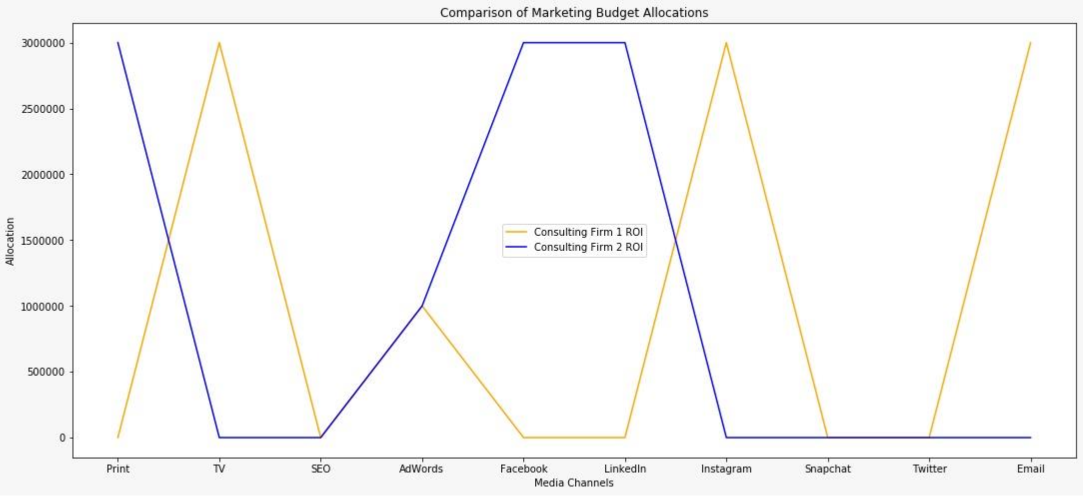
The budget allocation to obtain this return is illustrated below:

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
Type										
Variable	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9
ROI 2	0.04900	0.02300	0.02400	0.03900	0.04400	0.04600	0.02600	0.01900	0.03700	0.02600
Allocation 2	3000000.00000	0.00000	0.00000	1000000.00000	3000000.00000	3000000.00000	0.00000	0.00000	0.00000	0.00000

Comparison of Case 1 & Case 2:

1. **Are the allocations the same?**

   No, as seen from the results from the above two cases, we can conclude that the budget allocations aren't the same. The graph below illustrates the same and shows how greatly allocations have varied across the channels.



2. **Assuming the first ROI data is correct, if you were to use the second allocation (the allocation that assumed the second ROI data was correct) how much lower would the objective be relative to the optimal objective (the one that uses the first ROI data and the first allocation)?**

   The objective would be lower by **$204000.0** relative to the optimal objective.

3. **Assuming the second ROI data is correct, if you used the first allocation how much lower would the objective be relative to the optimal objective?**

   The objective would be lower by **$192000.0** relative to the optimal objective.

   Please refer to appendix section 1 for the code used to obtain the above-mentioned results.

4. **Do you think the third constraint above, based on your boss' experience, is useful?**

   Based on our analysis, we found that the third constraint isn't useful. We obtained an additional **$9000** by removing the 3rd constraint and re-running the analysis. Please refer to appendix section 2 for budget allocations post this analysis.

   But on the other hand, one could argue that is $9000 worth the risk of betting all our money on specific channels. If the stock market has taught us anything, it is anything that diversification of investment could lead to lesser risk. Now it all comes down to where we want to be on the risk-return trade-off spectrum.

Case 3: Sensitivity Analysis based on ROI data from the first consulting firm

It is a method of discovering how the optimal solution is altered by changes, within certain ranges of the objective function coefficients and the right-hand side values. From the comparison of market budget allocations based on the data from two consulting firms we noticed that the budgets changed drastically across channels based on the ROI data, hence the next logical step is to run a sensitivity analysis and see by how much would each advertising medium's ROI increase or decrease and still result in the same optimal allocation i.e., our overall return.

Result

	Lower Bound	Upper Bound
Print	-inf	0.049
TV	0.039	0.062
SEO	-inf	0.039
AdWords	0.033	0.046
Facebook	-inf	0.029
LinkedIn	-inf	0.039
Instagram	0.039	inf
Snapchat	-inf	0.039
Twitter	-inf	0.039
Email	0.029	inf

This gives us an insight into how sensitive each channel is to the ROI data. Elaborating further on this, "AdWords" ROI can be anywhere between 0.033 and 0.062 and we would still obtain the same optimal location given everything else doesn't change.

Case 4: Re-investing half of current months return in the next month

It would be interesting to look at how the allocations would change at a monthly level with additional investment. We did so by looping over the months and re-allocating the budget for the subsequent month by taking the current month's return. Please refer to appendix section 3 for the relevant code chunk and exact methodology.

Result

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	Budget	Return
January	3000000.0	0.0	0.0	1333333.3	0.0	0.0	2666666.7	0.0	0.0	3000000.0	10000000.0	373000.0
February	3000000.0	0.0	0.0	2395500.0	3000000.0	0.0	0.0	0.0	1791000.0	0.0	10186500.0	406296.0
March	0.0	0.0	0.0	3000000.0	0.0	3000000.0	1203148.0	0.0	3000000.0	0.0	10203148.0	407516.5
April	0.0	0.0	0.0	3000000.0	0.0	3000000.0	3000000.0	0.0	1203758.2	0.0	10203758.2	400335.3
May	1200167.6	0.0	0.0	0.0	0.0	0.0	3000000.0	0.0	3000000.0	3000000.0	10200167.6	411005.9
June	3000000.0	0.0	0.0	0.0	0.0	0.0	3000000.0	0.0	1205502.9	3000000.0	10205502.9	423809.1
July	0.0	0.0	0.0	3000000.0	1211904.6	0.0	3000000.0	0.0	3000000.0	0.0	10211904.6	428264.3
August	2714132.1	0.0	0.0	1500000.0	0.0	0.0	0.0	0.0	3000000.0	3000000.0	10214132.1	437993.5
September	609498.4	0.0	0.0	3000000.0	0.0	3000000.0	0.0	0.0	3000000.0	609498.4	10218996.8	402712.4
October	0.0	0.0	0.0	3000000.0	0.0	3000000.0	3000000.0	0.0	0.0	1201356.2	10201356.2	371443.4
November	3000000.0	0.0	0.0	1185721.7	0.0	0.0	3000000.0	0.0	0.0	3000000.0	10185721.7	441614.6
December	3000000.0	2110403.6	0.0	0.0	3000000.0	0.0	0.0	0.0	0.0	2110403.6	10220807.3	432501.1

We notice that the monthly budget allocation for the same channel has changed quite a bit over the months. This problem would be dealt with in the next case. For example, let's look at the **Print** medium, the range has gone from **0.0 to $3M.** Our overall annual return here is **$2684496.55** which is undoubtedly an upgrade from the previous scenarios and is intuitive as more investment would lead to higher returns.

Case 5: Stabilising Monthly Budget Allocations

We can see from the previous output that the allocation is not stable as the changes in many cases, we thought it would be wise to try and stabilize our allocations for each channel over the months. We did so by introducing a new constraint in the subsequent cases after January that limits the change to under $1M for the channel.

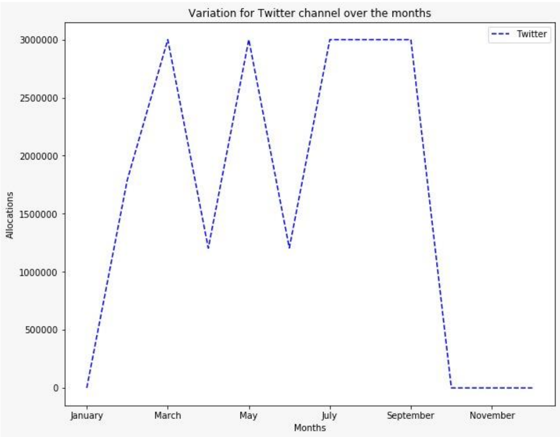
Result

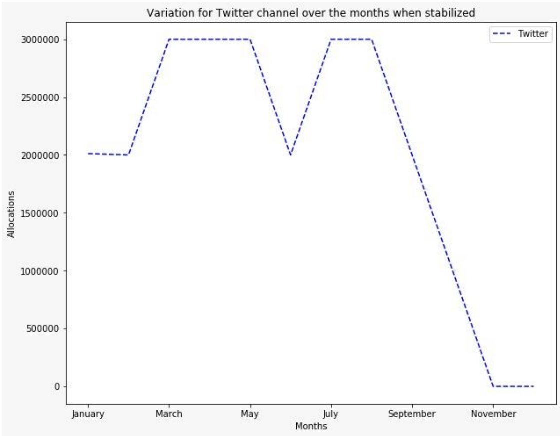
	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	Budget	Return
January	2592048.1	0.0	0.0	1804316.7	597046.2	0.0	1000000.0	0.0	2011587.2	1995001.8	10000000.0	368192.3
February	1597046.2	995001.8	0.0	2000000.0	1597046.2	1000000.0	0.0	0.0	2000000.0	995001.8	10184096.1	388185.0
March	597046.2	0.0	0.0	3000000.0	597046.2	2000000.0	1000000.0	0.0	3000000.0	0.0	10194092.5	394225.7
April	598556.4	0.0	0.0	3000000.0	0.0	1000000.0	2000000.0	0.0	3000000.0	598556.4	10197112.9	392103.3
May	597495.2	0.0	0.0	2000000.0	0.0	0.0	3000000.0	0.0	3000000.0	1598556.4	10196051.6	403256.0
June	1597495.2	0.0	0.0	1005576.4	0.0	0.0	3000000.0	0.0	2000000.0	2598556.4	10201628.0	415630.2
July	1601119.4	0.0	0.0	2005576.4	2562.9	0.0	2000000.0	0.0	3000000.0	1598556.4	10207815.1	417534.1
August	1770620.9	0.0	0.0	1839589.7	0.0	0.0	1000000.0	0.0	3000000.0	2598556.4	10208767.1	431366.8
September	2770620.9	0.0	0.0	1674441.6	348883.1	1000000.0	0.0	0.0	2000000.0	2421737.8	10215683.4	393378.7
October	2000000.0	0.0	0.0	2258317.2	1348883.1	1167751.3	1000000.0	0.0	1000000.0	1421737.8	10196689.3	346729.4
November	3000000.0	0.0	0.0	1583875.6	2000000.0	167751.3	1000000.0	0.0	0.0	2421737.8	10173364.7	414269.2
December	3000000.0	1000000.0	0.0	583875.6	3000000.0	0.0	0.0	0.0	0.0	2623259.0	10207134.6	428664.0

As expected, the budget allocation for a channel has stabilized over the months after putting an additional constraint of not changing the allocation by more than $1M from month to month. The annual return with the additional constraint is **$2611099.31**.

As we can see, the trade-off here is that it is leading to slightly lesser annual returns. The question posed earlier comes into play in this situation as well. Where do we want to be on the risk and return trade-off?

Comparison of Twitter Channel Marketing Budget allocation pre- and post-stabilizing the budget





Please note that this report is meant to analyze and highlight the results of each analysis. Additionally, give insights into the pros and cons of moving forward with a particular decision to aid the decision-makers in taking the optimal decision.

Appendix

1: Code to support the (3) & (4) point when comparing Case (1) and Case (2)

```
output_df_3 = output_df_1.append(output_df_2).set_index('Type')

optimum=sum(output_df_3.loc['ROI 1']*output_df_3.loc['Allocation 1'])
ROI_1_Allocation_2=sum(output_df_3.loc['ROI 1']*output_df_3.loc['Allocation 2'])
ROI_2_Allocation_1=sum(output_df_3.loc['ROI 2']*output_df_3.loc['Allocation 1'])

```

2: Budget allocation after removing the third constraint:

	Allocation
Print	5000000.0
TV	0.0
SEO	0.0
AdWords	0.0
Facebook	5000000.0
LinkedIn	0.0
Instagram	0.0
Snapchat	0.0
Twitter	0.0
Email	0.0

3. Code Chunk for Case 4

```
prev_month_return = 0
col_names = list(roi_mat.columns.values) + ["Budget", "Return"]
monthly_allocations = DataFrame(
    columns=col_names
)

for month in roi_mat.index.values:
    # 1. Objective
    obj = array(roi_mat.loc[month].values/100)
    n_var = len(roi_mat.loc[month].values)

    # 2. Matrix A - Coefficients
    A = zeros((3, n_var))
        # The amount invested in print and TV should be no more than the amount spent on Facebook and Email.
    A[0,indices_mat_cons_1] = cons_1
        # The total amount used in social media should be at least twice of SEO and AdWords
    A[1,indices_mat_cons_2] = cons_2
        # The total investment should not exceed the budget
    A[2,:] = [1.]*n_var

    # 3. Sense
    sense = array([cons_1_sense, cons_2_sense, '<'])

    # 4. RHS
    rhs = array([0, 0, budget + 0.5*prev_month_return])

    # 5. Upper bound - For each platform, the amount invested should be no more than $3M
    ubound = array([upper_bound_value]*n_var)


    # 6. Solve
    objModX, objModCon, objModel = solve_inequations(A, n_var, sense, rhs, obj, ubound=ubound)

    monthly_allocations.loc[month] = Series(list(objModX.x) + [budget + 0.5*prev_month_return, obj @ objModX.x], index=col_names)
    prev_month_return = obj @ objModX.x
```