```python
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
"""
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@author: muskan
"""
#Assignment 2 by Muskan Malik: 301585994, muskan_malik@sfu.ca
#Vanshika Goel : 301553598, vanshika_goel@sfu.ca
# Kusum Matlani: 301593345, kam51@sfu.ca



import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as stats
from arch import arch_model, unitroot
import statsmodels.api as sm_api
from statsmodels.stats import stattools
from datetime import datetime
import matplotlib.dates as mdates

################################################################################
#PART 1

#load and sort the data
index_data=pd.read_excel("IndexClosingPrices.xlsx",index_col="Date").\
    sort_index().dropna()

################################################################################
#PART 2

#Calculate return data
#return for S&P500
sp_return=(index_data["SP500"].pct_change())

#return for ASX
asx_return=(index_data["ASX"].pct_change())

#return for DAX
dax_return=(index_data["DAX"].pct_change())

combine_data=pd.concat([sp_return,asx_return,dax_return],axis=1).\
    sort_index().dropna()
combine_data.columns=["Return S&P500","Return ASX","Return DAX"]

################################################################################
#PART 3 ~ Price Series

#plot for PRICE SERIES
# Filter data for the desired time range (up to November 29, 2023)
index_data = index_data[index_data.index <= datetime(2023, 11, 29)]

# Function to plot Price series
```

```python
def plot_price_series(data, title):
    fig, ax = plt.subplots()
    data.plot(title=title, grid=True, legend=True, ax=ax)
    ax.xaxis.set_major_locator(mdates.YearLocator(4))
    ax.set_xlabel("Date")
    ax.set_ylabel("Return")

# Plotting return series for S&P500
plot_price_series(index_data["SP500"], "Price Series - S&P500")

# Plotting return series for ASX
plot_price_series(index_data["ASX"], "Price Series - ASX")

# Plotting return series for DAX
plot_price_series(index_data["DAX"], " Price Series - DAX")

plt.show()

###############################################################################
#PART 3 ~ Return Series

#plot and sort for RETURN SERIES
# Filter data for the desired time range (up to November 29, 2023)
combine_data = combine_data[combine_data.index <= datetime(2023, 11, 29)]

# Function to plot return series
def plot_return_series(data, title):
    fig, ax = plt.subplots()
    data.plot(title=title, grid=True, legend=True, ax=ax)
    ax.xaxis.set_major_locator(mdates.YearLocator(4))
    ax.set_xlabel("Date")
    ax.set_ylabel("Return")

# Plotting return series for S&P500
plot_return_series(combine_data["Return S&P500"], "Return Series - S&P500")

# Plotting return series for ASX
plot_return_series(combine_data["Return ASX"], "Return Series - ASX")

# Plotting return series for DAX
plot_return_series(combine_data["Return DAX"], "Return Series - DAX")

plt.show()

#The time series plots appear noisy as expected, aligning with our intended
#analysis. While visual inspection suggests stationarity in all three,
#it's crucial not to affirm stationarity without conducting the augmented
#Dickey-Fuller Test on the return data. We'll rely on the 5% confidence level
# benchmarks to assess data stationarity based on the test statistic values.


###############################################################################
#Part 4 ~ Dickey-Fuller test

adf_test1 = unitroot.ADF(combine_data["Return S&P500"])
```

```
print(adf_test1.summary())

adf_test2 = unitroot.ADF(combine_data["Return ASX"])
print(adf_test2.summary())

adf_test3 = unitroot.ADF(combine_data["Return DAX"])
print(adf_test3.summary())

# The series is weakly stationary, which is sufficient for modelled using GARCH


##############################################################################
# PART 5 ~ Fitting Models

# Fit GARCH(1,1) models with different distributions

# Use two different models for the mean
# Use a GARCH(1,1) process for the variance

# Note: the distribution for the residuals is specified by using
# input dist = "StudentsT"

# Fit GARCH(1,1) with Student's t distribution for S&P500 constant mean
sp500_con =  arch_model(100*combine_data["Return S&P500"]\
                        ,mean='constant',vol='GARCH', p=1, q=1,\
                            dist="StudentsT").fit()
print(sp500_con.summary())

# Fit GARCH(1,1) with Student's t distribution for S&P500 "AR" mean
sp500_ar =  arch_model(100*combine_data["Return S&P500"],\
                       mean='AR', lags=1,vol='GARCH', p=1, q=1,\
                       dist="StudentsT").fit()
print(sp500_ar.summary())

# Fit GARCH(1,1) with Student's t distribution for ASX constant mean
asx_con =  arch_model(100*combine_data["Return ASX"],\
                      mean='constant',vol='GARCH', p=1, q=1,\
                      dist="StudentsT").fit()
print(asx_con.summary())

# Fit GARCH(1,1) with Student's t distribution for ASX "AR" mean
asx_ar =  arch_model(100*combine_data["Return ASX"],\
                     mean='AR', lags=1,vol='GARCH', p=1, q=1,\
                     dist="StudentsT").fit()
print(asx_ar.summary())

# Fit GARCH(1,1) with Student's t distribution for DAX constant mean
dax_con =  arch_model(100*combine_data["Return DAX"],\
                      mean='constant',vol='GARCH', p=1, q=1,\
                      dist="StudentsT").fit()
print(dax_con.summary())

# Fit GARCH(1,1) with Student's t distribution for DAX "AR" mean
dax_ar =  arch_model(100*combine_data["Return DAX"],\
                     mean='AR', lags=1,vol='GARCH', p=1, q=1,\
```

```
                     dist="StudentsT").fit()
print(dax_ar.summary())
###############################################################################

# checking for serial correlation of the residuals using DURBIN WATSON test
# value below 2 indicates a positive auto correlation
# value higher than 2 indicates a negative serial correlation

# Return SP500

dw_1 = stattools.durbin_watson(sp500_con.resid.dropna())
print("\n\nFor sp500_con, the DW statistics is {0}".format(dw_1))

#The test gives us a result of around 2.108, which is not far off from 2.
#The residuals are not very serially correlated at all.


dw_2 = stattools.durbin_watson(sp500_ar.resid.dropna())
print("For sp500_ar, the DW statistics is {0}".format(dw_2))

#The test gives us a result of around 2.08, which is not far off from 2.
#The residuals are not very serially correlated at all.


# Return ASX

dw_3 = stattools.durbin_watson(asx_con.resid.dropna())
print("\n\nFor asx_con, the DW statistics is {0}".format(dw_3))

#The test gives us a result of around 1.95, which is not far off from 2.
#The residuals are not very serially correlated at all.


dw_4 = stattools.durbin_watson(asx_ar.resid.dropna())
print("For asx_ar, the DW statistics is {0}".format(dw_4))

#The test gives us a result of around 2.08, which is not far off from 2.
#The residuals are not very serially correlated at all.


# Return DAX

dw_5 = stattools.durbin_watson(dax_con.resid.dropna())
print("\n\nFor dax_con, the DW statistics is {0}".format(dw_5))

#The test gives us a result of around 2.005, which is not far off from 2.
#The residuals are not very serially correlated at all.

dw_6 = stattools.durbin_watson(dax_ar.resid.dropna())
print("For dax_ar, the DW statistics is {0}".format(dw_6))

#The test gives us a result of around 2.02, which is not far off from 2.
#The residuals are not very serially correlated at all.
```

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# The Durbin Watson test for all the models give a value close to 2,
# which does not show significant serial correlation,
#hence rejecting the null hypothesis

##############################################################################
# Perform log-likelihood ratio test
# and Compare the models using the likelihood ratio test

# sp500_con vs sp500_ar
LR_stat = -2*(sp500_con.loglikelihood-sp500_ar.loglikelihood)
dof = 1  # There is one less parameter in the smaller model
LR_pval = stats.chi2.sf(LR_stat, dof)

print("\nThe Likelihood ratio test statistics is {0}".format(LR_stat))
print("The corresponding p-value is {0}".format(LR_pval))

#The results seem to indicate that since the test statistic is less than the
#critical value, we have insufficient evidence to reject the null hypothesis
#so the simpler model seems to be good enough for our purposes.

# asx_con vs asx_ar
LR_stat = -2*(asx_con.loglikelihood-asx_ar.loglikelihood)
dof = 1
LR_pval = stats.chi2.sf(LR_stat, dof)

print("\nThe Likelihood ratio test statistics is {0}".format(LR_stat))
print("The corresponding p-value is {0}".format(LR_pval))

#The results seem to indicate that since the test statistic is greater than the
#critical value, we have sufficient evidence to reject the null hypothesis
#so the complex model seems to be a better fit.

# dax_con vs dax_ar
LR_stat = -2*(dax_con.loglikelihood-dax_ar.loglikelihood)
dof = 1
LR_pval = stats.chi2.sf(LR_stat, dof)

print("\nThe Likelihood ratio test statistics is {0}".format(LR_stat))
print("The corresponding p-value is {0}".format(LR_pval))

#The results seem to indicate that since the test statistic is greater than the
#critical value, we have sufficient evidence to reject the null hypothesis
#so the complex model seems to be a good fit for our use.


# Since the p-value is greater than 0.05 for SP&500 & ASX,
# with a 95% confidence the null
# hypothesis is accepted (or fails to be rejected). This implies that both
# models are equally good. Since this is the case, the simpler of the two
# models should be used.
# However, note that at a 94% confidence level, the null hypothesis of SP&500
# would be rejected, i.e. the larger model may be considered better.
# So in this case the likelihood ratio test is a little inconclusive.
# For DAX, p-value is substantially lower than 0.05, with a 95% confidence
# null hypothesis is rejected. This suggests that adding another additional
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# term (lag term), is considered better

#Based on the provided LRT results, there is evidence to suggest that the more
#complex AR-GARCH model is preferred over the constant mean GARCH model for
# each index.

# As part of any decision, the square of the (standardized) residuals should
# be inspected for GARCH effects.

##################################################################################
# plotting residuals

# Model 1 ~ sp500_con
sp500_con.plot()

sm_api.graphics.tsa.plot_acf(sp500_con.resid, lags=10)
plt.grid()
plt.title("sp500_con ACF Residuals")

sm_api.graphics.tsa.plot_acf(sp500_con.resid**2, lags=10)
plt.grid()
plt.title("sp500_con ACF Residuals Squared")

# The results_sp500_con plots of the squared residuals show very clearly that
#there are GARCH effects that the model doesn't seem to be handling correctly.

# Model 2 ~ sp500_ar
sp500_ar.plot()

sm_api.graphics.tsa.plot_acf(sp500_ar.resid, lags=10)
plt.grid()
plt.title("sp500_ar ACF Residuals")

sm_api.graphics.tsa.plot_acf(sp500_ar.resid**2, lags=10)
plt.grid()
plt.title("sp500_ar ACF Residuals Squared")

# The sp500_ar plots of the squared residuals seem to indicate that it
#deals withthe GARCH effects better than mdlA.

# Model 3 ~ asx_con
asx_con.plot()

sm_api.graphics.tsa.plot_acf(asx_con.resid, lags=10)
plt.grid()
plt.title("asx_con ACF Residuals")

sm_api.graphics.tsa.plot_acf(asx_con.resid**2, lags=10)
plt.grid()
plt.title("asx_con ACF Residuals Squared")

# The asx_con plots of the squared residuals show very clearly that
#there are GARCH effects that the model doesn't seem to be handling correctly.

# Model 4 ~ asx_ar
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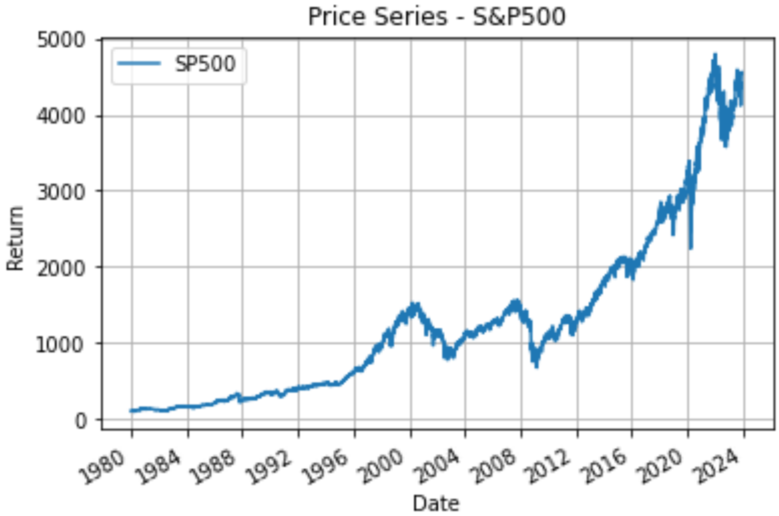
```
asx_ar.plot()

sm_api.graphics.tsa.plot_acf(asx_ar.resid, lags=10)
plt.grid()
plt.title("asx_ar ACF Residuals")

sm_api.graphics.tsa.plot_acf(asx_ar.resid**2, lags=10)
plt.grid()
plt.title("asx_ar ACF Residuals Squared")

# The asx_ar plots of the squared residuals seem to indicate that it deals
# with the GARCH effects better than results_asx_con.

# Model 5 ~ dax_con
dax_con.plot()

sm_api.graphics.tsa.plot_acf(dax_con.resid, lags=10)
plt.grid()
plt.title("rdax_con ACF Residuals")

sm_api.graphics.tsa.plot_acf(dax_con.resid**2, lags=10)
plt.grid()
plt.title("dax_con ACF Residuals Squared")

# The dax_con plots of the squared residuals show very clearly that there are
# GARCH effects that the model doesn't seem to be handling correctly.

# Model 6 ~ dax_ar
dax_ar.plot()

sm_api.graphics.tsa.plot_acf(dax_ar.resid, lags=10)
plt.grid()
plt.title("dax_ar ACF Residuals")

sm_api.graphics.tsa.plot_acf(dax_ar.resid**2, lags=10)
plt.grid()
plt.title("dax_ar ACF Residuals Squared")

# The dax_ar plots of the squared residuals seem to indicate that it deals
# with the GARCH effects better than results_dax_con.

################################################################################
# Plot each as a histogram

#to check for normality of residuals create histograms

#for sp500_con
fig, ax = plt.subplots()
plt.hist(sp500_con.resid,bins=100,axes=ax)
ax.set_title("sp500_con Residuals")
ax.grid(True)

#for sp500_ar
fig, ax = plt.subplots()
plt.hist(sp500_ar.resid, bins=100, axes=ax)
```

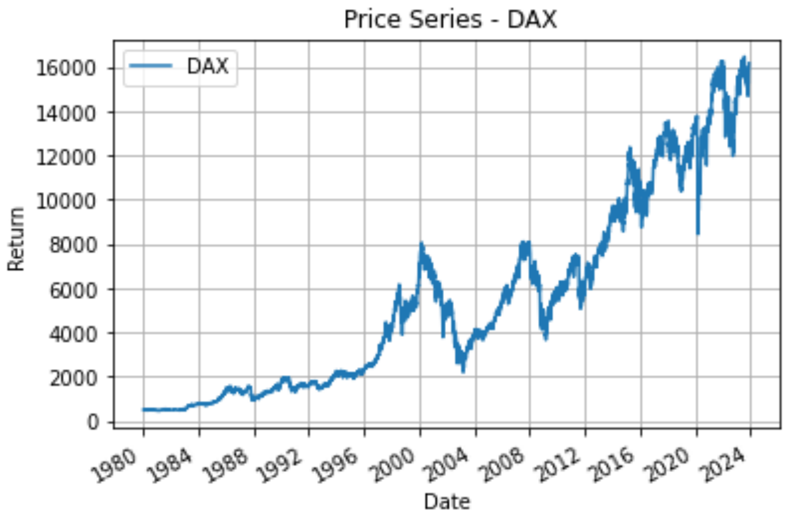
```
ax.set_title("sp500_ar Residuals")
ax.grid(True)

#for asx_con
fig, ax = plt.subplots()
plt.hist(asx_con.resid, bins=100, axes=ax)
ax.set_title("asx_con Residuals")
ax.grid(True)

#for asx_ar
fig, ax = plt.subplots()
plt.hist(asx_ar.resid, bins=100, axes=ax)
ax.set_title("rasx_ar Residuals")
ax.grid(True)

#for dax_con
fig, ax = plt.subplots()
plt.hist(dax_con.resid, bins=100, axes=ax)
ax.set_title("dax_con Residuals")
ax.grid(True)

#for dax_ar
fig, ax = plt.subplots()
plt.hist(dax_ar.resid, bins=100, axes=ax)
ax.set_title("dax_ar Residuals")
ax.grid(True)

################################################################################
# Plot each as a qq-plot

#to check for normality of residuals using qq-plots

#for sp500_con
fig, ax = plt.subplots()
stats.probplot(sp500_con.resid, dist=stats.norm,
               sparams=(np.mean(sp500_con.resid), np.std(sp500_con.resid)),
               plot=plt)
ax.set_title("sp500_con Residuals")
ax.grid(True)


# for sp500_ar
fig, ax = plt.subplots()
stats.probplot(sp500_ar.resid, dist=stats.norm,
               sparams=(np.mean(sp500_ar.resid), np.std(sp500_ar.resid)),
               plot=plt)
ax.set_title("sp500_ar Residuals")
ax.grid(True)

# for asx_con
fig, ax = plt.subplots()
stats.probplot(asx_con.resid, dist=stats.norm,
               sparams=(np.mean(asx_con.resid), np.std(asx_con.resid)),
               plot=plt)
ax.set_title("asx_con Residuals")
```

```
ax.grid(True)

# for asx_ar
fig, ax = plt.subplots()
stats.probplot(asx_ar.resid, dist=stats.norm,
               sparams=(np.mean(asx_ar.resid), np.std(asx_ar.resid)),
               plot=plt)
ax.set_title("asx_ar Residuals")
ax.grid(True)

# for dax_con
fig, ax = plt.subplots()
stats.probplot(dax_con.resid, dist=stats.norm,
               sparams=(np.mean(dax_con.resid), np.std(dax_con.resid)),
               plot=plt)
ax.set_title("dax_con Residuals")
ax.grid(True)

# for dax_ar
fig, ax = plt.subplots()
stats.probplot(dax_ar.resid, dist=stats.norm,
               sparams=(np.mean(dax_ar.resid), np.std(asx_ar.resid)),
               plot=plt)
ax.set_title("dax_ar Residuals")
ax.grid(True)

#NOTE ~
#The histogram and QQ plot data for all the models seems inconclusive
#since all the histogram plots seem to be showing kurtosis and mean peak
#similar to normal distributions while the QQ plots seemed to deviate at both
#ends indicating a lack of normality of all the models.


##########################################################################################
# Answer ~ 1

#summary offindings for each index:

# for all three naming S&P500, ASX and DAX:
#ADF Test: The process is weakly stationary.
#GARCH Coefficients: The coefficients seem reasonable.
#LRT: The AR-GARCH model is preferred over the constant mean GARCH model.

#The GARCH(1,1) models for the ASX and DAX indices seems to be an appropriate
#fit.

#For the S&P 500 index, the GARCH(1,1) model seemed to provide some
#inconclusive findings since the p-value from the log-likelihood test just
#barely exceeded the 0.05 benchmark necessary for a 5% confidence interval
#when comparing the constant model with the complex model. However,this would
#not be the case if we changed our guidelines to a 6% confidence level.

#The findings led to the conclusion that the squared residuals from lagged
# models for all three indices (sp500_ar, asx_ar, and dax_ar) consistently
#fall within acceptable ranges. This implies the effective capture of
```

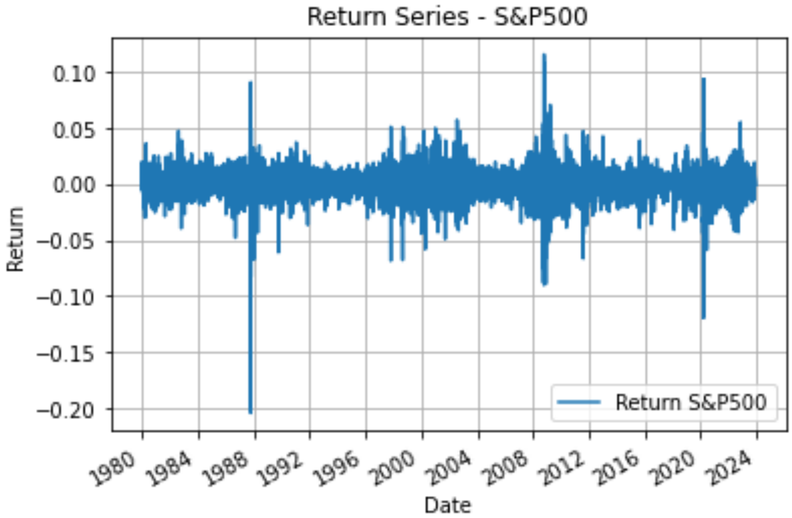
```
#time-varying variance effects, strongly suggesting the presence and capture
#of GARCH effects.


##############################################################################
# Answer ~ 2

#mu at t is dependent on returns at time t-1 so mu at t+1 is dependent on
#returns from time t
#coefficient of return lag is irrelavent since it was established that
#return at t is 0 in the question.

#S&P 500: Constant (mu): 0.0716
#ASX: Constant (mu): 0.0622
#DAX: Constant (mu): 0.0780

# Mu(t) = phi(0) + phi * R(t-1); where phi(0) is the constant

# Therefore, the index with the highest expected return at time t+1 when
#Rt = 0 is the DAX.
##############################################################################
```

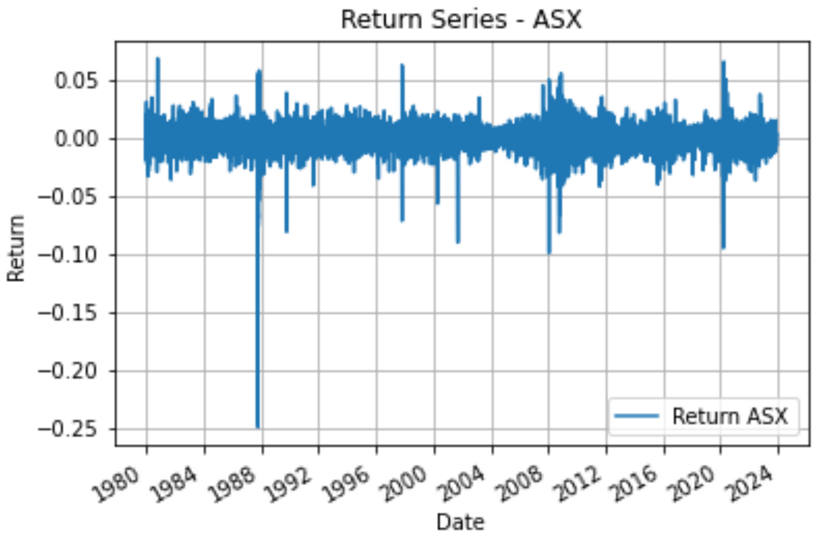
PLOTS OF RAW DATA

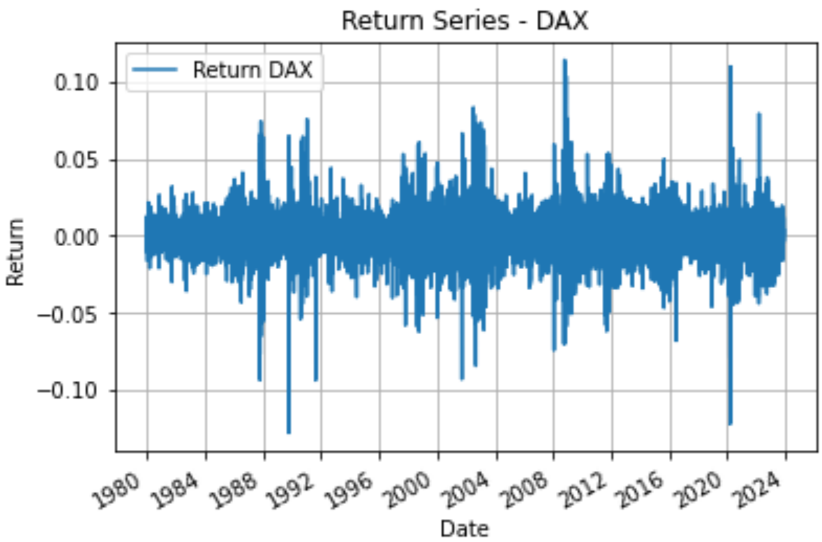




PLOTS OF COMPUTED RETURNS







CHECKING THE STATIONARITY OF DATA USING DICKEY-FULLER TEST

Augmented Dickey-Fuller Results

```
=====================================
Test Statistic          -17.774
P-value                   0.000
Lags                         35
-------------------------------------

Trend: Constant
Critical Values: -3.43 (1%), -2.86 (5%), -2.57 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

Augmented Dickey-Fuller Results

```
=====================================
Test Statistic          -23.474
P-value                   0.000
Lags                         17
-------------------------------------

Trend: Constant
Critical Values: -3.43 (1%), -2.86 (5%), -2.57 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

Augmented Dickey-Fuller Results

```
=====================================
Test Statistic          -39.974
P-value                   0.000
Lags                          6
-------------------------------------

Trend: Constant
Critical Values: -3.43 (1%), -2.86 (5%), -2.57 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

MODEL~SP500_CON

```
                    Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:           Return S&P500   R-squared:                       0.000
Mean Model:              Constant Mean   Adj. R-squared:                  0.000
Vol Model:                       GARCH   Log-Likelihood:               -14207.0
Distribution:  Standardized Student's t   AIC:                          28424.1
Method:             Maximum Likelihood   BIC:                          28460.4
                                         No. Observations:               10679
Date:                 Mon, Dec 04 2023   Df Residuals:                   10678
Time:                         21:13:50   Df Model:                           1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu             0.0716  7.291e-03      9.816  9.638e-23 [5.727e-02,8.585e-02]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega          0.0109  2.252e-03      4.857  1.192e-06 [6.525e-03,1.535e-02]
alpha[1]       0.0838  7.619e-03     10.997  3.951e-28 [6.886e-02,9.872e-02]
beta[1]        0.9101  8.056e-03    112.963      0.000     [  0.894,  0.926]
```

```
                     Distribution
==========================================================================
            coef    std err          t       P>|t|  95.0% Conf. Int.
--------------------------------------------------------------------------
nu          6.2250      0.395     15.760  5.826e-56 [  5.451,  6.999]
==========================================================================
```

MODEL~SP500_AR

```
                            AR - GARCH Model Results
====================================================================================
Dep. Variable:              Return S&P500   R-squared:                       0.001
Mean Model:                            AR   Adj. R-squared:                  0.000
Vol Model:                          GARCH   Log-Likelihood:               -14205.2
Distribution:      Standardized Student's t   AIC:                           28422.3
Method:                Maximum Likelihood   BIC:                           28466.0
                                            No. Observations:                10678
Date:                    Mon, Dec 04 2023   Df Residuals:                    10676
Time:                            21:13:50   Df Model:                            2
                                  Mean Model
================================================================================
                coef    std err          t       P>|t|       95.0% Conf. Int.
--------------------------------------------------------------------------------
Const          0.0727  7.434e-03      9.775  1.446e-22  [5.810e-02,8.724e-02]
Retu...500[1]   -0.0124  9.316e-03     -1.336      0.182 [-3.070e-02,5.815e-03]
                              Volatility Model
============================================================================
            coef    std err          t       P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega         0.0109  2.247e-03      4.840  1.296e-06 [6.473e-03,1.528e-02]
alpha[1]      0.0836  7.614e-03     10.977  4.945e-28 [6.865e-02,9.850e-02]
beta[1]       0.9103  8.047e-03    113.130      0.000     [  0.895,  0.926]
                              Distribution
==========================================================================
            coef    std err          t       P>|t|  95.0% Conf. Int.
--------------------------------------------------------------------------
nu          6.2076      0.394     15.772  4.831e-56 [  5.436,  6.979]
==========================================================================
```

MODEL~ASX_CON

```
                     Constant Mean - GARCH Model Results
====================================================================================
Dep. Variable:                 Return ASX   R-squared:                       0.000
Mean Model:                 Constant Mean   Adj. R-squared:                  0.000
Vol Model:                          GARCH   Log-Likelihood:               -13198.0
Distribution:      Standardized Student's t   AIC:                           26406.0
Method:                Maximum Likelihood   BIC:                           26442.4
                                            No. Observations:                10679
Date:                    Mon, Dec 04 2023   Df Residuals:                    10678
Time:                            21:13:50   Df Model:                            1
                                 Mean Model
============================================================================
            coef    std err          t       P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu          0.0622  7.154e-03      8.701  3.295e-18 [4.822e-02,7.627e-02]
                              Volatility Model
============================================================================
            coef    std err          t       P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
```

```
omega        0.0193  3.846e-03     5.009  5.474e-07 [1.173e-02,2.680e-02]
alpha[1]     0.0919  1.034e-02     8.882  6.579e-19  [7.159e-02,  0.112]
beta[1]      0.8868  1.308e-02    67.816      0.000    [  0.861,  0.912]
                             Distribution
========================================================================
              coef    std err        t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu          7.5337      0.631     11.941  7.272e-33 [  6.297,  8.770]
========================================================================
```

MODEL~ASX_AR

```
                           AR - GARCH Model Results
====================================================================================
Dep. Variable:                Return ASX   R-squared:                      -0.002
Mean Model:                           AR   Adj. R-squared:                 -0.002
Vol Model:                         GARCH   Log-Likelihood:               -13177.5
Distribution:      Standardized Student's t   AIC:                        26366.9
Method:                Maximum Likelihood   BIC:                          26410.6
                                           No. Observations:                10678
Date:                   Mon, Dec 04 2023   Df Residuals:                    10676
Time:                           21:13:50   Df Model:                            2
                                  Mean Model
==============================================================================
                   coef    std err          t      P>|t|      95.0% Conf. Int.
------------------------------------------------------------------------------
Const            0.0582  7.160e-03      8.123  4.549e-16 [4.413e-02,7.219e-02]
Return ASX[1]    0.0621  1.041e-02      5.963  2.475e-09 [4.166e-02,8.244e-02]
                              Volatility Model
============================================================================
              coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega        0.0204  4.057e-03      5.026  5.012e-07 [1.244e-02,2.834e-02]
alpha[1]     0.0955  1.074e-02      8.886  6.328e-19  [7.440e-02,  0.117]
beta[1]      0.8819  1.370e-02     64.351      0.000    [  0.855,  0.909]
                             Distribution
========================================================================
              coef    std err        t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu          7.6574      0.655     11.690  1.438e-31 [  6.374,  8.941]
========================================================================
```

MODEL~DAX_CON

```
                       Constant Mean - GARCH Model Results
====================================================================================
Dep. Variable:                Return DAX   R-squared:                       0.000
Mean Model:                Constant Mean   Adj. R-squared:                  0.000
Vol Model:                         GARCH   Log-Likelihood:               -16437.5
Distribution:      Standardized Student's t   AIC:                        32884.9
Method:                Maximum Likelihood   BIC:                          32921.3
                                           No. Observations:                10679
Date:                   Mon, Dec 04 2023   Df Residuals:                    10678
Time:                           21:13:51   Df Model:                            1
                                 Mean Model
============================================================================
              coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu          0.0780  9.214e-03      8.465  2.559e-17 [5.994e-02,9.606e-02]
                              Volatility Model
============================================================================
              coef    std err          t      P>|t|      95.0% Conf. Int.
```

```
---------------------------------------------------------------------------
omega        0.0189  3.464e-03     5.443  5.237e-08 [1.207e-02,2.564e-02]
alpha[1]     0.0854  8.157e-03    10.472  1.167e-25  [6.943e-02,  0.101]
beta[1]      0.9056  8.710e-03   103.962      0.000   [  0.888,  0.923]
                              Distribution
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu           7.3782      0.582     12.671  8.576e-37 [  6.237,  8.519]
========================================================================
```

MODEL~DAX_AR

```
                           AR - GARCH Model Results
==========================================================================================
Dep. Variable:               Return DAX   R-squared:                  -0.001
Mean Model:                          AR   Adj. R-squared:             -0.001
Vol Model:                        GARCH   Log-Likelihood:            -16434.0
Distribution:      Standardized Student's t   AIC:                    32880.1
Method:                Maximum Likelihood   BIC:                      32923.7
                                    No. Observations:             10678
Date:                  Mon, Dec 04 2023   Df Residuals:               10676
Time:                          21:13:51   Df Model:                       2
                                 Mean Model
=====================================================================================
                   coef    std err          t      P>|t|       95.0% Conf. Int.
-------------------------------------------------------------------------------------
Const           0.0777  9.336e-03     8.320  8.832e-17  [5.937e-02,9.597e-02]
Return DAX[1] 6.4312e-03  9.456e-03     0.680      0.496 [-1.210e-02,2.496e-02]
                              Volatility Model
================================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
---------------------------------------------------------------------------
omega        0.0190  3.469e-03     5.472  4.461e-08 [1.218e-02,2.578e-02]
alpha[1]     0.0857  8.173e-03    10.489  9.701e-26  [6.971e-02,  0.102]
beta[1]      0.9052  8.722e-03   103.788      0.000   [  0.888,  0.922]
                              Distribution
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
nu           7.3911      0.585     12.627  1.496e-36 [  6.244,  8.538]
========================================================================

Covariance estimator: robust
```

TESTING RESULTS FOR SERIAL CORRELATION

For sp500_con, the DW statistics is 2.107611179288561
For sp500_ar, the DW statistics is 2.0829317018731257

For asx_con, the DW statistics is 1.949987931738237
For asx_ar, the DW statistics is 2.075134960788002

For dax_con, the DW statistics is 2.0048652957247675
For dax_ar, the DW statistics is 2.017352945329236

TESTING RESULTS FOR LIKELIHOOD RATIO

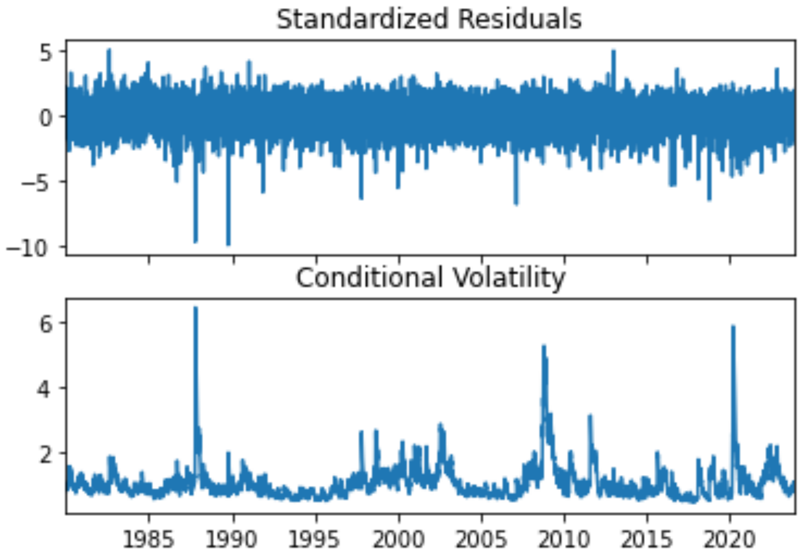
The Likelihood ratio test statistics is 3.761394375218515
The corresponding p-value is 0.05244882325629066

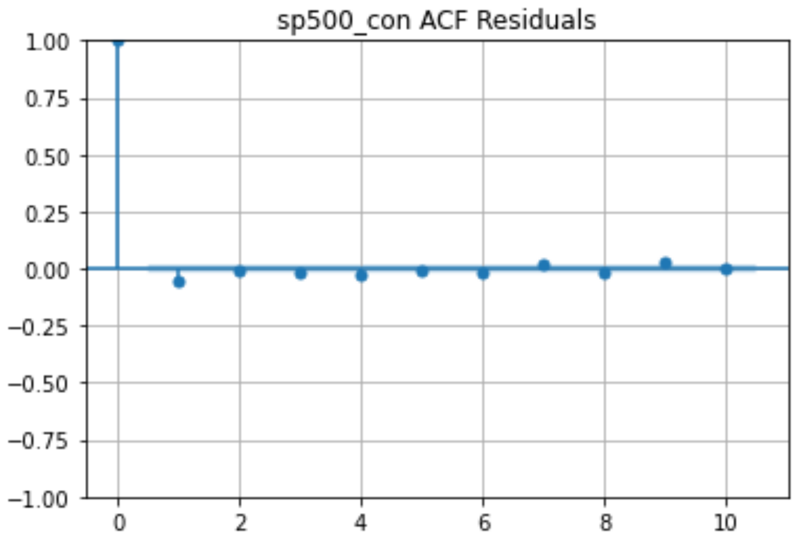
The Likelihood ratio test statistics is 41.08855085916002
The corresponding p-value is 1.4548607233366306e-10

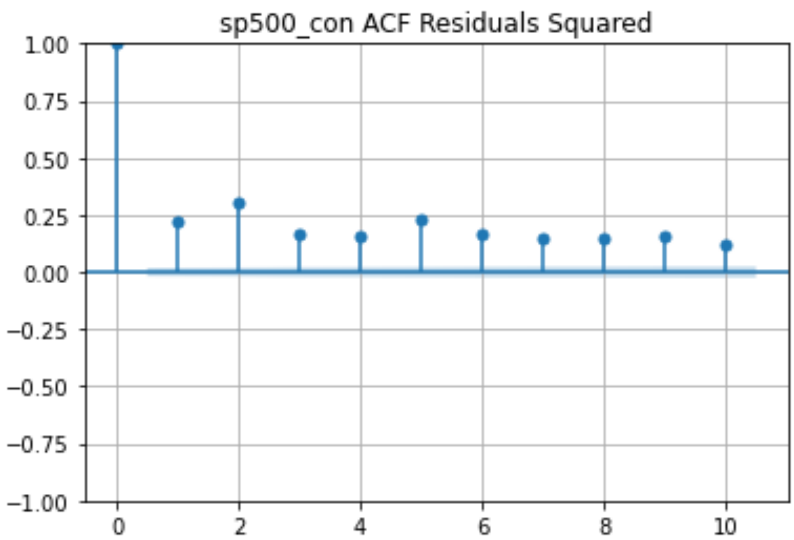
The Likelihood ratio test statistics is 6.85261521780194
The corresponding p-value is 0.008851161779641091

PLOTS OF RESIDUALS AND SQUARED RESIDUALS

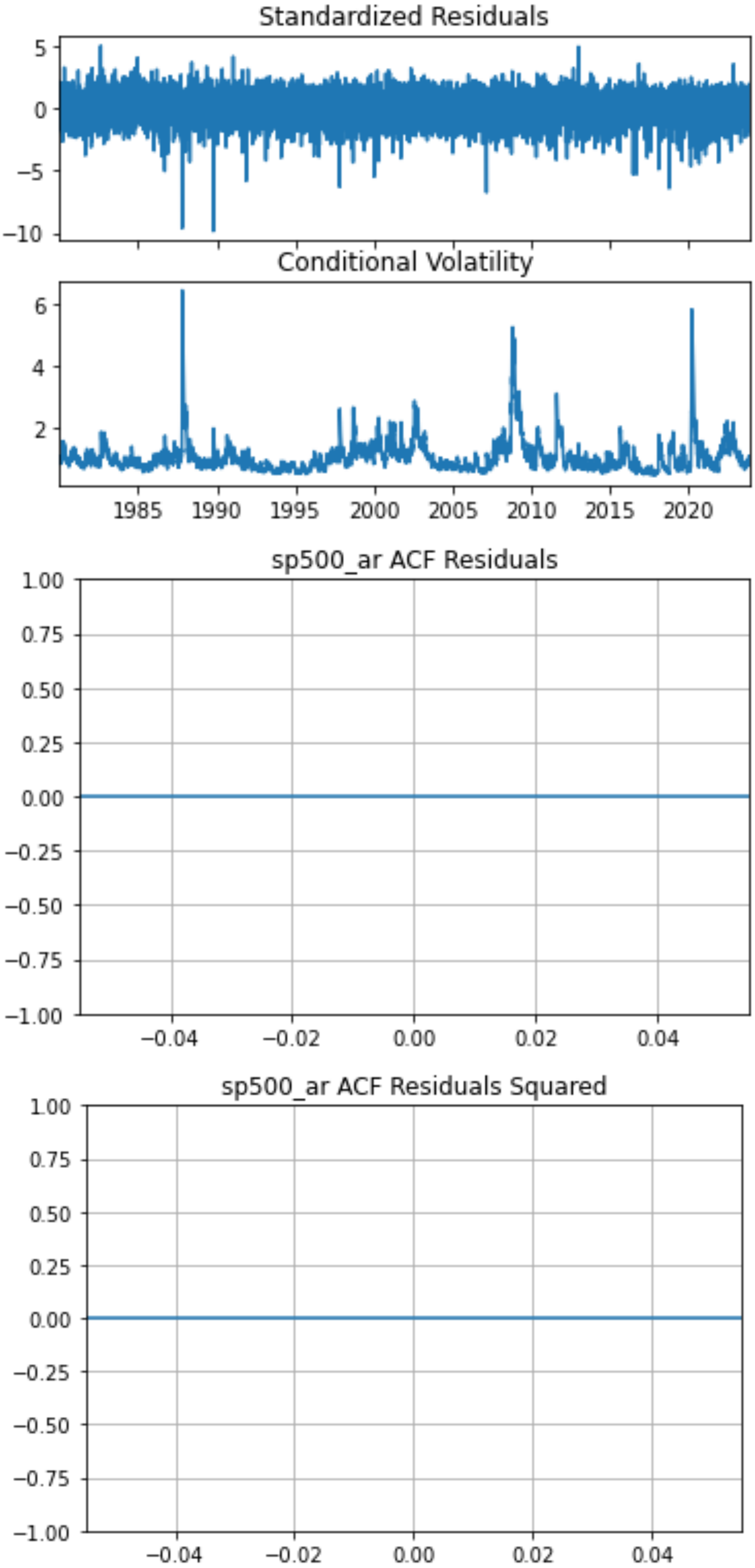
MODEL~ SP500_CON



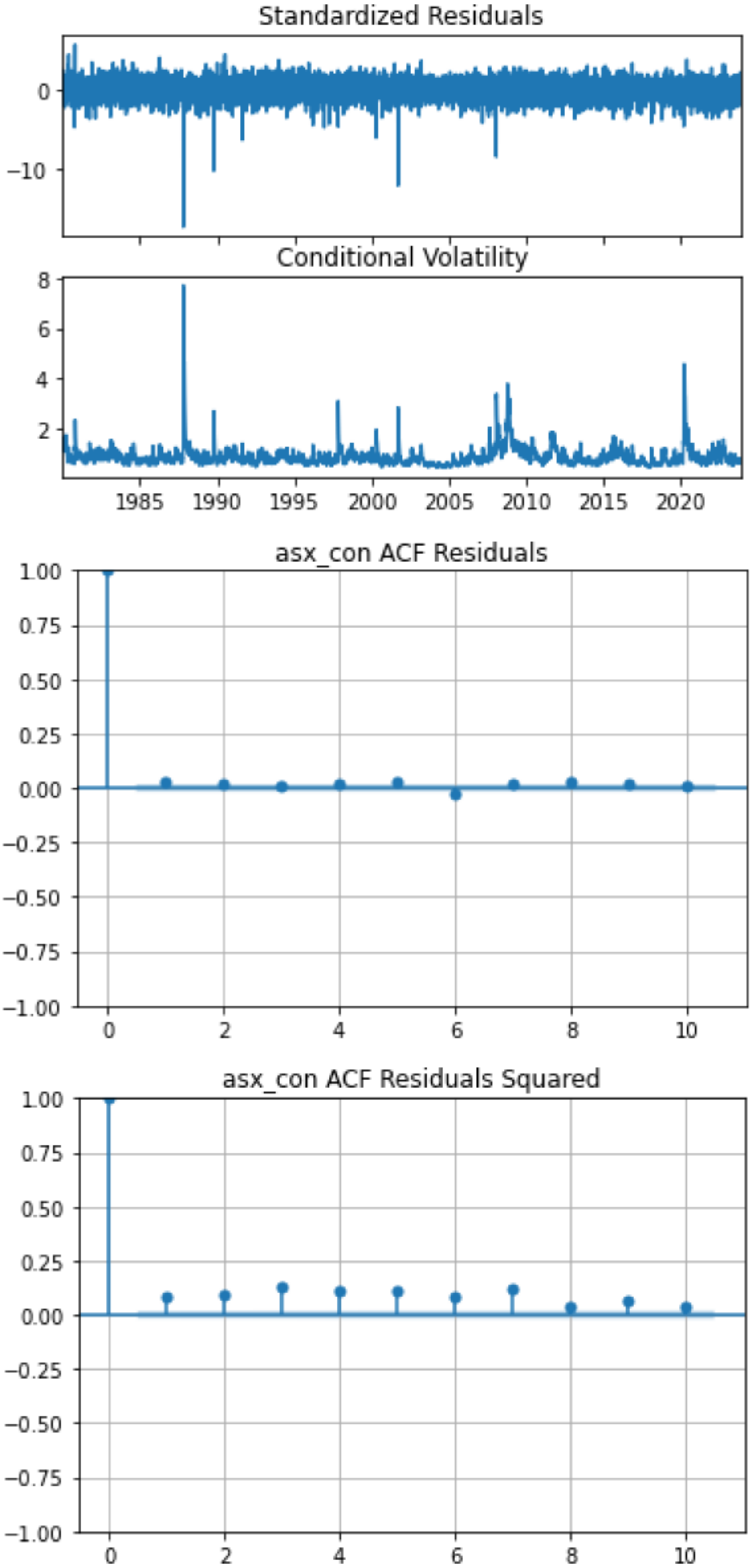




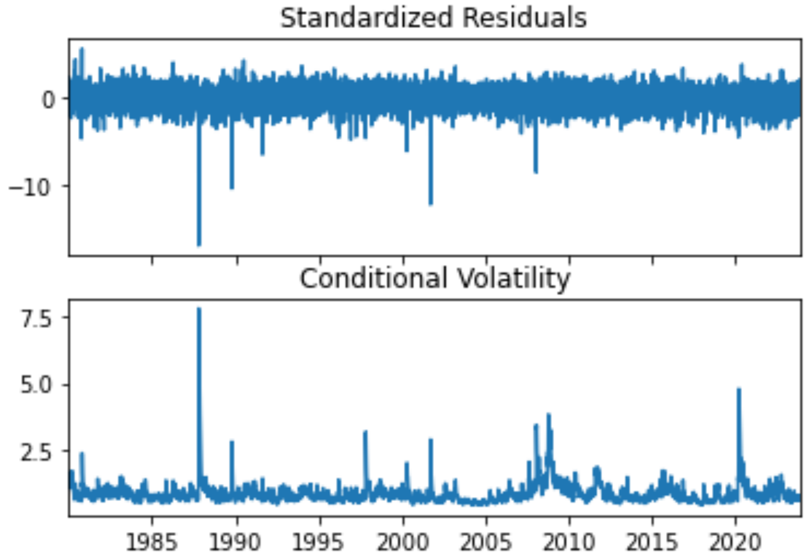
MODEL ~ SP500_AR

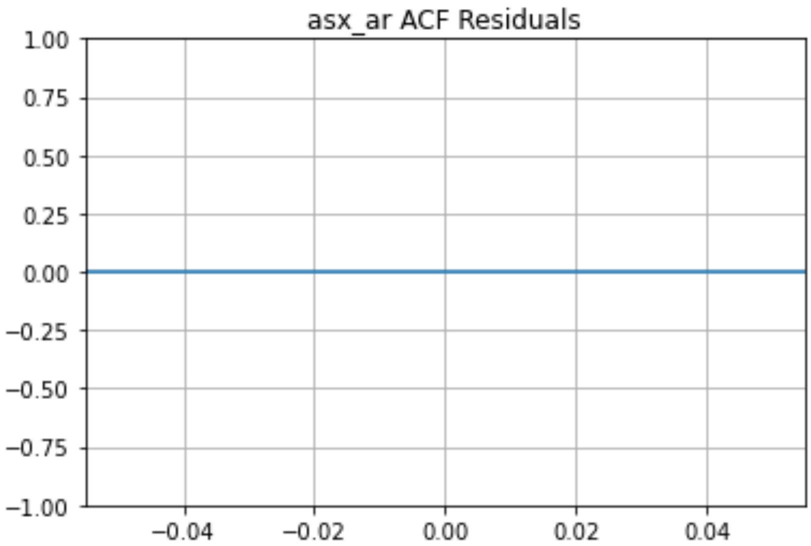


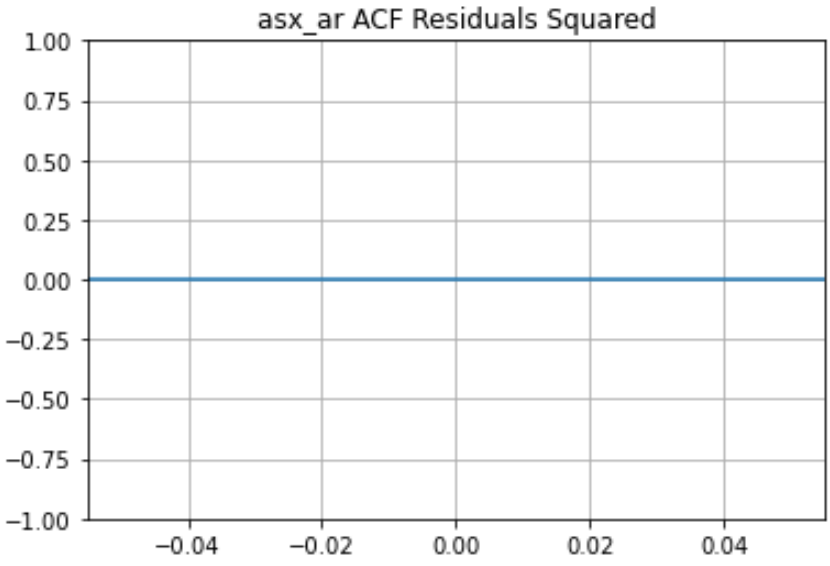
MODEL ~ ASX_CON



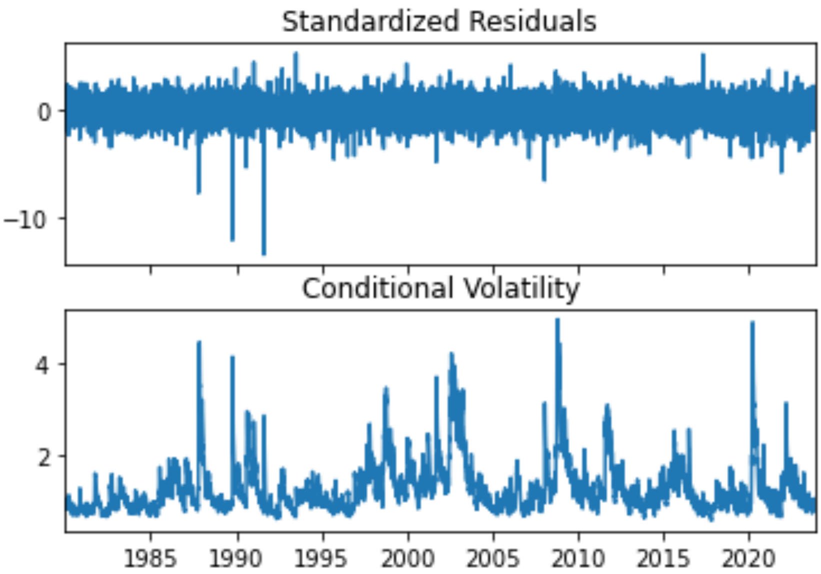
MODEL ~ ASX_AR

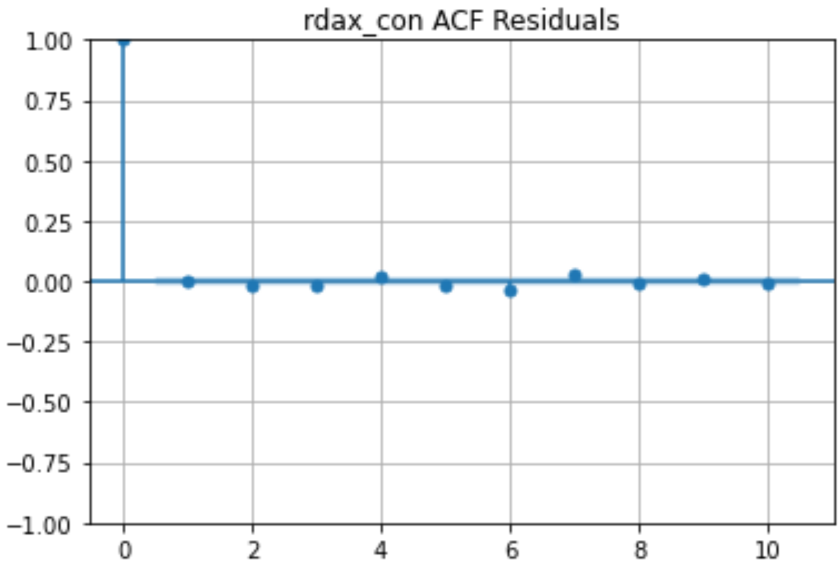

Standardized Residuals
0
−10
Conditional Volatility
7.5
5.0
2.5
1985
1990
1995
2000
2005
2010
2015
2020

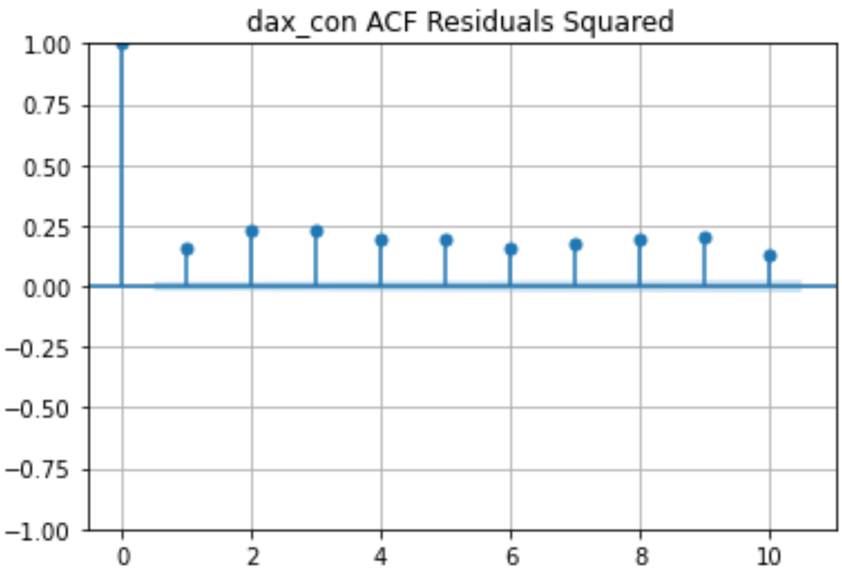

asx_ar ACF Residuals
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−0.04
−0.02
0.00
0.02
0.04


asx_ar ACF Residuals Squared
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−0.04
−0.02
0.00
0.02
0.04

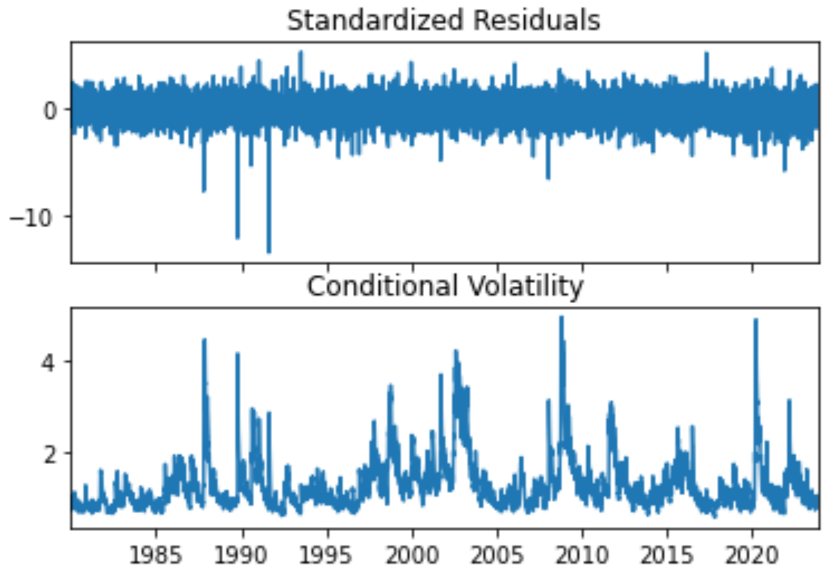
MODEL ~ DAX_CON

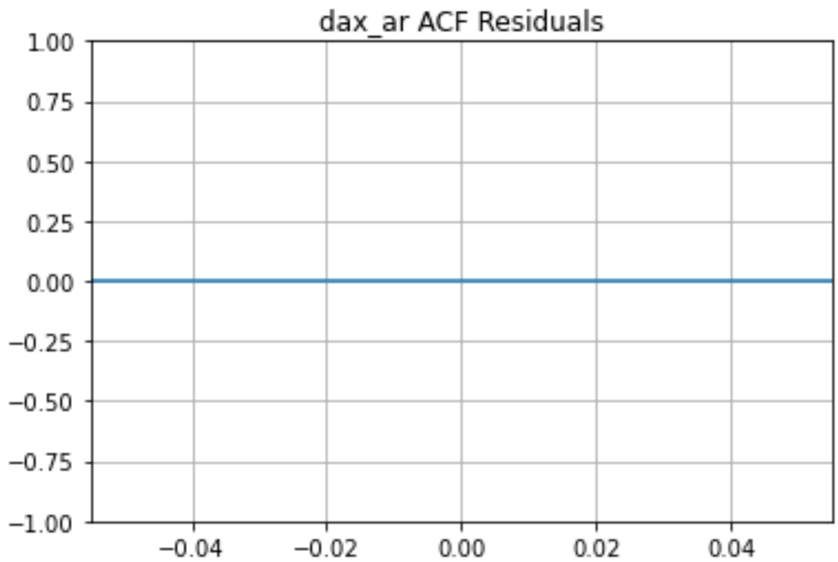


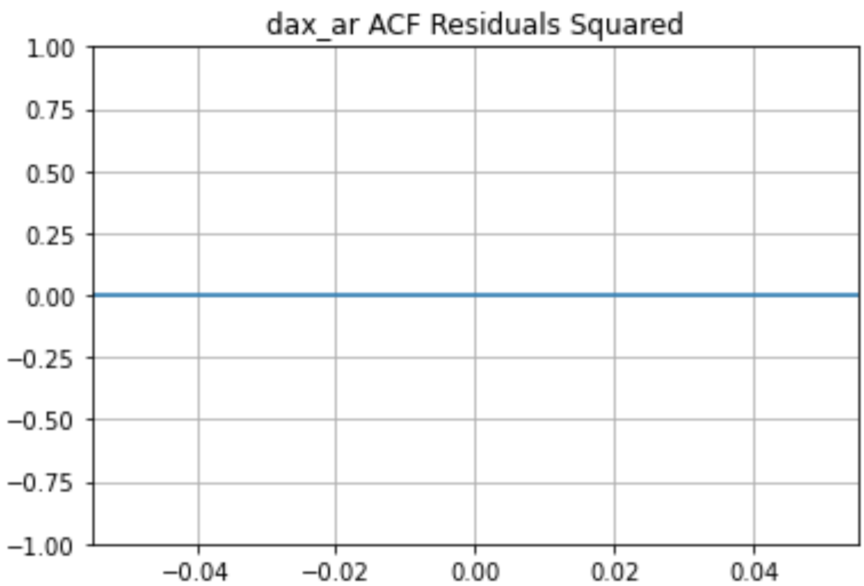




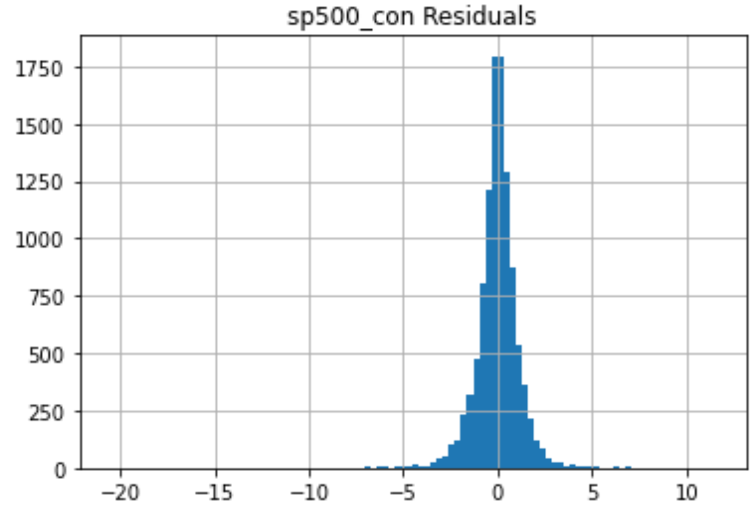
MODEL ~ DAX_AR

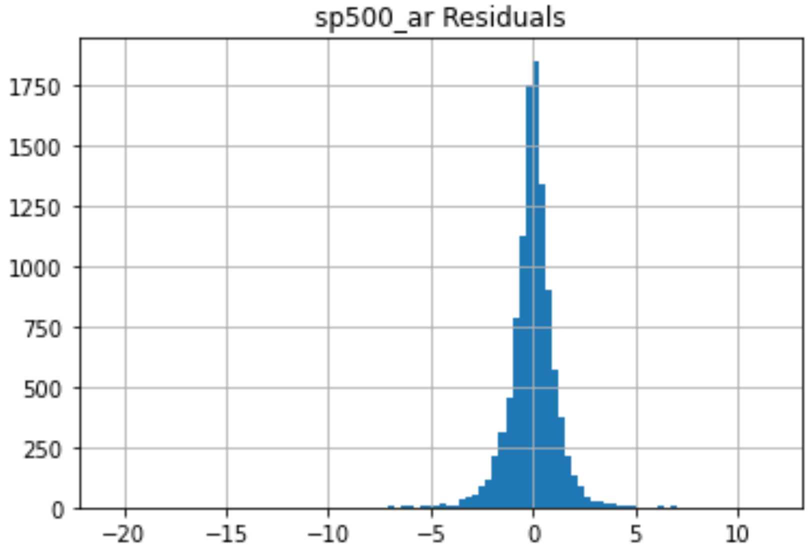

Standardized Residuals
0
−10
Conditional Volatility
4
2
1985
1990
1995
2000
2005
2010
2015
2020

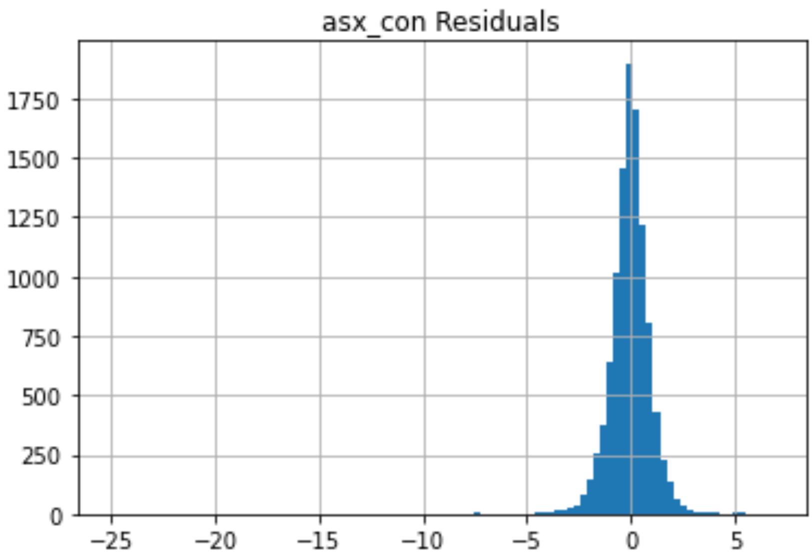

dax_ar ACF Residuals
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−0.04
−0.02
0.00
0.02
0.04

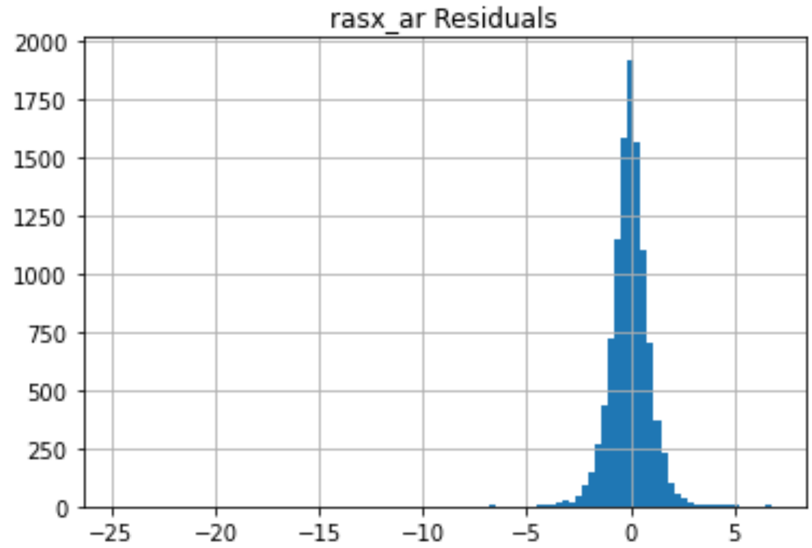

dax_ar ACF Residuals Squared
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−0.04
−0.02
0.00
0.02
0.04

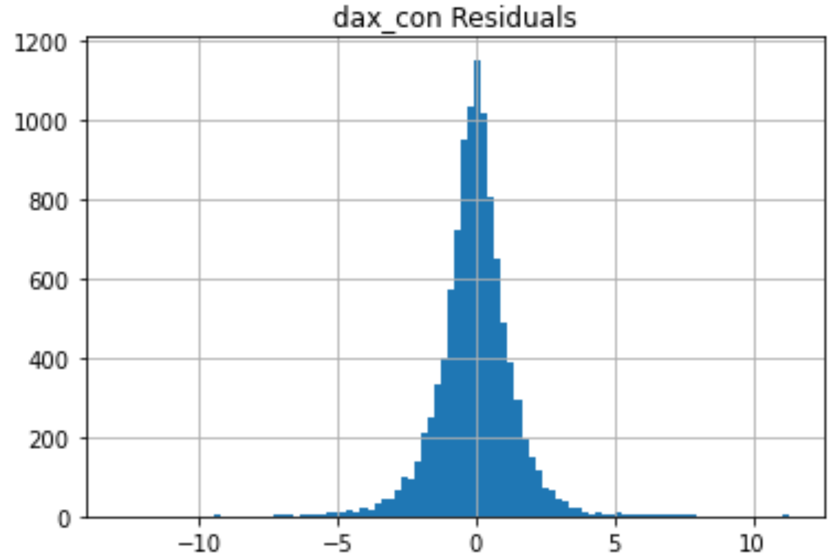
PLOTTING HISTOGRAMS AND QQ-PLOTS FOR EACH MODEL

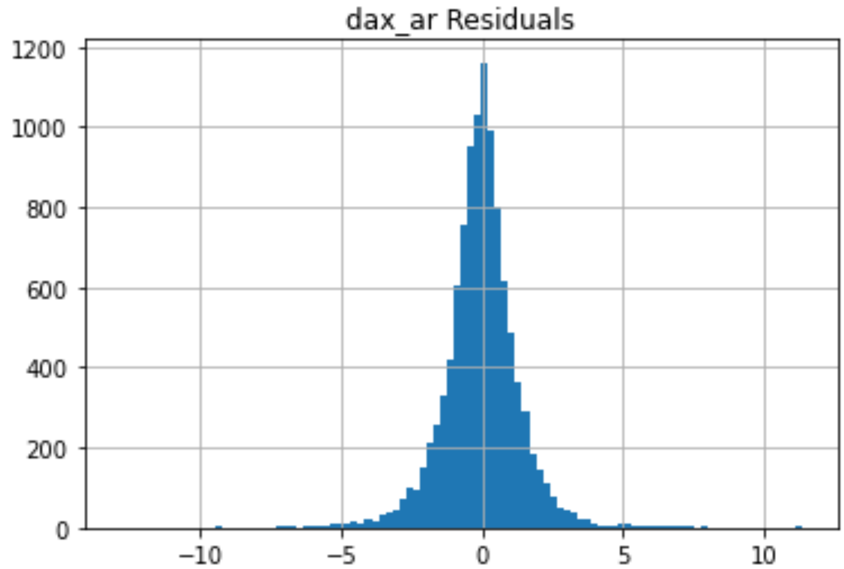


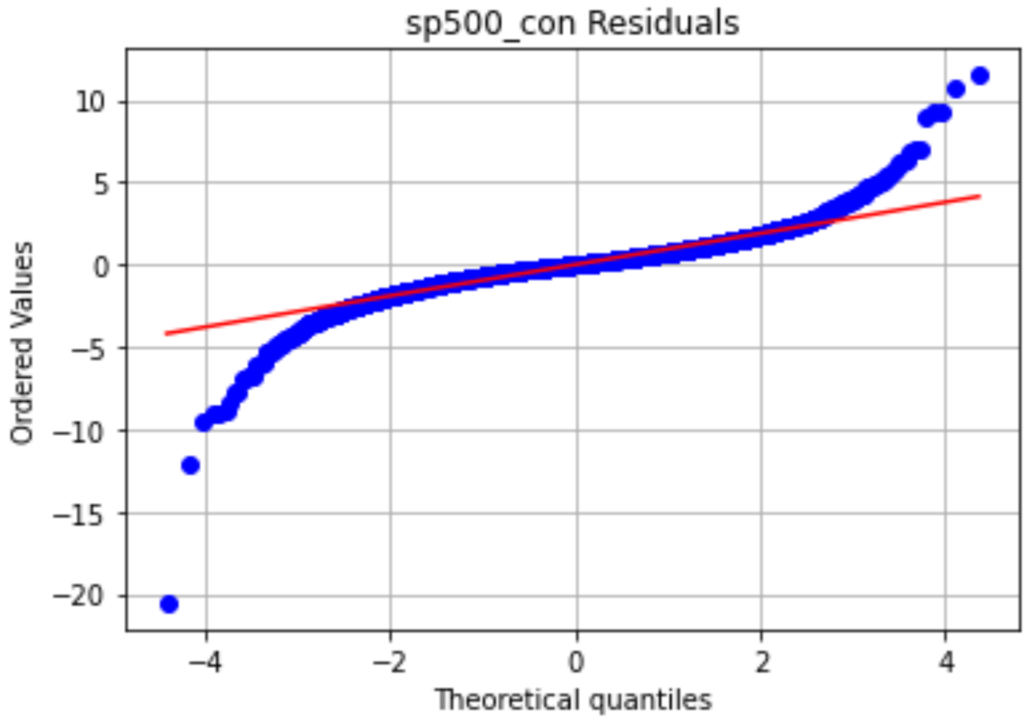


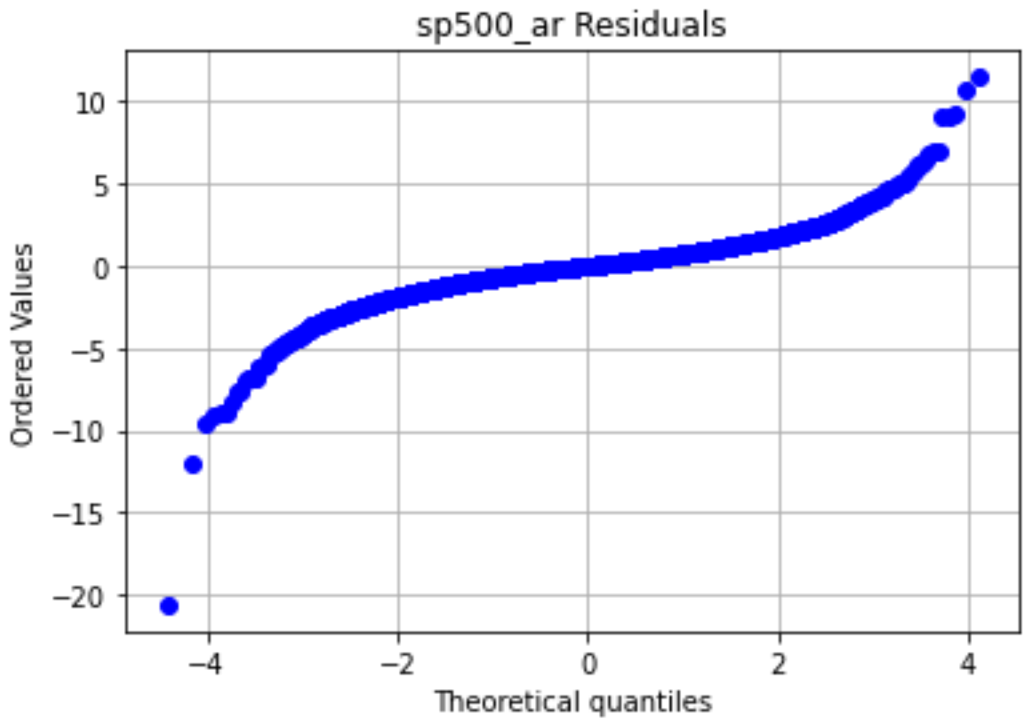


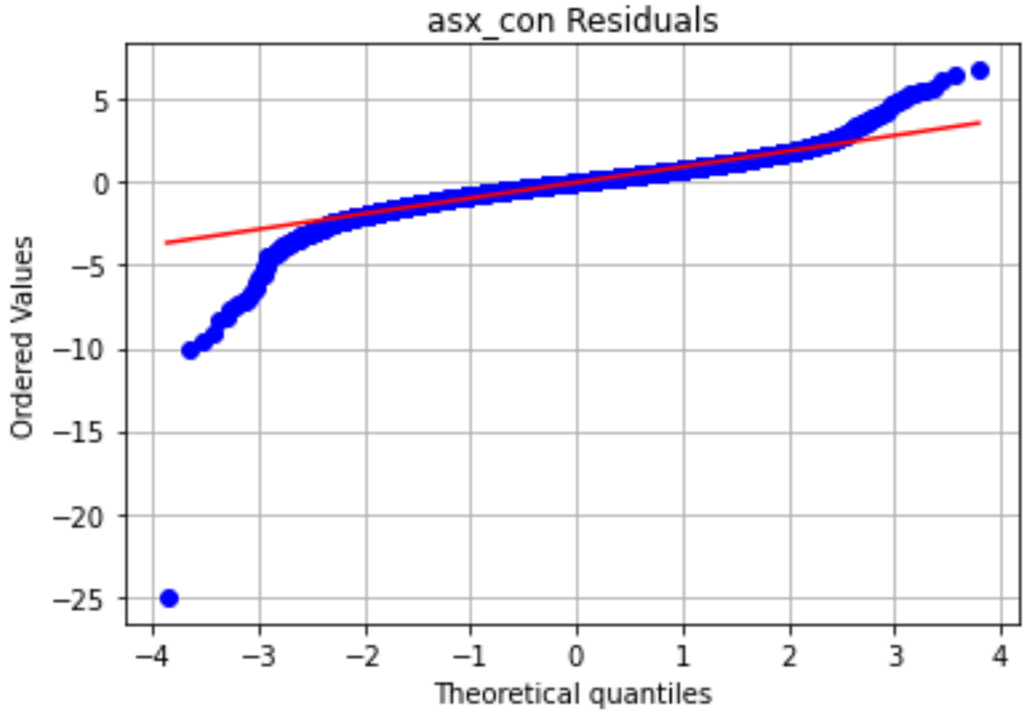

rasx_ar Residuals
2000
1750
1500
1250
1000
750
500
250
0
−25
−20
−15
−10
−5
0
5

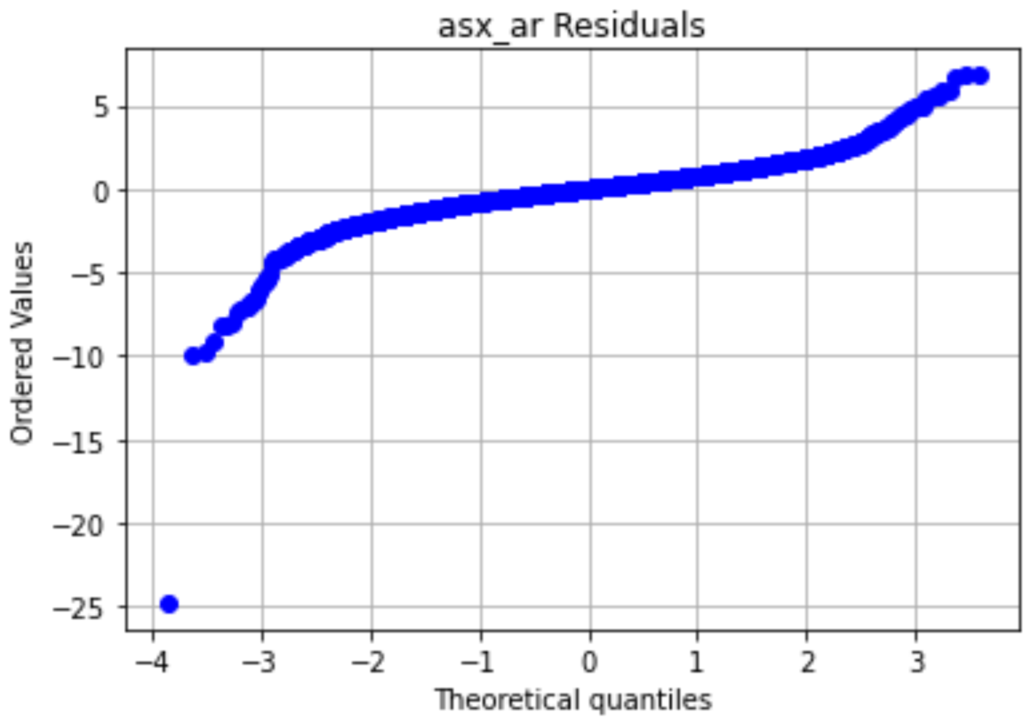

dax_con Residuals
1200
1000
800
600
400
200
0
−10
−5
0
5
10

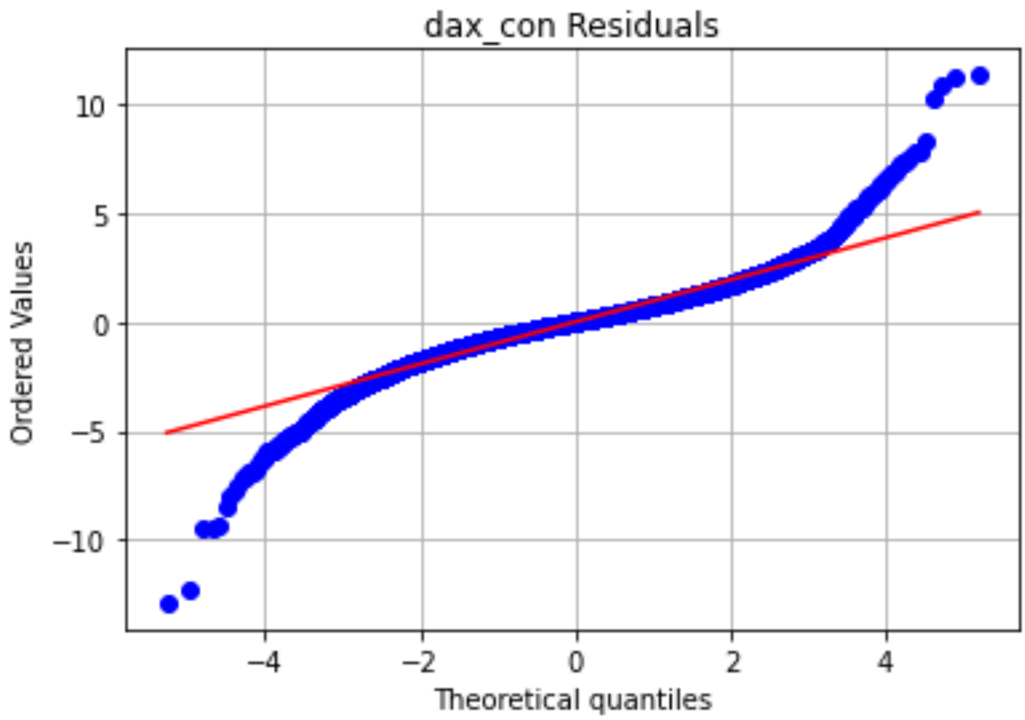

dax_ar Residuals
1200
1000
800
600
400
200
0
−10
−5
0
5
10

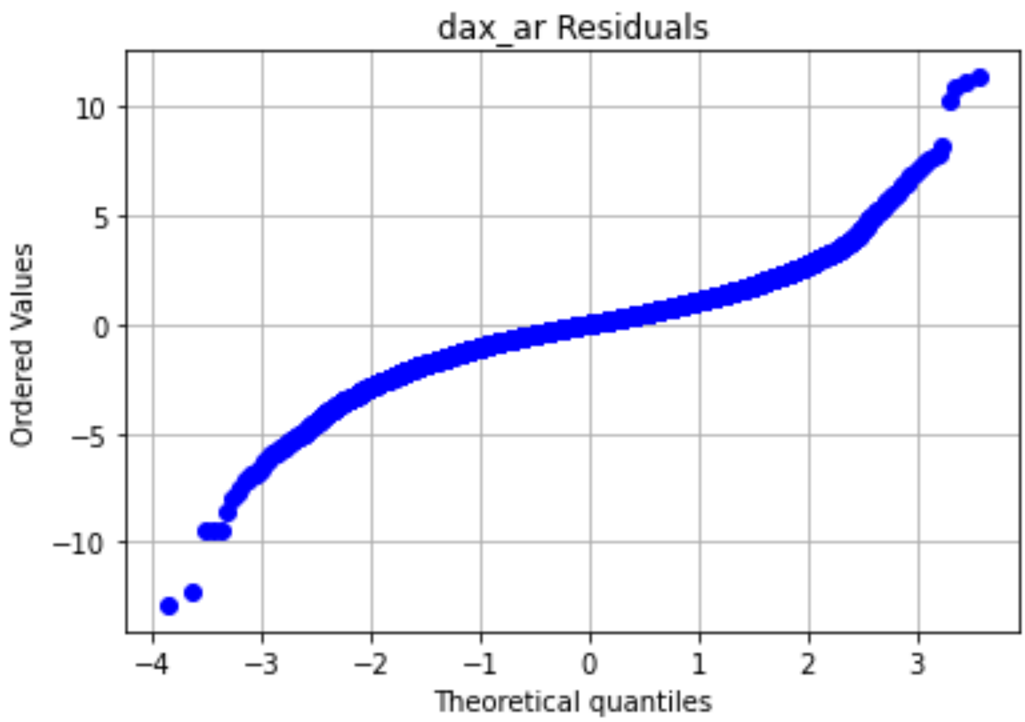

sp500_con Residuals
Ordered Values
Theoretical quantiles
10
5
0
−5
−10
−15
−20
−4
−2
0
2
4


sp500_ar Residuals
Ordered Values
Theoretical quantiles
10
5
0
−5
−10
−15
−20
−4
−2
0
2
4


asx_con Residuals
Ordered Values
Theoretical quantiles
5
0
−5
−10
−15
−20
−25
−4
−3
−2
−1
0
1
2
3
4


asx_ar Residuals
Ordered Values
Theoretical quantiles
5
0
−5
−10
−15
−20
−25
−4
−3
−2
−1
0
1
2
3





ANSWERS

ANS 1:

summary Of Findings for each index:

for all three naming S&P500, ASX and DAX:

- ADF Test: The process is weakly stationary.
- GARCH Coefficients: The coefficients seem reasonable.
- LRT: The AR-GARCH model is preferred over the constant mean GARCH model.

The GARCH(1,1) models for the ASX and DAX indices seems to be an appropriate fit.

For the S&P 500 index, the GARCH(1,1) model seemed to provide some inconclusive findings since the p-value from the log-likelihood test just barely exceeded the 0.05 benchmark necessary for a 5% confidence interval when comparing the constant model with the complex model.
However, this would not be the case if we changed our guidelines to a 6% confidence level.

The findings led to the conclusion that the squared residuals from lagged models for all three indices (sp500_ar, asx_ar, and dax_ar) consistently fall within acceptable ranges. This implies the effective capture of time-varying variance effects, strongly suggesting the presence and capture of GARCH effects.

ANS 2:
mu at t is dependent on returns at time t-1 so mu at t+1 is dependent on returns from time t coefficient of return lag is irrelevant since it was established that return at t is 0 in the question.

- S&P 500: Constant (mu): 0.0716
- ASX: Constant (mu): 0.0622
- DAX: Constant (mu): 0.0780

Mu(t) = phi(0) + phi * R(t-1); where phi(0) is the constant

Therefore, the index with the highest expected return at time t+1 when Rt = 0 is the DAX.