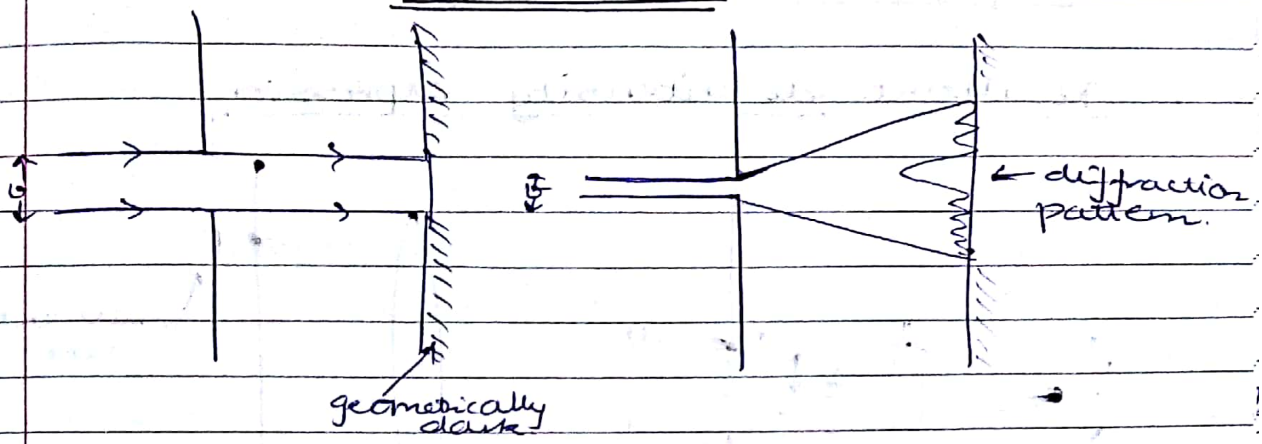
DIFFRACTION.

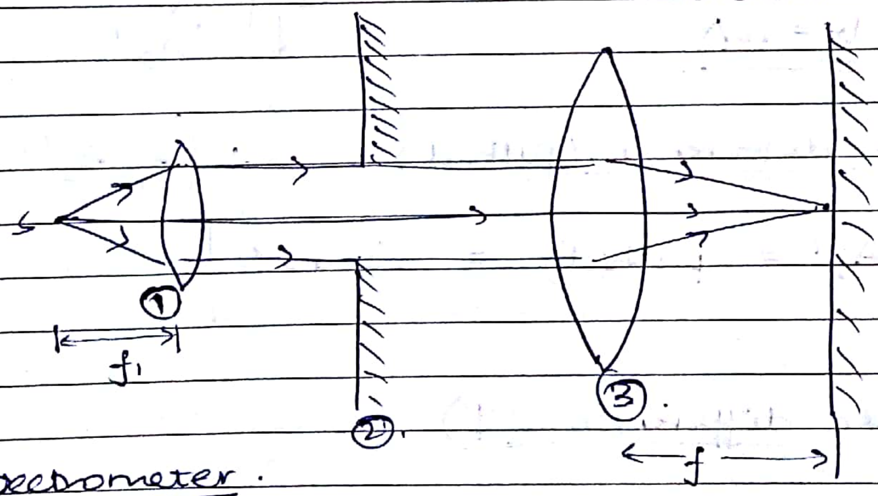


- when the slit of size is comparable to wavelength of light.

$$\theta \approx \frac{\lambda}{r}$$

- two types of diffraction.

① fresnel diffraction.	② fraunhoffer diffraction
• source and screen are at finite distance	• infinite distance
• diverging wavefront	• plane wavefront incident



spectrometer.

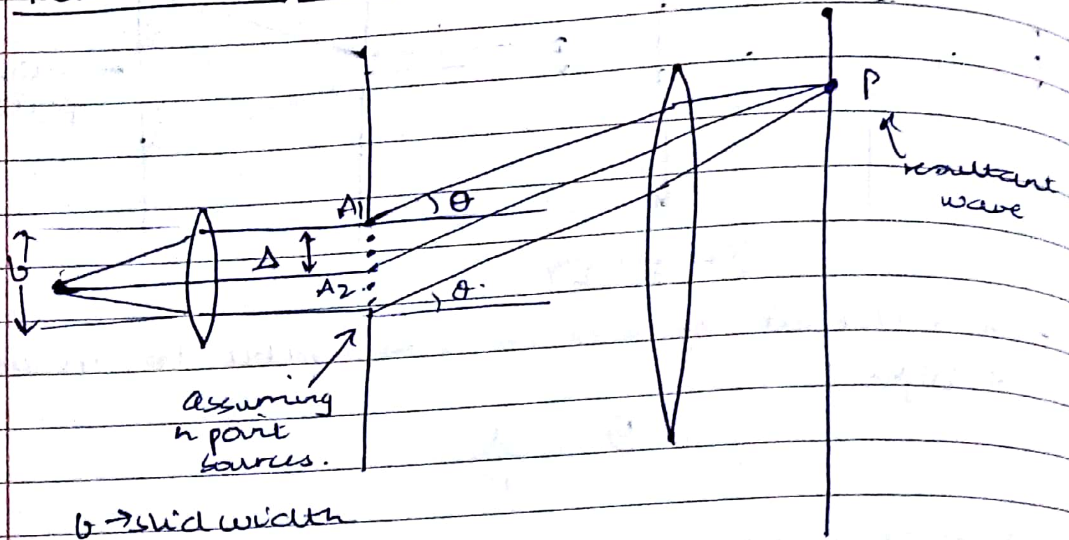
① collimater ② turn table / prismtable ③ telescope.

FRAUNHOFFER diffraction.

① single slit ② doubleslit ③ N parallel slits diffraction. (grating).

Single slit diffraction.

Resultant ~~at~~ intensity expression.

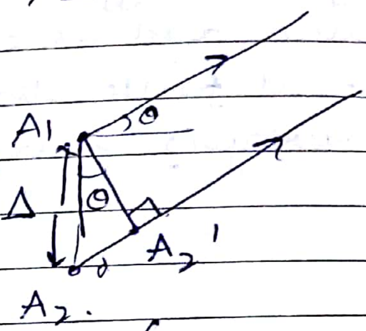


$b \rightarrow$ slit width

$(b = (n-1)\Delta)$ if $n \rightarrow \infty$, $\Delta \rightarrow 0$.

$\Rightarrow$ [$b = n\Delta$ finite]

$\Rightarrow$ $b = n\Delta$.



$\rightarrow$ Path difference (path d) $\therefore$ $(A_2A_2' = \Delta \sin\theta)$

$$\left[A_2A_2' = \text{path}(d) = \Delta\sin\theta\right]$$

$\rightarrow$ phase difference (ϕ)

$$\left[\phi = \cancel{2\pi}\ \frac{2\pi}{\lambda}(A_2A_2') = \frac{2\pi}{\lambda}\Delta\sin\theta\right]$$

$\Rightarrow$ Resultant.

* Since light is an electromagnetic wave resultant can be written in terms of electric wave vector or magnetic wave vector
- easier to represent as $\vec{E}$

$$\vec{E} = E_1 + E_2 + E_3 \cdots E_n$$

$$= a\cos\omega t + a\cos(\omega t + \phi) + a\cos(\omega t + 2\phi) \cdots\cdots a\cos(\omega t + (n-1)\phi)$$

$$= a \frac{\sin n\phi/2}{\sin \phi/2} \left[\cos\left(\omega t - \frac{(n-1)\phi}{2}\right)\right]$$

Resultant

$$\left[\vec{E} = \left[\frac{a\sin n\phi/2}{\sin\phi/2}\right]\cos\left(\omega t - \frac{(n-1)\phi}{2}\right).\right]$$

Now $\phi = \frac{2\pi}{\lambda}(\text{path d})$.

$$\Rightarrow \frac{n\phi}{2} = \frac{n\cancel{2}\pi}{\cancel{2}\lambda}(\text{path d}) = \frac{n\pi}{\lambda}(\text{path d})$$

$$= \frac{n\pi}{\lambda}\Delta\sin\theta \Rightarrow \frac{(n\Delta)\pi\sin\theta}{\lambda}$$

$$= \frac{b\pi\sin\theta}{\lambda} = (\beta)$$

$$\hookrightarrow \vec{E} = \frac{an\sin\left(\frac{\pi b\sin\theta}{\lambda}\right)}{\left(\frac{\pi b\sin\theta}{\lambda}\right)}\cos\left(\omega t - \frac{n\phi}{2}\right)$$

$$\left[\vec{E} = \frac{na\sin\beta}{\beta}\cos(\omega t - \beta)\right]$$

θ/π

$$\left(\beta = \frac{n\phi}{2} = \frac{b\pi\sin\theta}{\lambda}\right).$$

$$\therefore \left[\vec{E} = \frac{na\sin\beta}{\beta}\cos(\omega t - \beta)\right]$$

$$\star \left(\phi/2 = \frac{2\pi\Delta\sin\theta}{2\lambda} = \frac{\cancel{2}\pi}{\cancel{2}\lambda}\frac{(\Delta n)\sin\theta}{n} = \frac{\pi b\sin\theta}{\lambda n}\right)$$

$$\left(n \to \infty,\ \frac{\phi}{2} \to 0,\ \sin\phi/2 \to \phi/2\right)$$

$$I_0 = A^2 = (na)^2.$$

$$\left[I_{max} = \left(\frac{A \sin \beta}{\beta}\right)^2 , \quad A = na. \right]$$

Position of maxima/minima

$I = 0$ when $\sin \beta = 0$, $\beta \neq 0$

$\beta = m\pi$, $m = 1, 2, 3 \ldots$

$\frac{\pi b \sin \theta}{\lambda} = m\pi \Rightarrow \left[b \sin \theta = m\lambda \right]$

→ I is maximum $I = A^2$.
when $\beta = 0$.

$$\lim_{\beta \to 0} \frac{\sin \beta}{\beta} = 1.$$

∴ Single slit diffraction.

$$\left[I = \underbrace{I_0}_{\text{initial intensity}} \frac{\sin^2 \beta}{\underbrace{\beta^2}_{\text{angular fringe width.}}} \overbrace{\cos(\omega t - \beta)}^{\text{pattern}} \right]$$

① $I_{min} = 0$, when $\sin \beta = 0$
$\beta = m\pi$, $m = \pm 1, \pm 2, \pm 3 \ldots$

$\left(\underset{\text{minima}}{b \sin \theta = m\lambda}\right)$ $\beta \neq 0$ ∴ $\left(\frac{\sin \beta}{\beta} \text{ when } \beta \to 0 = 1\right)$

② $I_{max} = I_0$, when $\beta \to 0$.
$$\lim_{\beta \to 0} \frac{\sin \beta}{\beta} = 1.$$

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slit width

Minimas.

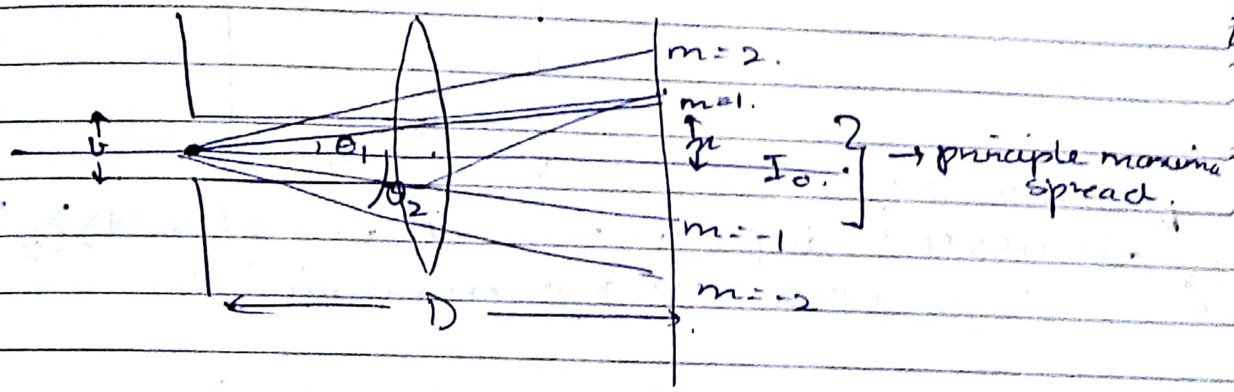
$$\boxed{b\sin\theta = m\lambda}$$

1st $m=1$.

$b\sin\theta_1 = \lambda \Rightarrow \boxed{\theta_1 = \sin^{-1}\lambda/b}$

2nd $m=2$.

$b\sin\theta_2 = 2\lambda \Rightarrow \boxed{\theta_2 = \sin^{-1}\frac{2\lambda}{b}}$



$$\therefore \boxed{\tan\theta_n = \frac{nx}{D}}$$

minima angle.

☆ Maxima condition.

$\Rightarrow \frac{dI}{d\beta} = 0 \longrightarrow I = \frac{I_0\sin^2\beta}{\beta^2}$

$\Rightarrow \frac{dI}{d\beta} = I_0 2\left(\frac{\sin\beta}{\beta}\right)\underbrace{\left[\frac{(\beta\cos\beta - \sin\beta)}{\beta^2}\right]}_{\text{must be zero.}} = 0$

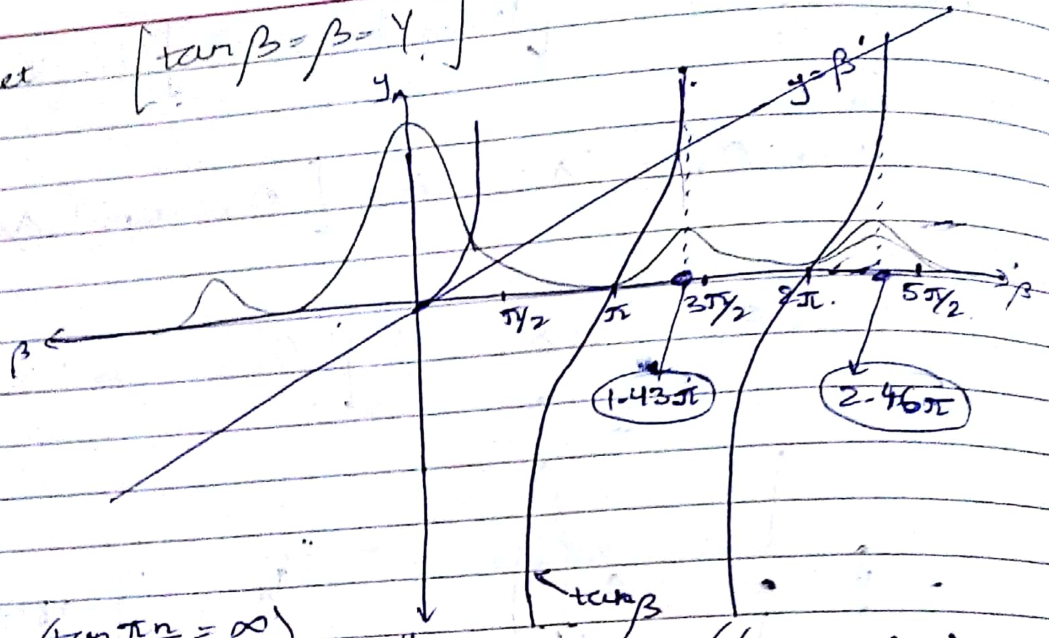
non zero

for maxima $\frac{\sin\beta}{\beta} = 1$.

$\Rightarrow \beta\cos\beta - \sin\beta = 0$

$\Rightarrow \beta\cos\beta = \sin\beta \Rightarrow \tan\beta = \beta$.

let $[\tan\beta = \beta = y]$



because $\left(\tan\frac{\pi n}{2} = \infty\right)$

calculated slightly less than $\left(\frac{(2n-1)\lambda}{2}\right)$

$\therefore \beta = 1.43\pi \rightarrow$ first maxima

$\beta = 2.46\pi \rightarrow$ second maxima

Hence.

1st maxima

$$I = I_0 \frac{\sin^2 1.43\pi}{(1.43\pi)^2}$$

$$= 0.0496\, I_0$$

$\therefore \frac{I_1}{I_0} \times 100 = 4.96\%$ (intensity of 1st maxima)

principle maxima

$$I = I_0$$

$$\left[\frac{I}{I_0} \times 100 = \underline{100\%}\right]$$

intensity of principle maxima.

2nd maxima

$$I = I_0 \frac{\sin^2 2.46\pi}{(2.46\pi)^2}$$

$$= 0.0164$$

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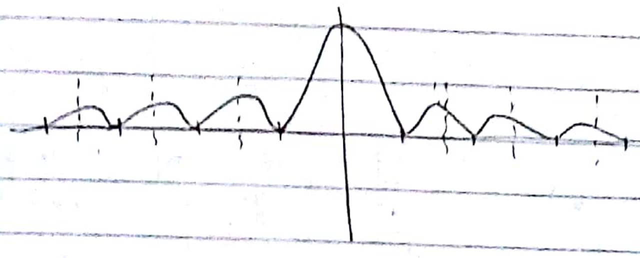
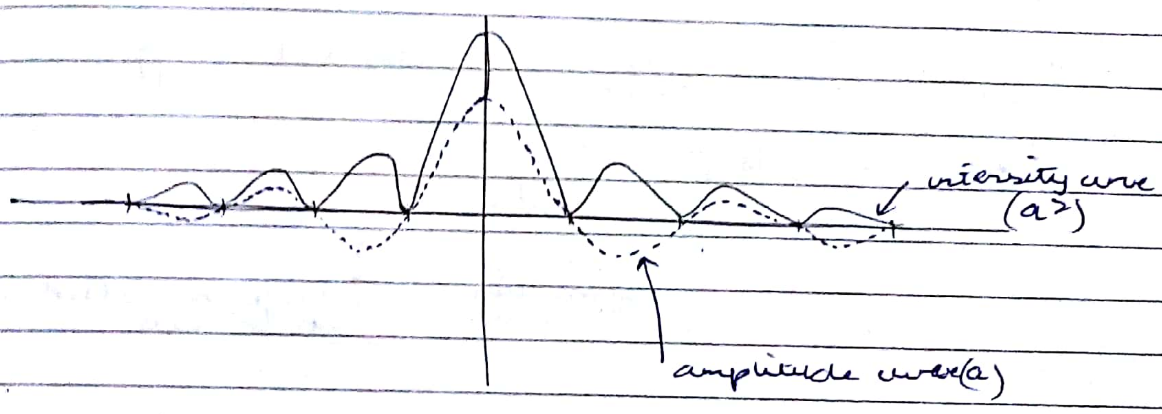
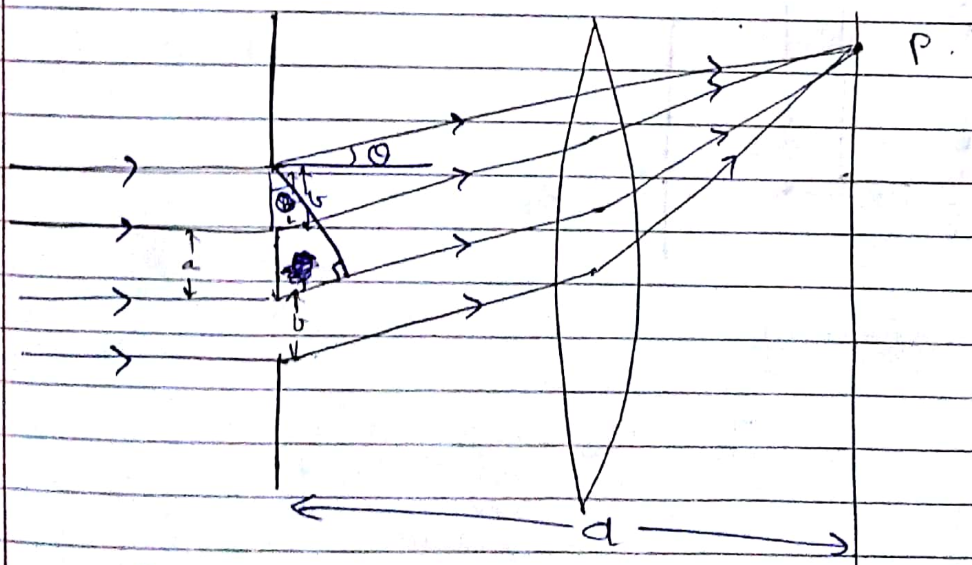
$$\therefore \left[\frac{I_2}{I_0} \times 100 = 1.64\,\% \right]$$ intensity of 2nd maxima.



Amplitude curve.



Double slit diffraction



$$\Rightarrow \phi = \frac{2\pi}{\lambda} \times (\text{path difference})$$

$$= \frac{2\pi}{\lambda} \times [(\sin\theta)(a+b)]$$

$$\therefore E = E_1 + E_2.$$

$$= \frac{na\sin\beta}{\beta}\left[\cos(\omega t - \beta) + \cos(\omega t - \beta - \phi_1)\right]$$

$$= 2na \frac{\sin\beta}{\beta}\left[\cos\frac{\phi_1}{2}\cos\left(\omega t - \beta - \frac{\phi_1}{2}\right)\right]$$

$$\left[E = 2na\frac{\sin\beta}{\beta}\cos\gamma\cos(\omega t - \beta - \gamma)\right]$$

phase difference, path difference Δ

$$\Rightarrow \left[\gamma = \frac{\phi_1}{2} = \frac{\pi(d\sin\theta)}{\lambda}\right]$$

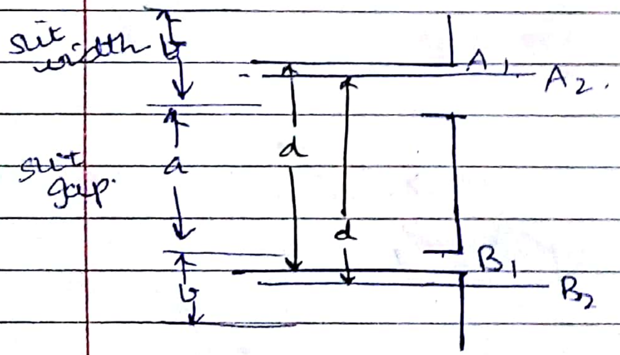
$$\frac{\phi}{2\pi} = \frac{pd}{\lambda}$$

$\rightarrow (d = a + b)$

→ Final intensity in double slit diff.

$$E \quad \left[I = 4I_0 \underbrace{\frac{\sin^2\beta}{\beta^2}}_{\text{diffraction curve}} \underbrace{\cos^2\gamma}_{\text{interfernce due to two slits}}\right]$$

→ Positions of maxima and minima



→ For minima

$$I = 4I_0 \underbrace{\frac{\sin^2\beta}{\beta^2}}_{\text{diffraction}} \underbrace{\cos^2\gamma}_{\text{interference}}$$

$\Rightarrow I = 0$ for minima

$\hookrightarrow I_0 \neq 0$, ① when $\cos^2\gamma = 0$

$$\Rightarrow \gamma = (n + 1/2)\pi$$

② when $\frac{\sin^2\beta}{\beta^2} = 0$.

$\Rightarrow \beta = m\pi \quad , m = 1, 2, 3 \ldots$

Two conditions for minima:

(diffraction pattern) minima

$$\boxed{\beta = m\pi}$$
$$\Downarrow$$
$$\boxed{b \sin\theta = m\lambda}$$

interference pattern minima

$$\boxed{\cancel{b \sin\theta = m} \quad d \sin\theta = \left(n + \frac{1}{2}\right)\lambda}$$
$$\parallel$$
$$\boxed{\gamma = (n + 1/2)\, 2\pi}$$

→ For maxima

$(I = I_{max}) \propto I_0$

↳ ~~I~~ ① when $\cos^2\gamma = 1$. interference

$\Rightarrow \gamma = n\pi \Rightarrow (d \sin\theta = n\lambda)$

② $\frac{\sin^2\beta}{\beta^2} = 1. \Rightarrow \beta = n\pi/2.$

$\therefore \left(b \sin\theta = \frac{n\lambda}{2}\right)$ diffraction.

Two conditions for maxima:

(diffractive pattern) ~~minima~~ maxima

$$\boxed{\beta = (n + 1/2)\pi}$$
$$\Downarrow$$
$$\boxed{b \sin\theta = (n + 1/2)\lambda}$$
$$\boxed{b \sin\theta = (n + 1/2)\lambda}$$

(interference pattern) ~~minima~~ maxima

$$\boxed{\gamma = n\pi}$$
$$\Downarrow$$
$$\boxed{d \sin\theta = n\lambda}.$$
$(a + b)\sin\theta = n\lambda.$

Now on the whole.

- maxima of ~~of~~ interference (rays) interfere destructively with minima of diffraction

$\therefore$ no maxima of interference obtained

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Now

minimas

interference.   diffraction.

$b\sin\theta = m\lambda$ $d\sin\theta = (n+\frac{1}{2})\lambda$

$\Rightarrow \left(\theta_1 = \sin^{-1}\frac{\lambda}{b}\right) \Rightarrow \left[\theta_2 = \sin^{-1}\frac{2\lambda}{d}\right]$

maximas.

interference.   diffraction.

$b\sin\theta = \frac{n}{2}\lambda$ $d\sin\theta = \frac{n}{2}\lambda$.

$\Rightarrow \left(\theta_1 = \sin^{-1}\frac{\lambda}{2b}\right) \Rightarrow \left(\theta_2 = \sin^{-1}\frac{\lambda}{d}\right)$.

$\hookrightarrow \theta_1 = \sin^{-1}\frac{\lambda}{b}(m) = \sin \frac{n}{2}\left(\frac{\lambda}{b}\right)$.

minima

$\left[\because \frac{n}{2} = m\right] \Rightarrow n = 2m$.

$(n = 2, 4, 6, 8 \ldots \quad m = 1, 2, 3 \ldots)$

case 1

$\Rightarrow$ let $a = b$ ← (slit gap)

$d = 2b$. ← (slit width)

$\left(\frac{n}{2} = m\right)$ $n = 2, 4, 6, \ldots$

missing order.

case 2

$\hookrightarrow$ let $a = 2b$.

$d = 3b$

$(b\sin\theta = m\lambda)$ minima ... (1)

$\Rightarrow d\sin\theta = n\lambda$ maxima.

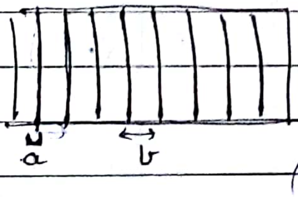
$3b\sin\theta = n\lambda$ ... (2)

(1)/(2) $\Rightarrow \frac{1}{3} = \frac{m}{n} \Rightarrow \boxed{n = 3m}$

($n = 3, 6, 9, 12 \ldots$ missing order).

N parallel slit diffraction.

Diffraction grating



$(a+b) = d$

$\Rightarrow$ 2700 lines per inch.

↳ grating element constant $= a+b = \frac{1}{(\text{no. of lines per inch})}$ = (d)

[Grating element constant $(d) = \frac{1}{\text{no. of lines}}$]

$= \frac{1}{2700}$ inch $= \frac{2.54\text{ cm}}{2700}$.

Now interference maxima $(d\sin\theta = n\lambda)$ ($n = 0, 1, 2 \ldots$ not infinite.)

$\Rightarrow \frac{d\sin\theta}{\lambda} = n.$

$\Rightarrow \frac{d}{\lambda} = \frac{n}{\sin\theta} \Rightarrow \boxed{\frac{d}{\lambda} = n_{max}}$

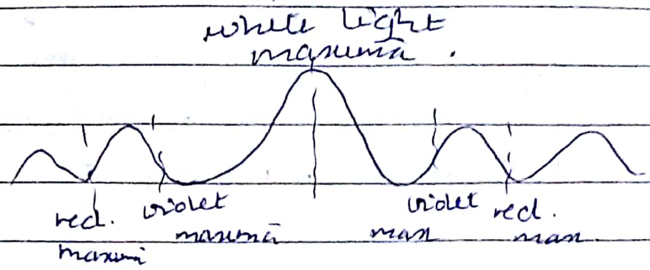
$\frac{d}{\lambda}$: can be calculated.

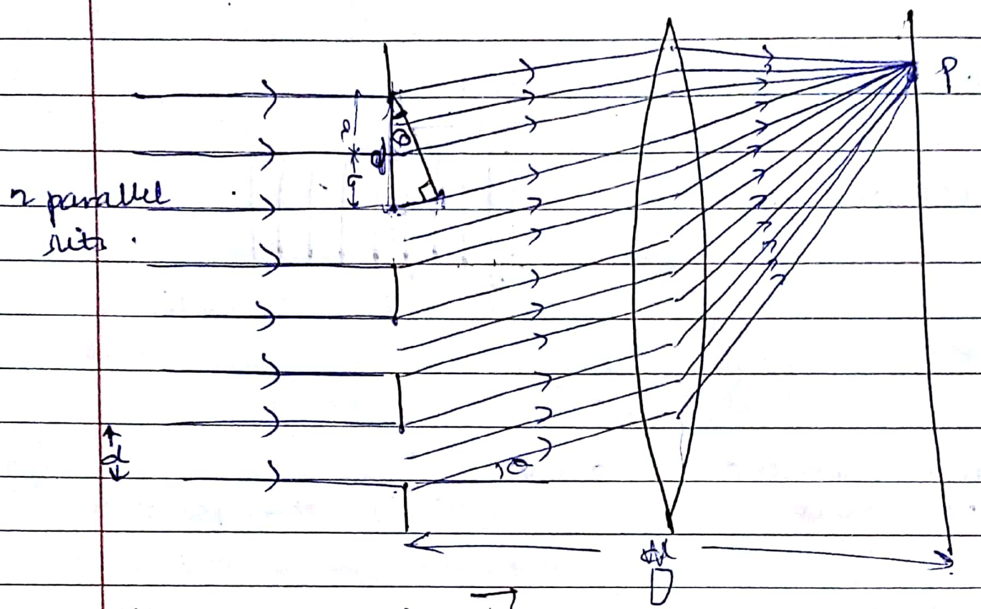
$\sin\theta$: can be experimentally calculated.

n_{max}: order of maximas

↳ d & λ (if one is known other can be calculated

☆ $\sin\theta \propto \lambda$.
∴ Larger the wavelength, farther spread out is its pattern.





N-parallel slit $\vec{E}$ vector & Intensity

phase difference: $\left(\phi = \frac{2\pi}{\lambda}(d\sin\theta)\right)$ $\qquad [\phi = 2\gamma]$

~~$\phi\lambda = 2\pi$ (path diff)~~

$$\left[\frac{\phi}{2\pi} = \frac{\text{path diff}}{\lambda}\right]$$

$\hookrightarrow E = E_1 + E_2 + E_3 \cdots$

$$= \frac{A\sin\beta}{\beta}\cos(\omega t - \beta) + \frac{A\sin\beta}{\beta}\cos(\omega t - \beta - \phi_1)$$

$$+ \ldots \frac{A\sin\beta}{\beta}\left[\cos(\omega t - \beta - (N-1)\phi_1)\right]$$

$$\boxed{E = \frac{A\sin\beta}{\beta}\frac{\sin n\gamma}{\sin\gamma}\cos\left(\omega t - \beta - \frac{1}{2}(N-1)\phi_1\right)}$$

$\vec{E}$ energy of N-parallel slits

$I_0 = A^2$

$$I = I_0 \underbrace{\frac{\sin^2\beta}{\beta^2}}_{\text{diffraction}} \underbrace{\frac{\sin^2 N\gamma}{\sin^2\gamma}}_{\text{interference}}$$

$$\Rightarrow \left[\gamma = \frac{\phi_1}{2} = \frac{\pi}{\lambda} d\sin\theta\right]$$

→ **conditions for maxima/minima.**

Minima

$I = 0$ when ① $\frac{\sin^2\beta}{\beta^2} = 0 \Rightarrow \sin\beta = 0$

$\Rightarrow [\beta = m\pi]$, $\Rightarrow [b\sin\theta = m\lambda]$

② $\frac{\sin^2 N\gamma}{\sin^2\gamma} = 0$ but $(\sin\gamma \neq 0)$ ∵ denominator cannot be zero.

$\therefore [N\gamma = p\pi]$ & $(p \neq 0, N)$

∵ because then $\sin\gamma = 0$ would be.

$\therefore$ p cannot be 0, or multiples of slit number N, 2N, 3N ...

Minima conditions.

~~diffraction~~ interference

$\therefore \left[\frac{\sin^2 N\gamma}{\sin^2\gamma} = 0\right]$

$\Downarrow$

$[N\gamma = p\pi]$.

$p \neq 0, N, 2N, 3N \ldots$

$N d\sin\theta = p\pi$

~~interference~~ diff

$[\beta = m\pi]$

$\Downarrow$

$[b\sin\theta = m\lambda]$

$m = 1, 2, 3 \ldots$

Maxima

$I = I_{max} \approx I_0$.

① $\frac{\sin^2\beta}{\beta^2} = 1. \Rightarrow \beta = \frac{m\lambda}{2}$

$\Rightarrow \left[b\sin\theta = \frac{m\lambda}{2} \right]$

② $\frac{\sin^2 N\gamma}{\sin^2\gamma} = 1 \Rightarrow \sin N\gamma = \sin\gamma$.

↳ $N\gamma = 2\pi \pm \gamma$.

primary.
~~principle~~ maximas.

$\left[d\sin\theta = n\lambda \right]$ $n = 0, 1, 2 \ldots$

interference max.

inter min.

~~Subsidiary~~ minima ~~maxima~~s.

$\left[d\sin\theta = \frac{p\lambda}{N} \right]$

$$\left[d\sin\theta = \frac{\lambda}{N}, \frac{2\lambda}{N}, \frac{3\lambda}{N}, \ldots \frac{(N-1)\lambda}{N}, \cancel{\frac{N\lambda}{N}}, \frac{(N+1)\lambda}{N} \ldots \frac{(2N-1)\lambda}{N} \ldots \cancel{\frac{2N\lambda}{N}} \right]$$

$\downarrow$

$d\sin\theta \neq \lambda$.

$\left[d\sin\theta = \gamma \neq m\lambda \text{ (integral multiples of } \lambda) \right]$

if ↳ minima will be adjacent to n^{th} order maxima

↳ $\left[Nd\sin\theta = p\lambda = (nN+1)\lambda \right]$

$N\gamma = p\pi$

✓ $(p \neq 0, N, 2N, 3N, \ldots)$

excluding integral multiples of N from p