ESC102T : Introduction to Electronics

Week3_L14: Transient Analysis of RL Circuits

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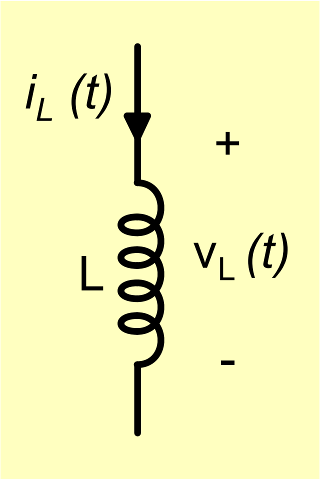
Acknowledgements
Prof. Baquer Mazhari for the lecture slides

Recall

Current through an inductor cannot change instantaneously



$$v_L = L\frac{di_L}{dt}$$

$$i_L = \frac{1}{L}\int v_L(t)\,dt$$

Instant change in voltage implies infinite voltage!

Recall: First Order Differential Equation

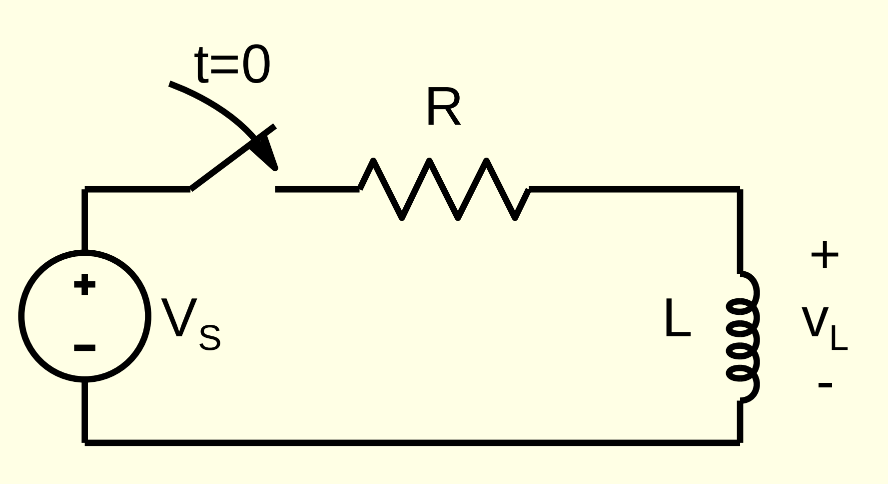
$$\frac{dy}{dt} = -a\, y$$

Solution: $$y(t) = y(0)\; e^{-at}$$

$$\frac{dx}{dt} = -a_1\, x + a_2$$

Solution: $$x(t) = x(\infty) + \{x(0) - x(\infty)\}\; e^{-a_1 t}$$

R-L Transients



$$v = L\frac{di}{dt}$$

$$Ri(t) + L\frac{di}{dt} = V_s$$

$$\frac{dx}{dt} = -a_1\, x + a_2$$

$$x(t) = x(\infty) + \{x(0) - x(\infty)\}\; e^{-a_1 t}$$

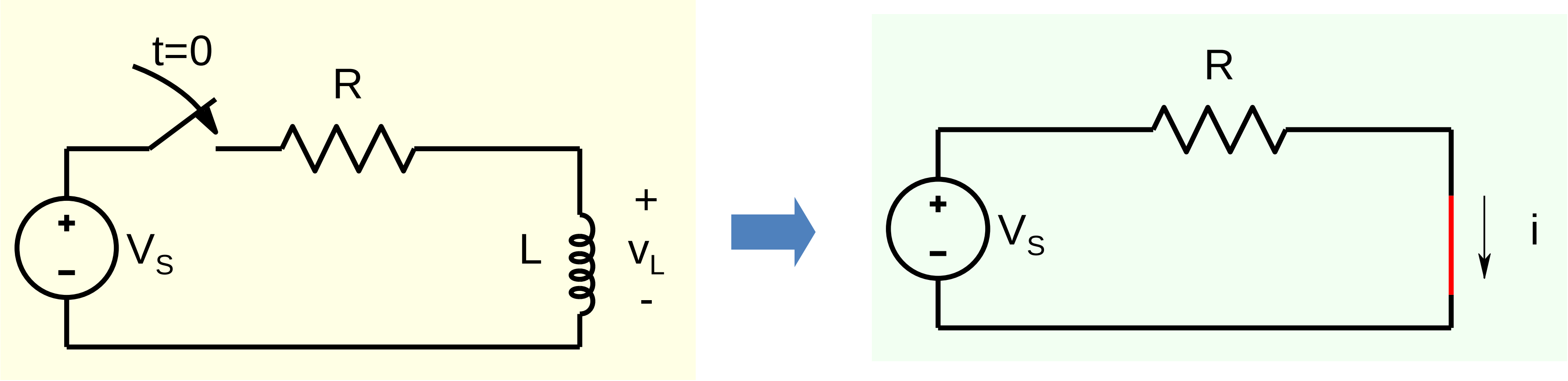
$$i(t) = i(\infty) + \{i(0) - i(\infty)\}\; e^{-\frac{R}{L}t}$$

$$e^{-\frac{t}{\tau}}$$

$$\text{Time Constant} : \tau = \frac{L}{R}$$

What is $i(\infty)$?

Remember that inductor in steady state is like a short circuit



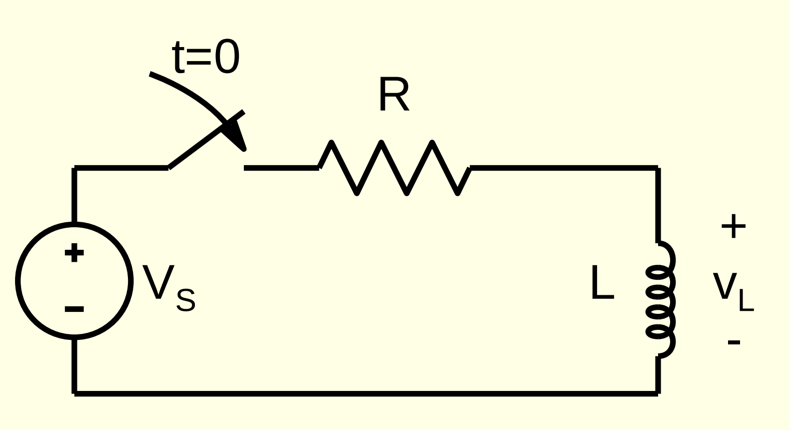
$$i(\infty) = \frac{V_S}{R}$$

$$i(t) = \frac{V_S}{R} + \{i(0) - \frac{V_S}{R}\}\, e^{-\frac{R}{L}t}$$

We also note that inductor current cannot change instantly

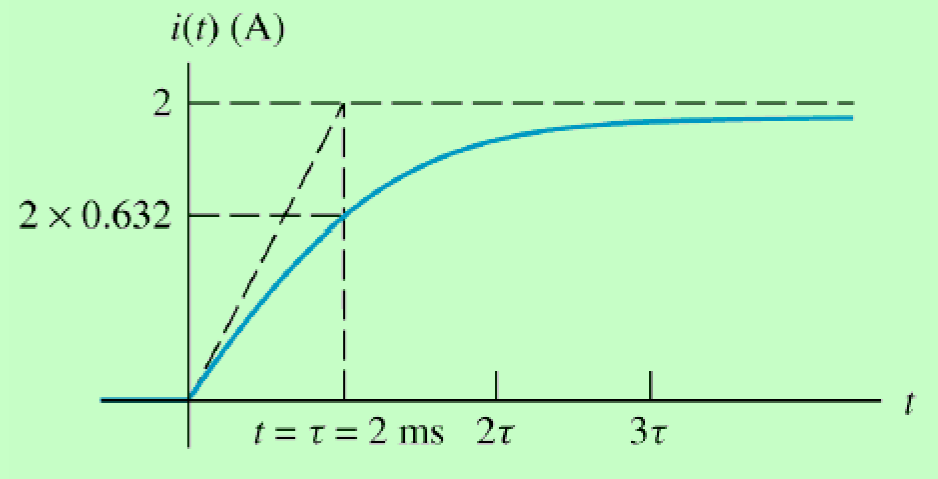
Current through an inductor cannot change instantaneously

$$i(0^+) = i(0^-)$$

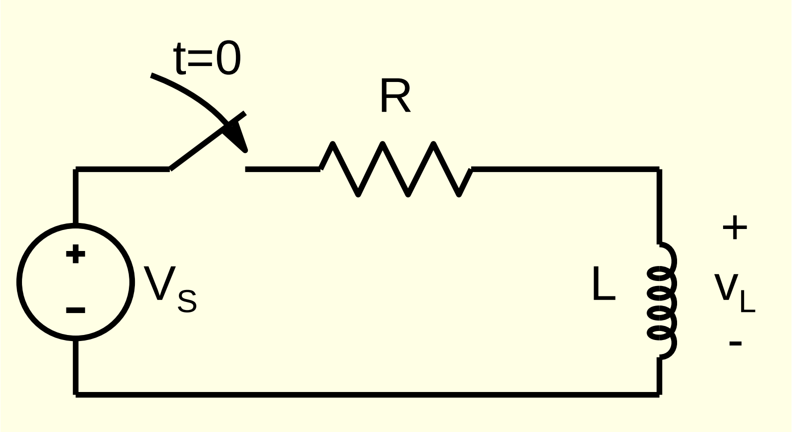


$$i(0^+) = i(0^-) = 0$$

$$i(t) = \frac{V_S}{R} \times (1 - e^{-\frac{t}{\tau}})$$

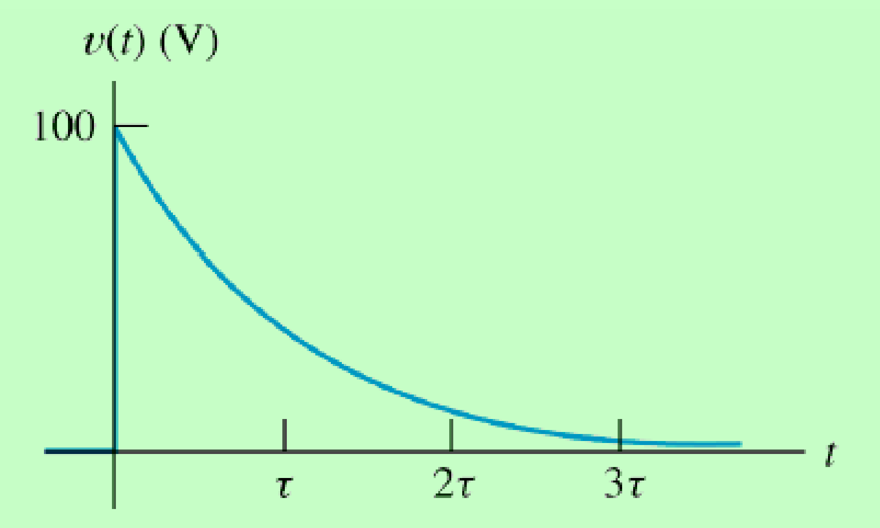


What about voltage across the Inductor?

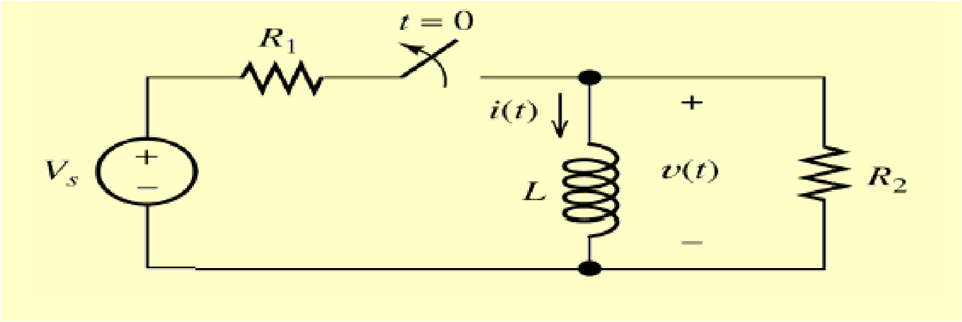


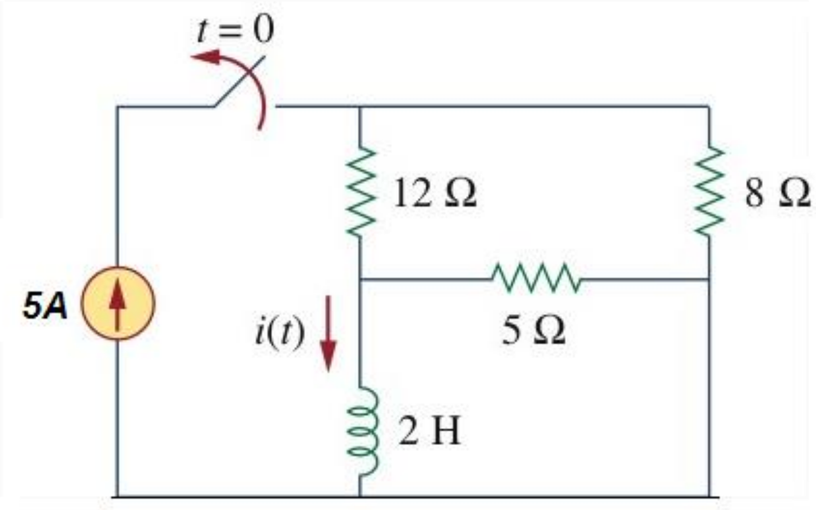
$$i(t) = \frac{V_S}{R} \times (1 - e^{-\frac{t}{\tau}})$$

$$v = L\frac{di}{dt} = \frac{L}{R} V_S \times e^{-\frac{t}{\tau}}$$



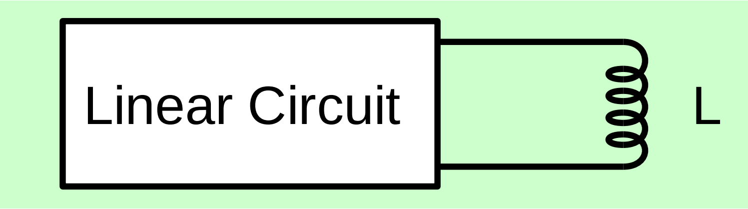
How do we solve more complex circuits containing a single inductor?

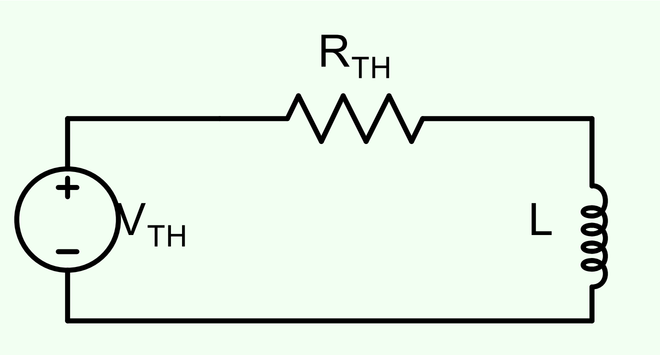




Method for circuits containing a single inductor

Circuit for t > 0

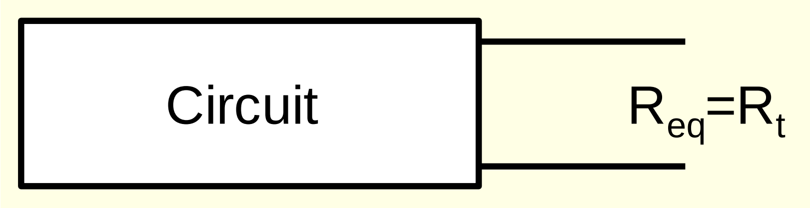




$$x(t) = x(\infty) + \{x(0^+) - x(\infty)\}e^{-\frac{t}{\tau}}$$

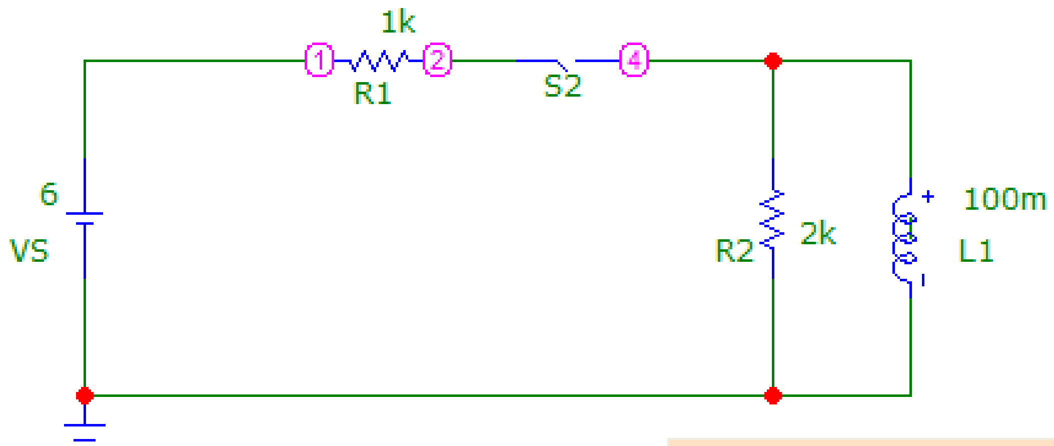
$$\tau = \frac{L}{R_{eq}}$$

Where x is inductor current



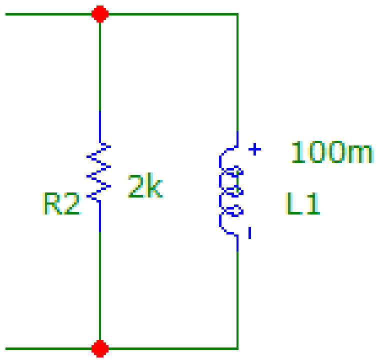
Example:

Switch is opened at t = 0



$$i(t) = i(\infty) + \{i(0^+) - i(\infty)\} \times e^{-\frac{t}{\tau}}$$

Circuit for t > 0



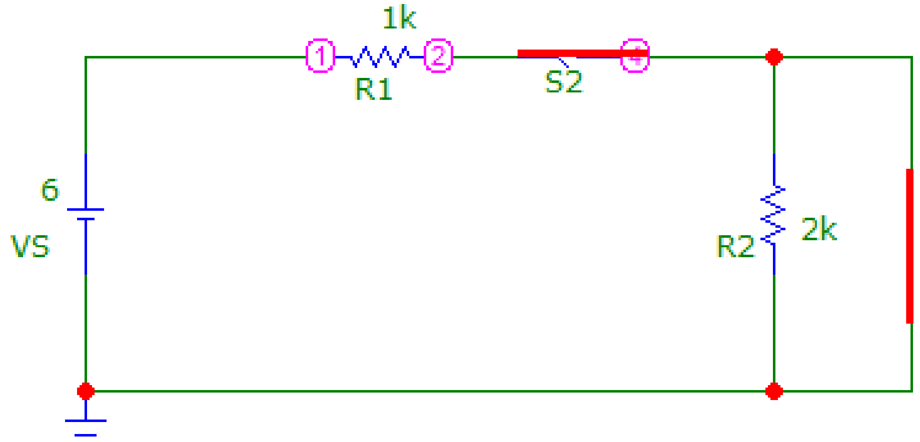
$$\tau = \frac{L}{R_2}$$

Steady state Solution: $i(t \rightarrow \infty) = 0$

$$i(t) = i(0^+) \times e^{-\frac{t}{\tau}}$$

Initial condition

Circuit for t < 0

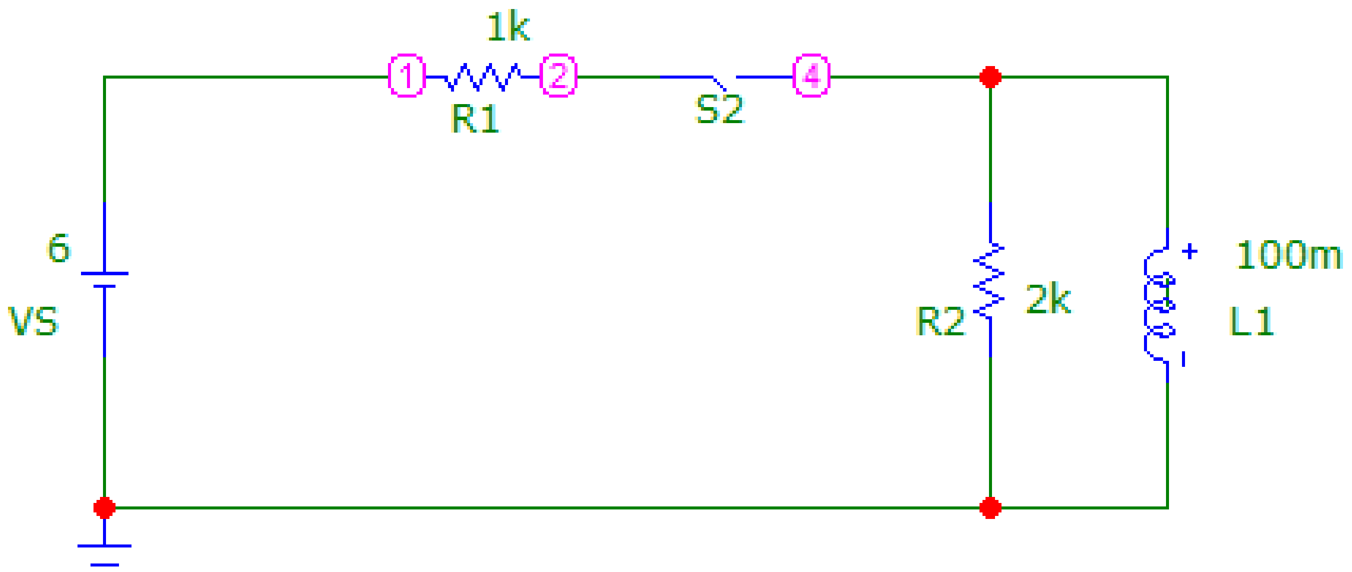


$$i(0^+) = i(0^-) = \frac{V_S}{R_1}$$

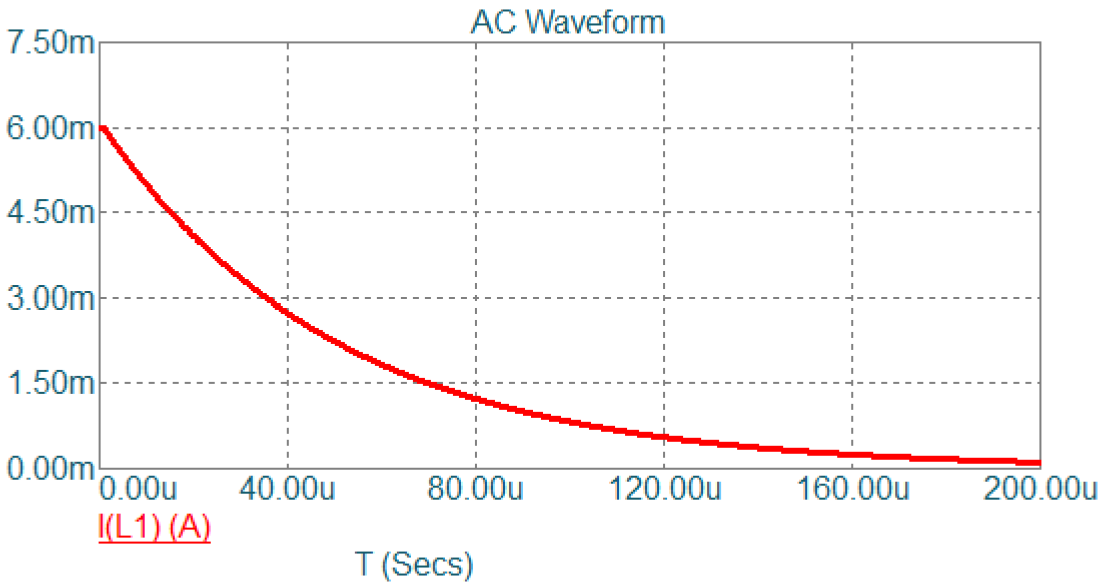
$$i(t \to \infty) = 0$$

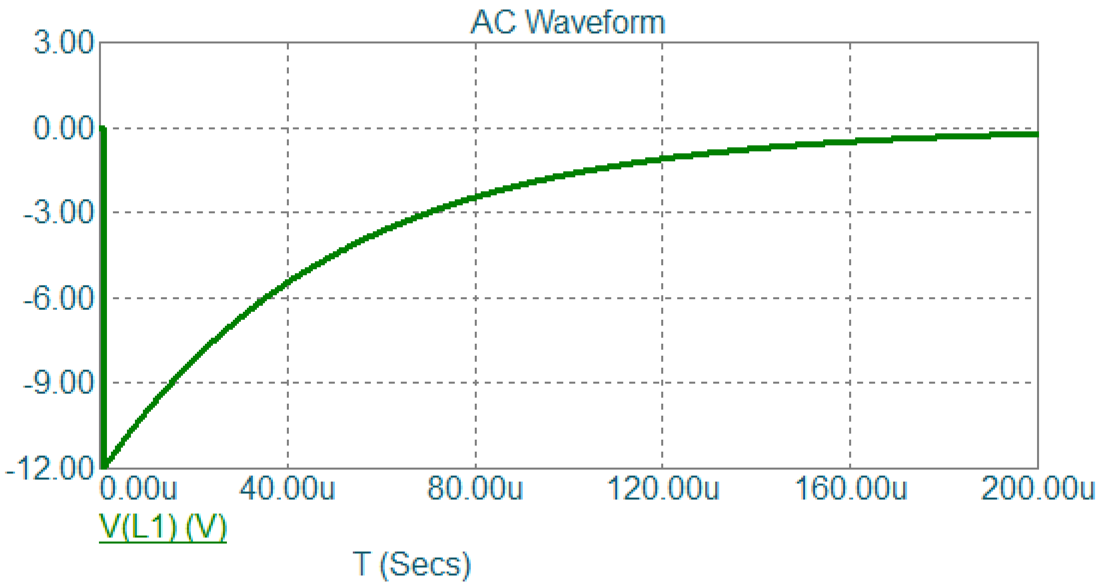
$$i(t) = i(\infty) + \{i(0^+) - i(\infty)\} \times e^{-\frac{t}{\tau}}$$

$$i(t) = \frac{V_S}{R_1} e^{-\frac{R_2}{L}t}$$

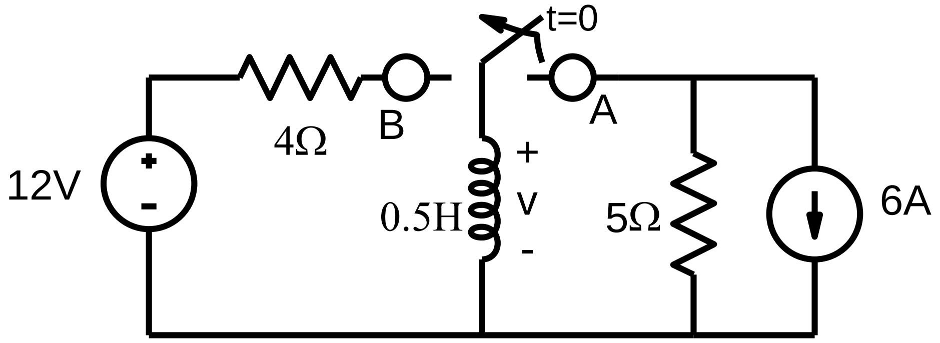


$$x(t) = x(\infty) + \{x(0^+) - x(\infty)\}e^{-\frac{t}{\tau}}$$





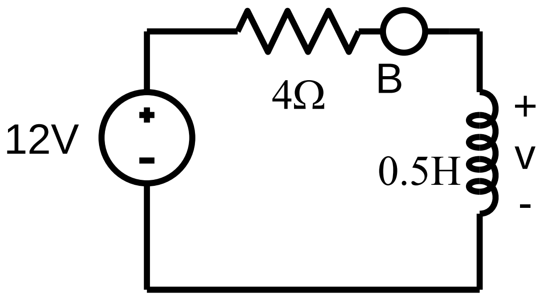
Example: Determine the current and voltage across the inductor as a function of time after the switch is connected to node B.



$$i(t) = i(\infty) + \{i(0^+) - i(\infty)\} \times e^{-\frac{t}{\tau}}$$

Circuit after switch is connected to node B (t > 0)

$$\tau = \frac{L}{R_{eq}} = \frac{0.5}{4} = 0.125$$



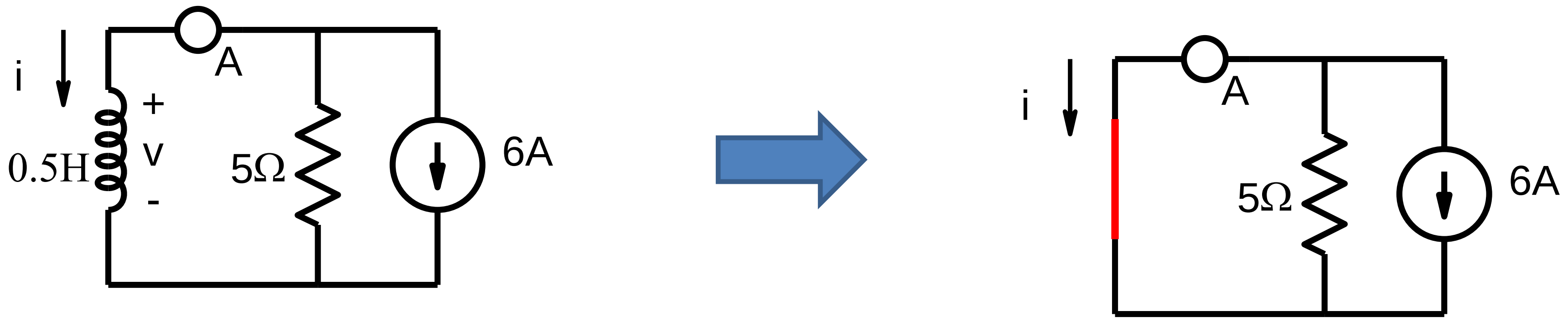
$$i(\infty) = \frac{12}{4} = 3A$$

Next find current $i(0^+)$

Inductor Current cannot change instantly

$$i(0^-) = i(0^+)$$

Circuit before switch is connected to node B (t < 0)

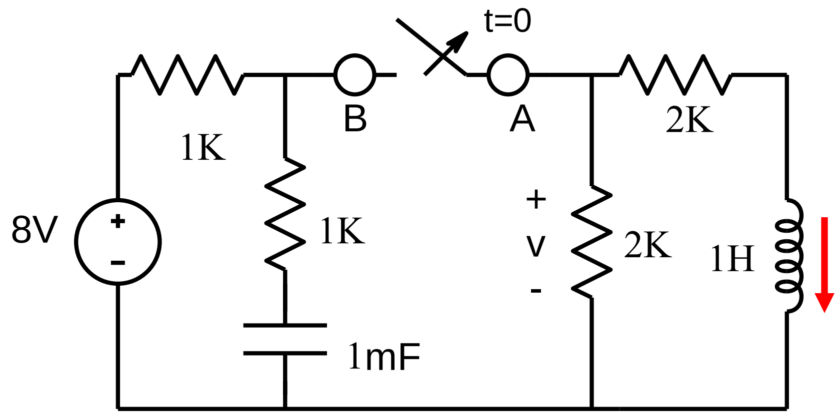


$$i(0^-) = i(0^+) = -6A$$

$$i(t) = 3 - 9 \times e^{-8t}$$

$$v = L\frac{di}{dt} = 36 \times e^{-8t}$$

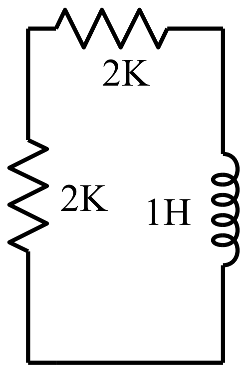
Example: For the circuit shown below, determine the voltage v across the 2K resistor as a function of time after the switch is opened at t= 0.



First find the inductor current

$$i_L(t) = i_L(\infty) + \{i_L(0^+) - i_L(\infty)\} \times e^{-\frac{t}{\tau}}$$

Circuit after opening the switch (t > 0)



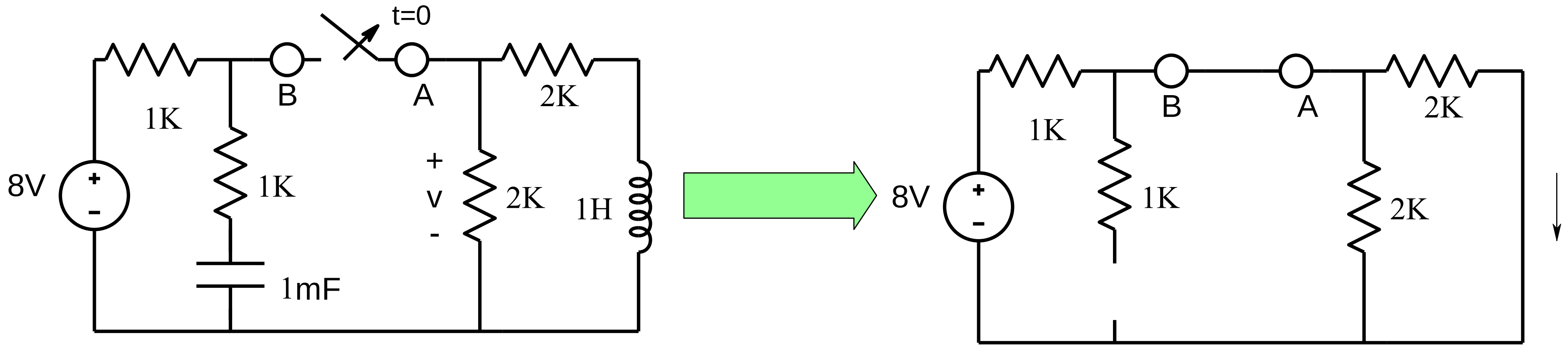
$$R_{eq} = 2K + 2K = 4K$$

$$\tau = \frac{L}{R_{eq}} = 0.25ms$$

One can also see that : $i_L(\infty) = 0$

$$i_L(0^+) = i_L(0^-)$$

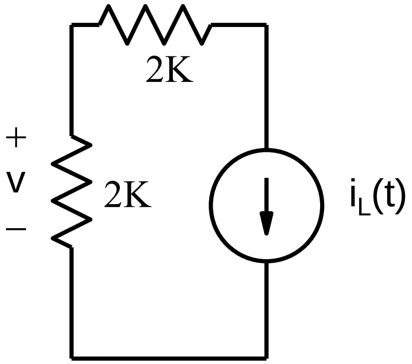
Circuit before opening the switch (t < 0) and assuming steady state condition:



$$i_L(0^+) = i_L(0^-) = \frac{8}{(2K\|2K)+1K} \times 0.5 = 2mA$$

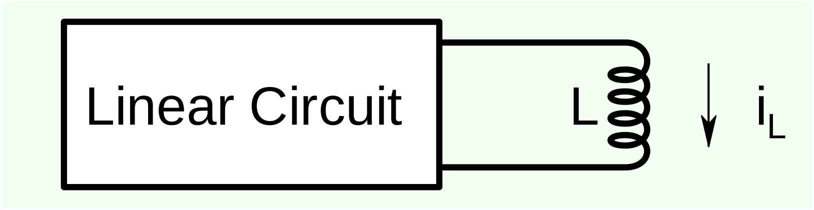
$$\Rightarrow i_L(t) = 2 \times e^{-4000t} mA$$

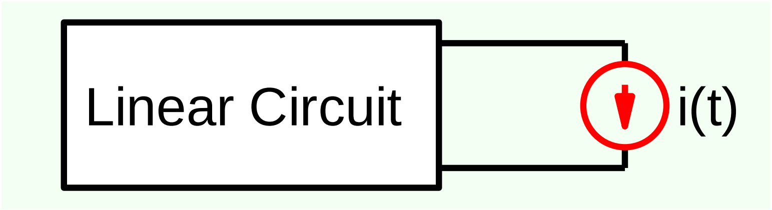
Voltage across the 2K resistor:



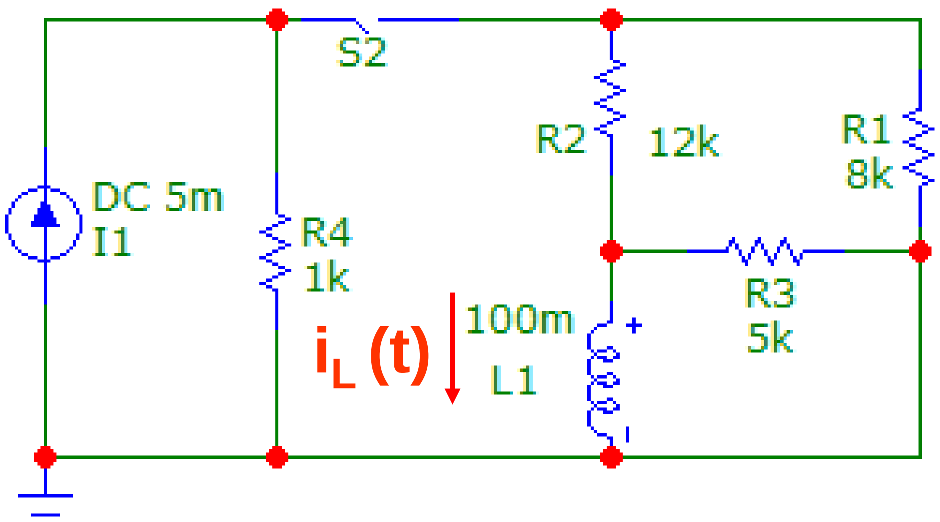
$$v(t) = -2 \times 10^3 \times i_L(t) = -4 \times e^{-4000t} V$$

How do we find voltages and currents elsewhere in the circuit?

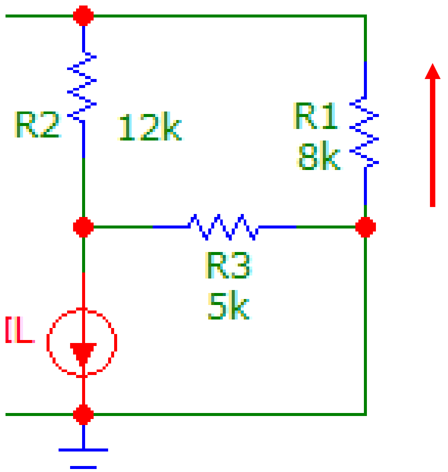




Example: Find current in 8 kΩ resistor as a function of time after the switch is opened



$$i_L(t) = 0.34mA \times e^{-\frac{t}{25\mu s}}$$



$$i_8 = i_L(t) \times \frac{5}{5+20} = 0.069mA \times e^{-\frac{t}{25\mu s}}$$