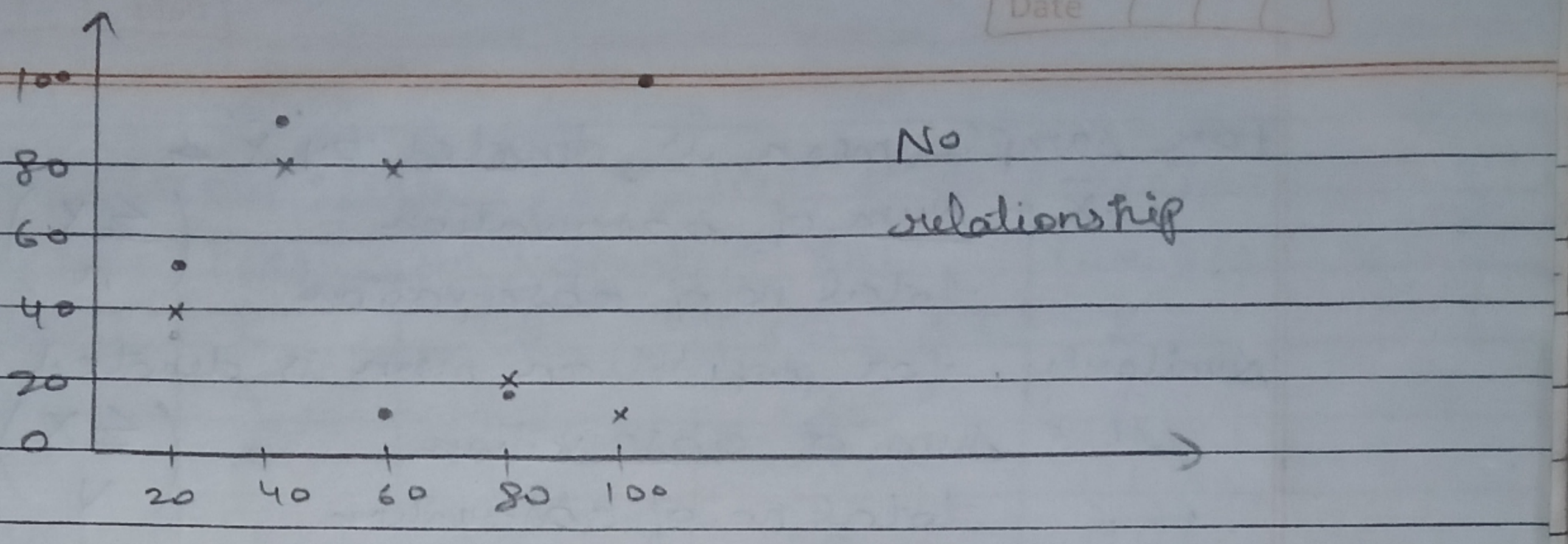
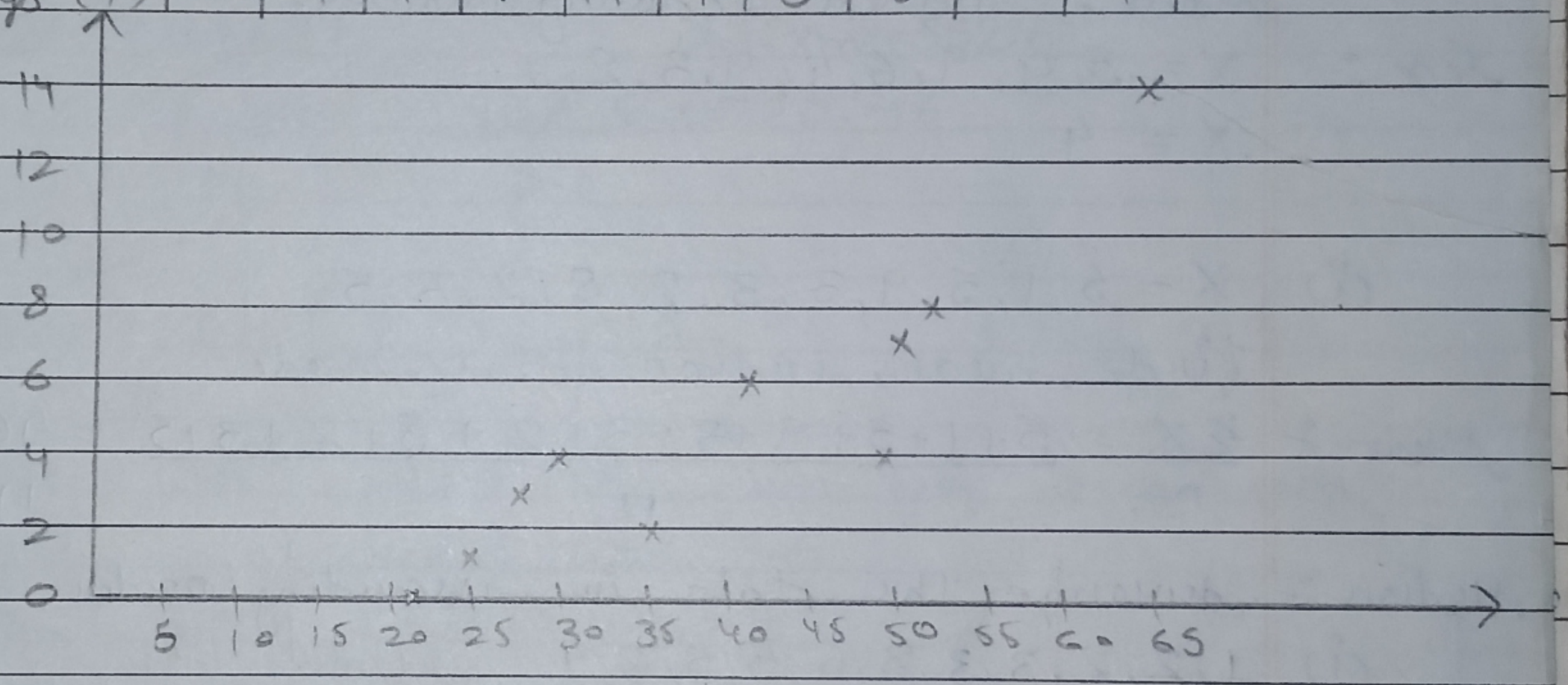


Age (x)	22	30	25	35	65	50	27	53	42	48
Days (y)	0	4	1	2	14	7	3	8	6	4



Unit = 3

Data description :-

* Measure of central tandency
→ Mean, Mode, Median for simple data & group data.
* Measure of dispersion.
→ Range, S.D, variance for population & sample
* Measure of position.
→ Percentage, Quartiles, Deciles, IQR, Kurtosis.

(i) Mean / Averages :- It can be defined as sum of the observation to the total no. of observations.
Mean (μ), Sample ($\bar{x}$)

for sample mean is denoted by '$\bar{x}$' &

$$\bar{x} = \frac{\text{sum of observation}}{\text{total no. of observation}} \quad \left(\frac{\Sigma x}{n}\right)$$

Similarly, for population mean is denoted by 'μ'

$$\mu = \frac{\text{sum of observation}}{\text{total no. of observation}} \quad \left(\frac{\Sigma x}{N}\right)$$

Median :- Central most element / item

Mode :- highest occuring number.

Ex :- $X = 5, 4, 9, 6, 4, 4, 3, 2, 1$

$X = 4$

(i) $X = 5, 1, 5, 9, 3, 3, 2, 8, 2, 3, 5$

find mean, median and mode

Mean :- $\frac{\Sigma X}{n} = \frac{5+1+5+9+3+3+2+8+2+3+5}{11} = \frac{46}{11} = 4.18$

Median :- Arrange the data in ascending order.

(odd) (i) $1, 2, 2, 3, 3, \underline{3}, 5, 5, 5, 8, 9.$ (Middle value)

3

(even) (ii) $1, 2, 2, 3, 3, \check{3} \mid \check{5}, 5, 5, 8, 9, 10.$

$\frac{3+5}{2} = \frac{8}{2} = 4$

Mode :- 3, 5 (Bimodal) There are two mode

Multimodal ⇒ There are more than 2 modes.

(iii)

Mean :- $\frac{\Sigma X}{n} = \frac{15+11+5+9+3+13+12+18+21+13}{10} = \frac{120}{10} = 12$

Median :- $3, 5, 9, 11, \check{12} \mid \check{13}, 13, 15, 18, 21$

$\frac{12+13}{2} = \frac{25}{2} = 12.5$

Mode:- 13

From Group Data, find Mean, Mode & Median.

Class-Int	Freq.	xm → Midpoint	f. xm	Cum. freq.	
5-10	2	7.5	2×7.5 = 15.0	2	
10-15	5	12.5	62.5	7	
15-20	4	17.5	70	(11)	cf
[20-25]	[8]	22.5	180	[19]	
25-30	7	27.5	192.5	26	
30-35	1	32.5	32.5	27	

$\Sigma f = 27$ $\Sigma f.xm = 552.5$

$\therefore$ Mean $= \frac{\Sigma f.xm}{N} = \frac{552.5}{27} = 20.46$

$\therefore$ Median $= l + \left[\frac{\frac{N}{2} - Cf}{f}\right] \times h$

$\frac{N}{2} = \frac{27}{2} = 13.5$ where the cum. freq. cross the value i.e: 13.5

Now, $l = 20$, $h = 5$, $Cf = 19$, $f = 8$

Median $= 20 + \left[\frac{13.5 - 19}{8}\right] \times 5$

$= 20 + \left[\frac{-5.5}{8}\right] \times 5 \Rightarrow 20 - 3.4375 \Rightarrow 16.5625$

(ii)

Class-Int	Freq.	Mid point	f. xm	Cum. freq.	
5-10	5	7.5	37.5	5	
10-15	10	12.5	120	15	
15-20	2 f_0	17.5	35	(17)	cf
[20-25]	[15] f_1	22.5	337.5	[32]	
25-30	9 f_2	27.5	247.5	41	
30-35	7	32.5	227.5	48	

$\Sigma f = 48$ $\Sigma f.xm = 1,010$

$\therefore$ Mean $= \frac{\Sigma f \cdot xm}{N} = \frac{1,010}{48} = 20.9375$ ~~21.0416~~

$\therefore$ Median $= \frac{N}{2} = \frac{48}{2} = 24$

Median $= l + \left[\frac{\frac{N}{2} - C.f}{f}\right] \times h \Rightarrow 20 + \left[\frac{24-32}{15}\right] \times 5$

$20 + \left[\frac{-8}{15}\right] \times 5 \Rightarrow 20 - 2.66 \Rightarrow 17.34$

$\therefore$ Mode $= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

$20 + \left[\frac{15-2}{30-2-9}\right] \times 5 \Rightarrow 20 + \left[\frac{13}{19}\right] \times 5$

$20 + 3.42 \Rightarrow 23.42$

0.005

C.f	(iii)	Class-Int	Class-boundary	Freq	Mid-point	f.xm
7		2.48-7.48	2.475-7.485	7	4.98	34.86
10		7.49-12.49	7.485-12.495	3	9.99	29.97
11		12.50-17.50	12.495-17.505	1 f_0	15	15
18		17.51-22.51	17.505-22.515	7 f_1	20.01	140.07
23		22.52-27.52	22.515-27.525	5 f_2	25.02	125.1
28		27.53-32.53	27.525-32.535	5	30.03	150.15

$\Sigma f = 28$ $\Sigma f xm = 495.15$

Mean $= \frac{\Sigma f xm}{N} = \frac{495.15}{28} = 17.6$

Median $= \frac{N}{2} = \frac{28}{2} = 14$

$l + \left[\frac{\frac{N}{2} - C.f}{f}\right] \times h \Rightarrow 17.505 + \left[\frac{14-18}{7}\right] \times 5.01$

$17.505 \left[\frac{-4}{7}\right] \times 5.01 \Rightarrow 17.505 - 2.862 \Rightarrow 14.643$

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h = 17.505 + \left[\frac{7-1}{14-1-5}\right] \times 5.01$$

$$17.505 + \left[\frac{6}{8}\right] \times 5.01 \Rightarrow 17.505 + 3.7575 = 21.2625$$

(iv)

Class int	Freq.	Mid-point	$f \times m$	C.f.
0.8 - 4.4	26 f_1	2.6	67.6	26
4.5 - 8.1	11 f_2	6.3	69.3	37
8.2 - 11.8	4	10	40	41
11.9 - 15.5	5	13.7	68.5	46
15.6 - 19.2	2	17.4	34.8	48
19.3 - 22.9	1	21.1	21.1	49
23.0 - 26.6	1	24.8	24.8	50

$\Sigma f = 50$ $\Sigma f.xm = 326.1$

$$\text{Mean} = \frac{\Sigma f xm}{N} = \frac{326.1}{50} = 6.522$$

$$\text{Median} = \frac{N}{2} = \frac{50}{2} = 25$$

$$0.8 + \left[\frac{25-26}{26}\right] \times 3.6 \Rightarrow 0.8 + \left[\frac{-1}{26}\right] \times 3.6$$

$$0.8 + (-0.13) = +0.67$$

$$\text{Mode} = 0.8 + \left[\frac{26}{52-0-11}\right] \times 3.6 \Rightarrow 0.8 + \left[\frac{26}{41}\right] \times 3.6$$

$$0.8 + 2.28 \Rightarrow 3.08$$

2. Measure of Dispersion :-

(i) Variance, Standard deviation and Range.

① S.D for population $= \sigma = \sqrt{\sigma^2} = \sqrt{\frac{\Sigma(x-\mu)^2}{N}}$

→ Variance $= \sigma^2 = \frac{\Sigma(x-\mu)^2}{N}$

Date / / /

Q. 1 5, 9, 3, 8, 6, 12, 15, 1 Calculate variance & S.D.

We know that $\sigma^2 = \frac{\Sigma(x-\mu)^2}{N}$

Step I :-

$\therefore \quad \mu = \frac{\Sigma x}{N} = \frac{59}{8} = 7.3 \simeq 7$

Step II :- $\underline{(x-\mu)^2} = (5-7)^2, (9-7)^2, (3-7)^2, (8-7)^2, (6-7)^2, (12-7)^2, (15-7)^2, (1-7)^2$

Step III :- $\underline{\Sigma(x-\mu)^2} = (-2)^2 + (2)^2 + (-4)^2 + (1)^2 + (-1)^2 + (5)^2 + (8)^2 + (-6)^2$

$4+4+16+1+1+25+64+36 = 151$

Also, we know $\sigma^2 = \left(\frac{\Sigma(x-\mu)^2}{N}\right) = \frac{151}{8} = 18.8 \simeq 19$

$\therefore$ variance $= \sigma^2 = 19.$

Also, we know that S.D $= \sigma = \sqrt{\sigma^2}$

$\therefore$ S.D $= \sigma = \sqrt{\sigma^2} = \sqrt{19} = 4.35 \simeq 4.$

Q. 2 15, 29, 31, 18, 16, 12, 15, 19, 8, 13, 17, 14

$\longrightarrow \quad \mu = \frac{\Sigma x}{N} = \frac{207}{12} = 17.25 \simeq 17$

$\underline{(x-\mu)^2} = (15-17)^2, (29-17)^2, (31-17)^2, (18-17)^2, (16-17)^2, (12-17)^2, (15-17)^2, (19-17)^2, (8-17)^2, (13-17)^2, (17-17)^2, (14-17)^2.$

$\Sigma(x-\mu)^2 \quad (-2)^2 + (12)^2 + (14)^2 + (1)^2 + (-1)^2 + (-5)^2 + (-2)^2 + (+2)^2 + (-9)^2 + (-4)^2 + (0) + (-3)^2$

$= 4 + 144 + 196 + 1 + 1 + 25 + 4 + 4 + 81 + 16 + 9$

$= 485$

$\sigma^2 = \frac{\Sigma(x-\mu)^2}{N} = \frac{485}{12} = 40.41 \simeq 40$

S.D $= \sqrt{\sigma^2} = \sqrt{40} = 6.32$

Sample $= S^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$ (variance)

$$S.D = \sqrt{S^2} = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n-1}}$$

(i) $X = 5, 9, 4, 3, 8, 2, 1, 6.$

I $\mu = \frac{\Sigma X}{N} = \frac{38}{8} = 4.75 = 5$

II $(x-\bar{x})^2 = (5-5)^2, (9-5)^2, (4-5)^2, (3-5)^2, (8-5)^2, (2-5)^2, (1-5)^2, (6-5)^2$

III $\Sigma(x-\bar{x})^2 = (0)^2 + (4)^2 + (-1)^2 + (-2)^2 + (3)^2 + (-3)^2 + (-4)^2 + (1)^2$

IV $0+16+1+4+9+9+16+1 = 56$

V $\frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{56}{7} = 8$

VI $\sqrt{S^2} = \sqrt{8} = 2.82$

(ii) S.D & variance for grouped data.

Procedure for S.D & variance

I Make a table as shown and find the mid-point of each class.

A	B	C	D	E
Class	Frequency	mid-point	$f.xm$	$f.(xm)^2$

II Multiply the frequency by the mid points & place the product in column D.

III Multiply the freq. by the square of mid point & place the product in column E.

IV Substitute in the ~~sample~~ formula & solve to get the variance.

$$S^2 = \frac{n(\Sigma f.(xm)^2) - (\Sigma f.xm)^2}{n(n-1)}$$

Q Find the variance & S.D for the freq. dist. of the

Page No.
Date

data. The data represents the number of miles that 20 runners ran during one week.

Class	Freq.	Mid-point
5.5 - 10.5	1	8
10.5 - 15.5	2	13
15.5 - 20.5	3	18
20.5 - 25.5	5	23
25.5 - 30.5	4	28
30.5 - 35.5	3	33
35.5 - 40.5	2	38

A Class	B Freq.	mid point C $(xm)^2$	D $f \cdot xm$	$f \cdot (xm)^2$ E
5.5 - 10.5	1	8 64	8	$1 \times (8)^2 = 64$
10.5 - 15.5	2	13 169	26	338
15.5 - 20.5	3	18 324	54	972
20.5 - 25.5	5	23 529	115	2645
25.5 - 30.5	4	28 784	112	3136
30.5 - 35.5	3	33 1089	99	3267
35.5 - 40.5	2	38 1444	76	2888

$\Sigma(f \cdot xm) = 490$ $\Sigma f(xm)^2 = 13310$

$$s = \frac{n(\Sigma f \cdot (xm)^2) - (\Sigma f \cdot xm)^2}{n(n-1)}$$

$$= \frac{20(13310) - (490)^2}{20(20-1)}$$

$$= 68.68$$

(iii)

Class int	freq	xm	xm^2	f.xm	$f.xm^2$
5 - 10	2	7.5	56.2	15	112.4
10 - 15	3	12.6	156.2	37.5	468.6
15 - 20	5	17.5	306.2	87.5	1531
20 - 25	1	22.5	506.2	22.5	506.2

					4537.2
25-30	6	27.5	756.2	165	2730.2
30-35	2	32.5	1056.2	65	2112.2
				$\Sigma f \cdot xm = 392.5$	9268.75

$$s^2 = \frac{n\Sigma(f \cdot (xm)^2) - \Sigma(f \cdot xm)^2}{n(n-1)}$$

$$= \frac{19(9268.75) - (154056.2)}{19(18)} = 64.47.$$

$$\rightarrow s = \sqrt{s^2} = \sqrt{64.47} = 8.029.$$

(iii)

Class int	freq.	xm	xm^2	f.xm	$f.xm^2$
0-10	22	5	25	110	550
10-20	31	15	225	465	6975
20-30	15	25	625	375	9375
30-40	10	35	1225	350	12250
40-50	6	45	2025	270	12150
50-60	12	55	3025	660	36300
	$\Sigma = 96$			$\Sigma = 2230$	$\Sigma = 77600$

$$s^2 = \frac{96(77600) - (4972900)}{96(95)} = \frac{2476700}{9120} = 271.56$$

(iv)

NL	Freq.	xm	xm^2	f.xm	$f.xm^2$
0.252-0.256	4	0.254	0.064516	1.016	0.258064
0.257-0.261	6	0.259	0.067081	1.554	0.402486
0.262-0.266	1	0.264	0.069696	0.264	0.069696
0.267-0.271	4	0.269	0.072361	1.076	0.289444
0.272-0.276	1	0.274	0.075076	0.274	0.075076
				$\Sigma = 4.184$	$\Sigma = 1.094766$

$$= \frac{16(1.094766) - (17.505856)}{16(15)} = \frac{17.516256 - 17.505856}{240}$$

$$\frac{0.0104}{240} = 0.000043$$

* Measure of position
→ Z-score
→ Decile
→ Quartile
→ Percentile.

- Z-score :- Value - mean / SD.

For sample = $\frac{X - \bar{X}}{S}$

For pop- = $\frac{X - \mu}{\sigma}$

1. A student score 65 on calculus test that have a mean of 50 and a SD of 10. She score 30 on a history test with a mean of 25 & SD of 5. Compare her relative positions on the two tests.

→ $\frac{65 - 50}{10} = \frac{15}{10} = 1.5$ (calculus)

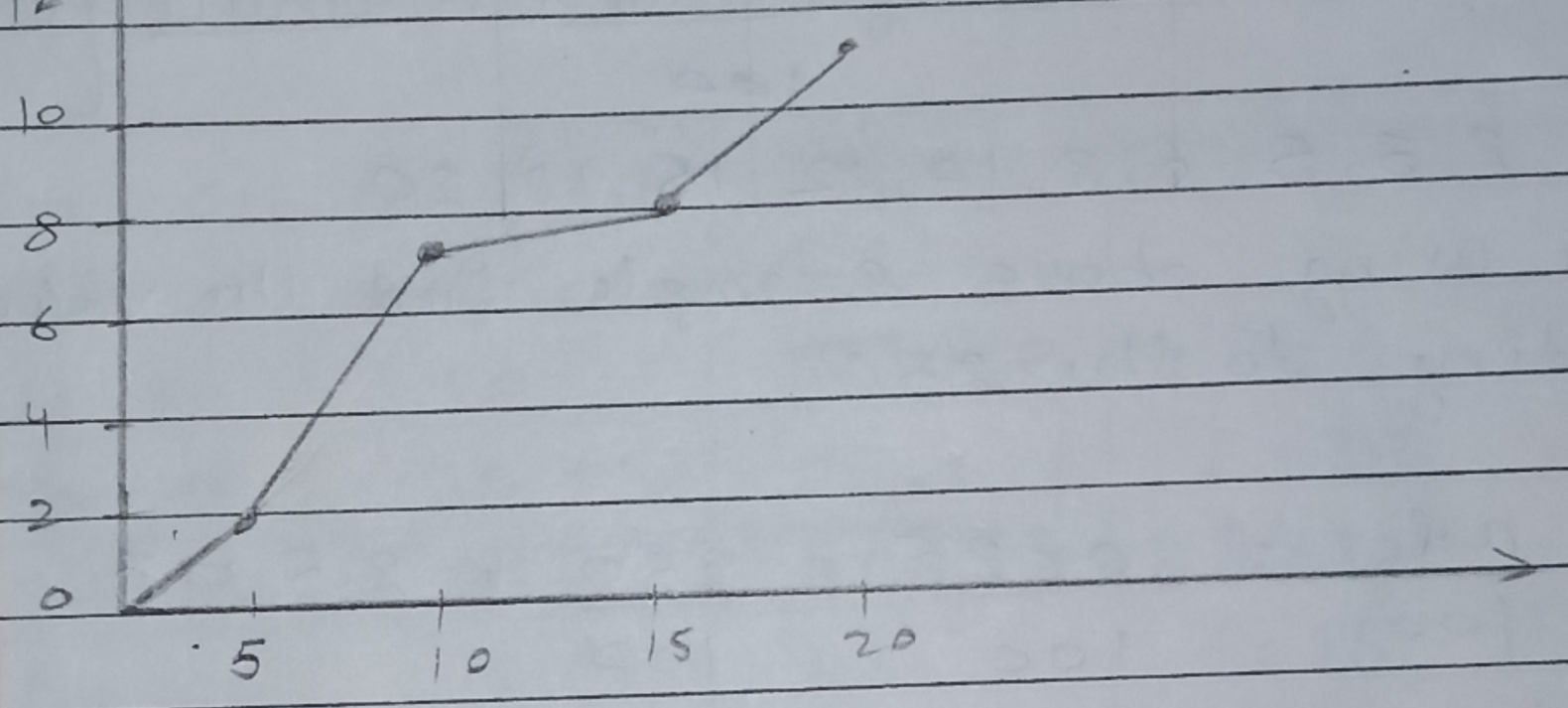
→ $\frac{30 - 25}{5.} = \frac{5}{5} = 1$ (History)

Since the z-score for calculus is large, her relative position in the calculus test is higher than her relative position in the history test.

- Percentile :- $\frac{\text{obtained m}}{\text{total m}} \times 100$

P. rank = $\frac{(\text{no. of value below } X) + 0.5}{\text{total no. of value}} \times 100$

7. Class boundary	Freq.	Cum. freq.	Cum %
0 - 5	2	2	(2/11) x 100 = 18.18
5 - 10	5	7	(7/11) x 100 = 63.63
10 - 15	1	8	(8/11) x 100 = 72.72
15 - 20	3	11	(11/11) x 100 = 100
	11		



P. rank = $\frac{(\text{below value of } x + 0.5)}{\text{total no of value}} \times 100$

$x = 1, 2, 4, [5], 6, 8, 9$ (Ascending order)

P. rank = $\frac{3 + 0.5}{7} \times 100$

1. A teacher gives a 20 point test to 10 student. The scores are shown here find the Percentile rank of a score of 12.

→ $x = 18, 15, 12, 6, 8, 2, 3, 5, 20, 10$

Arranged the data in ascending order.

2, 3, 5, 6, 8, 10, (12), 15, 18, 20

P. rank = $\frac{6 + 0.5}{10} \times 100 = 65.0$

Thus the student who score was 12 did better than

65 % of the class.

→ X = 2, 3, 5, (6), 8, 10, 12, 15, 18, 20

$= \frac{3+05}{10} \times 100 = 35.0$

2. Combination = $\frac{\text{no of values} \times \text{Percentile}}{100}$ $\left[c = \frac{n \cdot p}{100}\right]$

→ X = 2, 3, 5, 6, 8, 10, 12, 15, 18, 20

The using above example find the value corresponding to the 25 %

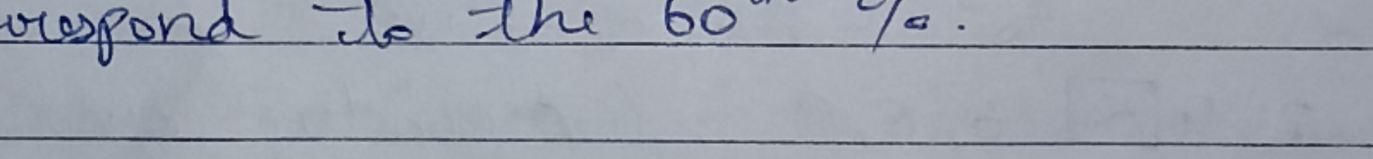
$c = \frac{n \cdot p}{100} = \frac{10 \times 25}{100} = \frac{250}{100} = 2.5 = 3$

→ Using the same example find the value that correspond to the 60th %.

$c = \frac{n \cdot p}{100} = \frac{10 \times 60}{100} = \frac{600}{100} = 6$ (whole no.)

$c = \frac{c + (c+1)}{2} = \frac{10+12}{2} = 11$

② Quartile :- Divide into four equal parts.



1. X = 25, 9, 4, 6, 8, 2, 18, 19. (Ungrouped data)

X = 2, 4, 6, 8, 9, 18, 19, 25

n = 8

$n = \frac{8}{2} = 4$

(IQR) $Q_2 = \frac{8+9}{2} = 8.5$

$Q_1 = \frac{4+6}{2} = 5$

$Q_3 = \frac{18+19}{2} = 18.5$

$Q_2 = 8.5$, $Q_1 = 5$, $Q_3 = 18.5$

2. for grouped data

Age	No. of workers (f)	C.f	
20-30	10	10 c.f.	
30-40	15 f.	25	$l = 30$
45-50	25 f.	50	$h = 10$
50-60	12	62	$f = 15$
60-70	9	71	$c.f = 10$
70-80	3	74	
	$\Sigma f = 74$		

$Q(x) = l + \frac{h}{f}\left(\frac{xn}{4} - cf\right)$ $\quad n = \frac{74}{4} = 18.5$

$Q_1 = l + \frac{h}{f}\left(\frac{n}{4} - f\right)$ $\quad \frac{2n}{4} = 37$

$30 + \frac{10}{15}(18.5 - 10) = 35.1$

$Q_2 = l + \frac{h}{f}\left(2 \times \frac{n}{4} - cf\right)$

$Q(x)$:

$30 + \frac{10}{15}(37 - 10) = 30 + 16.2 = 46.2$

$Q_3 = 30 + \frac{10}{15}(3 \times 18.5 - 10) = 30 + 27.3 = 57.3$