ESC201T : Introduction to Electronics

Week1_L3: Circuit Analysis Using Kirchhoff's Laws

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Acknowledgements
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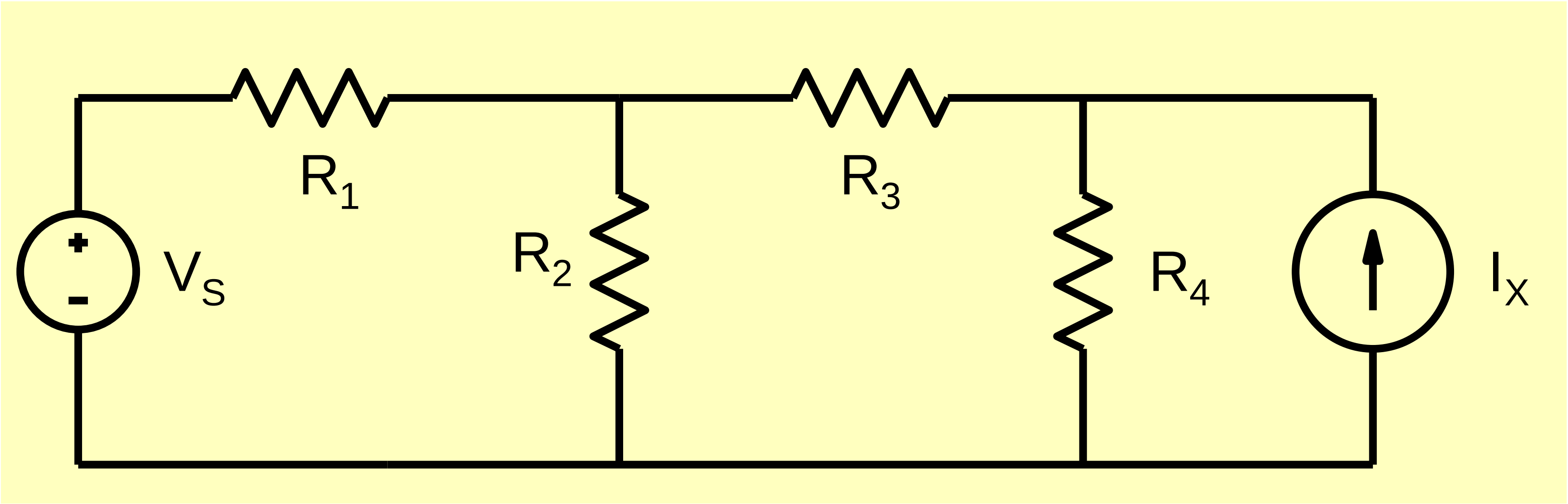
Circuit Analysis



What is current in R_2 ?

Procedure:

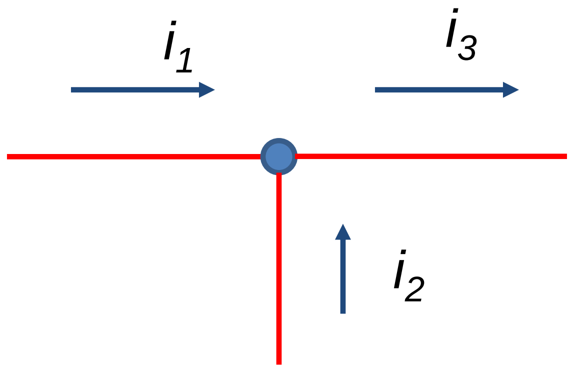
Use Kirchhoff's voltage law (KVL) and Kirchhoff's Current law (KCL) to transform the circuit into a set of equations whose solution gives the required voltage or current value

Kirchhoff's Current Law (KCL)

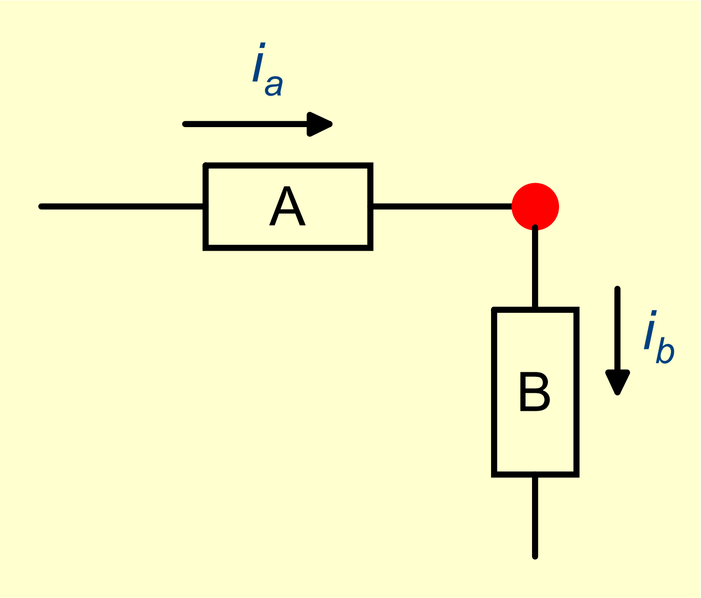
Net current entering a node is zero

$$\sum_{1}^{N} i_j = 0$$

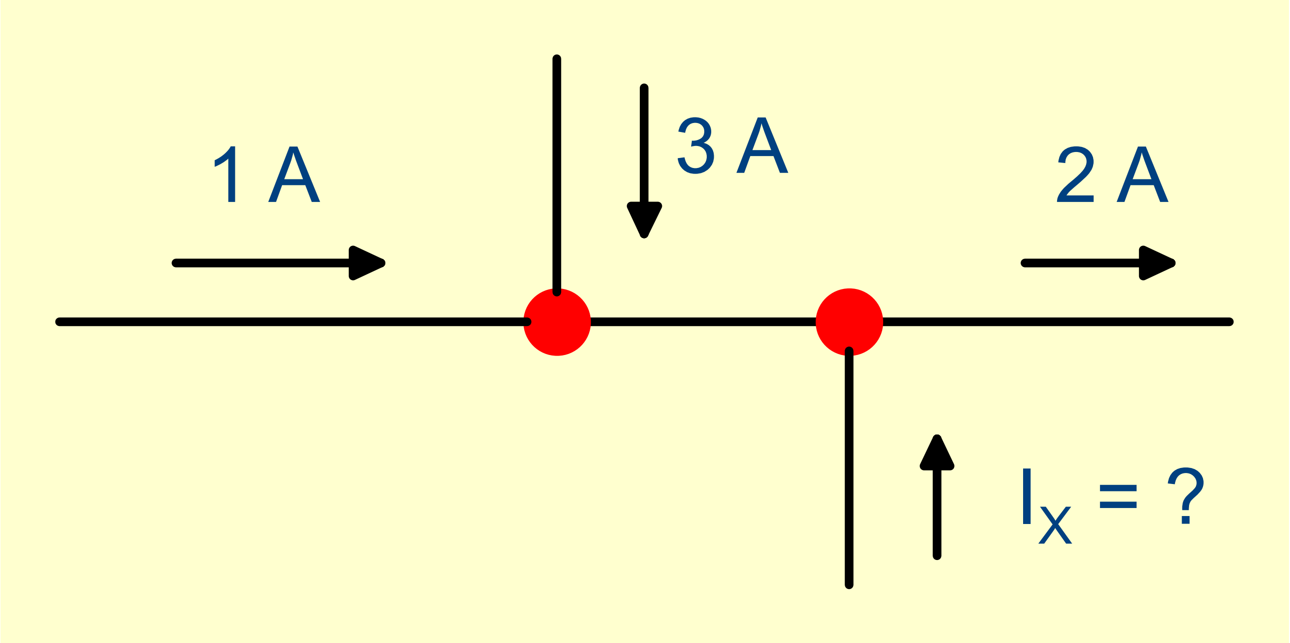
Current entering a node is considered positive and current leaving a node is considered as negative



$$i_1 + i_2 - i_3 = 0$$



$$i_a = i_b$$

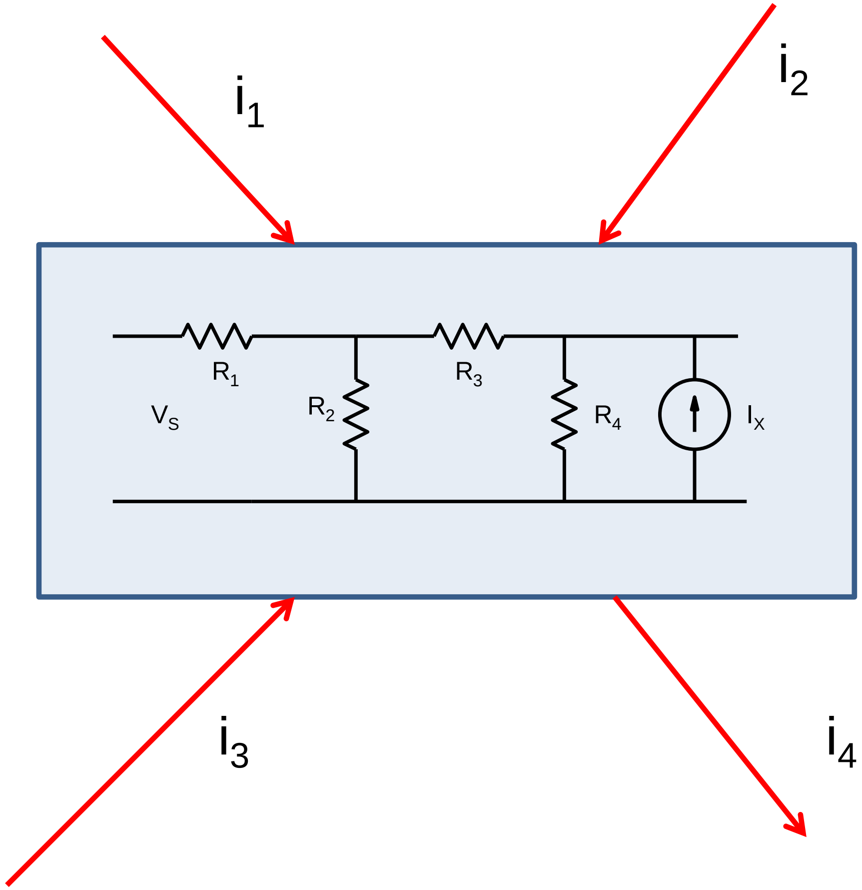


$$1 + 3 + I_x - 2 = 0$$

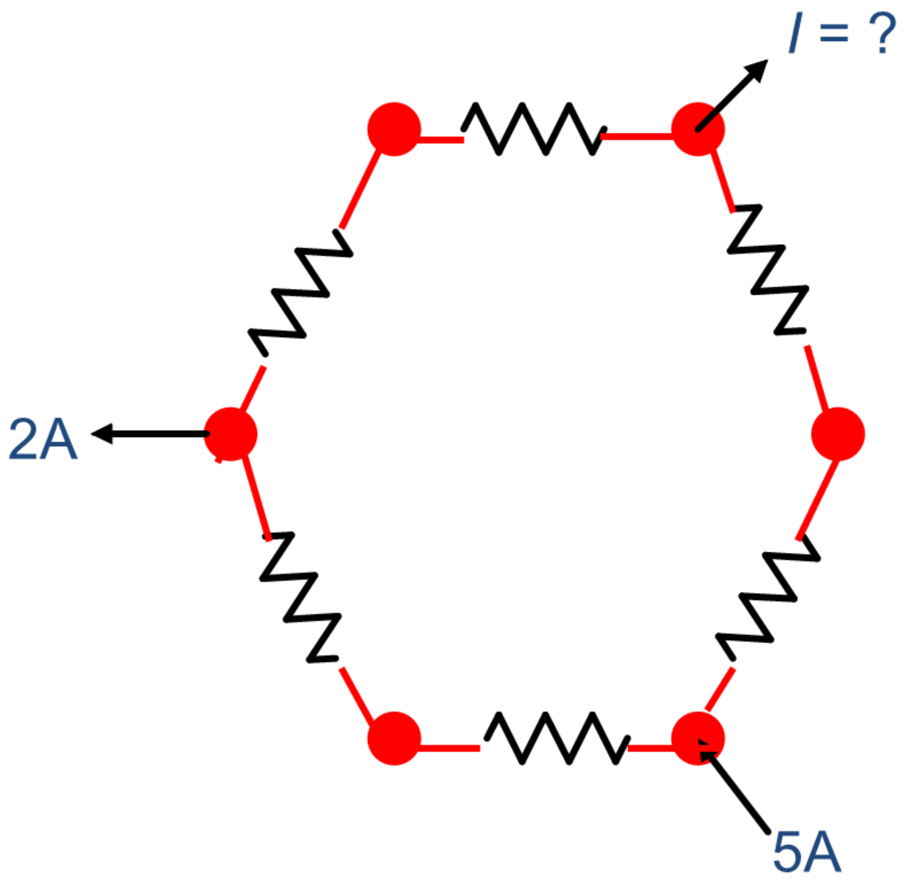
$$I_x = -2$$

KCL: More general formulation

The sum of currents entering/leaving a **closed surface is zero.**

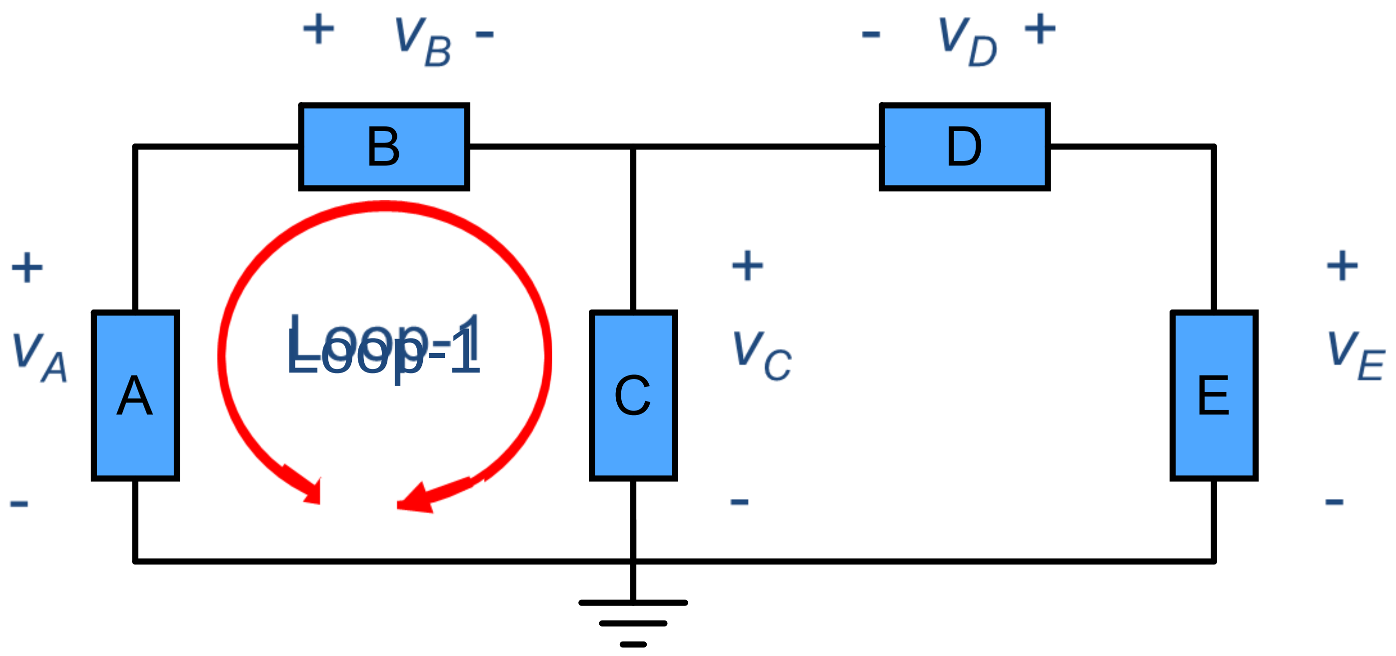


$$i_1 + i_2 + i_3 - i_4 = 0$$



Kirchhoff's Voltage Law (KVL)

The sum of the voltages for any closed path (loop) in an electrical circuit equals zero

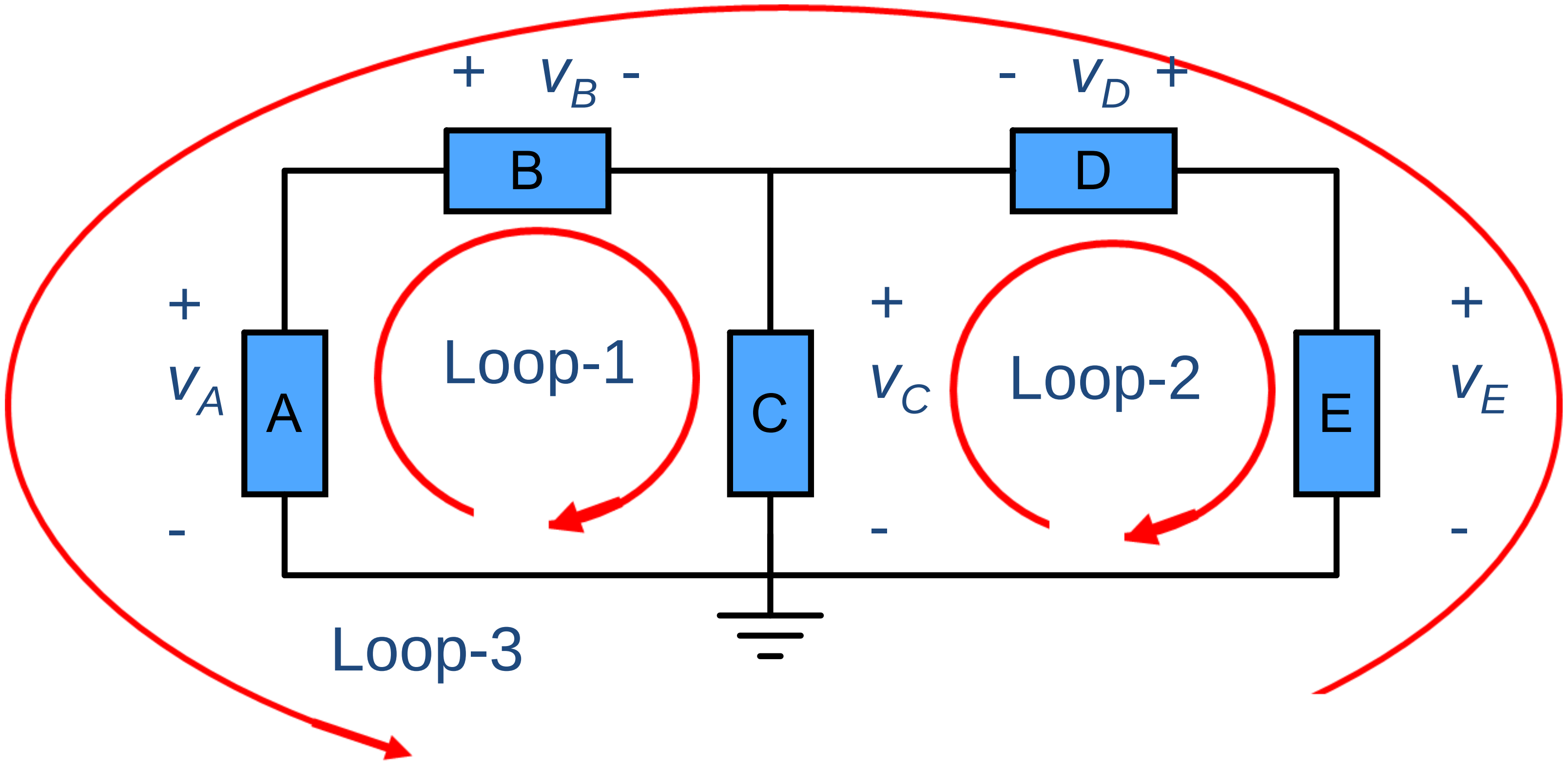


$$\text{Loop1}: -v_a + v_b + v_c = 0$$

$$\text{Loop1}: -v_c - v_b + v_a = 0$$

Voltages are added or subtracted depending on their reference polarities relative to direction of travel around the loop

Example

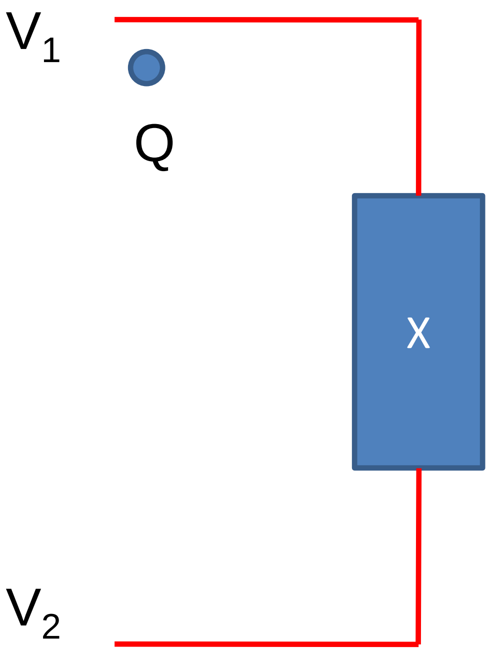


$$\text{Loop1}: -v_a + v_b + v_c = 0$$

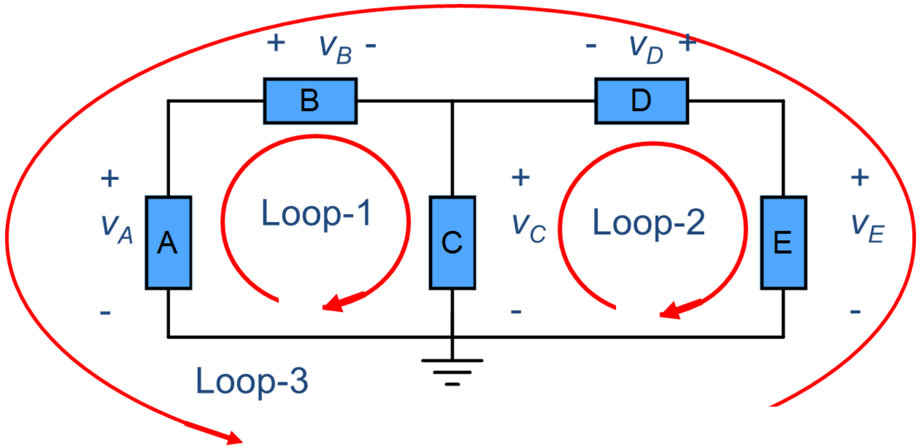
$$\text{Loop2}: -v_c - v_d + v_e = 0$$

$$\text{Loop3}: \ -v_e + v_d - v_b + v_a = 0$$

KVL and Conservation of Energy



The charge loses energy = Q x (V_1-V_2) Joules



$$\text{Loop1}: -v_a + v_b + v_c = 0$$

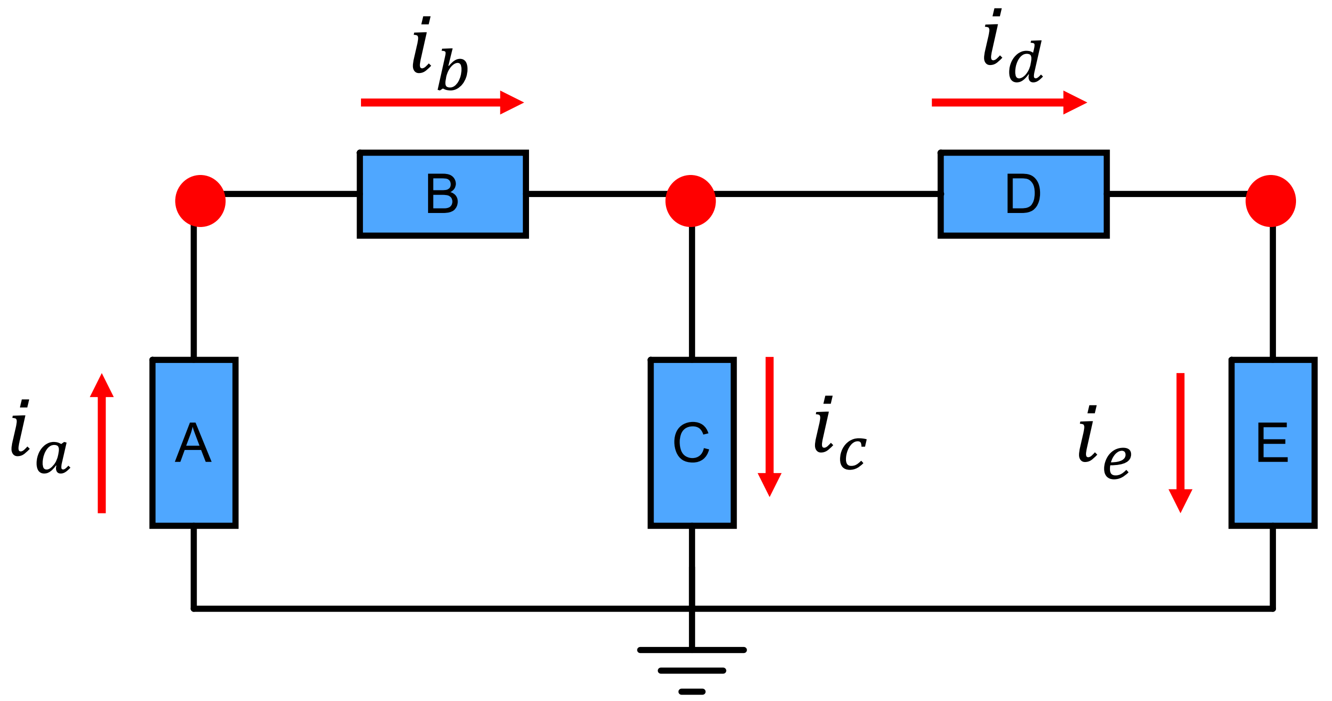
KVL: law of conservation of Energy

Energy gained

Energy lost

Series Connected Elements

Two elements are connected in series if there is no other element connected to the node joining them



A, B are in series

The elements have the same current going through them

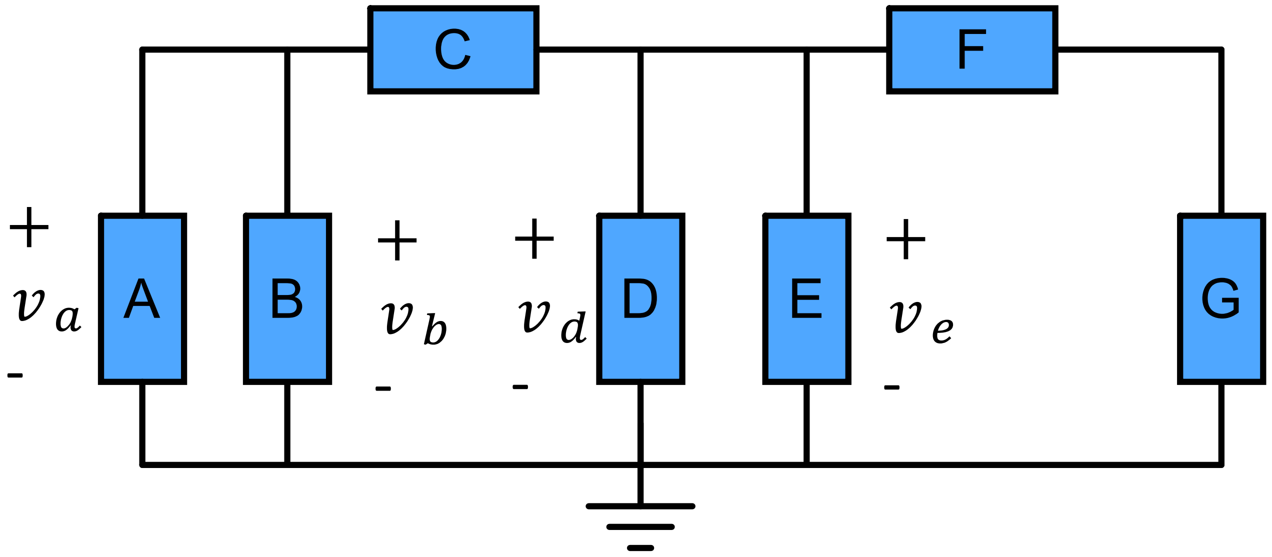
$$i_a = i_b$$

E and D are in series $i_d = i_e$

B and C are not in series $i_b \neq i_c$

Parallel Connected Circuit Elements

Two elements are connected in parallel if both ends of one element are connected directly to corresponding ends of the other



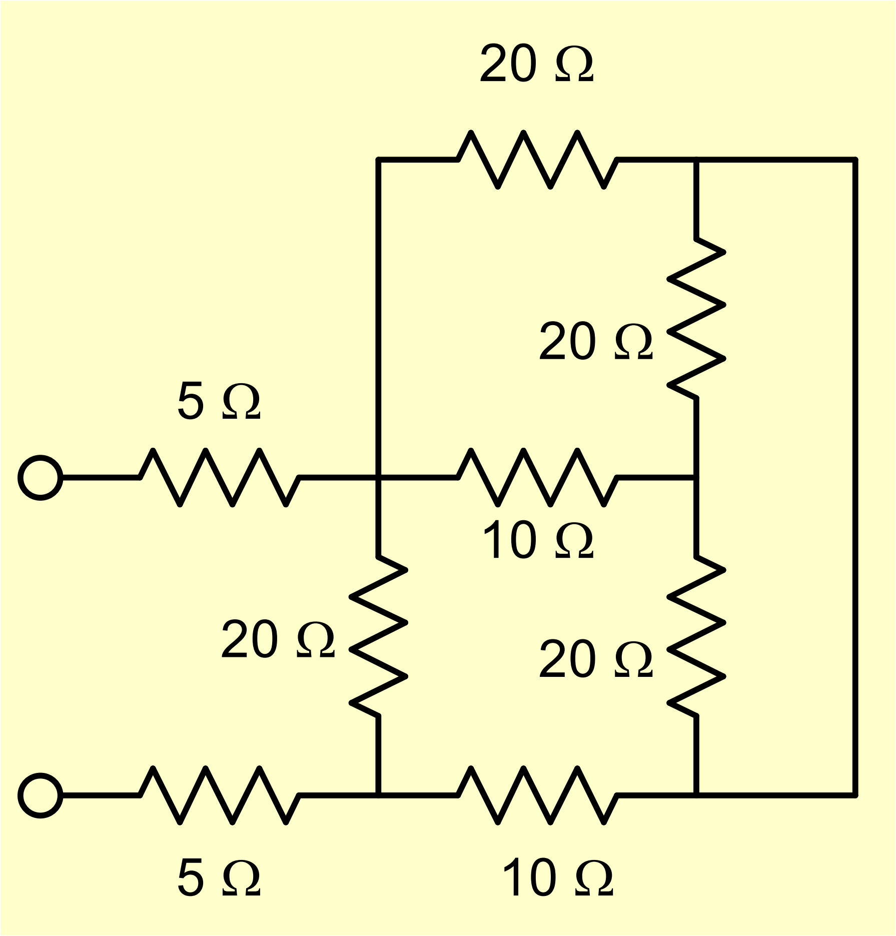
A and B are connected in parallel

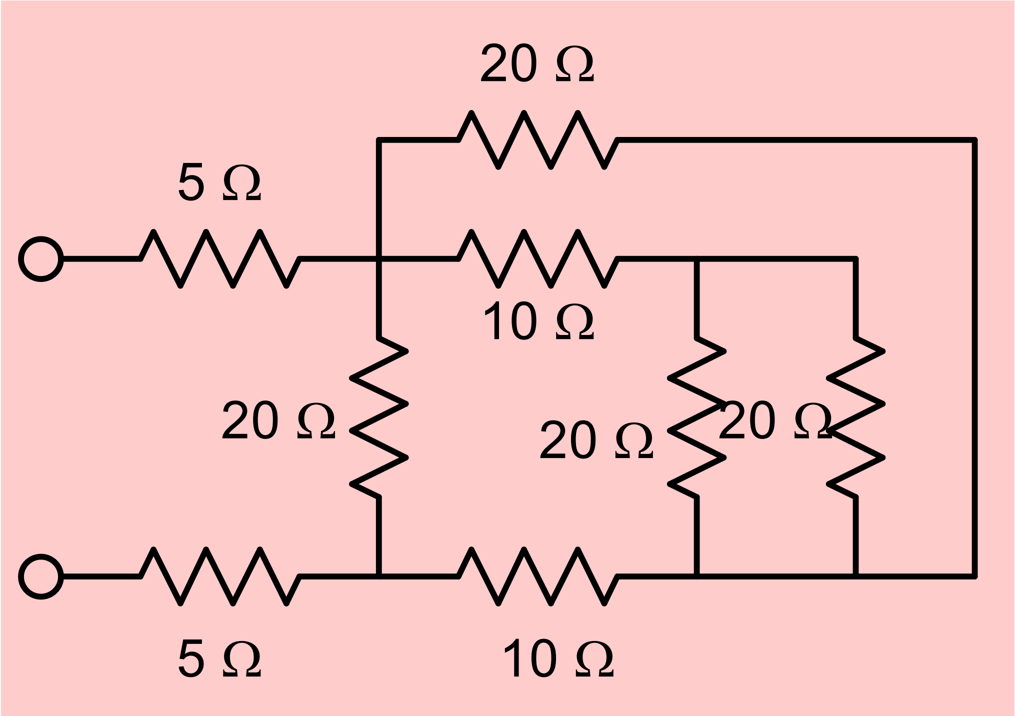
D, E are connected in parallel

$$v_a = v_b \qquad v_d = v_e$$

The voltage across parallel elements are equal (both magnitude and polarity)

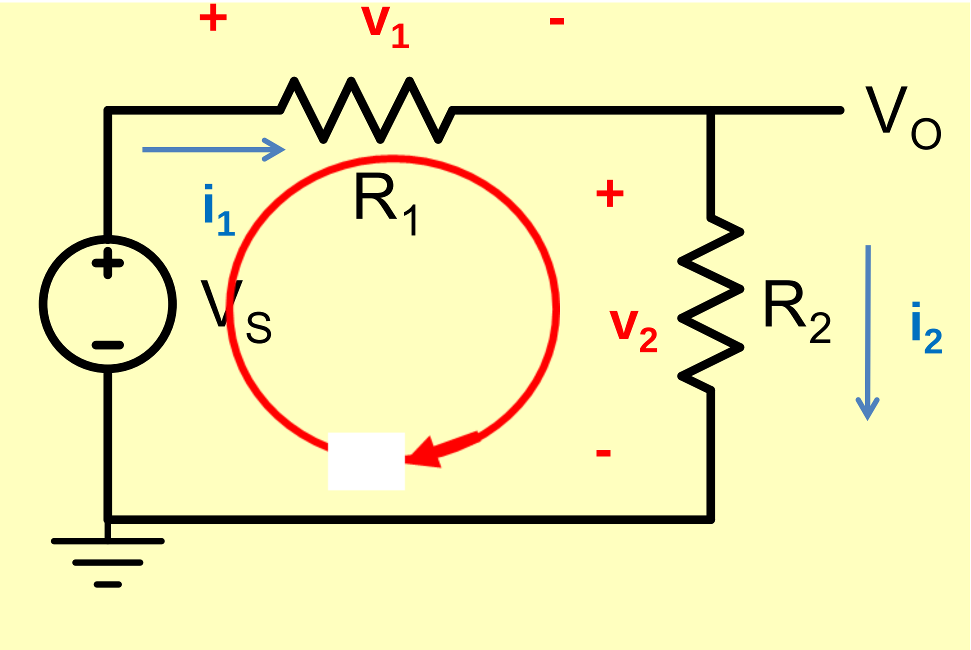
Identification of series or parallel connected elements may not be straightforward





Re-drawing of circuit often helps in visualization of simplification opportunities

Circuit Analysis



What is expression for current I_2 ?

Procedure:

Use Kirchhoff's voltage law (KVL) and Kirchhoff's Current law (KCL) to transform the circuit into a set of equations whose solution gives the required voltage or current value

$$KVL: \quad -V_S + v_1 + v_2 = 0 \Rightarrow V_S = v_1 + v_2$$

$$KCL: \quad i_1 - i_2 = 0 \Rightarrow i_1 = i_2 = i$$

$$Model\ of\ Resistor: \quad v_1 = i_1 \times R_1; v_2 = i_2 \times R_2$$

$$V_S = i \times (\mathrm{R}_1 + \mathrm{R}_2) \qquad V_O = i \times \mathrm{R}_2$$

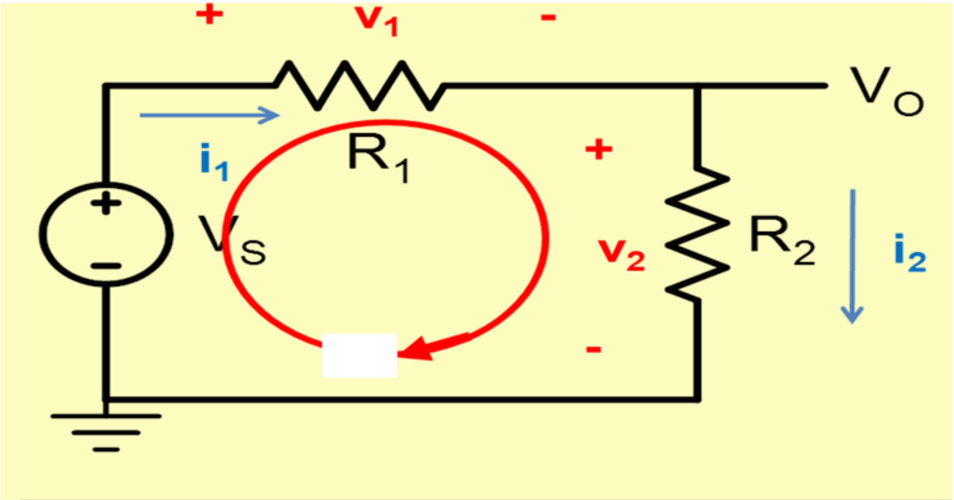
$$V_O = V_S \times \frac{R_2}{R_2 + R_1}$$

Circuit Analysis

Apply KVL and KCL

Use mathematical model of circuit elements

Solve the resulting system of Equations



$KVL: \quad -V_S + v_1 + v_2 = 0 \Rightarrow V_S = v_1 + v_2$

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$Model\ of\ Resistor: \quad v_1 = i_1 \times R_1;\ v_2 = i_2 \times R_2$

$V_S = i \times (R_1+R_2) \qquad V_O = i \times R_2$

$$V_O = V_S \times \frac{R_2}{R_2+R_1}$$

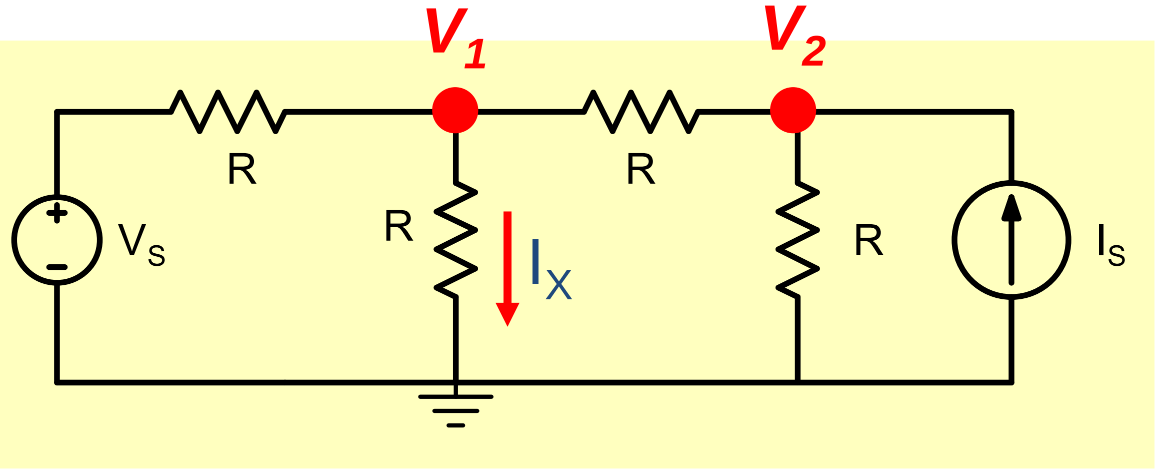
Find I_X in the circuit shown:

Circuit Analysis

Apply KVL and KCL

Use mathematical model of circuit elements

Solve the resulting system of Equations

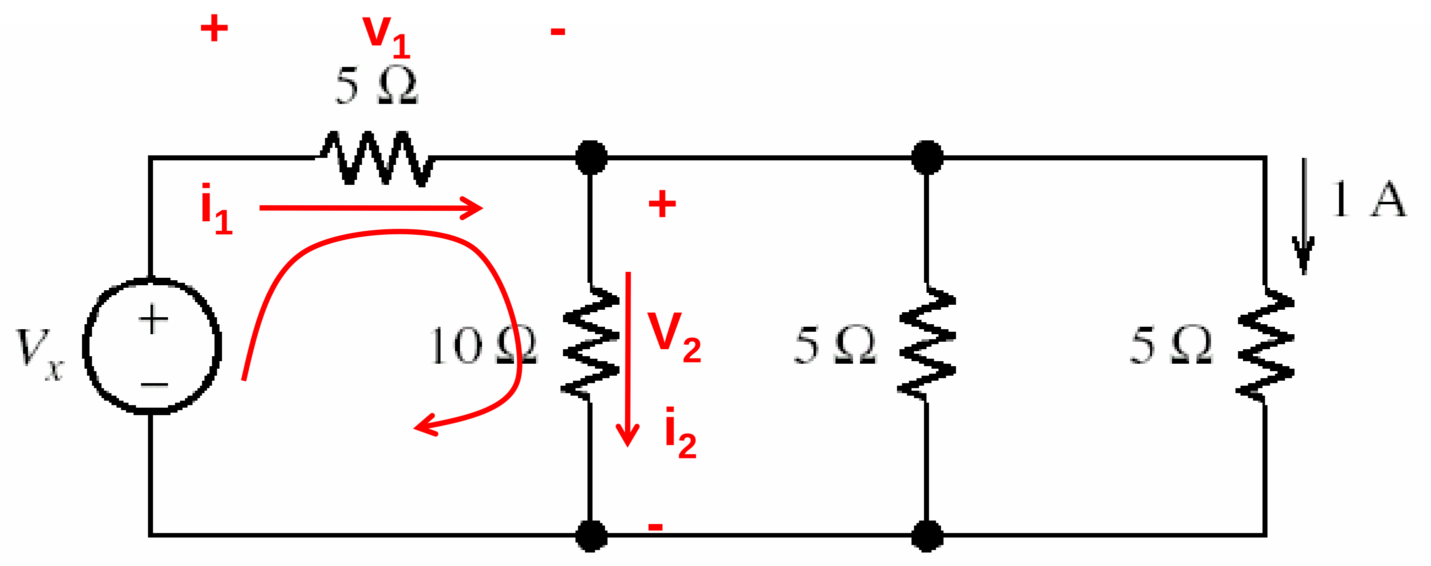


$$\frac{V_S - V_1}{R} - \frac{V_1}{R} + \frac{V_2 - V_1}{R} = 0$$

$$\frac{V_1 - V_2}{R} - \frac{V_2}{R} + I_S = 0$$

Use KVL , KCL and Ohm's law to solve the given problem

Find V_x

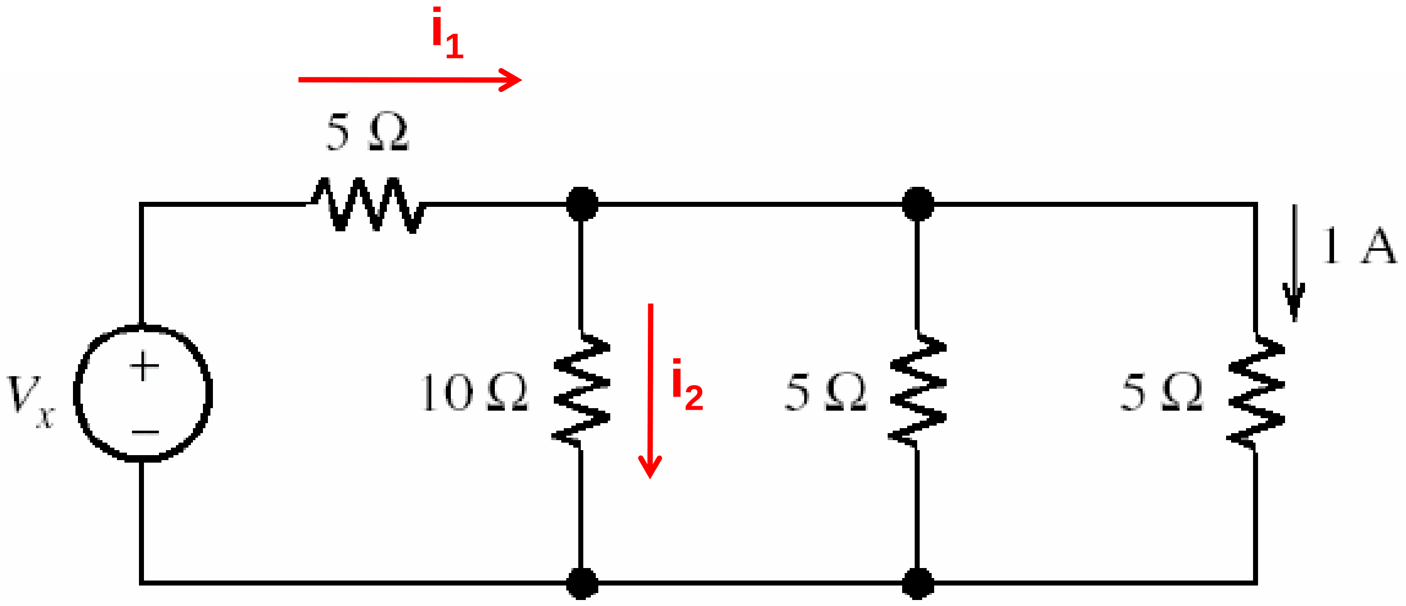


$$-v_x + v_1 + v_2 = 0 \Rightarrow v_x = v_1 + v_2$$

$$v_1 = i_1 \times 5$$

$$v_2 = i_2 \times 10$$

Next Problem: Find currents i_1 and i_2



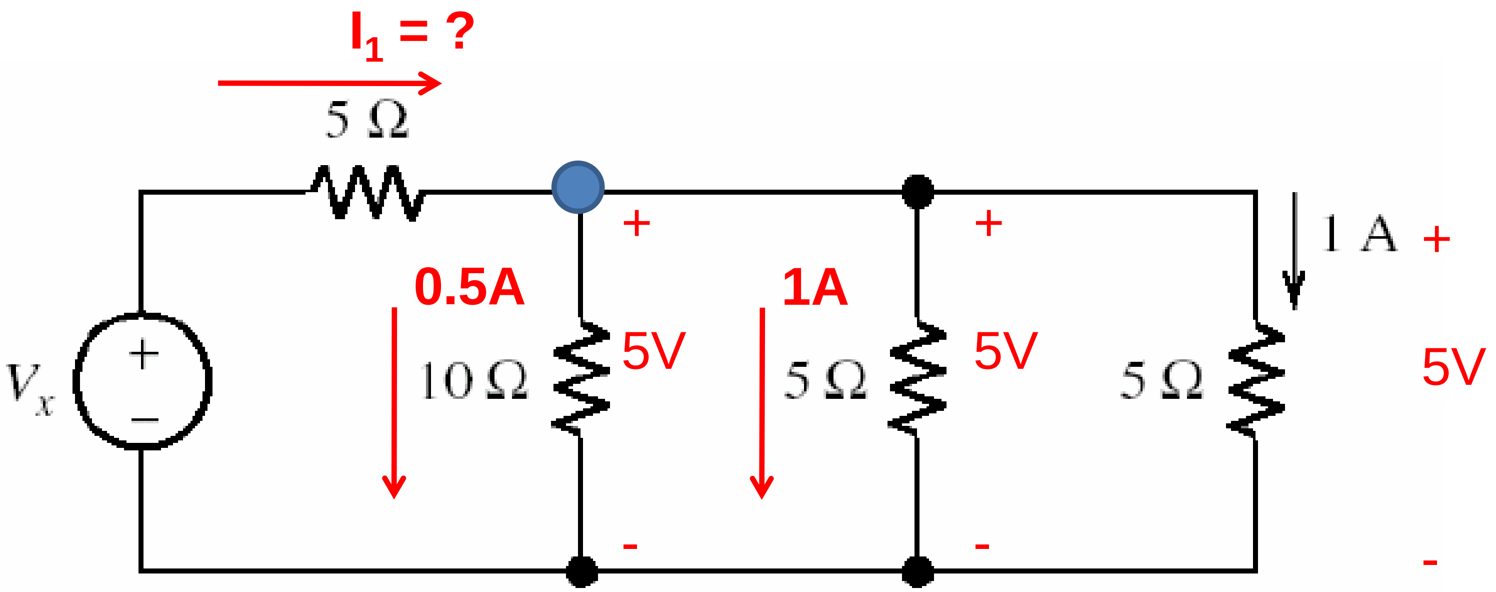
$$v_x = v_1 + v_2$$

$$v_1 = i_1 \times 5$$

$$v_2 = i_2 \times 10$$

$$v_x = (i_1 + 2i_2) \times 5$$

Use ohm's law : v = I x R



Apply KCL at the indicated node

$$i_1 - 0.5 - 1 - 1 = 0 \Rightarrow i_1 = 2.5A$$

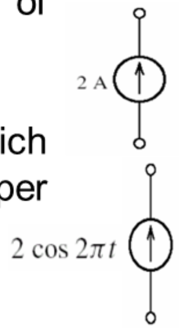
$$v_1 = i_1 \times 5 = 12.5V$$

$$v_x = v_1 + v_2 = 12.5 + 5 = 17.5V$$

Summary

Current: The time rate of flow of electrical charge $i(t) = \frac{dq(t)}{dt}$

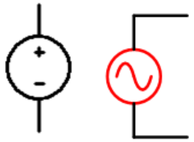
– The units are amperes (A), which are equivalent to coulombs per second (*C/s*)



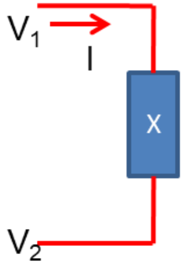
Direction of current flow is opposite to direction of electron flow

Voltage difference is the Source of current flow

Units of Voltage: Volts (V)



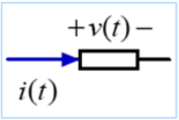
Power



$$P = (V_1 - V_2) \times I$$

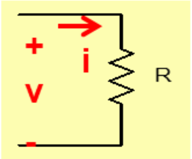
$$P(t) = \frac{dw}{dt} \Rightarrow \quad w = \int_{t1}^{t2} p(t)\,dt$$

Ohm's law



$$v(t) = R \times i(t)$$

$$i(t) = \frac{v(t)}{R} = G \times v(t)$$



$$P = i^2 \times R$$

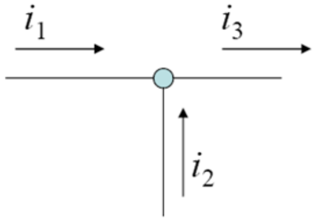
$$P = \frac{v^2}{R}$$

Two elements are connected in series if there is no other element connected to the node joining them. Same current flows

Two elements are connected in parallel if both ends of one element are connected directly to corresponding ends of the other. **Same voltage**

Kirchhoff's Current Law (KCL)

Sum of currents entering a node is equal to sum of currents leaving a node



$$i_1 + i_2 = i_3$$

Kirchhoff's Voltage Law (KVL)

The algebraic sum of the voltages equals zero for any closed path (loop) in an electrical circuit