# Unit-IV

## Probability (chance)

Page No.
Date

1. Prob. ("that sun will rise in the East") = 1
2. Prob. ("that sun will rise in the West") = 0

$0 \le P(X) \le 1$

* Prob ("that I win chess game") = 50% (1/2)
* Prob ("that I win the race with 3 friend") = 1/4 (25%)

- Probability Experiment :- (Random Experiment)

→ Well defined result.

- Outcome :- 1 coin (H, T)

2 coin (HH, HT, TH, TT)

Set of all outputs / outcomes are called sample space. It is denoted by S.

(i) $S = \{HH, HT, TH, TT\}$

$n(S) = 4$

(ii) While roll a dice

Prob ("that one ('1') will upper on upper side")

Event = {1}

$n(S) = 6$

$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$ [Classical Prob.]

# Prob. is a general concept and can be defined as chance of an event occuring.

# Prob. experiment is a chance process that leads to well-defined results.

Ex:- Process such as flipping a coin, rolling dice or drawing a card are called prob. exp.

# Outcome is the result of single trial of a prob exp. while flipping a coin there are two possible outcomes H, T. and set of all posible outcomes is called as sample space.

∴ Prob of an event 'E' = $\frac{\text{no. of outcomes in E}}{\text{Total no. of out comes in S. space}}$

$$P(E) = \frac{n(E)}{n(S)}$$

This prob is called as classical probability.

(i) Find the sample space for rolling of two dice.

| | Dice 1 | | | | | |
|---|---|---|---|---|---|---|
| Dice 2 | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) |
| 2 | (2,1) | (2,2) | (2,3) | (2,4) | (2,5) | (2,6) |
| 3 | (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3,6) |
| 4 | (4,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) |
| 5 | (5,1) | (5,2) | (5,3) | (5,4) | (5,5) | (5,6) |
| 6 | (6,1) | (6,2) | (6,3) | (6,4) | (6,5) | (6,6) |

(ii) Three children :-
BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG

Empirical Probability :-

$$P(E) = \frac{f}{n} \quad \frac{(\text{frequency})}{(\text{total no.})}$$

Probability Distribution :-

* Variable → X, Y, Z   [Discrete, continuous]

X = 10

| Page No. | |
|---|---|
| Date | / / |

[Discrete, Continuous Rand. var]

* Random variable :- X, Y, Z ....

① $S = \{H, T\}$

(Variable) $X = \{H\}$

(Frequency) $P(X) = \frac{1}{2}$

P.D. Table

| X | {H} |
|---|---|
| P(x) | 1/2 |

② $S = \{HH, HT, TH, TT\}$

$X = 2, 1, 0$ (Head)

$P(X) = \frac{1}{4}, \frac{1}{2}, \frac{1}{4}$

| X | 2 | 1 | 0 |
|---|---|---|---|
| P(x) | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ |

* Discrete RV

→ Pmf (Prob. mass function)

→ cdf (Cum. distribution function)

* Continuous RV

→ Pdf (Prob. density function)

①

| X | 2 | 1 | 0 |
|---|---|---|---|
| P(x) | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ |

| cdf | $\frac{1}{4}$ | $\frac{3}{4}$ | 1 |
|---|---|---|---|

If the sum of row is 1 then this is called Pmf

sum = 1 (Pmf)

$\frac{1}{4} + \frac{1}{2} + \frac{1}{4} = \frac{4}{4} = 1$

* Expected mean - (Expectation (E]) ⇒

Mean $(\mu) = \Sigma x_i \cdot P(x_i)$

1. Find the mean of the no. of steps that appears when a dice is tossed

| Outcome (X) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Prob P(X) | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

2. $S = \{GG, GB, BG, BB\}$
$\quad 2 \quad 1 \quad 1 \quad 0$

Page No.
Date

We know that
Mean $(\mu) = \Sigma x_i \cdot P(x_i)$

$1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$

$\frac{1}{6}[1+2+3+4+5+6] = \frac{21}{6} = \frac{7}{3}$ or $\underline{3.5}$

2. In a family with 2 children find the mean of the no. of children who will be girls.

| Outcome (x) | 0 | 1 | 2 |
|---|---|---|---|
| Prob. (P(x)) | $\frac{1}{4}$ | $\frac{2}{4} = \frac{1}{2}$ | $\frac{1}{4}$ |

$\mu = 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4}$

$= \frac{1}{2} + \frac{2}{4} = \frac{2+2}{4} = \frac{4}{4} = 1$

3. $S = \{\overset{0}{BBB}, \overset{1}{BBG}, \overset{1}{BGB}, \overset{2}{BGG}, \overset{1}{GBB}, \overset{2}{GBG}, \overset{2}{GGB}, \overset{3}{GGG}\}$

| (x) | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(x) | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

$\mu = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$

$= \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = \frac{3}{2}$ or $\underline{1.5}$

4. Variance $(\sigma^2) = \Sigma[x_i^2 \cdot P(x_i)] - \mu^2$

| Outcome (x) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Prob. P(x) | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

$1 \times \frac{1}{6} + 4 \times \frac{1}{6} + 9 \times \frac{1}{6} + 16 \times \frac{1}{6} + 25 \times \frac{1}{6} + 36 \times \frac{1}{6} - 3.5$

Page No.
Date

$$\frac{1}{6}+\frac{4}{6}+\frac{9}{6}+\frac{16}{6}+\frac{25}{6}+\frac{36}{6}-3.5^2$$

$$\frac{91}{6}-3.5^2 \Rightarrow \cancel{2.95} \text{ or } 2.91$$

$$\sigma^2 = 2.91$$

$$S.D = \sqrt{2.91} = 1.70$$

5.

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(X) | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

$$0\times\frac{1}{8}+1\times\frac{3}{8}+4\times\frac{3}{8}+9\times\frac{1}{8}-2.25$$

$$\frac{0}{8}+\frac{3}{8}+\frac{12}{8}+\frac{9}{8}-2.25$$

$$\frac{24}{8}-2.25 = 3-2.25 = 0.75$$

$$S.D = \sqrt{0.75} = 0.86$$

V.V Important

# Binomial distribution

P.D

Discrete P.D. → Binomial D, Pioson D.

Continuous P.D → Normal D.

→ Swiss mathematician → James Bernoulle in 1700
Bernoulle distribution
2 outcomes :-
Prob of success (p), Prob of failure (q)

- There should be finite no of trials.
- Trials should be independent

Page No.
Date

- Trials should have exactly 2 outcomes
- Prob. of success remain same in each trials.

# $P(X=r)$ $^nC_r (p)^r \cdot (q)^{n-r}$

OR

# $P(X=r)$ $\frac{n!}{r!(n-r)!} (p)^r \cdot (q)^{n-r}$

where $n$ = no. of trials, $p$ = prob of success
$r$ = random variable, $q$ = prob of failure
where $p = 1-q$, $q = 1-p$

# Mean $= np$

Variance $\sigma^2 = npq$
$\sigma = \sqrt{npq}$
If $p = q = \frac{1}{2}$ (Symmetric dist)

If $p \neq q$ (skewed dist.)

# Bionomial dist.

$(q+p)^n$

(1) Determine the bionomial dist. whose mean is 9 & S.D $= 3/2$

Sol => We know that $B.D = [q+p]^n$ —— (1)
Also mean is given $\mu = np = 9$ —— (2)
S.D is also given $\sigma = \sqrt{npq} = 3/2$
Square both sides
$\sigma = (\sqrt{npq})^2 = \left(\frac{3}{2}\right)^2$

$npq = \frac{9}{4}$ —— (3)

Divide $= n$ III by $n$ II
$\frac{\cancel{np}q}{\cancel{np}} = \frac{\cancel{9}}{4} \times \frac{1}{\cancel{9}} = q = \frac{1}{4}$

Page No.
Date

Also we know that $p = 1 - q$

$$1 - \frac{1}{4} \Rightarrow \frac{4-1}{4} = \frac{3}{4}$$

$np = 9$

$n\left(\frac{3}{4}\right) = 9$

$$n = \frac{9}{1} \times \frac{4}{3} = \frac{36}{3} = 12$$

$B.D = (q+p)^n$

$$\left(\frac{1}{4} + \frac{3}{4}\right)^{12} = \frac{4}{4} = 1$$

(2) An unbiased coin is tossed 10 times. find using binomial dist.

→ a) Exactly 6 heads.

$P(X=r) = {}^nC_r \, p^r q^{n-r}$

Given $n = 10$

$p(\text{success}) = \frac{1}{2}$, $q(\text{failure}) = \frac{1}{2}$

$P(X=6) = {}^nC_r \, p^r q^{n-r}$

here $r = 6$

$p = \frac{1}{2}$, $q = \frac{1}{2}$, $n = 10$

$$P(X=6) = {}^{10}C_6 \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^{10-6}$$

$$\frac{10!}{6!\,4!} \times \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^4$$

$$\frac{10 \times 9 \times 8 \times 7 \times \cancel{6!}}{\cancel{6!}\,4!} \times \left(\frac{1}{2}\right)^{10}$$

$$\frac{\cancel{10}^{5} \times \cancel{9}^{3} \times \cancel{8}^{2} \times 7}{\cancel{4} \times \cancel{3} \times \cancel{2}} \times \left(\frac{1}{2}\right)^{10} = 5 \times 3 \times 2 \times 7 \times \frac{1}{1024}$$

Page No.
Date

$\frac{210}{1024} = 0.2050$

(b) Atleast 7 heads

$P(X=r) = {}^{n}C_r \cdot p^r \cdot q^{n-r}$

- $P(X \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$P(X=7) = {}^{10}C_7 \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^{10-7}$

$\frac{10 \times \cancel{9}^{3} \times \cancel{8}^{4} \times \cancel{7!}}{\cancel{7!}\,\cancel{3} \times \cancel{2}} \times \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^3$

$= 10 \times 3 \times 4 \times \left(\frac{1}{2}\right)^{10} \Rightarrow 10 \times 3 \times 4 \times \frac{1}{1024} = \underline{\underline{0.117}}$

- $P(X=8) = {}^{10}C_8 \left(\frac{1}{2}\right)^8 \times \left(\frac{1}{2}\right)^{10-8}$

$\frac{\cancel{10}^{5} \times 9 \times \cancel{8!}}{\cancel{8!}\,\cancel{2}} \times \left(\frac{1}{2}\right)^{10} = \underline{\underline{0.043}}$

- $P(X=9) = {}^{10}C_9 \times \left(\frac{1}{2}\right)^9 \times \left(\frac{1}{2}\right)^1$

$\frac{10 \times \cancel{9!}}{\cancel{9!}} \times \frac{1}{1024} = \underline{\underline{0.009}}$

- $P(X=10) = {}^{10}C_{10} \times \left(\frac{1}{2}\right)^{10} \times \left(\frac{1}{2}\right)^0$

$1 \times \frac{1}{1024} = \underline{\underline{0.0009}}$

- $0.117 + 0.043 + 0.009 + 0.0009 \Rightarrow 0.1699$

(c) $P(X \geq 2)$

$1 - [P(X=0) + P(X=1)]$

$1 - {}^{10}C_0 \times \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^{10} + {}^{10}C_1 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^9$

$1 - \left(\frac{1}{2}\right)^{10} [1 + 10] \Rightarrow 1 - \frac{11}{1024} = 0.9892$

Page No.
Date

(3) The Prob of a man hitting a target is $\frac{1}{4}$. He fires 7 times. what is the prob of his hitting atleast twice the target

→ $p = \frac{1}{4}$, $q \Rightarrow 1-p \Rightarrow 1-\frac{1}{4} = \frac{3}{4}$

$P(X \geq 2) = 1 - [P(X=0) + P(X=1)]$

$1 - \left[{}^7C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^7 + {}^7C_1\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^6\right]$

$1 - \left[\left(\frac{3}{4}\right)^7 + 7\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^6\right] = 1 - \left[\frac{2187}{16384} + \frac{5103}{16384}\right]$

$1 - \frac{7290}{16384} \Rightarrow 1 - 0.44 \Rightarrow 0.55.$

# Formula for $Y = \dfrac{e^{-\frac{(X-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$

$X$ = random variable
$\mu$ = population mean
$\sigma$ = population standard deviation

→ $\mu = 0$, $\sigma = 1$ (Standard normal dist.)

# Mutually Exclusive Events are events that can not occur at the same time. In other words, the occurance of one event means that the other event cannot happen simultaneously. (They have no outcome as common).

Ex (i) Determine which events are mutually exclusive, when a single dice is roll.

(a) Getting an odd no & even no.

→ $E_1 = 1, 3, 5$ , $E_2 = 2, 4, 6$

These events are mutually exclusive, since the first event can be 1,3,5 and the second event can 2,4,6 and they have no common outcome.

(b) Getting a 3 and an odd no.

$E_1 \rightarrow 3$ , $E_2 \rightarrow 1, 3, 5$ (No)

(c) Getting an odd no. and getting an no. less than 4.

$E_1 \rightarrow 1, 3, 5$ $E_2 \rightarrow 1, 2, 3$ (No)

(d) Getting a no greater than 4 & less than 4.

$E_1 \rightarrow 5, 6$ $E_2 \rightarrow 1, 2, 3$

# Addition rule 1 = The Prob. of two or more events can be determined by the addition rule. The first addition rule is used when the events are mutually exclusive.

→ The addition rule 1 states, "when two events A and B are mutually exclusive the prob. of A or B will occur is $P(A \text{ or } B) = P(A) + P(B)$."

(i) A city has 9 coffee shops, three starbucks, two carbou and 4 crazy mocho. If a person select 1 shop at random to buy a cup of coffee, find the prob that it is either a starbuck or crazy mocho.

$A = 3$ SB , $B = 2$ Carbou, $C = 4$ Crazy mocho

$P(A \text{ or } C) = P(A) + P(C)$

$$\frac{3}{9} + \frac{4}{9} = \frac{7}{9}$$

Since there are 3 starbucks & 4 crazy mocho coffee shop, and a total of 9 shops. Therefore acc to addition rule 1, we have

$P(\text{Starbucks or Crazy mocho}) = \frac{3}{9} + \frac{4}{9} = \frac{7}{9}$